Using Net Refinement to Compute the Fixpoint of a Recursive Expression

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Abstract

The talk illustrates the general Petri net semantics of equations such as $X = term(X)$ using a basic CCS-like process algebra without restriction, synchronisation and relabelling.

1 Summary

On a syntactic level, substitution of (sub)expressions is instrumental for defining the operational semantics of recursion in process algebras such as CCS [4]. In Petri nets, transition refinement plays a similar role. More precisely, let a recursive equation of the form

$$X = term(X) \tag{1}$$

be given, where $term(X)$ may contain variable X freely. A Petri net solving this equation can be constructed in two steps. First, the right hand side $term(X)$ of (1) is turned into a Petri net having 'holes' at transitions corresponding to the free occurrences of X. Second, a net N is constructed which, when refined into these 'holes', yields itself; that is, N is a fixpoint with respect to transition refinement. The construction of such a net leads to a unique result only in the case of guarded recursion. However, even in the unguarded case it can be shown that a (countable) solution always exists, and that every solution can be approximated in a canonical way.

Simultaneous transition refinement, which is at the heart of this construction, is a graph manipulation operation involving both node duplication and node replacement. It can be formalised using tree-like names for the nodes of graphs. Using this device, constructing the solutions of (1) can be reduced to solving equations on sets of trees. It is shown that such equations have unique minimal and maximal solutions. The minimal solution may be empty and thus unsuitable for constructing a net solving (1). However, the maximal solution is nonempty and, therefore, always suitable for obtaining a solution of (1).

The talk illustrates the general Petri net semantics of equations such as (1) using a basic CCS-like process algebra without restriction, synchronisation and

relabelling. This allows the principle of the construction to be explained easily. Moreover, operations such as restriction, synchronisation and relabelling are orthogonal to recursion, in the sense that they can be treated without needing to change the construction of the solution of a recursive equation (although refinement needs to be generalised by allowing multiset addition and/or deletion of nodes of a graph, in addition to duplication and replacement).

The talk will be based on [1–3]. This work is the outcome of a cooperation between the two authors, Raymond Devillers, and Javier Esparza, which was started in the Esprit BRA 3148 DEMON and continued in the Esprit Basic Research Working Group CALIBAN (Causal Calculi Based on Nets).

References

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