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## New 2D-Experiments and Numerical Simulations on Stress-State-Dependence of Ductile Damage and Failure

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### Abstract

The paper deals with a series of new experiments and corresponding numerical simulations to be able to study the effect of stress state on damage and failure behavior of ductile metals. The material behavior is modeled by a continuum approach based on free energy functions defined in damaged and corresponding fictitious undamaged configurations leading to elastic material laws which are affected by damage. Inelastic behavior of ductile materials is modeled by continuum plasticity and continuum damage model, respectively. The present approach takes into account the effect of stress state on damage and failure conditions expressed in terms of the stress intensity, the stress triaxiality and the Lode parameter. Previous studies have shown that it will not be possible to propose the stress-state-dependent functions for damage and failure criteria only based on tests with uniaxially loaded specimens. Therefore, new experiments with carefully designed and two-dimensionally loaded specimens have been developed. Corresponding numerical simulations of these tests show that they cover a wide range of stress states allowing validation of stress-state-dependent functions for the damage criterion and evolution laws for the damage strains.

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## 1. Introduction

Modeling of inelastic behavior, damage and fracture of engineering materials is an important topic in solid mechanics. Based on many experiments it is well known that inelastic deformations due to complex loading of structures with ductile metals are accompanied by damage and local failure mechanisms acting on different scales leading to macro-cracks in structural elements. For example, during tension tests (high positive stress triaxialities) damage in ductile metals is mainly caused by nucleation, growth and coalescence of voids. However, during shear and compression tests (small positive or negative stress triaxialities) formation and growth of micro-shear-cracks are the prevailing damage mechanisms on the micro-level. Thus, it is important to analyze and to understand these complex stress-state-dependent damage and fracture processes as well as its corresponding mechanisms acting on different scales to be able to propose an accurate and realistic phenomenological model. In this context, uniaxial tension tests with differently prenotched flat specimens and corresponding numerical simulations have been performed, for example, by Bao and Wierzbicki (2004), Bonora et al. (2005), Brünig et al. (2008, 2011), Gao et al. (2010), and Dunand and Mohr (2011). However, these experiments with unnotched and differently notched specimens only cover a small region of positive stress triaxialities between 0.33 and 0.6. In addition, new geometries of specimens have been designed to analyze stress states with nearly zero triaxiality. Uniaxial tests and corresponding numerical calculations with this kind of specimens have been discussed by Bao and Wierzbicki (2004), Brünig et al. (2008), Gao et al. (2010) and Driemeier et al. (2010). Furthermore, for other regions of stress triaxialities butterfly specimens have been developed by Mohr and Henn (2007), Bai and Wierzbicki (2008) and Dunand and Mohr (2011). Using special experimental equipment these specimens can be tested in different directions. Corresponding numerical simulations have shown that these tests lead to stress triaxialities between -0.33 and 0.60.

In the present paper, new experiments with biaxially loaded specimens will be presented covering a wide range of stress states corresponding to different damage and failure mechanisms acting on the micro-level. The geometry of the 2D-specimen allows combination of shear/tension or shear/compression mechanisms with various ratios of the respective loads. Corresponding numerical simulations of these tests will show that they cover a wide range of stress triaxialities with different Lode parameters allowing validation of stress-state-dependent functions for damage and failure criteria as well as damage evolution laws.

## 2. Continuum damage model

The continuum damage model proposed by Brünig (2003) is used to predict the elastic-plastic deformations including damage. The approach takes into account information of the mechanisms on the micro-level of individual micro-defects and their interactions detected by numerical analyses discussed by Brünig et al. (2013). The phenomenological model is based on the consideration of damaged and corresponding fictitious undamaged configurations. The kinematic approach leads to the additive decomposition of the strain rate tensor into elastic, plastic and damage parts.

The effective undamaged configurations are used to model the elastic-plastic behavior of the undamaged matrix material. Undamaged elastic behavior is modeled by Hooke's law and plastic yielding of the aluminum alloy under investigation is governed by the yield condition

$$f^{\text{pl}} = \sqrt{\bar{J}_2} - c \left( 1 - \frac{a}{c} \bar{I}_1 \right) = 0 \quad (1)$$

where  $\bar{I}_1$  and  $\bar{J}_2$  are the first and the second deviatoric invariants of the effective stress tensor  $\bar{\mathbf{T}}$  (referred to the undamaged configurations),  $c$  denotes the strength coefficient and  $a$  represents the hydrostatic stress coefficient of the matrix material where  $a/c$  is a constant material parameter. In elastic-plastically deformed and damaged metals, irreversible volumetric strains are mainly caused by damage and, in comparison, volumetric plastic strains are negligible. Thus, the isochoric effective plastic strain rate

$$\dot{\mathbf{H}}^{\text{pl}} = \dot{\gamma} \frac{1}{\sqrt{2J_2}} \text{dev}\bar{\mathbf{T}} \quad (2)$$

is assumed to describe the evolution of plastic deformations in an accurate manner. In Eq. (2),  $\dot{\gamma}$  is a non-negative scalar-valued factor representing the equivalent plastic strain rate measure used in the present constitutive model.

Furthermore, the formulation of constitutive equations characterizing the elastic and damage behavior of the damaged aggregate is performed with respect to the anisotropically damaged configurations (Brünig, 2003). In particular, onset and continuation of damage during further loading of a material sample is described by a stress-state-dependent damage criterion. In addition, the generalized elastic-damage constitutive law is a function of elastic and damage strain tensors to be able to model deterioration of elastic material properties due to formation and growth of micro-defects. In this context, different damage and failure mechanisms acting on the micro-level are taken into account (see Bao and Wierzbicki, 2004, and Brünig et al., 2011, 2013, for further details). This leads to consideration of a wide range of stress triaxialities with different branches: damage and failure are caused by nucleation and growth of nearly spherical voids for large positive stress triaxialities, by formation and growth of micro-shear-cracks in the negative stress triaxiality regime and by combination of both mechanisms acting on the micro-scale for low positive stress triaxialities. In addition, for stress states with remarkable hydrostatic pressure a cut-off value has been proposed below which damage and fracture do not occur. This stress-triaxiality-dependent concept is taken into account in the present investigation and will be generalized by additional consideration of the Lode parameter.

Based on these aspects, the onset and continuation of continuum damage is assumed to be governed by the damage condition

$$f^{\text{da}} = \alpha I_1 + \beta \sqrt{J_2} - \sigma = 0 \quad (3)$$

Where  $\sigma$  is the damage threshold and  $\alpha$  and  $\beta$  represent the damage mode parameters taking into account the different branches depending on the stress triaxiality

$$\eta = \sigma_m / \sigma_{\text{eq}} = I_1 / (3\sqrt{3J_2}) \quad (4)$$

defined as the ratio of the mean stress  $\sigma_m$  and the von Mises equivalent stress  $\sigma_{\text{eq}}$  as well as on the Lode parameter

$$\omega = \frac{2\tilde{T}_2 - \tilde{T}_1 - \tilde{T}_3}{\tilde{T}_1 - \tilde{T}_3} \quad \text{with} \quad \tilde{T}_1 \geq \tilde{T}_2 \geq \tilde{T}_3 \quad (5)$$

expressed in terms of the principal Kirchhoff stress components  $\tilde{T}_1$ ,  $\tilde{T}_2$  and  $\tilde{T}_3$  with respect to the damaged configurations.

The damage evolution law models the increase in macroscopic damage strains caused by the simultaneous growth of voids, their coalescence as well as the evolution of micro-cracks and micro-shear-cracks leading to anisotropic damage behavior. This will be adequately described by the damage rule

$$\dot{\mathbf{H}}^{\text{da}} = \dot{\mu} \left( \bar{\alpha} \frac{1}{\sqrt{3}} \mathbf{1} + \bar{\beta} \mathbf{N} + \bar{\delta} \mathbf{M} \right) \quad (6)$$

predicting the rate of irreversible strains due to evolution and growth of micro-defects where  $\bar{\alpha}$ ,  $\bar{\beta}$  and  $\bar{\delta}$  are stress-state-dependent kinematic variables and  $\dot{\mu}$  represents the rate of the equivalent damage strain. In addition,

$\mathbf{N} = (\sqrt{2J_2})^{-1} \text{dev}\tilde{\mathbf{T}}$  and  $\mathbf{M} = (\|\text{dev}\tilde{\mathbf{S}}\|)^{-1} \text{dev}\tilde{\mathbf{S}}$  are normalized deviatoric stress tensors where  $\tilde{\mathbf{T}}$  denotes the stress tensor work-conjugate to the damage strain rate (6) and

$$\text{dev}\tilde{\mathbf{S}} = \text{dev}\tilde{\mathbf{T}} \text{dev}\tilde{\mathbf{T}} - \frac{2}{3} J_2 \mathbf{1} . \quad (7)$$

It is worthy to note that it is not possible to identify all parameters appearing in the constitutive equations given above as well as their stress-state-dependence only by uniaxial tests and, therefore, additional three-dimensional unit cell model calculations have been performed by Brünig et al. (2013) covering a wide range of stress triaxialities and Lode parameters. Based on their numerical results the stress-state-dependent parameters  $\alpha$  and  $\beta$  (Eq. (3)) as well as  $\bar{\alpha}$ ,  $\bar{\beta}$  and  $\bar{\delta}$  (Eq. (6)) have been identified for an aluminum alloy. However, the respective functions for these parameters are only based on numerical calculations on the micro-level and, therefore, to be able to validate the

stress-state-dependent damage criteria (3) and damage rules (6) series of new tests with biaxially loaded specimens and corresponding numerical simulations of these experiments have been developed and performed.

### 3. Experiments and numerical simulations

Experiments and corresponding numerical simulations on the macro-scale have to be performed to be able to identify material parameters appearing in the constitutive equations. In particular, experimental results from tension tests with unnotched specimens are used to determine basic elastic-plastic material parameters. Equivalent stress-equivalent strain curves can be easily obtained from load-displacement curves as long as the uniaxial stress state is homogeneous in the central part of the specimen. For the aluminum alloy investigated in the present paper fitting of experimental and numerical curves leads to Young's modulus  $E = 75000\text{MPa}$  and Poisson's ratio is taken to be  $\nu = 0.3$ . In addition, a power-law function for the equivalent stress

$$c = c_0 \left( \frac{H\gamma}{nc_0} + 1 \right)^n \quad (8)$$

is used to model the work-hardening behavior with the initial yield strength  $c_0 = 325\text{MPa}$ , the hardening modulus  $H = 3125\text{MPa}$  and the hardening exponent  $n = 0.135$ .

Furthermore, biaxially loaded specimens are tested in a 2D-tension/compression machine to achieve different stress states in the center of the specimens. Figure 1 shows the geometry of the specimen and the biaxial loading conditions. It should be noted that there is an additional notch in thickness direction in the center of the specimen. The load  $F_1$  leads to shear mechanisms in the specimen's center whereas the load  $F_2$  leads to additional tension or compression modes. These experiments with simultaneous loading in horizontal and vertical direction may be seen as generalization of the tests performed by Driemeier et al. (2010) performing subsequent shear and tension tests. In addition, load-displacement curves for different loading conditions leading to shear-tension, shear-compression and shear mechanisms in the specimen's center are also shown in Fig. 1. It can be seen that the additional force  $F_2$  only slightly affects the vertical load  $F_1$ .

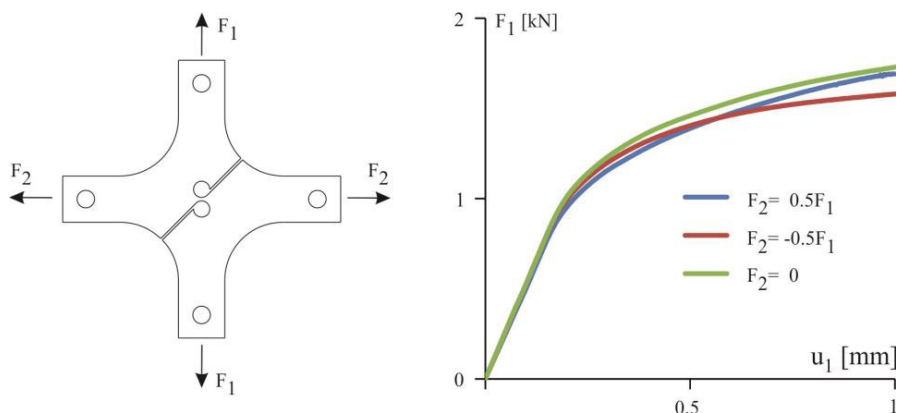


Fig. 1 2D-loaded shear-tension specimen and load-displacement curves

However, the additional forces have remarkable effect on the stress states in the center of the specimens. These have been detected by numerical simulations of the respective experiments. Following the ideas given by Brünig et al. (2011) onset of damage is identified by comparison of experimental load-displacement curves with those obtained by numerical calculations using an elastic-plastic analysis. For example, Fig. 2 shows the distribution of the stress triaxiality  $\eta$  in the specimen's center (vertical section) at onset of damage for different loading ratios. In particular, in uniaxial tension loading ( $F_1:F_2 = 0:1$ ) remarkable high stress triaxiality of  $\eta = 0.84$  is numerically predicted in the midpoint of the specimen which will lead to void growth modes. This high stress triaxiality is a consequence of the notches in horizontal and thickness direction. On the other hand, nearly zero stress triaxiality exists when the specimen is only loaded by  $F_1$  ( $F_1:F_2 = 1:0$ ). This will lead to formation of micro-shear-cracks.

Additional forces  $F_2$  in horizontal direction will lead to different stress triaxialities which are nearly constant over the central area shown in Fig. 2. The loading ratio  $F_1:F_2=1:1$  leads to  $\eta = 0.25$ , the ratio  $F_1:F_2=1:0.5$  to  $\eta = 0.14$ , and the ratio  $F_1:F_2=1:-0.5$  to  $\eta = -0.14$  corresponding to different damage modes.

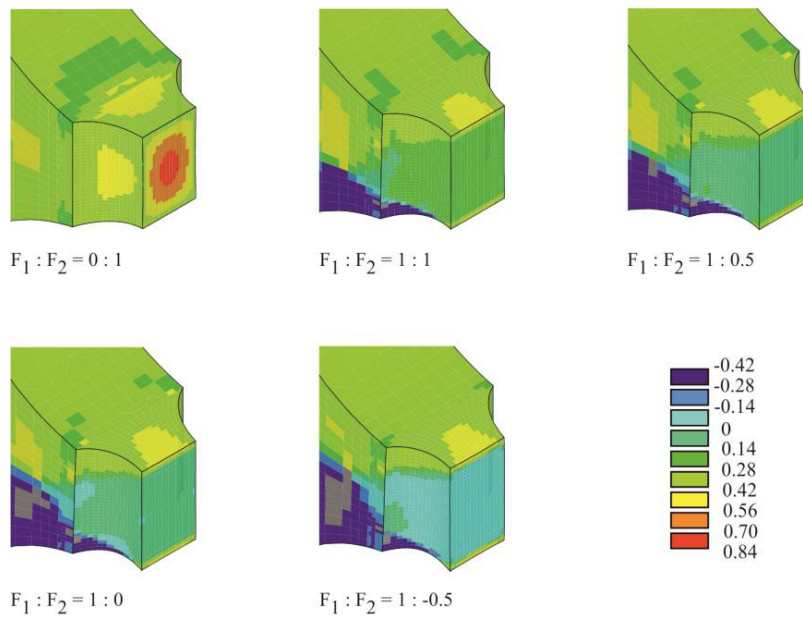


Fig. 2 Numerically predicted distribution of the stress triaxiality  $\eta$  for different load ratios

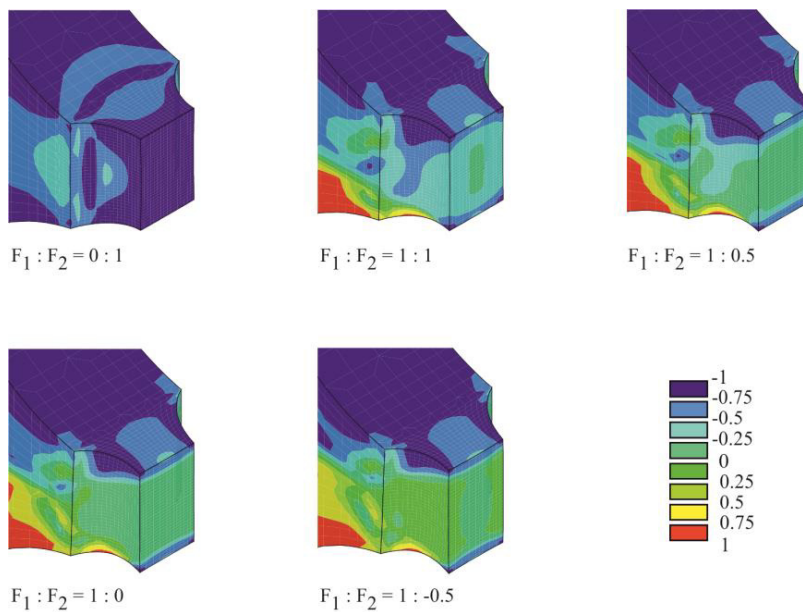


Fig. 3 Numerically predicted distribution of the Lode parameter  $\omega$  for different load ratios

Furthermore, Fig. 3 shows the distribution of the Lode parameter  $\omega$  in the specimen's center at onset of damage for different loading ratios. For example, in uniaxial tension loading ( $F_1:F_2=0:1$ ) the Lode parameter of  $\omega=-1.0$  is numerically predicted in the central cross section of the specimen. On the other hand, nearly zero Lode parameter exist in the shear mode regime when the specimen is only loaded by  $F_1$  ( $F_1:F_2=1:0$ ). Additional forces  $F_2$  in horizontal direction will lead to different Lode parameters which then become more inhomogeneous over the cross section shown in Fig. 3. For example, the loading ratio  $F_1:F_2=1:1$  leads in this area to various Lode parameters between  $\omega=-0.50$  and  $\omega=-0.10$ , the ratio  $F_1:F_2=1:0.5$  to nearly homogeneous  $\omega=-0.10$ , and the ratio  $F_1:F_2=1:-0.5$  to inhomogeneous distribution up to  $\omega=0.20$ . These different stress states allow validation of the stress-state-dependent damage criterion proposed by Brünig et al. (2013) based on series of numerical calculations on the micro-level.

#### 4. Conclusions

A phenomenological continuum damage model has been discussed taking into account the effect of stress state on damage and failure mechanisms acting on the micro-level. To be able to propose stress-state-dependent damage and fracture criteria as well as evolution equations for inelastic strains series of new 2D experiments with biaxially loaded specimens have been developed. Corresponding numerical simulations have shown that a wide range of stress triaxialities and Lode parameters can be considered and their effect on onset and propagation of damage can be studied in detail. Numerical results are used to validate or to modify constitutive functions depending on the stress triaxiality and the Lode parameter.

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