Hairy AdS solitons

Andrés Anabalón [a], Dumitru Astefanesei [b], David Choque [b,c,*]

[a] *Departamento de Ciencias, Facultad de Artes Liberales and Facultad de Ingeniería y Ciencias, Universidad Adolfo Ibáñez, Av. Padre Hurtado 750, Viña del Mar, Chile*
[b] *Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla 4059, Valparaíso, Chile*
[c] *Universidad Técnica Federico Santa María, Av. España 1680, Valparaíso, Chile*

A B S T R A C T

We construct exact hairy AdS soliton solutions in Einstein-dilaton gravity theory. We examine their thermodynamic properties and discuss the role of these solutions for the existence of first order phase transitions for hairy black holes. The negative energy density associated to hairy AdS solitons can be interpreted as the Casimir energy that is generated in the dual filed theory when the fermions are antiperiodic on the compact coordinate.

© 2016 Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/). Funded by SCOAP³.

1. Introduction

We construct analytic neutral hairy soliton solutions in Anti de Sitter (AdS) spacetime and discus their properties. This analysis is important in the context of AdS/CFT duality [1] because bulk solutions correspond to 'phases' of the dual field theory [2].

There is by now a huge literature on (locally) asymptotically AdS solutions in both phenomenological models and consistent embedding in supergravity. We will consider theories of gravity coupled to a scalar field with potential $V(\phi)$. AdS spacetime is not globally hyperbolic, which means that the evolution is well defined if the boundary conditions are imposed. In particular, since for the same self-interaction there exist many boundary conditions for the scalar field (that may or may not break the conformal symmetry in the boundary), one can 'design' a specific field theory [3] with a given effective potential [3–5].

Different foliations of AdS spacetime lead to different definitions of time and so to distinct Hamiltonians of the dual field theory. Since the classical (super)gravity background, with possible α' corrections, is equivalent to the full quantum gauge theory on the corresponding slice, one expects physically inequivalent dual theories for different foliations. Indeed, when the horizon topology of the black hole is Ricci flat and there are no compact directions, there are no first order phase transitions similar to the Hawking–Page [6] phase transitions that exist for the spherically symmetric black holes. However, when some of the spatial directions are compactified on a circle asymptotically, one expects the existence of a negative Casimir energy of the non-supersymmetric field theory that 'lives' on the corresponding topology. Horowitz and Myers have shown in [7] that, indeed, there exists a (bulk) gravity solution dubbed 'AdS soliton' with a lower energy than AdS itself. This solution was obtained by a double analytic continuation (in time and one of the compactified angular directions) of the planar black hole. This fits very nicely with the proposal of Witten [2] that a non-supersymmetric Yang–Mills gauge theory can be described within AdS/CFT duality by compactifying one direction and imposing anti-periodic boundary conditions for the fermions around the circle.

Hairy neutral AdS solitons were previously analysed (see, e.g. [8–14]), though most of these studies are using numerical methods. Hence, it would be interesting to find examples of analytic hairy AdS solitons and investigate their generic properties. In recent years, analytic regular neutral hairy black holes in AdS were constructed, e.g. [15–21] and so one expects that constructing analytic soliton solutions could be also possible. We use some particular exact planar hairy black hole solutions in four and five dimensions of [15,16] and obtain the corresponding solitons by using a double analytic continuation as in [7]. The hairy AdS solitons are the ground state candidates of the theory [22].

Since the AdS soliton is the solution with the minimum energy within these boundary conditions [23,24], it is natural to investigate the existence of phase transitions with respect to this thermal background. In the nice work [25], it was shown that there exist first order phase transitions between planar black holes and the AdS soliton. We construct the hairy AdS soliton and compute their

* Corresponding author.
E-mail addresses: andres.anabalon@uai.cl (A. Anabalón), dumitru.astefanesei@pucv.cl (D. Astefanesei), brst1010123@gmail.com (D. Choque).

mass by using the counterterm method of Balasubramanian and Kraus [26] supplemented with extra counterterms for the scalar field as was proposed in [27]. We investigate then the existence of first order phase transitions with respect to the hairy AdS soliton and discuss the effect of 'hair' on the thermodynamical behaviour.

2. Hairy AdS soliton

In this section we construct exact hairy AdS soliton solutions in four and five dimensions and compute their energy. In five dimensions [15,16], we obtain a new hairy black hole solution, which corresponds to a parameter ν that at first sight makes the moduli potential divergent. However, by taking the right limit, we show that the theory is in fact well defined and the solution is regular.

2.1. AdS soliton

We start with a short review of [25], though, to connect this analysis with the rest of the paper, the computations are done by using the counterterm method of Balasubramanian and Kraus [26].

We consider the usual AdS gravity action supplemented with the gravitational counterterm proposed in [26]

$$I[g_{\mu\nu}] = \int_{\mathcal{M}} d^4x (R - 2\Lambda)\sqrt{-g} + 2\int_{\partial\mathcal{M}} d^3x K\sqrt{-h}$$
$$- \int_{\partial\mathcal{M}} d^3x \frac{4}{l}\sqrt{-h} \tag{1}$$

where $\Lambda = -3/l^2$ is the cosmological constant (l is the radius of AdS), $16\pi G_N = 1$ with G_N the Newton gravitational constant, the second term is the Gibbons–Hawking boundary term, and the last term is the gravitational counterterm. Here, h is the determinant of the induced boundary metric and K is the trace of the extrinsic curvature. The planar black hole solution is

$$ds^2 = -\left(-\frac{\mu_b}{r} + \frac{r^2}{l^2}\right)dt^2 + \left(-\frac{\mu_b}{r} + \frac{r^2}{l^2}\right)^{-1}dr^2$$
$$+ \frac{r^2}{l^2}(dx_1^2 + dx_2^2) \tag{2}$$

where μ_b is the mass parameter and we consider the compactified coordinates $0 \leq x_1 \leq L_b$ and $0 \leq x_2 \leq L$. The normalization is such that the time coordinate and the coordinates x_1 and x_2 have the same dimension and so the analytic continuation for obtaining the AdS soliton produces the same boundary geometry.

The role of the counterterm is to cancel the infrared divergence of the action so that the final result is finite:

$$I_b^E = \frac{2LL_b\beta_b}{l^4}\left(-r_b^3 + \frac{\mu_b l^2}{2}\right) = -\frac{LL_b\beta_b r_b^3}{l^4} \tag{3}$$

The horizon radius is denoted by r_b and β_b is the periodicity of the Euclidean time that is related to the temperature of the black hole by:

$$T = \beta_b^{-1} = \frac{(-g_{tt})'}{4\pi}\bigg|_{r=r_b} = \frac{3r_b}{4\pi l^2} \tag{4}$$

Using the usual thermodynamic relations and free energy $F = I_b^E/\beta_b$, we obtain the energy and entropy of the planar black hole:

$$E = -T^2\frac{\partial I_b^E}{\partial T} = \frac{2LL_b\mu_b}{l^2} \tag{5}$$

$$S = -\frac{\partial(I_b^E T)}{\partial T} = \frac{LL_b r_b^2}{4l^2 G_N} = \frac{\mathcal{A}}{4G_N} \tag{6}$$

The AdS soliton solution was obtained in [7]

$$ds^2 = -\frac{r^2}{l^2}d\tau^2 + \left(-\frac{\mu_s}{r} + \frac{r^2}{l^2}\right)^{-1}dr^2$$
$$+ \left(-\frac{\mu_s}{r} + \frac{r^2}{l^2}\right)d\theta^2 + \frac{r^2}{l^2}dx_2^2 \tag{7}$$

by using a double analytic continuation $t \to i\theta$, $x_1 \to i\tau$ of the planar black hole metric (3). To distinguish from the black hole solution, we denote by μ_s the mass parameter of the AdS soliton and, in the Euclidean section ($\tau \to i\tau_E$), the periodicity is $0 \leq \tau_E \leq \beta_s$. To obtain a regular Lorentzian solution, the coordinate r is restricted to $r_s \leq r$, where

$$-\frac{\mu_s}{r_s} + \frac{r_s^2}{l^2} = 0 \tag{8}$$

and to avoid the conical singularity in the plane (r, θ), we impose the following periodicity for θ:

$$L_s = \frac{4\pi\sqrt{g_{\theta\theta}g_{rr}}}{(g_{\theta\theta})'}\bigg|_{r=r_s} = \frac{4\pi l^2}{3r_s} \tag{9}$$

The finite on-shell Euclidean action and mass of the AdS soliton can be obtained in a similar way (but we do not present the details here):

$$I_s^E = -\frac{LL_s\beta_s\mu_s}{l^2} \tag{10}$$

and the mass can be obtained by using the thermodynamical relations with the free energy $F = I_s^E/\beta_s = M$ (or from the quasilocal stress tensor) and the result is

$$M = -\frac{LL_s\mu_s}{l^2} \tag{11}$$

The mass of the AdS soliton corresponds to a Casimir energy associated to the compact directions of the dual boundary theory, and so it is negative.

With this information it is straightforward to check the existence of first order phase transitions. To compare the Euclidean solutions, one should impose the same periodicity conditions, which become in the boundary ($r \to \infty$), $\beta_b = \beta_s$ and $L_s = L_b$. Let us know compare the actions (free energies):

$$\Delta I = I_b^E - I_s^E = \frac{L}{l^4}\left(\frac{4\pi l^2}{3}\right)^3 L_b\beta_b[L_s^{-3} - \beta_b^{-3}] \tag{12}$$
$$= \frac{L}{l^4}\left(\frac{4\pi l^2}{3}\right)^3 L_b\beta_b\left[\frac{1}{L_s^3} - T^3\right]$$

The change of sign is an indication of a first order phase transition between the planar black hole and the AdS soliton. It was shown in [25] that the small hot black holes (with respect to r_s) are unstable and decay to small hot solitons, but the large cold black holes are stable. Note that the phase transition is controlled by the dimensionless parameter $z = TL_s$.

2.2. Hairy AdS soliton in 4-dimensions

We consider the exact regular hairy black hole solutions with a planar horizon [15,16,28]. The action is

$$I[g_{\mu\nu}, \phi] = \int_{\mathcal{M}} d^4x\sqrt{-g}\left[R - \frac{(\partial\phi)^2}{2} - V(\phi)\right]$$
$$+ 2\int_{\partial\mathcal{M}} d^3x K\sqrt{-h} \tag{13}$$

and we are interested in the following moduli potential[1]:

$$V(\phi) = \frac{\Lambda(v^2 - 4)}{3v^2}\left[\frac{v-1}{v+2}e^{-\phi l_v(v+1)} + \frac{v+1}{v-2}e^{\phi l_v(v-1)}\right. \tag{14}$$
$$+ 4\frac{v^2-1}{v^2-4}e^{-\phi l_v}\Bigg]$$
$$+ \frac{2\alpha}{v^2}\left[\frac{v-1}{v+2}\sinh\phi l_v(v+1)\right.$$
$$\left. - \frac{v+1}{v-2}\sinh\phi l_v(v-1) + 4\frac{v^2-1}{v^2-4}\sinh\phi l_v\right]$$

We focus on the concrete case of $v = 3$, though hairy AdS solitons for other values of v probably also exist but the analysis is technically more involved and we do not investigate them in the present work. In this case, the scalar field potential becomes

$$V(\phi) = \frac{2\Lambda}{27}\left(5e^{-\phi\sqrt{2}} + 10e^{\phi\sqrt{2}/2} + 16e^{-\phi\sqrt{2}/4}\right) \tag{15}$$
$$+ \frac{4\alpha}{45}\left[\sinh\left(\phi\sqrt{2}\right) - 10\sinh\left(\phi\sqrt{2}/2\right)\right.$$
$$\left. + 16\sinh\left(\phi\sqrt{2}/4\right)\right]$$

The potential has two parts that are controlled by the parameters Λ and α. Asymptotically, where the scalar field vanishes, just the parameter Λ survives and it relates to the AdS radius as $\Lambda = -3l^{-2}$.

Using the following metric ansatz

$$ds^2 = \Omega(x)\left[-f(x)dt^2 + \frac{\eta^2 dx^2}{f(x)} + \frac{dx_1^2}{l^2} + \frac{dx_2^2}{l^2}\right] \tag{16}$$

the equations of motion can be integrated for the conformal factor [15,16,28,33,34]

$$\Omega(x) = \frac{9x^2}{\eta^2(x^3 - 1)^2} \tag{17}$$

With this choice of the conformal factor, it is straightforward to obtain the expressions for the scalar field

$$\phi(x) = 2\sqrt{2}\ln x \tag{18}$$

and metric function

$$f(x) = \frac{1}{l^2} + \alpha\left[\frac{1}{5} - \frac{x^2}{9}\left(1 + x^{-3} - \frac{x^3}{5}\right)\right] \tag{19}$$

where η is the only integration constant. The parameter α is positive for $x < 1$ and negative otherwise. We shall focus below on the case $x < 1$.

The conformal boundary is at $x = 1$, where the metric becomes

$$ds^2 = \frac{R^2}{l^2}\left[-dt^2 + dx_1^2 + dx_2^2\right] \tag{20}$$

and we use the following notation for the conformal factor:

$$R^2 \equiv \frac{1}{\eta^2(x-1)^2} \tag{21}$$

[1] For some particular values of the parameters, it becomes the one of a truncation of ω-deformed gauged $\mathcal{N} = 8$ supergravity [29], see [30–32].

The geometry where the dual field theory 'lives' has the metric

$$ds_{dual}^2 = \frac{l^2}{R^2}ds^2 = \gamma_{ab}dx^a dx^b = -dt^2 + dx_1^2 + dx_2^2 \tag{22}$$

The regularized Euclidean action for these black holes was obtained in [27] (see, also, [35]) (in what follows we use the same notations as in the previous section for β_b and L_b):

$$I_{BH}^E = \beta_b\left(-\frac{\mathcal{A}T}{4G_N} + \frac{2LL_b}{l^2}\frac{\alpha}{3\eta^3}\right) = -\frac{LL_b\alpha\beta_b}{3l^2\eta^3} \tag{23}$$

where the area of the horizon and black hole temperature are

$$\mathcal{A} = \frac{LL_b\Omega(x_h)}{l^2}, \qquad T = \frac{\alpha}{4\pi\eta^3\Omega} \tag{24}$$

The mass of the hairy black hole is [27,36]

$$M_b = \frac{2LL_b\mu_b}{l^2}, \qquad \mu_b = \frac{\alpha}{3\eta^3} \tag{25}$$

as can be also checked by using the usual thermodynamical relations. Using this expression of the mass, one can also easily check the first law of thermodynamics.

Let us now construct the hairy AdS soliton. By using again a double analytical continuation $x_1 \to i\tau$ and $t \to i\theta$ in (16), the metric becomes

$$ds^2 = \Omega_s(x)\left[-\frac{d\tau^2}{l^2} + \frac{\lambda^2 dx^2}{f(x)} + f(x)d\theta^2 + \frac{dx_2^2}{l^2}\right]. \tag{26}$$

Similarly with the hairy black hole case, the conformal factor (17) is

$$\Omega_s(x) = \frac{9x^2}{\lambda^2(x^3 - 1)^2} \tag{27}$$

but now we denote the integration constant with λ to distinguish it from the integration constant η of the black hole. To get rid of the conical singularity in the plane (x, θ), we have to impose the periodicity:

$$L_s = \left.\frac{4\pi\lambda}{f'}\right|_{x=x_s} = \frac{4\pi\lambda^3\Omega_s}{\alpha} \tag{28}$$

where x_s is the minimum value of x, namely the biggest root of $f(x_s) = 0$. After imposing the right periodicity on θ and restricting the coordinate x so that the metric is Lorentzian, we obtain a well-defined regular solution.

We use the method of [27] to compute the regularized Euclidean action and the result is

$$I_{soliton}^E = -\frac{L\beta_s\Omega_s(x_s)}{4l^2 G_N} + \frac{2LL_s\beta_s}{l^2}\frac{\alpha}{3\lambda^3} = -\frac{LL_s\beta_s}{l^2}\left(\frac{\alpha}{3\lambda^3}\right) \tag{29}$$

from which the mass can be immediately read off:

$$M_{soliton} = -\frac{LL_s\mu_s}{l^2}, \qquad \mu_s = \frac{\alpha}{3\lambda^3} \tag{30}$$

As a check, we have also obtained the quasilocal stress tensor for this case and then computed the mass, but we do not present the details here.

2.3. Hairy AdS soliton in 5-dimensions

Let us now construct an exact hairy AdS soliton solution in five dimensions. We consider the solutions in [16], but we investigate the case $v = 5$. In this case, at first sight the potential of [16] is not well defined. However, by taking the limit carefully, we obtain that

the theory (potential) and solution are regular. The ansatz metric is

$$ds^2 = \Omega(x)\left[-f(x)dt^2 + \frac{\eta^2 dx^2}{f(x)} + \frac{dx_1^2}{l^2} + \frac{dx_2^2}{l^2} + \frac{dx_3^2}{l^2} \right] \quad (31)$$

and, for $\nu = 5$, we obtain

$$\Omega(x) = \frac{25x^4}{\eta^2 (x^5 - 1)^2} \quad (32)$$

and

$$f(x) = \frac{1}{l^2} + \frac{\alpha}{3^2 10^4}\left(x^{10} - 6x^5 + 30\ln x + 3 + \frac{2}{x^5} \right) \quad (33)$$

The black hole temperature is

$$T = \beta_b^{-1} = \left. \frac{|\alpha|}{288\pi\,\eta^4\,|\Omega|^{3/2}} \right|_{x=x_h} \quad (34)$$

where $f(x_h) = 0$. We shall consider the below the case when $\alpha < 0$. The black hole entropy can be also easily computed and we obtain

$$S = \frac{L_b L_2 L_3 \Omega^{3/2}}{4l^3 G_N} = \frac{\mathcal{A}}{4G_N}, \qquad \mathcal{A} = \frac{L_b L_2 L_3}{l^3}\Omega^{3/2} \quad (35)$$

To regularize the Euclidean action we choose the following counterterm for the scalar field:

$$I_\phi^{ct} = \int d^4 x^E \sqrt{h^E}\left(\frac{\phi^2}{2l} - \frac{\phi^3}{36l} + \frac{7\phi^4}{864l} \right) \quad (36)$$

$$= \frac{3L_b L_2 L_3}{l^3 T}\left[\frac{6}{\eta^4 l^2 (x-1)^2} - \frac{8}{\eta^4 l^2 (x-1)} - \frac{12}{\eta^4 l^2} \right] + O(x-1)$$

The finite action is

$$I_{BH}^E = \beta_b\left[-\frac{\mathcal{A}T}{4G_N} + \frac{3L_b L_2 L_3}{l^3}\left(-\frac{\alpha}{288\eta^4} \right) \right] \quad (37)$$

$$= \frac{L_b L_2 L_3 \beta_b}{l^3}\left(\frac{\alpha}{288\eta^4} \right)$$

and the mass of the hairy black hole is

$$M_{bh} = -\frac{3L_b L_2 L_3 \mu_b}{l^3}, \qquad \mu_b = \frac{\alpha}{288\eta^4} \quad (38)$$

We again construct the hairy AdS soliton by using a double analytical continuation $x_1 \to i\tau$ and $t \to i\theta$:

$$ds^2 = \Omega(x)\left[-\frac{d\tau^2}{l^2} + \frac{\eta^2 dx^2}{f(x)} + f(x)d\theta^2 + \frac{dx_2^2}{l^2} + \frac{dx_3^2}{l^2} \right] \quad (39)$$

The conformal factor for the hairy soliton is

$$\Omega(x) = \frac{25x^4}{\lambda^2 (x^5 - 1)^2} \quad (40)$$

and, to get rid of the conical singularity in the plane (x, θ), we have to impose the following periodicity of the angular coordinate:

$$L_s = \left| \frac{4\pi\lambda}{f'} \right|_{x=x_s} = \frac{288\pi\lambda^4 \Omega^{3/2}}{|\alpha|} \quad (41)$$

We again consider $\alpha < 0$, to be consistent with the black hole case. To complete the analysis, we compute the Euclidean action

$$I_{soliton}^E = \beta_s\left[\frac{L_s L_2 L_3}{l^3}\left(\frac{\alpha}{72\lambda^4} \right) + \frac{3L_s L_2 L_3}{l^3}\left(-\frac{\alpha}{288\lambda^4} \right) \right] \quad (42)$$

$$= \frac{L_s L_2 L_3 \beta_s}{l^3}\left(\frac{\alpha}{288\lambda^4} \right)$$

and the mass of the hairy AdS soliton

$$I_{soliton}^E \beta_s^{-1} = M_{soliton} = \frac{L_s L_2 L_3}{l^3}\left(\frac{\alpha}{288\lambda^4} \right) \quad (43)$$

3. Implications for phase transitions

Within AdS/CFT duality, the black holes are interpreted as thermal states in the dual field theory. We are going to show that there exist first order phase transitions between the planar hairy black hole and the hairy AdS soliton.

With the results from the previous sections, we are ready to investigate the existence of phase transitions.[2] Let us focus on $D = 4$. Before comparing the actions, we would like to point out that from the definitions of x_s and x_h we obtain that they are equal, $x_s = x_h$. At first sight, this may be a bit strange because in general it is expected that they depend on the mass parameters λ and η for the soliton and black hole. However, in these unusual coordinates, x_s and x_h are defined by (19), but the true are of the horizon and 'center' of the soliton are determined by the conformal factor in front of the metric. This conformal factor depends on the mass parameter and we define:

$$r_b^2 = \frac{\Omega(x_h, \eta)}{l^2}, \qquad r_s^2 = \frac{\Omega(x_s, \lambda)}{l^2} \quad (44)$$

As before (13), we have to compare the free energies of solutions in the same theory and so we have to impose the same periodicity conditions at the boundary $\beta_b = \beta_s$ and $L_s = L_b$. The hairy AdS soliton has a negative energy (the AdS space in planar coordinates has zero mass) and it is the ground state of the theory. Hence, the energy of the hairy black hole should be computed with respect to the ground state and we obtain

$$E = M_{bh} - M_{soliton} = \frac{LL_b}{l^2}[2\mu_b + \mu_s] \quad (45)$$

with μ_b and μ_s defined in (25) and (30).

The same periodicity of the Euclidean time implies the same temperature and we consider the hairy soliton solution as thermal background:

$$\Delta F = \beta_b^{-1}(I_{BH}^E - I_{soliton}^E) = \frac{TL\alpha}{3l^2}\left(\frac{L_s\beta_s}{\lambda^3} - \frac{L_b\beta_b}{\eta^3} \right) \quad (46)$$

Using the expressions of the black hole temperature T and periodicity L_s, we can rewrite the difference of the free energies as

$$\Delta F = \frac{4\pi LL_s}{3l^2}\left[\frac{\Omega(\lambda, x_s)}{L_s} - T\Omega(\eta, x_h) \right] \quad (47)$$

$$= \frac{4\pi L}{3l^2}\Omega(\lambda, x_s)\left[1 - \frac{r_b^3}{r_s^3} \right]$$

Written in terms of the temperature, there is a drastic change compared with the no-hair case because the conformal factor appears explicitly. Clearly, the sign of this expression is controlled by the ratio r_b/r_s. Interestingly enough, despite the appearance of the conformal factor, the critical point where $\Delta F = 0$ it is again for the temperature $T_c = 1/L_s$ (that is because when $\Delta F = 0$, $\mu_b = \mu_s$

---

[2] The case $k = 1$, when the horizon topology is spherical, was studied in [37].

and so $\eta = \lambda$). This is what one expects for a conformal field theory because the phase transition should depend on the ratio of the scales.

Writing the area of the black hole in terms of β_b and β_s, we find that

$$\frac{\mathcal{A}}{Tl^3} = \frac{\alpha L}{4\pi l^5} \frac{\beta_b^2 L_s}{\eta^3} = \frac{L\mathcal{L}}{l}\left(\frac{\lambda}{\eta}\right) \qquad (48)$$

where

$$\mathcal{L} = \frac{16\pi^2}{\alpha^2 l^4}\left[\frac{9x_h^2}{(x_h^3 - 1)^2}\right]^3 \qquad (49)$$

However, since x_h satisfies $f(x_h) = 0$, it can be computed as a function of the parameter α of the moduli potential, which implies that the coefficient $\mathcal{L}(\alpha, l)$ is a function only of α and l. From the definition (44), one can easily obtain $r_b/r_s = \lambda/\eta$ and so (48) can be rewritten in this useful form:

$$\frac{\mathcal{A}}{Tl^3} = \frac{L\mathcal{L}(\alpha, l)}{l}\frac{r_b}{r_s} \qquad (50)$$

There is an important difference by comparing with the no hair case, namely the appearance of the function $\mathcal{L}(\alpha, l)$. When α is very small so behave $\mathcal{L}$ and, in this case, one can still keep the radius of the horizon of the same size as r_s. Therefore, for small α, not only the small hot black holes, but also the large hot black holes are unstable and decay to hairy AdS solitons. We are going to comment more on this new feature in 'Conclusions' section. When α parameter is large, the thermodynamical behaviour of hairy black holes is similar to the one of no-hairy planar black holes.

4. Conclusions

Hawking and Page have shown that there exists a phase transition between spherical AdS (Schwarzschild) black hole and global ($k = 1$) AdS spacetime. As is well known, the phase transition, both on the gravity side and on the gauge theory side, is sensitive to the topology of the AdS foliation. For AdS black holes with planar horizon geometry, there exists no Hawking–Page transition with respect to AdS spacetime. In other words, the planar black hole phase is always dominant for any non-zero temperature.

Interestingly, it was shown that when one (or more directions) are compact there exist also Hawking–Page phase transitions between the planar black holes and the AdS soliton, which is obtained by a double analytic continuation from the black hole. We have obtained a similar behaviour for the hairy black holes, but now the ground state corresponds to a hairy soliton. One important difference with the no hair case is that the phase transition is also controlled by the parameter α in the scalar potential. Once α is fixed, the theory is fixed, but for very small α the theory contains hot black holes (small or large) that are unstable and decay to hairy AdS solitons. This drastic change is related to the fact that when α vanishes, the hairy black hole solutions become naked singularities. The self interaction of the scalar field is very weak and so a large temperature can destabilize the system regardless of the size of the black hole.

As a future direction, it will be interesting to understand the physics of this instability in the dual field theory. It will also be interesting to investigate the general phase diagram for an arbitrarily parameter ν in the moduli potential and the embedding in supergravity [38]. When the effective cosmological constant vanishes, one can also obtain hairy black holes in flat space (stationary hairy black holes were also obtained, but only numerically). The thermodynamics and phase diagram of asymptotically flat hairy black holes [28,39–41] can be also studied with a similar counterterm method [42–44].

References

[1] J.M. Maldacena, The large N limit of superconformal field theories and supergravity, Int. J. Theor. Phys. 38 (1999) 1113, http://dx.doi.org/10.1023/A:1026654312961, Adv. Theor. Math. Phys. 2 (1998) 231, arXiv:hep-th/9711200.

[2] E. Witten, Anti-de Sitter space, thermal phase transition, and confinement in gauge theories, Adv. Theor. Math. Phys. 2 (1998) 505, arXiv:hep-th/9803131.

[3] T. Hertog, G.T. Horowitz, Designer gravity and field theory effective potentials, Phys. Rev. Lett. 94 (2005) 221301, http://dx.doi.org/10.1103/PhysRevLett.94.221301, arXiv:hep-th/0412169.

[4] T. Hertog, G.T. Horowitz, Towards a big crunch dual, J. High Energy Phys. 0407 (2004) 073, http://dx.doi.org/10.1088/1126-6708/2004/07/073, arXiv:hep-th/0406134.

[5] A. Anabalon, D. Astefanesei, J. Oliva, Hairy black hole stability in AdS, quantum mechanics on the half-line and holography, J. High Energy Phys. 1510 (2015) 068, http://dx.doi.org/10.1007/JHEP10(2015)068, arXiv:1507.05520 [hep-th].

[6] S.W. Hawking, D. Page, Thermodynamics of black holes in Anti-de Sitter space, Commun. Math. Phys. 87 (1983) 577.

[7] G.T. Horowitz, R.C. Myers, The AdS/CFT correspondence and a new positive energy conjecture for general relativity, Phys. Rev. D 59 (1998) 026005, http://dx.doi.org/10.1103/PhysRevD.59.026005, arXiv:hep-th/9808079.

[8] Y. Brihaye, B. Hartmann, S. Tojiev, AdS solitons with conformal scalar hair, Phys. Rev. D 88 (2013) 104006, http://dx.doi.org/10.1103/PhysRevD.88.104006, arXiv:1307.6241 [gr-qc].

[9] N. Ogawa, T. Takayanagi, Higher derivative corrections to holographic entanglement entropy for AdS solitons, J. High Energy Phys. 1110 (2011) 147, http://dx.doi.org/10.1007/JHEP10(2011)147, arXiv:1107.4363 [hep-th].

[10] H.q. Shi, D.f. Zeng, Geodesic motions in AdS soliton background space–time, arXiv:1603.08624 [hep-th].

[11] B. Kleihaus, J. Kunz, E. Radu, B. Subagyo, Axially symmetric static scalar solitons and black holes with scalar hair, Phys. Lett. B 725 (2013) 489, http://dx.doi.org/10.1016/j.physletb.2013.07.051.

[12] Y. Brihaye, B. Hartmann, S. Tojiev, Formation of scalar hair on Gauss–Bonnet solitons and black holes, Phys. Rev. D 87 (2) (2013) 024040, http://dx.doi.org/10.1103/PhysRevD.87.024040, arXiv:1210.2268 [gr-qc].

[13] I. Smolić, Symmetry inheritance of scalar fields, Class. Quantum Gravity 32 (14) (2015) 145010, http://dx.doi.org/10.1088/0264-9381/32/14/145010, arXiv:1501.04967 [gr-qc].

[14] M. Cadoni, S. Mignemi, M. Serra, Black brane solutions and their solitonic extremal limit in Einstein-scalar gravity, Phys. Rev. D 85 (2012) 086001, http://dx.doi.org/10.1103/PhysRevD.85.086001, arXiv:1111.6581 [hep-th].

[15] A. Aceña, A. Anabalón, D. Astefanesei, R. Mann, Hairy planar black holes in higher dimensions, J. High Energy Phys. 1401 (2014) 153, http://dx.doi.org/10.1007/JHEP01(2014)153, arXiv:1311.6065 [hep-th].

[16] A. Acena, A. Anabalon, D. Astefanesei, Exact hairy black brane solutions in AdS_5 and holographic RG flows, Phys. Rev. D 87 (12) (2013) 124033, http://dx.doi.org/10.1103/PhysRevD.87.124033, arXiv:1211.6126 [hep-th].

[17] X.H. Feng, H. Lu, Q. Wen, Scalar hairy black holes in general dimensions, Phys. Rev. D 89 (4) (2014) 044014, http://dx.doi.org/10.1103/PhysRevD.89.044014, arXiv:1312.5374 [hep-th].

[18] Q. Wen, Strategy to construct exact solutions in Einstein-scalar gravities, Phys. Rev. D 92 (10) (2015) 104002, http://dx.doi.org/10.1103/PhysRevD.92.104002, arXiv:1501.02829 [hep-th].

[19] H. Lü, Y. Pang, C.N. Pope, AdS dyonic black hole and its thermodynamics, J. High Energy Phys. 1311 (2013) 033, http://dx.doi.org/10.1007/JHEP11(2013)033, arXiv:1307.6243 [hep-th].

[20] Z.Y. Fan, H. Lu, Static and dynamic hairy planar black holes, Phys. Rev. D 92 (6) (2015) 064008, http://dx.doi.org/10.1103/PhysRevD.92.064008, arXiv:1505.03557 [hep-th].

[21] Z.Y. Fan, B. Chen, Exact formation of hairy planar black holes, Phys. Rev. D 93 (8) (2016) 084013, http://dx.doi.org/10.1103/PhysRevD.93.084013, arXiv:1512.09145 [hep-th].

[22] E. Woolgar, The rigid Horowitz–Myers conjecture, arXiv:1602.06197 [math.DG].

[23] G.J. Galloway, S. Surya, E. Woolgar, A uniqueness theorem for the AdS soliton, Phys. Rev. Lett. 88 (2002) 101102, http://dx.doi.org/10.1103/PhysRevLett.88.101102, arXiv:hep-th/0108170.

[24] G.J. Galloway, S. Surya, E. Woolgar, On the geometry and mass of static, asymptotically AdS space–times, and the uniqueness of the AdS soliton , Com-

mun. Math. Phys. 241 (2003) 1, http://dx.doi.org/10.1007/s00220-003-0912-7, arXiv:hep-th/0204081.

[25] S. Surya, K. Schleich, D.M. Witt, Phase transitions for flat AdS black holes, Phys. Rev. Lett. 86 (2001) 5231, http://dx.doi.org/10.1103/PhysRevLett.86.5231, arXiv:hep-th/0101134.

[26] V. Balasubramanian, P. Kraus, A stress tensor for Anti-de Sitter gravity, Commun. Math. Phys. 208 (1999) 413, http://dx.doi.org/10.1007/s002200050764, arXiv:hep-th/9902121.

[27] A. Anabalon, D. Astefanesei, D. Choque, C. Martinez, Trace anomaly and counterterms in designer gravity, J. High Energy Phys. 1603 (2016) 117, http://dx.doi.org/10.1007/JHEP03(2016)117, arXiv:1511.08759 [hep-th].

[28] A. Anabalon, D. Astefanesei, R. Mann, Exact asymptotically flat charged hairy black holes with a dilaton potential, J. High Energy Phys. 1310 (2013) 184, http://dx.doi.org/10.1007/JHEP10(2013)184, arXiv:1308.1693 [hep-th].

[29] G. Dall'Agata, G. Inverso, M. Trigiante, Evidence for a family of SO(8) gauged supergravity theories, Phys. Rev. Lett. 109 (2012) 201301, http://dx.doi.org/10.1103/PhysRevLett.109.201301, arXiv:1209.0760 [hep-th].

[30] A. Anabalon, D. Astefanesei, Black holes in ω-defomed gauged $N = 8$ supergravity, Phys. Lett. B 732 (2014) 137, http://dx.doi.org/10.1016/j.physletb.2014.03.035, arXiv:1311.7459 [hep-th].

[31] A. Guarino, On new maximal supergravity and its BPS domain-walls, J. High Energy Phys. 1402 (2014) 026, http://dx.doi.org/10.1007/JHEP02(2014)026, arXiv:1311.0785 [hep-th].

[32] J. Tarrío, O. Varela, Electric/magnetic duality and RG flows in AdS$_4$/CFT$_3$, J. High Energy Phys. 1401 (2014) 071, http://dx.doi.org/10.1007/JHEP12(2015)068. Addendum: J. High Energy Phys. 1512 (2015) 068, arXiv:1311.2933 [hep-th].

[33] A. Anabalon, Exact black holes and universality in the backreaction of nonlinear sigma models with a potential in (A)dS4, J. High Energy Phys. 1206 (2012) 127, http://dx.doi.org/10.1007/JHEP06(2012)127, arXiv:1204.2720 [hep-th].

[34] A. Anabalón, D. Astefanesei, On attractor mechanism of AdS_4 black holes, Phys. Lett. B 727 (2013) 568, http://dx.doi.org/10.1016/j.physletb.2013.11.013, arXiv:1309.5863 [hep-th].

[35] A. Anabalon, D. Astefanesei, R. Mann, Holographic equation of state in fluid/gravity duality, arXiv:1604.05595 [hep-th].

[36] A. Anabalon, D. Astefanesei, C. Martinez, Mass of asymptotically Anti-de Sitter hairy spacetimes, Phys. Rev. D 91 (4) (2015) 041501, http://dx.doi.org/10.1103/PhysRevD.91.041501, arXiv:1407.3296 [hep-th].

[37] A. Anabalon, D. Astefanesei, D. Choque, On the thermodynamics of hairy black holes, Phys. Lett. B 743 (2015) 154, http://dx.doi.org/10.1016/j.physletb.2015.02.024, arXiv:1501.04252 [hep-th].

[38] A. Anabalon, D. Astefanesei, D. Choque, J. Edelstein, Phase transitions in ω-deformed Gauged Supergravity, to appear.

[39] C.A.R. Herdeiro, E. Radu, Asymptotically flat black holes with scalar hair: a review, Int. J. Mod. Phys. D 24 (09) (2015) 1542014, http://dx.doi.org/10.1142/S0218271815420146, arXiv:1504.08209 [gr-qc].

[40] C. Herdeiro, E. Radu, Construction and physical properties of Kerr black holes with scalar hair, Class. Quantum Gravity 32 (14) (2015) 144001, http://dx.doi.org/10.1088/0264-9381/32/14/144001, arXiv:1501.04319 [gr-qc].

[41] C.A.R. Herdeiro, E. Radu, Kerr black holes with scalar hair, Phys. Rev. Lett. 112 (2014) 221101, http://dx.doi.org/10.1103/PhysRevLett.112.221101, arXiv:1403.2757 [gr-qc].

[42] D. Astefanesei, E. Radu, Quasilocal formalism and black ring thermodynamics, Phys. Rev. D 73 (2006) 044014, http://dx.doi.org/10.1103/PhysRevD.73.044014, arXiv:hep-th/0509144.

[43] R.B. Mann, D. Marolf, Holographic renormalization of asymptotically flat spacetimes, Class. Quantum Gravity 23 (2006) 2927, http://dx.doi.org/10.1088/0264-9381/23/9/010, arXiv:hep-th/0511096.

[44] D. Astefanesei, R.B. Mann, M.J. Rodriguez, C. Stelea, Quasilocal formalism and thermodynamics of asymptotically flat black objects, Class. Quantum Gravity 27 (2010) 165004, http://dx.doi.org/10.1088/0264-9381/27/16/165004, arXiv:0909.3852 [hep-th].