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On decays of light unflavoured pseudoscalar mesons

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Abstract

The ongoing and planned experimental activities with direct reference to light unflavoured pseudoscalar mesons motivate a new theoretical study regarding their properties. An overview including details on new precise calculations is presented.

Keywords: Chiral symmetries, Decays of π mesons, Light mesons

1. Introduction

The subjects of this work, light pseudoscalar mesons, play a prominent role in hadronic physics. Our even more focused interest into the unflavoured particles, namely π^0, η and eventually η', is motivated by the wish to avoid a discussion on K^0 decays. Of course, not because they are not interesting, but such decays violate hypercharge conservation and are suppressed by Fermi coupling constant G_F^2, meaning that studying only unflavoured ones enables to reduce standard model to QCD and simplifies the problem. For studying QCD at low energy region, in our case enlarged at most only by QED corrections, a standard tool successfully developed in recent years is called chiral perturbation theory (ChPT). In this contribution we will mainly discuss properties of π^0 as many radiative η decays are technically very similar and what one obtains for π^0 can be simply converted also for η decay prediction. On the other hand, π^0 being the lightest meson cannot decay into other hadronic states, therefore a hadronic discussion for η decays is inevitable. As an important example of such processes we will briefly mention $\eta \to 3\pi$ decays.

The decay modes of π^0 were subjects of many experiments in the past (including e.g. SINDRUM coll. at PSI), present (e.g. KTeV, or PrimEx at JLab) or future (NA62 at CERN). Using the conserved vector current hypothesis we can connect the vector form factor (i.e. charged pion) to the lifetime of the neutral pion (cf. PIBETA [1]). Experiments have reached (or plan to reach) a level of precision which makes it mandatory to reopen previous theoretical calculations to achieve appropriate order (NLO or NNLO). This can, on one hand, help us to verify and fix the underlying structure of the low energy effective theory of QCD - ChPT (e.g. pion decay constant, low energy constants, power-counting, etc.). On the other hand it can set a framework for studying new physics beyond the SM (e.g. KTeV's discrepancy with a theory for $\pi^0 \to e^+e^-$).

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As mentioned the η meson can be treated technically very similarly. However, due to its mass one can also study different leptonic variants and combination (for example μ in place of e) and last but not least its hadronic decays. This can provide us with important information on isospin breaking effects and again test the internal consistency of ChPT.

We will mainly focus on four most important allowed decay modes of π^0: $\gamma\gamma$, $e^+e^-\gamma$, $e^+e^-e^+e^-$, e^+e^- (with branching ratios [2]: 0.98798(32), 0.01198(32), $3.14(30) \times 10^{-5}$, $6.46(33) \times 10^{-8}$, respectively). For this purpose one can use two-flavour chiral perturbation theory (ChPT, for a review see e.g.[3]) which can simply incorporate corrections to the current algebra result attributed either to $m_{u,d}$ masses or electromagnetic corrections with other effects hidden in the low energy constants (LECs), denoted by c_i^W at next-to-leading order (NLO). However, phenomenologically richer $SU(3)$ ChPT must be also employed in order to obtain a numerical prediction. This is especially true for the studied anomalous processes as in this case the initial symmetry for the two flavour case must be extended and the number of monomials in $SU(2)$ increases [4].

We will not limit our focus only on "standard on-shell" decays. It is clear that both on-shell and off-shell or semi-of-shell vertices, especially $\pi^{0(*)}\gamma^*\gamma^{(*)}$, play a crucial role in many other experiments, from the famous $g - 2$ (cf. [5]) via virtual photons stemming from e^+e^- (see e.g. the recent paper [6]) to astrophysics. Our first aim is the common formulation of these interrelated processes in the given formalism at the given order (either NLO or NNLO) motivated by the precision of the appropriate experiments.

2. $\pi^0 \rightarrow \gamma\gamma$

The π^0 meson has a prominent position among all hadron particles as being the lightest state of them. Its primary decay mode is thus $\pi^0 \rightarrow \gamma\gamma$ which is connected with the famous Adler-Bell-Jackiw triangle anomaly [7]. It saturates the decay width with almost 99% and plays an important role in the further decay modes (see the following sections). The history of $\pi^0 \rightarrow \gamma\gamma$ is going back to Steinberger's calculation [8], for a review see e.g. [9], and also [10]. The prediction estimated from the chiral anomaly using current algebra agrees surprisingly very well with experiments. A first attempt to explain the small existing deviation from the measurement was made by Y. Kitazawa [11]. At that time a new experimental prediction from CERN-NA030 [12] suggested a smaller value for the partial width 7.25 ± 0.23 eV (statistical and systematic errors combined in quadrature). Older experiments (Tomsk, Desy and Cornell [13]), seemed not to be so precise (7.23 ± 0.55, 11.7 ± 1.2, 7.92 ± 0.42 eV, respectively); they relied on the so-called Primakoff effect [14] that is based on measuring the cross section for the photoproduction of the meson in the Coulomb field. The more precise number from the direct measurement at CERN motivated Y. Kitazawa to explain the $8 \sim 9\%$ discrepancy by including QED correction and the η/η' contribution. These corrections were not, however, large enough to explain the discrepancy which was attributed by the author to a possible $\pi(1300)$ contribution. Furthermore, it was found out in this work that the contribution from multi-pion states must be small. This was verified explicitly also within ChPT with the remarkable observation [15] that at one-loop order there are no chiral logarithms (either from pions or kaons). The π-η-η' mixing and electromagnetic correction were reconsidered relatively recently in [16].

The spread in the data basis of the PDG, summarized in the previous paragraph, shows, however, that the quoted errors seem to be underestimated [17]. The present situation fortunately looks more optimistic as the world average accuracy of 8 % is planned to be improved to the level of one or two percents in ongoing experiment PrimEx at JLab. The official Run-I result quoted in [18] is $\Gamma(\pi^0 \rightarrow \gamma\gamma) = 7.82$ eV $\pm 2.8\%$. This was the main motivation for a new study of $\pi^0 \rightarrow \gamma\gamma$ in [19]. The correction to the chiral anomaly due to the finite mass of light quarks was reconsidered using strict two-flavour ChPT at NNLO. We will summarize here this remarkably simple result (note that it involves a two-loop calculation and that it represents formally a full $O(p^8)$ result). Defining a reduced T amplitude

$$A = e^2 \varepsilon_{\mu\nu\alpha\beta}\epsilon_1^{*\mu}\epsilon_2^{*\nu}k_1^\mu k_2^\nu \, T \, , \tag{1}$$

we have for the partial decay width

$$\Gamma_{\gamma\gamma} = \frac{\pi}{4}\alpha^2 m_{\pi^0}^3 |T|^2 \, . \tag{2}$$

Up to and including next-to-next-to-leading order corrections

$$
\begin{aligned}
F_\pi T_{NNLO} ={}& \frac{1}{4\pi^2} + \frac{16}{3}m_\pi^2\left(-4c_3^{Wr} - 4c_7^{Wr} + c_{11}^{Wr}\right) + \frac{64}{9}B(m_d - m_u)(5c_3^{Wr} + c_7^{Wr} + 2c_8^{Wr}) \\
&+ \frac{M^4}{16\pi^2 F^4}L_\pi\left[\frac{3}{256\pi^4} + \frac{32F^2}{3}\left(2c_2^{Wr} + 4c_3^{Wr} + 2c_6^{Wr} + 4c_7^{Wr} - c_{11}^{Wr}\right)\right] \\
&+ \frac{32M^2 B(m_d - m_u)}{48\pi^2 F^4}L_\pi\left[-6c_2^{Wr} - 11c_3^{Wr} + 6c_4^{Wr} - 12c_5^{Wr} - c_7^{Wr} - 2c_8^{Wr}\right] \\
&- \frac{M^4}{24\pi^2 F^4}\left(\frac{1}{16\pi^2}L_\pi\right)^2 + \frac{M^4}{F^4}\lambda_+ + \frac{M^2 B(m_d - m_u)}{F^4}\lambda_- + \frac{B^2(m_d - m_u)^2}{F^4}\lambda_{--}\,,
\end{aligned}
\tag{3}
$$

where the chiral logarithm is denoted by $L_\pi = \log\frac{m_\pi^2}{\mu^2}$ and λ_+, λ_-, λ_{--} can be expressed as follows in terms of renormalized chiral coupling constants (d^{Wr} refer to combinations of couplings from the NNLO Lagrangian, i.e. of order p^8 in the anomalous sector),

$$
\begin{aligned}
\lambda_+ ={}& \frac{1}{\pi^2}\left[-\frac{2}{3}d_+^{Wr}(\mu) - 8c_6^r - \frac{1}{4}(l_4^r)^2 + \frac{1}{512\pi^4}\left(-\frac{983}{288} - \frac{4}{3}\zeta(3) + 3\sqrt{3}\,\mathrm{Cl}_2(\pi/3)\right)\right] \\
&+ \frac{16}{3}F^2\left[8l_3^r(c_3^{Wr} + c_7^{Wr}) + l_4^r(-4c_3^{Wr} - 4c_7^{Wr} + c_{11}^{Wr})\right] \\
\lambda_- ={}& \frac{64}{9}\left[d_-^{Wr}(\mu) + F^2 l_4^r(5c_3^{Wr} + c_7^{Wr} + 2c_8^{Wr})\right] \\
\lambda_{--} ={}& d_{--}^{Wr}(\mu) - 128F^2 l_7(c_3^{Wr} + c_7^{Wr})\,,
\end{aligned}
\tag{4}
$$

with Riemann zeta and Clausen function: $\zeta(3) = 1.202..$ and $\mathrm{Cl}_2(\pi/3) = 1.014..$, respectively.

All effects that were carefully studied e.g. in [11] and [16] are now hidden in the LECs and chiral logarithms L_π (with the exception of QED corrections that must be added by hand to the latter formula, see [19]). For a detailed phenomenological study of the previous formula see [19], our best estimate has led to

$$
\Gamma_{\gamma\gamma} = (8.09 \pm 0.11)\ \text{eV}.
\tag{5}
$$

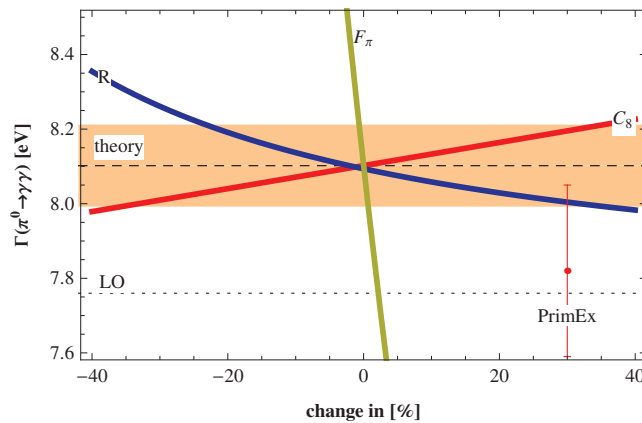The importance of various input parameters is depicted in Fig. 1. One can see immediately the importance of F_π in

Figure 1: The dependence of $\pi^0 \to \gamma\gamma$ decay width on various parameters, namely $SU(3)$ LEC C_8^W, R and F_π (see main text).

$\pi^0 \to \gamma\gamma$ decay. The F_π, on the other hand, is determined from the weak decay of π^+ based on the standard $V - A$ interaction. The new proposed variant of this interaction beyond SM assumes contributions of right-handed current which would lead to a change of F_π [20]. Determination of this constant directly from π^0 lifetime can provide constraints

on such contributions. This again put big effort to minimize the uncertainty stemming from other parameters as for example visualized in Fig. 1. One of them, a quark mass ratio R will be subject of Sec. 7, the second one C_8^W reflects the intrinsic connection with $\eta \to \gamma\gamma$ decay as calculated in three-flavour ChPT. However, better understanding of $\eta \to \gamma\gamma$ again in full two-loop calculation is important and probably necessary as motivated and explained in [21].

3. $\pi^0 \to e^+e^-\gamma$

If $\pi^0 \to \gamma\gamma$ represented 99% of all decay modes, $\pi^0 \to e^+e^-\gamma$ represents again more than 99% of the rest modes. It is thus the second most important decay mode with a branching ratio $\sim 1.174 \pm 0.035\%$ and nowadays called Dalitz decay, after R.H.Dalitz who first realized its connection with two-photon production [22]. Knowing the branching ratio one can also use, at least in principle, the Dalitz decay to extrapolate the total decay width, which can serve as an independent possibility how to measure the life-time of π^0 (however, handicapped by the larger dependence on error of this ratio). This year's change of the official PDG number is due to ALEPH archived data on hadronic Z boson decay which has enabled to reconstruct 12,490 Dalitz decays with a result

$$\text{ALEPH:} \qquad \Gamma_{\pi^0 \to e^+e^-\gamma}/\Gamma_{\gamma\gamma} = (1.140 \pm 0.041)\%, \qquad (6)$$

that led to the update

$$\text{PDG 10:} \qquad \Gamma_{\pi^0 \to e^+e^-\gamma}/\Gamma_{\gamma\gamma} = (1.188 \pm 0.035)\%. \qquad (7)$$

To conclude let us also summarize a theoretical prediction:

$$\text{theory:} \qquad \Gamma_{\pi^0 \to e^+e^-\gamma}/\Gamma_{\gamma\gamma} = (1.1851 + 0.0104 + 0.0018)\% = (1.1973 \pm 0.0055)\%, \qquad (8)$$

where the first number stands for the leading order, predicted already by Dalitz [22], the second represents the radiative corrections (numerically first done in [23]) and the last number [24] stands for the prediction of QCD corrections and two-photon exchange contribution (which was neglected previously in QED corrections). The final error was estimated as a half of all QED corrections.

The Dalitz decay is very often used experimentally as the normalization mode not only for rare pion modes (see also below) but also for kaon decay modes, and thus its precise value has impact on these measurements. Its uncertainty has in fact a direct effect on external systematic error and different central values can produce substantial shifts in the final predictions.

The motivation for studying the decay width of $\pi^0 \to e^+e^-\gamma$ is thus two-fold. The precise and well-understood theoretical prediction with model-dependent QCD contribution suppressed by phase-space integration can serve as a calibration of the experiment. On the other hand, if theory is well under control and still some measurement would signalize some discrepancy the theory is missing something new and important.

The total decay rate is, however, not the whole story. It turns out that the corrections to the differential decay which were taken as negligible are indeed important. The reason is that there is a part of the phase space where, roughly-speaking, the correction to the differential decay width is positive and a part where it is negative; and only summing these parts together gives us the small number in (8). It is clear now, that in physically relevant applications, when we have to cut some parts of the phase space, these corrections can become important. Let us discuss this a little bit more in detail (for details see [24]). First it is useful to define two kinematic variables that represent the normalized di-lepton invariant mass and the difference of energy for positron and electron (normalized to the photon energy in the pion rest frame):

$$x = \frac{(p_+ + p_-)^2}{M_{\pi^0}^2}, \qquad \nu^2 \le x \le 1, \qquad \nu^2 = \frac{4m^2}{M_{\pi^0}^2},$$

$$y = \frac{2P \cdot (p_+ - p_-)}{M_{\pi^0}^2(1-x)}, \qquad -\sigma_e(M_{\pi^0}^2 x) \le y \le \sigma_e(M_{\pi^0}^2 x), \qquad \sigma_e(s) = \sqrt{1 - \frac{4m^2}{s}}$$

(m denotes the electron mass, $p_-^2 = p_+^2 = m^2$ and P stands for the pion momentum). The next-to-leading corrections to the differential decay rates can be described as

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}x\mathrm{d}y} = \delta(x, y)\, \frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x\mathrm{d}y},$$
$$\frac{\mathrm{d}\Gamma}{\mathrm{d}x} = \delta(x)\, \frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x},$$
(9)

where the corresponding LO partial decay rates have a relatively simple form

$$\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x\mathrm{d}y} = \frac{\alpha^3}{(4\pi)^4} \frac{M_{\pi^0}}{F_\pi^2} \frac{(1-x)^3}{x^2} [M_{\pi^0}^2 x(1+y^2) + 4m^2],$$
$$\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x} = \frac{\alpha^3}{(4\pi)^4} \frac{8}{3} \frac{M_{\pi^0}}{F_\pi^2} \frac{(1-x)^3}{x^2} \sigma_e(xM_{\pi^0}^2)(xM_{\pi^0}^2 + 2m^2)$$
(10)

(in fact, integrating the last equation one can verify the Dalitz result, i.e. the first number in (8)). With these quantities in hand we can extract information on the QCD part of the form factor $\mathcal{F}_{\pi^0\gamma\gamma^*}(q^2)$, which is related to the doubly off-shell $\pi\gamma\gamma$ transition form factor defined as

$$\int d^4x\, e^{il\cdot x} \langle 0|T(j^\mu(x)j^\nu(0)|\pi^0(P)\rangle = -\mathrm{i}\varepsilon^{\mu\nu\alpha\beta} l_\alpha P_\beta\, \mathcal{F}_{\pi^0\gamma^*\gamma^*}(l^2, (P-l)^2),$$
(11)

by

$$\mathcal{F}_{\pi^0\gamma\gamma^*}(q^2) = \mathcal{F}_{\pi^0\gamma^*\gamma^*}(0, q^2).$$

The Dalitz decay can provides us with the information on the transition form factor in the time-like region. This is usually specified by its slope parameter a_π

$$\mathcal{F}_{\pi^0\gamma\gamma^*}(q^2) = \mathcal{F}_{\pi^0\gamma\gamma^*}(0)\left[1 + a_\pi \frac{q^2}{M_{\pi^0}^2} + \cdots\right],$$
(12)

and from the experiment can be obtained by subtracting the QED corrections via

$$\frac{\mathrm{d}\Gamma^{exp}}{\mathrm{d}x} - \delta_{QED}(x)\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x} = \frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x}[1 + 2x\,a_\pi].$$
(13)

The direct measurements in the time-like region is endowed with large errors

$$\begin{aligned}
\text{Saclay [25]:} \quad a_\pi &= -0.11 \pm 0.03 \pm 0.08 \\
\text{TRIUMF [26]:} \quad a_\pi &= +0.026 \pm 0.024 \pm 0.0048 \\
\text{PSI [27]:} \quad a_\pi &= +0.025 \pm 0.014 \pm 0.026,
\end{aligned}$$

whereas the values extracted from the extrapolation of data at higher energies in the space-like region, $Q^2 = -q^2 > 0.5$ GeV^2 are more precise:

$$\begin{aligned}
\text{CELLO [28]:} \quad a_\pi &= +0.0326 \pm 0.0026 \pm 0.0026 \\
\text{CLEO [29]:} \quad a_\pi &= +0.0303 \pm 0.0008 \pm 0.0009 \pm 0.0012
\end{aligned}$$

The theoretical calculation [24] (here without isospin correction for simplicity) is given by

$$a_\pi = -\frac{32\pi^2 M_{\pi^0}^2}{3} c_{13}^{Wr}(M_V^2) - \frac{M_{\pi^0}^2}{96\pi^2 F_\pi^2}\left(1 + 2\ln\frac{M_{\pi^0}^2}{M_V^2}\right) - \frac{1}{360}\frac{\alpha}{\pi}$$
(14)

We can see that the EM corrections are very small[1] which signalized its tight connection with a low energy constant c_{13}^W (or via $SU(2) - SU(3)$ relations [19] to $SU(3)$ LEC C_{22}^W. The LMD prediction for c_{13}^W leads to

$$\text{theory [24]:} \qquad a_\pi = +0.029 \pm 0.005 \tag{15}$$

Let us also note that the off-shell form factor $\mathcal{F}_{\pi\gamma^*\gamma^*}$ plays an important role in the π^0-exchange contribution to a hadronic light-by-light scattering contribution in $g - 2$ and thus more experimental information on this factor can help us to understand the consistency in different approaches.

4. $\pi^0 \rightarrow e^+e^-e^+e^-$

Having the well established decay of π^0 into two photons it is clear that the pion cannot be a $J = 1$ state (so-called Young-Landau theorem [30]). However, using directly $\pi^0 \rightarrow \gamma\gamma$ to verify whether it is a (pseudo)scalar is experimentally impossible. It was thus suggested in [31] to use the double-internal conversion, the so-called double-Dalitz decay. The experiment was performed at Nevis Lab [32] in a bubble chamber with the following result for the branching ratio:

$$\frac{\Gamma_{e^+e^-e^+e^-}^{\text{PDG'08}}}{\Gamma_{tot}} = (3.14 \pm 0.30) \times 10^{-5} \tag{16}$$

which was used as only relevant measurement for almost half century. The experiment also confirmed the negative parity of π^0 known from the previous indirect studies via the cross-section of π^- capture on deuterons. However, the significance of this direct measurement was only $3.6\,\sigma$. Recently the long standing experimental gap was filled with a new measurement in the KTeV-E799 experiment at Fermilab [33] giving a branching ratio (including the radiative final states above a certain cut as tacitly assumed for all Dalitz modes)

$$\frac{\Gamma_{e^+e^-e^+e^-}^{\text{KTeV}}}{\Gamma_{tot}} = (3.46 \pm 0.19) \times 10^{-5}\,, \tag{17}$$

which is in good agreement with the previous experiment. In addition to the precisely verified parity of π^0 (which represents its best direct determination) this experiment sets the first limits on the parity and CPT violation for this decay. More precisely, having a $\pi^0\gamma^*\gamma^*$ vertex $C_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}\pi^0$ we can study, using the following decomposition (for details see [34])

$$C_{\mu\nu\rho\sigma} = \cos\zeta\,\varepsilon_{\mu\nu\rho\sigma} + \sin\zeta\mathrm{e}^{i\delta}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})\,,$$

the parameters ζ and δ which represent parity mixing and CPT violation parameters. For details see [33]; for example their limit on the mixing assuming CPT conservation is $\zeta < 1.9°$.

A detailed analysis of the radiative corrections in [34] showed that they seem to be very important in extracting physically relevant quantities. This motivates us to reopen this subject [35] in the same manner as was done in [24]. The simply looking task of attaching another Dalitz pair on the virtual photon line is complicated (in the defined power-counting) by the necessity to include a pentagonal diagram [34]. This strengthens the need of a correct description of the off-shell $\pi^0\gamma^*\gamma^*$ vertex, which can be, on the other hand, directly studied in the next mode.

5. $\pi^0 \rightarrow e^+e^-$

In the previous decay modes the fully off-shell $\pi^0\gamma^*\gamma^*$ vertex was suppressed by the dominant semi-on-shell contributions. As this is not true anymore for $\pi^0 \rightarrow e^+e^-$ it naturally represents the simplest and cleanest candidate for studying not-well understood effects of QCD, i.e. $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(k^2 \neq 0, l^2 \neq 0)$, even though the process itself is suppressed by approximate helicity conservation and two extra powers of α. This is supported by the existing experiment at

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[1]Let us note that in previous experimental analyses the two-photon contribution were neglected and their omission would lead to approximately 0.005 correction into the right direction towards the independent CELLO or CLEO result.

Fermilab (KTeV E799-II) [36]. Comparing with the previous measurements their result has increased significantly the precision and provide thus the most important contribution to the present PDG's average

$$\frac{\Gamma^{\text{PDG}}_{e^+e^-}}{\Gamma_{tot}} = (6.46 \pm 0.33) \times 10^{-8} \, .$$

This process was first calculated in [37] and proceeds, as mentioned, via two intermediate photons and at LO is thus represented by an one-loop (triangle) diagram. One can get (for details see e.g. [38])

$$\frac{\Gamma_{e^+e^-}}{\Gamma_{\gamma\gamma}} = 2\Big(\frac{\alpha m}{\pi M_{\pi^0}}\Big)^2 \sigma(M^2_{\pi^0})|\mathcal{A}(M^2_{\pi^0})|^2 \, , \tag{18}$$

with the amplitude given by

$$\mathcal{A}(s) = \chi(\mu) - \frac{5}{2} + \frac{3}{2} \ln \frac{m^2}{\mu^2} + C(s) \, , \tag{19}$$

where C represents the scalar one-loop triangle. The imaginary part can be calculated in a model independent way by cutting the photon lines and knowing the on-shell form factor $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,0)$ (in fact as we have a normalization to two-photon decays it has to be equal 1) with the result

$$\text{Im}\,\mathcal{A} = \frac{\pi}{2\sigma} \ln \frac{1-\sigma}{1+\sigma} \, , \tag{20}$$

implying thus a unitarity bound on the ratio $\Gamma_{e^+e^-}/\Gamma_{\gamma\gamma} \gtrsim 5 \times 10^{-8}$. The real part can be now calculated via a dispersive integral leaving us with one unknown subtraction, or equivalently by $\chi(\mu)$. The techniques of large N_C together with the LMD approximation (V represents the ρ meson) leads to [38]

$$\chi(\mu = M_V) = \frac{11}{4} - 4\pi^2 \frac{F_0^2}{M_V^2} = 2.2 \pm 0.9 \quad \Rightarrow \quad \frac{\Gamma^{\text{th.no-rad}}_{e^+e^-}}{\Gamma_{\gamma\gamma}} = (6.2 \pm 0.3) \times 10^{-8} \, . \tag{21}$$

This should be compared with the experiment after removing the effects of final state radiation:

$$\frac{\Gamma^{\text{KTeV.no-rad}}_{e^+e^-}}{\Gamma_{\gamma\gamma}} = (7.57 \pm 0.39) \times 10^{-8} \, , \tag{22}$$

which is 3.5σ off from the mentioned theoretical prediction.

At present the theoretical activities concerning this process focus on two main directions: *i)* understanding the discrepancy within the SM – i.e. calculating radiative corrections or employing a different model for the QCD part [39]. *ii)* It naturally represents an ideal candidate for testing new models beyond SM, it can set valuable limit on light dark matter scenarios, supersymmetric extensions (axion), etc...

Before concluding let us also mention the weak contribution. This process has in fact a direct tree level contribution mediated via $\pi^0 \to Z^* \to e^+e^-$. The amplitude would be proportional to the Fermi constant:

$$\mathcal{A}^{\text{weak}}_{\pi^0 \to e^+e^-} \sim G_F F_\pi m \bar{u} \gamma_5 v \, , \tag{23}$$

which makes it three orders of magnitude smaller than the dominant EM contribution. Beyond-SM scenarios would use a similar relation (with different coupling G_F) for introducing the effect of a light vector particle (e.g. U-boson).

6. $\pi^0 \to \nu\bar{\nu}$, invisible, extra-light particles

As studied for example by [40] there is a tight connection for $\pi^0 \to \nu\bar{\nu}$ with cosmology so that the strong limit on this decay obtained are much higher than those in the laboratory: 2.7×10^{-7} (E949 based on $K^+ \to \pi^+\pi^0$ [41]). This decay mode represents not only two neutrino decay modes but all possible combination and weakly coupled exotics and also even more generally $\pi^0 \to$ invisible.

Within the SM (extended by massive neutrinos) one can use the calculation of the weak sector for electron from previous section with a prediction

$$\mathcal{A}^{\text{weak}}_{\pi^0 \to \nu^+\nu^-} = \sqrt{2}G_F F_\pi m_\nu \bar{u}\gamma_5 \nu, \tag{24}$$

with one subtlety – we don't know the mass of the neutrino and its nature (for the Majorana type the amplitude is twice bigger). The relative branching ratio is then (in the Dirac case)

$$\frac{\Gamma_{\nu\bar{\nu}}}{\Gamma_{\gamma\gamma}} = \left(\frac{4\pi F_\pi^2 G_F}{\alpha} \frac{m_\nu}{M}\right)^2 \sqrt{1 - \frac{4m_\nu^2}{M^2}} \tag{25}$$

With a direct limit on the absolute tau neutrino mass $m_\nu^\tau < 18.2$ MeV we get a reasonably high limit on branching ratio $< 5 \times 10^{-10}$ (and twice bigger for Majorana case). Note the maximum for the ratio for $m_\nu = M/\sqrt{6}$, which is, however, ruled out.

The helicity suppression for $\pi^0 \to \nu\bar{\nu}$ can be avoided in the decay mode $\pi^0 \to \nu\bar{\nu}\gamma$. This decay mode is also interesting as it depends on the actual number of light neutrinos.

A general possibility to set the constraints on an extra-light long-lived (/non-interacting/weakly interacting) neutral vector particle X via decays of known particles opens naturally this question also for the exotic π^0 decay.

7. $\eta \to 3\pi$

We have selected this representative of η decay modes as the most important example of the hadronic decays for the studied particles. A brief look into the history of this mode ([42] for NLO and [43] for NNLO) shows even after two-loop calculation a discrepancy between theory and experiments. This could be attributed either to bad convergence of the ChPT series and necessity to somehow (at least partially) re-sum higher orders or to some problems in the calculation at NNLO (e.g. to a wrong estimate of LECs). The new ongoing efforts represent in some sense combinations of these two possibilities ([44], [45], [46]).

Apart from the obvious motivation to better understand how to perform NNLO calculation in ChPT in the three flavour case, which seems to be a problematic subject we have in hands a process which vanishes in the isospin limit. It means that a precise measurement together with a good understanding of the theory should provide us with parameters which can quantify the isospin breaking, for example:

$$R = \frac{m_s - \hat{m}}{m_d - m_u} \qquad Q^2 = \tfrac{1}{2}\left(1 + \frac{m_s}{\hat{m}}\right)R.$$

The absolute value of the partial decay width for $\eta \to 3\pi$ is experimentally obtained via normalization to $\eta \to \gamma\gamma$. A change in one decay width has thus influence in other (a change by 1% in $\Gamma(\eta \to \gamma\gamma)$ input shifts R by ≈ 0.2).

8. Summary

The new experimental activities in the low energy physics that concern directly π^0 or η decay modes call for a more detailed theoretical study in this area. We have discussed some allowed decay modes that represent important tools for studying basic phenomena of the underlying theory: QCD. Namely, $\pi^0 \to \gamma\gamma$ and $\eta \to \gamma\gamma$ played an important role in understanding a symmetry pattern of the theory as they are directly connected with the so-called $U(1)_A$ anomaly. QCD enlarged by photons possesses, however, two such anomalies. The first one, internal, connected with QCD only, proportional to gluonic term $G_{\mu\nu}\tilde{G}^{\mu\nu}$, dubbed $U(1)$-problem and the resulting strong CP problem is still an open issue. As a remnant of the anomaly, the η' plays a more important role than naively expected and has to be included in a theoretical consideration. The second anomaly, external, in our case connected with electromagnetic interaction (or $F_{\mu\nu}\tilde{F}^{\mu\nu}$) explains why $\pi^0 \to \gamma\gamma$ can decay so quickly even it should be suppressed due to Sutherland's theorem. Furthermore, $\eta \to \gamma\gamma$ and $\eta \to 3\pi$ represent 95% of all η-decay modes and are thus perfectly suited to study directly properties of the η. Simultaneous treatment of two-photon π^0 and η decays, apart from testing or fixing our understanding of η', can provide valuable information on the decay constants F_π and F_η or quark mass ratio.

A common treatment is thus useful and important in order to understand all phenomena. In this work we have mainly focused on chiral and QED corrections in order to prepare the ground for the discussion of non-perturbative effects or eventual new physics.

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