Physics Letters B 736 (2014) 86–92

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Cosmological perturbations in warm-tachyon inflationary universe model with viscous pressure

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ARTICLE INFO

Article history:
Received 30 November 2013
Received in revised form 1 June 2014
Accepted 3 July 2014
Available online 9 July 2014
Editor: J. Hisano

ABSTRACT

We study the warm-tachyon inflationary universe model with viscous pressure in high-dissipation regime. General conditions which are required for this model to be realizable are derived in the slow-roll approximation. We present analytic expressions for density perturbation and amplitude of tensor perturbation in longitudinal gauge. Expressions of tensor-to-scalar ratio, scalar spectral index and its running are obtained. We develop our model by using exponential potential, the characteristics of this model are calculated for two specific cases in great details: 1. Dissipative parameter Γ and bulk viscous parameter ζ are constant parameters. 2. Dissipative parameter is a function of tachyon field ϕ and bulk viscous parameter is a function of matter-radiation mixture energy density ρ. The parameters of the model are restricted by recent observational data from the nine-year Wilkinson microwave anisotropy probe (WMAP9), Planck and BICEP2 data.

1. Introduction

Big Bang model has many long-standing problems (monopole, horizon, flatness, etc.). These problems are solved in a framework of inflationary universe models [1]. Scalar field as a source of inflation provides a causal interpretation of the origin of the distribution of Large-Scale Structure (LSS), and also observed anisotropy of cosmological microwave background (CMB) [2–4]. The standard models for inflationary universe are divided into two regimes, slow-roll and reheating regimes. In the slow-roll period, kinetic energy remains small compared to the potential term. In this period, all interactions between scalar fields (inflatons) and other fields are neglected and as a result the universe inflates. Subsequently, in reheating epoch, the kinetic energy is comparable to the potential energy that causes inflaton to begin an oscillation around the minimum of the potential while losing their energy to other fields present in the theory. After the reheating period, the universe is filled with radiation.

In warm inflation scenario the radiation production occurs during inflationary period and reheating is avoided [5]. Thermal fluctuations may be generated during warm inflationary epoch. These fluctuations could play a dominant role in producing initial fluctuations which are necessary for Large-Scale Structure (LSS) formation. In this model, density fluctuation arises from thermal rather than quantum fluctuation [6]. Warm inflationary period ends when the universe stops inflating. After this period, the universe enters in the radiation phase smoothly [5]. Finally, remaining inflatons or dominant radiation fields create matter components of the universe. Some extensions of this model are found in Ref. [7].

In the warm inflation models there has to be a continuous particle production. For this to be possible, the microscopic processes that produce these particles must occur at a timescale much faster than Hubble expansion. Thus the decay rates Γ_i (not to be confused with the dissipative coefficient) must be bigger than H. Also these produced particles must thermalize. Thus the scattering processes among these produced particles must occur at a rate bigger than H. These adiabatic conditions were outlined since the early warm inflation papers, such as Ref. [8]. More recently there has been considerable explicit calculation from Quantum Field Theory (QFT) that explicitly computes all these relevant decay and scattering rates in warm inflation models [9,10].

In warm inflation models, for simplicity, particles which are created by the inflaton decay are considered as massless particles (or radiation). Existence of massive particles in the inflationary fluid model as a new model of inflation was studied in Ref. [11]. Perturbation parameters of this model were obtained in Ref. [12]. In this scenario the existence of massive particles alters the dynamic of the inflationary universe models by modification

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http://dx.doi.org/10.1016/j.physletb.2014.07.008

of the fluid pressure. Using the random fluid hydrodynamic fluctuation theory which is generalized by Landau and Lifshitz [13], we can describe the cosmological fluctuations in the system with radiation and tachyon scalar field. Decay of the massive particles within the fluid is an entropy-producing scalar phenomenon. In the other hand, "bulk viscous pressure" has entropy-producing property. Therefore, the decay of particles may be considered by a bulk viscous pressure $\Pi = -3\zeta H$, where H is Hubble parameter and ζ is phenomenological coefficient of bulk viscosity [14]. This coefficient is positive-definite by the second law of thermodynamics and in general depends on the energy density of the fluid.

The Friedmann–Robertson–Walker (FRW) cosmological models in the context of string/M-theory have been related to brane–antibrane configurations [15]. Tachyon fields, associated with unstable D-branes, are responsible of inflation in early time [16]. The tachyon inflation is a k-inflation model [17] for scalar field ϕ with a positive potential $V(\phi)$. Tachyon potentials have two special properties, first a maximum of these potential is obtained where $\phi \to 0$ and second property is the minimum of these potentials is obtained where $\phi \to \infty$. If the tachyon field starts to roll down the potential, then universe, which is dominated by a new form of matter, will smoothly evolve from inflationary universe to an era which is dominated by a non-relativistic fluid [18]. So, we could explain the phase of acceleration expansion (inflation) in terms of tachyon field.

Cosmological perturbations of warm inflation model (with viscous pressure) have been studied in Refs. [19,12]. Warm tachyon inflationary universe model has been studied in Ref. [20], also warm inflation on the brane (with viscous pressure) has been studied in Refs. [21,22]. To the best of our knowledge, a model in which warm tachyon inflation with viscous pressure has not been yet considered. In the present work we will study warm-tachyon inspired inflation with viscous pressure. The paper organized as follows: In the next section, we will describe warm-tachyon inflationary universe model with viscous pressure and the perturbation parameters for our model. In Section 3, we study our model using the exponential potential in high dissipative regime. Finally in Section 4, we close by some concluding remarks.

2. The model

In this section, we will obtain the parameters of the warm tachyon inflation with viscous pressure. This model may be described by an effective tachyon fluid and matter-radiation imperfect fluid. Tachyon fluid in a spatially flat Friedmann Robertson Walker (FRW) is recognized by these parameters [18,22]

$$T^{\nu}_{\mu} = \mathrm{diag}(-\rho_\phi, P_\phi, P_\phi, P_\phi) \tag{1}$$

$$P_\phi = -V(\phi)\sqrt{1-\dot{\phi}^2},$$

$$\rho_\phi = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}.$$

Important characteristics of the potential are $\frac{dV}{d\phi} < 0$, and $V(\phi \to \infty) \to 0$ [23]. The imperfect fluid is a mixture of matter and radiation of adiabatic index γ which has energy density $\rho = Ts(\phi, T)$ (T is temperature and s is entropy density of the imperfect fluid) and pressure $P + \Pi$ where, $P = (\gamma - 1)\rho$. $\Pi = -3\zeta H$ is bulk viscous pressure [14], where ζ is phenomenological coefficient of bulk viscosity. The dynamic of the model in background level is given by the Friedmann equation,

$$3H^2 = \rho_T = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} + \rho, \tag{2}$$

the conservation equations of tachyon field and imperfect fluid

$$\dot{\rho}_\phi + 3H(P_\phi + \rho_\phi) = -\Gamma\dot{\phi}^2$$

$$\Rightarrow \quad \frac{\ddot{\phi}}{1-\dot{\phi}^2} + 3H\dot{\phi} + \frac{V'}{V} = -\frac{\Gamma}{V}\sqrt{1-\dot{\phi}^2}\dot{\phi}, \tag{3}$$

and

$$\dot{\rho} + 3H(\rho + P + \Pi) = \dot{\rho} + 3H(\gamma\rho + \Pi) = \Gamma\dot{\phi}^2, \tag{4}$$

where we have used the natural units ($c = \hbar = 1$) and $\frac{8\pi}{m_p^2} = 1$. Γ is the dissipative coefficient with the dimension m_p^5. Dissipation term denotes the inflaton decay into the imperfect fluid in the inflationary epoch. In the above equations dots "." mean derivative with respect to cosmic time, prime denotes derivative with respect to the tachyon field ϕ. The energy density of radiation and the entropy density increase by the bulk viscosity pressure Π (see Fig. 1 and Fig. 2) [22].

During slow-roll inflation epoch the energy density (1) is the order of potential, i.e. $\rho_\phi \sim V$, and dominates over the imperfect fluid energy density, i.e. $\rho_\phi > \rho$. Using slow-roll approximation when $\dot{\phi} \ll 1$, and $\ddot{\phi} \ll (3H + \frac{\Gamma}{V})$ [5] the dynamic equations (2) and (3) are reduced to

$$3H^2 = V,$$

$$3H(1+r)\dot{\phi} = -\frac{V'}{V}, \tag{5}$$

where $r = \frac{\Gamma}{3HV}$. From above equations and Eq. (4), when the decay of the tachyon field to imperfect fluid is quasi-stable, i.e. $\dot{\rho} \ll 3H(\gamma\rho + \Pi)$, and $\dot{\rho} \ll \Gamma\dot{\phi}^2$, ρ may be written as

$$\rho = \frac{1}{\gamma}(rV\dot{\phi}^2 - \Pi) = \frac{1}{\gamma}\left(\frac{r}{3(1+r)^2}\left(\frac{V'}{V}\right)^2 - \Pi\right). \tag{6}$$

In the present work, we will restrict our analysis in high dissipative regime, i.e. $r \gg 1$, where the dissipation coefficient Γ is much greater than $3HV$ [22]. Dissipation parameter Γ may be a constant parameter or a positive function of inflaton ϕ by the second law of thermodynamics. There are some specific forms for the dissipative coefficient, with the most common which are found in the literatures being the $\Gamma \sim T^3$ form [24–26,9]. In some works Γ and potential of the inflaton have the same form [20,22]. In Ref. [12], perturbation parameters for warm inflationary model with viscous pressure have been obtained where $\Gamma = \Gamma(\phi) = V(\phi)$ and $\Gamma = \Gamma_0 = const$. In this work we will study the warm-tachyon inflationary universe model with viscous pressure in this two cases.

The slow-roll parameters of the model are presented by

$$\epsilon = -\frac{\dot{H}}{H^2} \simeq \frac{1}{2(1+r)V}\left(\frac{V'}{V}\right)^2,$$

$$\eta = -\frac{\ddot{H}}{H\dot{H}} \simeq \frac{1}{(1+r)V}\left(\frac{V''}{V} - \frac{1}{2}\left(\frac{V'}{V}\right)^2\right). \tag{7}$$

From Eqs. (6) and (7) we find

$$\rho = \frac{1}{\gamma}\left(\frac{2}{3}\frac{r}{1+r}\epsilon\rho_\phi - \Pi\right). \tag{8}$$

The condition of slow-roll is $\epsilon < 1$, therefore from above equation, warm-tachyon inflation with viscous pressure could take place when

$$\rho_\phi > \frac{3(1+r)}{2r}[\gamma\rho + \Pi]. \tag{9}$$

Inflation period ends when $\epsilon \simeq 1$ which implies

$$\rho_\phi \simeq \frac{3(1+r)}{2r}[\gamma\rho+\Pi], \left[\frac{V'_f}{V_f}\right]^2\frac{1}{V_f} \simeq 2(1+r_f),$$

where the subscript f denotes the end of inflation. The number of e-folds is given by

$$N = \int_{\phi_*}^{\phi_f} H dt = \int_{\phi_*}^{\phi_f} \frac{H}{\dot\phi} d\phi = -\int_{\phi_*}^{\phi_f} \frac{V^2}{V'}(1+r)d\phi. \qquad (10)$$

where the subscript $*$ denotes the epoch when the cosmological scale exits the horizon.

We will study inhomogeneous perturbations of the FRW background by using the linear perturbation equation of warm inflation scenario [13]. These scalar perturbations in the longitudinal gauge, may be described by the perturbed FRW metric

$$ds^2 = (1+2\Phi)dt^2 - a^2(t)(1-2\Psi)\delta_{ij}dx^i dx^j, \qquad (11)$$

where Φ and Ψ are gauge-invariant metric perturbation variables [27]. All perturbed quantities have a spatial sector of the form $e^{i\mathbf{kx}}$, where k is the wave number. Following Ref. [13], we introduce the stress–energy tensor as

$$T_{ab} = (\rho+P)n_a n_b + P g_{ab} + n_a q_b + n_b q_b + \Pi_{ab} \qquad (12)$$

where the trace-free tensor Π_{ab} and q_a are orthogonal to the unit vector n_a (n^a is the unit normal to the constant-time surface [13]). For the linear perturbation theory ρ and P are replaced by $\rho+\delta\rho$ and $P+\delta P$ respectively. We also define the perturbation parameters

$$q_i = (\rho+P)\nabla_i\delta V \qquad \delta\Pi_{ij} = \nabla_i\nabla_j\delta\Pi - \frac{1}{3}g_{ij}\nabla^2\delta\Pi \qquad (13)$$

So, the perturbed Einstein field equation of motion in momentum space have these forms

$$\Phi = \Psi,$$
$$\dot\Phi + H\Phi = \frac{1}{2}\left[-\frac{(\gamma\rho+\Pi)av}{k} + \frac{V\dot\phi}{\sqrt{1-\dot\phi^2}}\delta\phi\right], \qquad (14)$$

$$\frac{\ddot{\delta\phi}}{1-\dot\phi^2} + \left[3H+\frac{\Gamma}{V}\right]\dot{\delta\phi} + \left[\frac{k^2}{a^2} + (\ln V)'' + \dot\phi\left(\frac{\Gamma}{V}\right)'\right]\delta\phi$$
$$-\left[\frac{1}{1-\dot\phi^2}+3\right]\dot\phi\dot\Phi - \left[\dot\phi\frac{\Gamma}{V} - 2(\ln V)'\right]\Phi = 0. \qquad (15)$$

The fluid equations obtain from the stress-energy tensor [13].

$$\dot{(\delta\rho)} + 3\gamma H\delta\rho + ka(\gamma\rho+\Pi)v + 3(\gamma\rho+\Pi)\dot\Phi$$
$$-\dot\phi^2\Gamma'\delta\phi - \Gamma\dot\phi[2\dot{(\delta\phi)} + \dot\phi\Phi] = 0, \qquad (16)$$

$$\dot v + 4Hv + \frac{k}{a}\left[\Phi + \frac{\delta P}{\rho+P} + \frac{\Gamma\dot\phi}{\rho+P}\delta\phi\right] = 0. \qquad (17)$$

where

$$\delta P = (\gamma-1)\delta\rho + \delta\Pi, \qquad \delta\Pi = \Pi\left[\frac{\zeta_{,\rho}}{\zeta}\delta\rho + \Phi + \frac{\dot\Phi}{H}\right].$$

The above equations are obtained for Fourier components $e^{i\mathbf{kx}}$, where the subscript k is omitted. v in the above set of equations is given by the decomposition of the velocity field ($\delta u_j = -\frac{iak_j}{k}ve^{i\mathbf{kx}}$, $j=1,2,3$) [21]. Warm inflation model may be considered as a hybrid-like inflationary model where the inflaton field interacts with imperfect fluid [19,28]. Entropy perturbation may be related to dissipation term [29]. In slow-roll approximation the set of perturbed equations are reduced to [22]

$$\Phi \simeq \frac{1}{2H}\left[-\frac{4(\gamma\rho+\Pi)av}{k} + V\dot\phi\delta\phi\right], \qquad (18)$$

$$\left[3H+\frac{\Gamma}{V}\right]\dot{\delta\phi} + \left[(\ln V)'' + \dot\phi\left(\frac{\Gamma}{V}\right)'\right]\delta\phi \simeq \left[\dot\phi\frac{\Gamma}{V} - 2(\ln V)'\right]\Phi, \qquad (19)$$

$$\delta\rho \simeq \frac{\dot\phi^2}{3\gamma H}\left[\Gamma'\delta\phi + \Gamma\Phi\right], \qquad (20)$$

and

$$v \simeq -\frac{k}{4aH}\left(\Phi + \frac{(\gamma-1)\delta\rho+\delta\Pi}{\gamma\rho+\Pi} + \frac{\Gamma\dot\phi}{\gamma\rho+\Pi}\delta\phi\right). \qquad (21)$$

Using Eqs. (18), (20) and (21), perturbation variable Φ is determined

$$\Phi \simeq \frac{\dot\phi V}{2H}\frac{\delta\phi}{G(\phi)}\left[1+\frac{\Gamma}{4HV} + \left([\gamma-1]+\Pi\frac{\zeta_{,\rho}}{\zeta}\right)\frac{\dot\phi\Gamma'}{12\gamma H^2 V}\right], \qquad (22)$$

where

$$G(\phi) = 1 - \frac{1}{8H^2}\left[2\gamma\rho + 3\Pi + \frac{\gamma\rho+\Pi}{\gamma}\left(\Pi\frac{\zeta_{,\rho}}{\zeta} - 1\right)\right].$$

In Eq. (22), for $\Pi \to 0$ and $\gamma = \frac{4}{3}$ case, we may obtain the perturbation variable Φ of warm tachyon inflation model without viscous pressure effect [20]. (In this case, we find $G(\phi) \to 1$ because of the inequality $\frac{\rho}{V} \ll 1$.) Using Eq. (5), we find

$$\left(3H+\frac{\Gamma}{V}\right)\frac{d}{dt} = \left(3H+\frac{\Gamma}{V}\right)\dot\phi\frac{d}{d\phi} = -\frac{V'}{V}\frac{d}{d\phi}. \qquad (23)$$

From above equation, Eq. (19) and Eq. (22), the expression $\frac{(\delta\phi)'}{\delta\phi}$ is obtained

$$\frac{(\delta\phi)'}{\delta\phi} = \frac{1}{(\ln V)'}\left[(\ln V)'' + \dot\phi\left(\frac{\Gamma}{V}\right)' + \left(2(\ln V)' - \dot\phi\frac{\Gamma}{V}\right)\left(\frac{V\dot\phi}{2GH}\right)\right.$$
$$\left.\times\left(1+\frac{\Gamma}{4HV} + \left[(\gamma-1)+\Pi\frac{\zeta_{,\rho}}{\zeta}\right]\frac{\dot\phi\Gamma'}{12\gamma H^2 V}\right)\right]. \qquad (24)$$

We will return to the above relation soon. Following Refs. [20–22] and [29], we introduce auxiliary function χ as

$$\chi = \frac{\delta\phi}{(\ln V)'}\exp\left[\int \frac{1}{3H+\frac{\Gamma}{V}}\left(\frac{\Gamma}{V}\right)' d\phi\right]. \qquad (25)$$

From above definition we have

$$\frac{\chi'}{\chi} = \frac{(\delta\phi)'}{\delta\phi} - \frac{(\ln V)''}{(\ln V)'} + \frac{(\frac{\Gamma}{V})'}{3H+\frac{\Gamma}{V}}. \qquad (26)$$

Using above equation, Eqs. (24) and (5)

$$\frac{\chi'}{\chi} = -\frac{9}{8G}\frac{2H+\frac{\Gamma}{V}}{(3H+\frac{\Gamma}{V})^2}\left[\Gamma + 4HV - \left([\gamma-1]+\Pi\frac{\zeta_{,\rho}}{\zeta}\right)\right.$$
$$\left.\times\frac{\Gamma'(\ln V)'}{3\gamma H(3H+\frac{\Gamma}{V})}\right]\frac{(\ln V)'}{V}. \qquad (27)$$

A solution for the above equation is

$$\chi(\phi) = C\exp\Big(-\int\Big\{-\frac{9}{8G}\frac{2H+\frac{\Gamma}{V}}{(3H+\frac{\Gamma}{V})^2}\times\Big[\Gamma+4HV-\Big([\gamma-1]+\Pi\frac{\zeta_{,\rho}}{\zeta}\Big)\frac{\Gamma'(\ln V)'}{3\gamma H(3H+\frac{\Gamma}{V})}\Big]\times\frac{(\ln V)'}{V}\Big\}d\phi\Big), \tag{28}$$

where C is integration constant. From above equation and Eq. (26) we find small change of variable $\delta\phi$

$$\delta\phi = C(\ln V)'\exp\big(\Im(\phi)\big), \tag{29}$$

where

$$\Im(\phi) = -\int\Big[\frac{(\frac{\Gamma}{V})'}{3H+\frac{\Gamma}{V}}+\frac{9}{8G}\frac{2H+\frac{\Gamma}{V}}{(3H+\frac{\Gamma}{V})^2}\times\Big[\Gamma+4HV-\Big([\gamma-1]+\Pi\frac{\zeta_{,\rho}}{\zeta}\Big)\frac{\Gamma'(\ln V)'}{3\gamma H(3H+\frac{\Gamma}{V})}\Big]\times\frac{(\ln V)'}{V}\Big]d\phi \tag{30}$$

Finally the density perturbation is given by [30]

$$\delta_H = \frac{16\pi}{5}\frac{\exp(-\Im(\phi))}{(\ln V)'}\delta\phi = \frac{16\pi}{15}\frac{\exp(-\Im(\phi))}{H r\dot{\phi}}\delta\phi. \tag{31}$$

By inserting $\Gamma = 0$ and $\xi = 0$, the above equation reduces to $\delta_H \simeq \frac{H}{\dot\phi}\delta\phi$ which agrees with the density perturbation in cool inflation model [1]. In warm inflation model the fluctuations of the scalar field in high dissipative regime ($r \gg 1$) may be generated by thermal fluctuation instead of quantum fluctuations [31] as

$$(\delta\phi)^2 \simeq \frac{k_F T_r}{2\pi^2}, \tag{32}$$

where in this limit freeze-out wave number $k_F = \sqrt{\frac{\Gamma H}{V}} = H\sqrt{3r} \geq H$ corresponds to the freeze-out scale at the point when, dissipation damps out to thermally excited fluctuations ($\frac{V''}{V'} < \frac{\Gamma H}{V}$) [31]. With the help of the above equation and Eq. (31) in high dissipative regime ($r \gg 1$) we find

$$\delta_H^2 = \frac{64}{225\sqrt{3}}\frac{\exp(-2\Im(\phi))}{r^{\frac{1}{2}}\tilde\epsilon}\frac{T_r}{H}, \tag{33}$$

where

$$\tilde\Im(\phi) = -\int\Big[\frac{1}{3Hr}\Big(\frac{\Gamma}{V}\Big)'+\frac{9}{8G}\Big(1-\Big[(\gamma-1)+\Pi\frac{\zeta_{,\rho}}{\zeta}\Big]\times\frac{(\ln\Gamma)'(\ln V)'}{9\gamma r H^2}\Big)(\ln V)'\Big]d\phi, \tag{34}$$

and

$$\tilde\epsilon = \frac{1}{2r}\frac{V'^2}{V^3}. \tag{35}$$

An important perturbation parameter is scalar index n_s which in high dissipative regime is given by

$$n_s = 1+\frac{d\ln\delta_H^2}{d\ln k} \approx 1-\frac{5}{2}\tilde\epsilon-\frac{3}{2}\tilde\eta+\tilde\epsilon\Big(\frac{2V}{V'}\Big)\Big(\frac{r'}{4r}-2\tilde\Im(\phi)'\Big), \tag{36}$$

where

$$\tilde\eta = \frac{1}{rV}\Big[\frac{V''}{V}-\frac{1}{2}\Big(\frac{V'}{V}\Big)^2\Big]. \tag{37}$$

In Eq. (36) we have used a relation between small change of the number of e-folds and interval in wave number ($dN = -d\ln k$). The Planck measurement constraints the spectral index as [3]:

$$n_s = 0.96 \pm 0.0073 \tag{38}$$

Running of the scalar spectral index may be found as

$$\alpha_s = \frac{dn_s}{d\ln k} = -\frac{dn_s}{dN} = -\frac{d\phi}{dN}\frac{dn_s}{d\phi} = \frac{1}{rV}\Big(\frac{V'}{V}\Big)n_s'. \tag{39}$$

This parameter is one of the interesting cosmological perturbation parameters which is approximately -0.0134 ± 0.0090, by using Planck observational results [3].

During inflation epoch, there are two independent components of gravitational waves ($h_{\times +}$) with action of massless scalar field are produced by the generation of tensor perturbations. The amplitude of tensor perturbation is given by

$$A_g^2 = 2\Big(\frac{H}{2\pi}\Big)^2\coth\Big[\frac{k}{2T}\Big] = \frac{V^2}{6\pi^2}\coth\Big[\frac{k}{2T}\Big], \tag{40}$$

where, the temperature T in extra factor $\coth[\frac{k}{2T}]$ denotes the temperature of the thermal background of gravitational wave [32]. Spectral index n_g may be found as

$$n_g = \frac{d}{d\ln k}\Big(\ln\Big[\frac{A_g^2}{\coth(\frac{k}{2T})}\Big]\Big) \simeq -2\tilde\epsilon, \tag{41}$$

where $A_g \propto k^{n_g}\coth[\frac{k}{2T}]$ [32]. Using Eqs. (33) and (40) we write the tensor–scalar ratio in high dissipative regime

$$R(k) = \frac{A_g^2}{P_R}\Big|_{k=k_0} = \frac{54\sqrt{3}}{5}\frac{r^{\frac{1}{2}}\tilde\epsilon H^3}{T_r}\exp\big(2\Im(\phi)\big)\coth\Big[\frac{k}{2T}\Big]\Big|_{k=k_0}, \tag{42}$$

where k_0 is referred to pivot point [32] and $P_R = \frac{25}{4}\delta_H^2$. An upper bound for this parameter is obtained by using Planck data, $R < 0.11$ [2]. Non-Gaussianity of the warm-tachyon inflation model is presented in Ref. [20] as

$$f_{NL} = -\frac{5}{3}\frac{\dot\phi}{H}\Big[\frac{1}{H}\ln\Big(\frac{k_F}{H}\Big)\Big(\frac{V'''+2k_F^2V'}{\Gamma}\Big)\Big]. \tag{43}$$

In high dissipative regime ($r \gg 1$), f_{NL} parameter has the following form

$$f_{NL} = \frac{5}{9}\Big(\frac{V'}{V}\Big)^2\Big(\frac{\ln r}{r}\Big). \tag{44}$$

In the above equation, we have used Eq. (5) and definition $k_F = \sqrt{\frac{\Gamma H}{V}} = H\sqrt{3r}$.

We note that, the $\Im(\phi)$ factor (30) which is found in perturbation parameters (33), (36), (39) and (42) in high energy limit ($V \gg \lambda$), for tachyonic warm-viscous inflation model has an important difference with the same factor which was obtained for non-viscous tachyonic warm inflation model [20]

$$\Im(\phi) = -\int\Big[\frac{(\frac{\Gamma}{V})'}{3H+\frac{\Gamma}{V}}+\frac{9}{8G}\frac{2H+\frac{\Gamma}{V}}{(3H+\frac{\Gamma}{V})^2}\times\Big[\Gamma+4HV-\frac{\Gamma'(\ln V)'}{36H(3H+\frac{\Gamma}{V})}\Big]\frac{(\ln V)'}{V}\Big]d\phi.$$

The bulk viscous pressure effect leads to this difference. Therefore, the perturbation parameters P_R, R, n_s and α_s which may be found by WMAP and Planck observational data, for our model with viscous pressure, are modified due to the effect of this additional pressure.

3. Exponential potential

In this section we consider our model with the tachyonic effective potential

$$V(\phi) = V_0 \exp(-\alpha\phi), \tag{45}$$

where parameter $\alpha > 0$ (with unit m_p) is related to mass of the tachyon field [33]. The exponential form of potential have characteristics of tachyon field ($\frac{dV}{d\phi} < 0$, and $V(\phi \to 0) \to V_{max}$). We develop our model in high dissipative regime, i.e. $r \gg 1$, for two cases: 1. Γ and ζ are constant parameters, 2. Γ as a function of tachyon field ϕ and ζ as a function of energy density ρ of imperfect fluid.

3.1. $\Gamma = \Gamma_0$, $\zeta = \zeta_0$ case

From Eq. (35), the slow-roll parameter $\tilde{\epsilon}$ in the present case has the form

$$\tilde{\epsilon} = \frac{\sqrt{3}}{2}\frac{\alpha^2\sqrt{V_0}}{\Gamma_0}\exp\left(-\alpha\frac{\phi}{2}\right). \tag{46}$$

Dissipation parameter $r = \frac{\Gamma}{3HV}$ in this case is given by

$$r = \frac{\Gamma_0}{\sqrt{3}V_0^{\frac{3}{2}}}\exp\left(\frac{3}{2}\alpha\phi\right) \gg 1. \tag{47}$$

We find the evolution of tachyon field with the help of Eq. (5)

$$\phi(t) = \frac{1}{\alpha}\ln\left[\frac{\alpha^2 V_0}{\Gamma_0}t + e^{\alpha\phi_i}\right], \tag{48}$$

where $\phi_i = \phi(t = 0)$. Hubble parameter for our model has the form

$$H = \sqrt{\frac{V_0}{3}}\exp\left(-\frac{\alpha\phi}{2}\right). \tag{49}$$

At the end of inflation ($\tilde{\epsilon} \simeq 1$) the tachyon field becomes

$$\phi_f = \frac{2}{\alpha}\ln\left[\frac{\sqrt{3V_0}\alpha^2}{2\Gamma_0}\right], \tag{50}$$

so, by using the above equation and Eq. (48) we may find time at which inflation ends

$$t_f = \frac{3}{4}\frac{\alpha^2}{\Gamma_0} - \frac{\Gamma_0}{\alpha^2 V_0}e^{\alpha\phi_i}. \tag{51}$$

Using Eqs. (8) and (46), the energy density of the radiation-matter fluid in high dissipative limit becomes

$$\rho = \sqrt{\frac{V_0}{3\gamma^2}}\exp\left(-\frac{\alpha\phi}{2}\right)\left[\frac{\alpha^2}{\Gamma_0}V_0\exp(-\alpha\phi) + 3\zeta_0\right], \tag{52}$$

and, in terms of tachyon field energy density ρ_ϕ becomes

$$\rho = \frac{\rho_\phi^{\frac{1}{2}}}{\sqrt{3}\gamma}\left(\frac{\alpha^2}{\Gamma_0}\rho_\phi + 3\zeta_0\right). \tag{53}$$

For this example, the entropy density in terms of energy density of inflaton may be obtained from above equation

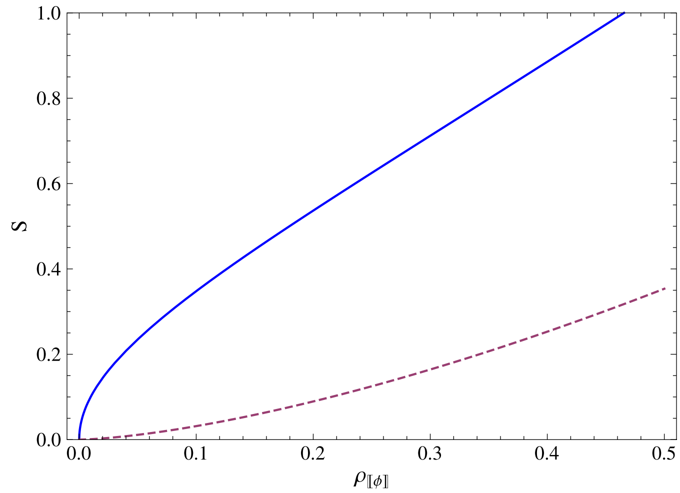

Fig. 1. We plot the entropy density s in terms of energy density of tachyon field ρ_ϕ where, $\Pi = 0$ (dashed curve) and $\Pi = -3\zeta_0 H$ (blue curve) ($T = 5.47 \times 10^{-5}$, $\Gamma = \Gamma_0 = 7.9 \times 10^3$, $\zeta_0 = 4.21 \times 10^{-5}$, $\alpha = 1$, $\gamma = \frac{4}{3}$). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$Ts = \frac{\rho_\phi^{\frac{1}{2}}}{\sqrt{3}\gamma}\left(\frac{\alpha^2}{\Gamma_0}\rho_\phi + 3\zeta_0\right). \tag{54}$$

In Fig. 1, we plot the entropy density in terms of inflaton energy density. It may be seen that the entropy density increases by the bulk viscous effect [24]. From Eq. (10), the number of e-folds at the end of inflation, by using the potential (45), for our inflation model is given by

$$N_{total} = \frac{2\Gamma_0}{\alpha^2\sqrt{3V_0}}\left[\exp\left(\frac{\alpha\phi_f}{2}\right) - \exp\left(\frac{\alpha\phi_i}{2}\right)\right]. \tag{55}$$

where $\phi_f > \phi_i$. Using Eqs. (33) and (42), we could find the scalar spectrum and scalar–tensor ratio

$$\delta_H^2 = \frac{128\sqrt{\Gamma_0}}{225\sqrt[4]{3}\alpha^2}\left[\frac{V^2(\phi_0)}{(\sqrt{V(\phi_0)} + A)^{\frac{9}{2}}}\right]\frac{T_r}{\sqrt[4]{V(\phi_0)}}, \tag{56}$$

where $A = \frac{3\sqrt{3}\zeta_0}{8}(-3 + \frac{5}{\gamma})$, and

$$R = \frac{9\sqrt[4]{3}}{5\sqrt{\Gamma_0}}\frac{(\sqrt{V(\phi_0)} + A)^{\frac{9}{2}}}{V^2(\phi_0)}\frac{V(\phi_0)^{\frac{5}{4}}}{T_r}\coth\left[\frac{k}{2T}\right], \tag{57}$$

respectively, where the subscript 0 denotes the time, when the perturbation was leaving the horizon. In the above equation we have used Eq. (34) where

$$\tilde{\Im}(\phi) = \ln\left(\frac{[\sqrt{V(\phi_0)} + A]^{\frac{9}{4}}}{V(\phi_0)}\right). \tag{58}$$

These parameters may be restricted by WMAP9 and Planck data [2,3]. Based on these data, an upper bound for $V(\phi_0)$ may be found

$$V(\phi_0) < 2.28 \times 10^{-4}.$$

In the above equation we have used these data: $R < 0.11$, $P_R = 2.28 \times 10^{-9}$ [2,3]. From Eqs. (44) and (47), non-Gaussianity for our model is presented as

$$f_{NL} = \frac{5\sqrt{3}}{9}\frac{\alpha^2 V_0^{\frac{3}{2}}}{\Gamma_0}\frac{\ln(\frac{\Gamma_0}{\sqrt{3}V_0^{\frac{3}{2}}}) + \frac{3}{2}\alpha\phi(t_F)}{\exp(\frac{3}{2}\alpha\phi(t_F))}, \tag{59}$$

where the freeze-out time t_F is the time when the last three wave-vectors k thermalize [20].

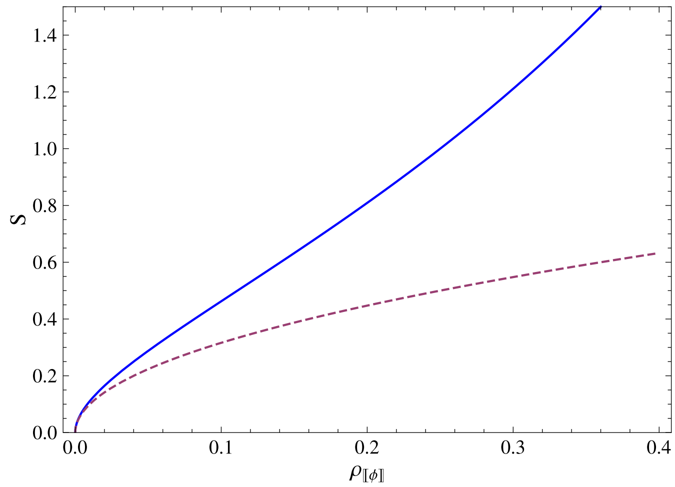

Fig. 2. We plot the entropy density s in terms of scalar field energy density ρ_ϕ where, $\Pi = 0$ (dashed line) and $\Pi = -3\zeta_1\rho H$ (blue curve) ($T = 5.47\times10^{-5}$, $\gamma = \frac{4}{3}$, $\alpha_1 = 1.38\times10^4$, $\zeta_1 = 0.76$, $\alpha = 1$). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

3.2. $\Gamma = \Gamma(\phi)$, $\zeta = \zeta(\rho)$ case

Now we assume $\zeta = \zeta(\rho) = \zeta_1\rho$, and $\Gamma = \Gamma(\phi) = \alpha_1 V(\phi) = \alpha_1 V_0 \exp(-\alpha\phi)$, where α_1 and ζ_1 are positive constants. By using exponential potential (45), Hubble parameter, r parameter and slow-roll parameter $\tilde\epsilon$ we have these forms

$$H(\phi) = \sqrt{\frac{V_0}{3}}\exp\left(-\frac{\alpha\phi}{2}\right), \qquad r = \frac{\alpha_1}{\sqrt{3V_0}}\exp\left(\alpha\frac{\phi}{2}\right),$$
$$\tilde\epsilon = \sqrt{\frac{3}{V_0}}\frac{\alpha^2}{2\alpha_1}\exp\left(\alpha\frac{\phi}{2}\right), \tag{60}$$

respectively. Using Eq. (5), we find the scalar field ϕ in terms of cosmic time

$$\phi(t) = -\frac{\alpha}{\alpha_1}t + \phi_i. \tag{61}$$

The energy density of imperfect fluid ρ in terms of the inflaton energy density ρ_ϕ, is given by the expression

$$\rho = \frac{\alpha^2}{\alpha_1}\frac{\rho_\phi^{\frac{1}{2}}}{\sqrt{3}}\left(\gamma - \sqrt{3}\xi_1\rho_\phi^{\frac{1}{2}}\right)^{-1}. \tag{62}$$

We can find the entropy density s in terms of energy density ρ_ϕ

$$Ts = \frac{\alpha^2}{\alpha_1}\frac{\rho_\phi^{\frac{1}{2}}}{\sqrt{3}}\left(\gamma - \sqrt{3}\xi_1\rho_\phi^{\frac{1}{2}}\right)^{-1}. \tag{63}$$

The entropy density and matter-radiation energy density of our model in this case increase by the bulk viscosity effect (see Fig. 2).

From Eq. (61) the scalar field and effective potential at the end of inflation where $\tilde\epsilon \simeq 1$, become

$$\phi_f = \frac{1}{\alpha}\ln\left[\frac{V_0}{3}\left(\frac{2\alpha_1}{\alpha^2}\right)^2\right], \qquad V_f = \frac{3}{4}\frac{\alpha^4}{\alpha_1^2}. \tag{64}$$

By using the above equation and Eq. (61) we may find time at which inflation ends

$$t_f = \frac{3}{4}\frac{\alpha^2}{\Gamma_0} - \frac{\Gamma_0}{\alpha^2 V_0}e^{\alpha\phi_i}. \tag{65}$$

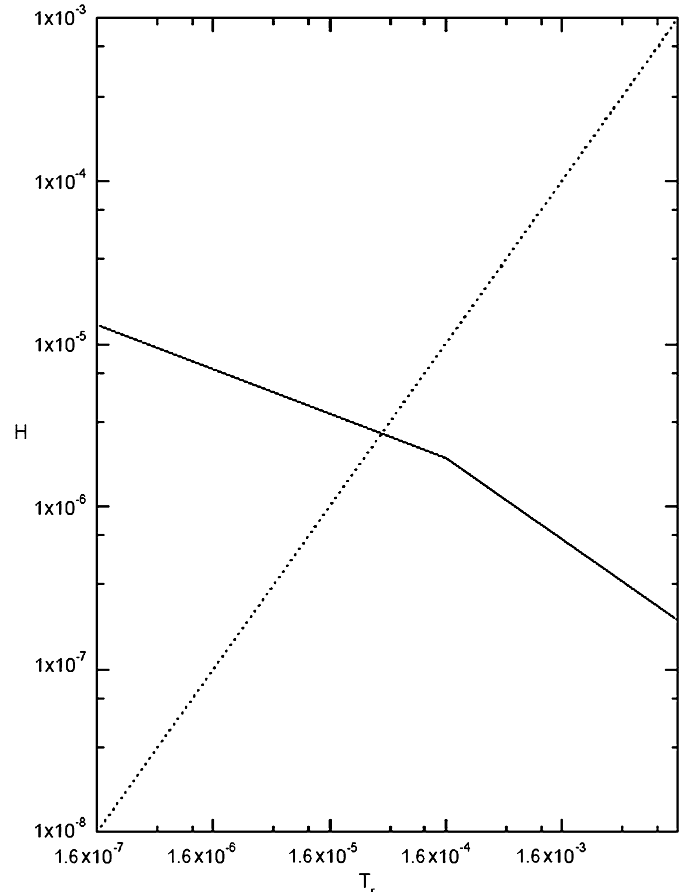

Fig. 3. In this graph we plot the Hubble parameter H in term of the temperature T_r. We can find the minimum amount of temperature $T_r = 5.47\times10^{-5}$ in order to have the necessary condition for warm inflation model ($T_r > H$).

Number of e-folds in this case is related to V_i and V_f by using Eq. (10)

$$V_i = (2N-1)^2 V_f. \tag{66}$$

At the beginning of the inflation r parameter is given by

$$r = r_i = \frac{2}{3}\frac{\alpha_1^2}{(2N-1)\alpha^2}. \tag{67}$$

High dissipative condition ($r \gg 1$), leads to $\alpha_1 \gg \alpha(N-1)^{\frac{1}{2}}$ which is agree with the warm-tachyon inflation model without viscous pressure [20]. From Eqs. (44) and (60), the non-Gaussianity for our model in this case ($\Gamma = \Gamma(\phi)$, $\xi = \xi(\rho)$) is given by

$$f_{NL} = \frac{5\sqrt{3}}{9}\frac{\alpha^2\sqrt{V_0}}{\alpha_1}\frac{\ln(\frac{\alpha}{\sqrt{3V_0}}) + \frac{\alpha\phi}{2}}{\exp(\frac{\alpha\phi}{2})}. \tag{68}$$

By using Eqs. (33) and (42) scalar power spectrum and tensor–scalar ratio result to be

$$\delta_H^2 = \frac{128\sqrt{\alpha_1}}{225\sqrt[4]{3}\alpha^2}\left[\sqrt{V(\phi_0)} + B\right]^{-\frac{9}{2}(1+\frac{\zeta_1\alpha^2}{\gamma\alpha_1})}$$
$$\times\exp\left(\frac{9}{8}[\gamma-1]\frac{\alpha^2}{\sqrt{3}\gamma\alpha_1}V(\phi_0)^{\frac{1}{2}}\right)\sqrt[4]{V(\phi_0)}T_r, \tag{69}$$

and

$$R=\frac{9\sqrt[4]{3}\alpha^2}{5\sqrt{\alpha_1}}\left[\sqrt{V(\phi_0)}+B\right]^{\frac{9}{2}(1+\frac{\zeta_1\alpha^2}{\gamma\alpha_1})}\times\exp\left(-\frac{9}{8}[\gamma-1]\frac{\alpha^2}{\sqrt{3}\gamma\alpha_1}V(\phi_0)^{\frac{1}{2}}\right)\frac{V(\phi_0)^{\frac{3}{4}}}{T_r},\qquad(70)$$

respectively, where $B=\frac{3\sqrt{3}\zeta_1^2V_0\alpha^2}{8\gamma\alpha_1}$. In the above equations we have used Eq. (34) where

$$\tilde{\Im}(\phi_0)=\frac{9}{16}(1-\gamma)\left(\frac{\alpha^2}{\sqrt{3}\gamma\alpha_1}\right)V^{-\frac{1}{2}}(\phi_0)+\frac{9}{4}\left[1+\frac{\zeta_1\alpha^2}{\gamma\alpha_1}\right]\ln\left(V^{\frac{1}{2}}(\phi_0)+B\right).\qquad(71)$$

These parameters may be restricted, using WMAP9, Planck and BICEP2 data [2–4]. Using WMAP9 (BICEP2) data, $P_R(k_0)=\frac{25}{4}\delta_H^2\simeq 2.28\times10^{-9}$, $R(k_0)\simeq 0.11$ ($R(k_0)\simeq 0.22$) and the characteristic of warm inflation, $T>H$ [5], we may restrict the values of temperature $T_r>5.47\times10^{-5}$ ($T_r>7.73\times10^{-5}$), using Eqs. (33), (42), or the corresponding equations (56), (57), (69), (70), in our coupled examples (see Fig. 3). We have chosen $k_0=0.002$ Mpc^{-1} and $T\simeq T_r$. Using BICEP2 data, we have found the new minimum of T_r (see for example [34]).

4. Conclusion

Warm-tachyon inflation model with viscous pressure, using overlasting form of potential $V(\phi)=V_0\exp(-\alpha\phi)$, which agrees with the tachyon potential properties, has been studied. The main problem of the inflation theory is how to attach the universe to the end of the inflation period. One of the solutions of this problem is the study of inflationary epoch in the context of warm inflation scenario [5]. In this model radiation is produced during inflation period where its energy density is kept nearly constant. This is phenomenologically fulfilled by introducing the dissipation term Γ. Warm inflation model with viscous pressure is an extension of warm inflation model where instead of radiation field we have radiation-matter fluid. The study of warm inflation model with viscous pressure as a mechanism that gives an end for tachyon inflation are motivated us to consider the warm tachyon inflation model with viscous pressure. In the slow-roll approximation the general relation between energy density of radiation-matter fluid and energy density of tachyon field is found. In longitudinal gauge and slow-roll limit the explicit expressions for the tensor–scalar ratio R scalar spectrum P_R index, n_s and its running α_s have been obtained. We have developed our specific model by exponential potential for two cases: 1. Constant dissipation coefficient Γ_0 and constant bulk viscous pressure coefficient ζ_0. 2. Γ as a function of tachyon field ϕ and ζ as a function of imperfect fluid energy density ρ. In these two cases we have found perturbation parameters and constrained these parameters by WMAP9 and Planck data.

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