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ORIGINAL ARTICLE

Crack width evaluation for flexural RC members

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Abstract Some building code equations and equations developed by researchers are used for the calculation of the crack width in reinforced concrete flexural members. To investigate codes' provisions beside some equations found in the literature concerning the crack width calculation of reinforced concrete members subjected to flexure, five reinforced concrete rectangular models were investigated theoretically. The models include different parameters such as reinforcement steel ratio, steel rebar arrangement and reinforcement grade. Also, to verify the accuracy of the building code equations and the equations developed by researchers a comparison against some experimental data available in the literature was carried out. The experimental data include some variables affecting the crack width such as steel stress, concrete cover, flexural reinforcement ratio and rebar arrangement. The study showed a large scatter among the different code equations, however, most of the code equations overestimate the effect of concrete cover on the calculated values of the crack width. Also, the Egyptian code equation should limit the value of the mean steel stress as given by Eurocode equation to overcome the underestimated values obtained in the case of sections having low steel ratio. Moreover, the reinforcement detailing (bars distribution) is an important factor affecting the crack width.

1. Introduction

Crack width calculation is one of the serviceability requirements in the structural concrete elements. The occurrence of cracks in reinforced concrete elements is expected under service loads, due to the low tensile strength of concrete. Control of cracking is important for obtaining acceptable appearance and for long-term durability of concrete structures, especially those subjected to aggressive environments. Excessive crack width may reduce the service life of the structure by permitting more rapid penetration of corrosive factors such as high humidity, repeated saturation with moisture, vapor, salt-water spray and gases with chemicals, to reach the reinforcement. Generally, cracking should not induce reinforcement steel corrosion or spoil the appearance of the structure. In addition, cracking in reinforced concrete structures has an effect on structural performance including stiffness, energy absorption, capacity, and ductility. Consequently, there is an increased interest in the control of cracking by building codes and scientific organizations. With the use of ultimate strength

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methodology and high strength reinforcement steel, researchers and designers recognized the need for providing a mechanism by which crack width would be minimized. Therefore, researches were undertaken in 1960s to quantify the above concept and develop design tools [1].

The crack width of a flexural member is obtained by multiplying the maximum crack spacing by the mean strain of the flexural steel reinforcement. Therefore, the crack width depends on the nature and the arrangement of the reinforcing steel crossing the cracks and the bond between the steel bars found in the tension zone of concrete. Many research work found in the literature predicted the crack width of a flexural member based on theoretical models and experimental data. Saliger [2] and Tomas [3] used Bond-Slip model, Borms [4] and Base et al. [5] used No-Slip model, however Welch and Janjua [6] and Leonhardt [7] used Localized Bond-Slip model to predict the crack width. Gergely and Lutz [8] used the results of experimental data to formulate an equation to calculate the crack width. Based on the experimental work, Oh and Kang [9] proposed a formulation for predicting the maximum crack width. Frosch [10] developed a simple theoretical equation to predict the crack width based on a physical model. Besides the research work carried out for crack width formulation, other research work experimentally investigated the factors affecting the crack width. Makhlouf and Malhas [11] investigated the effect of thick concrete cover on the maximum flexural crack width under service load. Beeby [12] and Nawy and Blair [13] showed that the transverse reinforcement had a strong influence on the crack spacing. Gilbert and Nejadi [14] tested six beams and six one-way slabs with different flexural reinforcement ratio and bar arrangement including various concrete cover.

In this paper, a study is carried out to investigate several formulas suggested by different building codes for the calculation of the crack width in reinforced concrete flexural members. Also, the formulas proposed by other researchers were also investigated and compared with codes' equations. In addition, the most prevalent building codes' equations are examined and tested against some experimental data available in the literature. Moreover, a comparison was carried out among the equations to discuss the various factors and parameters affecting the crack width.

2. Crack width prediction according to some building codes provisions

2.1. Eurocode2 1992-1 (2001)

Eurocode2 [15] gives the following equation for predicting the crack width of flexural members

$$W_k = S_{r,max}(\varepsilon_{sm} - \varepsilon_{cm}) \tag{1}$$

where W_k = the design crack width, mm.

The mean tensile strain $\varepsilon_{sm} - \varepsilon_{cm}$) is given by the following equation:

$$(\varepsilon_{sm} - \varepsilon_{cm}) = \frac{\left(f_s - K_t\left(\frac{f_{cteff}(1+n\rho_{eff})}{\rho_{eff}}\right)\right)}{E_s} \geqslant 0.6\frac{f_s}{E_s} \tag{2}$$

where ε_{sm} is the mean strain in the reinforcement under the relevant combination of loads, including the effect of imposed deformations and taking into account the effects of tensioning stiffening. Only the additional tensile strain beyond zero strain in the concrete is considered; ε_{cm} is the mean strain in concrete between cracks. K_t = factor expressing the duration of loading: K_t = 0.6 for short term loading and K_t = 0.4 for long term loading, f_s = the stress in the tension reinforcement computed on the basis of a cracked section, n = the modular ratio $\frac{E_s}{E_{cm}}$, f_{cteff} = the mean value of tensile strength of the concrete effective at the time when the cracks may first be expected to occur,

$$\rho_{eff} = \frac{A_s}{A_{ceff}}$$

A_{ceff} = effective tension area, is the area of concrete surrounding the tension reinforcement. $S_{r,max}$ = the maximum crack spacing, mm and is given by the following equation

$$S_{r,max} = 3.4c + 0.425k_1k_2\phi/\rho_{eff} \tag{3}$$

where c = concrete clear cover, k_1 = coefficient that takes into account the bond properties of the bonded reinforcement and equals to 0.8 for high bond reinforcing bars, and equals to 1.6 for plain reinforcing bars, ϕ = the bar diameter, mm; in case of using various diameters, the average diameter shall be used, k_2 = coefficient that takes into account the strain distribution and is equal to 0.5 for sections subjected to pure bending and equals to 1.0 for sections subjected to pure axial tension.

2.2. Egyptian code; ECP 203-2007

The Egyptian code ECP 203-2007 [16] gives the crack width by the following equation:

$$W_k = \beta\varepsilon_{sm}S_{rm} \tag{4}$$

where W_k = coefficient for checking crack width condition, mm; S_{rm} = average stabilized crack spacing, mm; ε_{sm} = mean steel strain under relevant combination of loads and allowing for the effect such as tension stiffening or shrinkage; β = coefficient relating the average crack width to the design value: β = 1.7 for cracks induced by loading and for cracking induced by restraining the deformation for cross section having width or depth (whichever smaller) in excess of 800 mm, and β = 1.3 for cracking induced by restraining the deformation for cross section having width or depth (whichever smaller) less than 300 mm.

The mean steel strain ε_{sm} is given as

$$\varepsilon_{sm} = \frac{f_s}{E_s}\left(1 - \beta_1\beta_2\left(\frac{f_{scr2}}{f_s}\right)^2\right) \tag{5}$$

where f_s = stress in the tension reinforcement calculated on the basis of a cracked section, N/mm^2; f_{scr2} = stress in the tension longitudinal reinforcement computed on the basis of a cracked section under loading conditions that cause the first crack, N/mm^2; β_1 = a coefficient accounting for bar bond characteristics, and is equal to 0.8 for deformed bars and 0.5 for plain smooth bars, β_2 = a coefficient accounting for load duration; is equal to 1.0 for single short-term loading and 0.5 for sustained or cyclic loading; E_s = Modulus of elasticity of the reinforcement, N/mm^2.

The Egyptian code gives the average stabilized mean crack spacing by the following equation:

$$S_{rm} = 50 + 0.25k_1k_2\phi/\rho_{eff} \tag{6}$$

However, the items of the above equation are similar to the items of the equation given by the Eurocode2 [15] with the assumption that the clear concrete cover is equal to 25 mm.

2.3. ACI 318

Prior to 1999, flexural crack control requirements in ACI were based on the so-called z-factor method developed be Gergely and Lutz [8]. Their work was based on extensive statistical analysis techniques on experimental data from several researchers. The equation proposed by the early version of ACI 318-95 [17] took the following form:

$$W_{\max} = 0.011\beta f_s \sqrt[3]{d_c A_o} * 10^{-3}\text{mm}, \tag{7}$$

where $\beta = \frac{h-x}{d-x}$ is the ratio of distance between neutral axis and extreme tension face to distance between neutral axis and centroid of reinforcing steel; β = 1.20 in beams may be used to compare the crack widths obtained in flexure and axial tension. A_o = the area of concrete surrounding each reinforcing bar = A_e/n_b, A_e = the effective area of concrete in tension, A_e can be defined as the area of concrete having the full width of the beam and having the same centroid of the main reinforcement; A_e = 2 $d_c b$, n_b = the number of tension reinforcing bars. d_c = the distance measured from the centroid of tensile steel to the extreme tensioned fiber

The flexural crack width expression in the above equation, with $\frac{h-x}{d-x} = 1.2$, is used in ACI 318-95 in the following form:

$$z = f_s \sqrt[3]{d_c A_o} \tag{8}$$

A maximum value of z = 3064.5 N/mm is permitted for interior exposure, corresponding to a limiting crack width of 0.41 mm. ACI 318-95 also limits the value of z to 2539.2 N/mm for exterior exposure, corresponding to a crack width of 0.33 mm. When structures are subjected to very aggressive exposure or designed to be watertight, ACI committee 350 [18], limits the value of z to 1700 N/mm corresponding to a crack width of 0.20 mm.

In the 1999 edition, ACI decided to greatly simplify crack control requirements due to increased evidence suggesting a reduced correlation between crack width and reinforcement corrosion. Beeby [19] and [20] showed that corrosion does not correlate with surface crack widths in the range normally found with reinforcement stresses at service load levels. Therefore, ACI introduced changes to the crack rules in which a maximum bar spacing, rather than a z-factor (related to crack width) is prescribed.

ACI 318-05 [21], ACI 31808 [22] proposed the following equation for crack control:

$$s = 380(280/f_s) - 2.5c \leq 300(280/f_s) \tag{9}$$

where s = maximum spacing of reinforcement closest to the tension face, mm; c = least distance from surface of reinforcement to tension face, mm.

However the equation does not make a distinction between interior and exterior exposure, i.e. the exposure conditions dependence was eliminated. Also, the equation is indirectly tied to a crack width equals to 0.4 mm. The value of f_s at service load shall be computed on the basis of service moment. ACI permits the use of f_s = 0.67 f_y.

2.4. British standards BS 8110-1997

According to BS 8110-1997 [23], the width of flexural cracks at a particular point on the surface of a member depends primarily on three factors:

(a) The proximity to the point considered of reinforcing bars perpendicular to the cracks; concrete cover.
(b) The proximity of the neutral axis to the point considered; h–x.
(c) The average surface strain at the point considered.

BS 8110-1997 recommends that the strain in the tension reinforcement is limited to 0.8 f_y/E_s (i.e. 0.8 * steel yield strain) and the design surface crack width should not exceed the appropriate value. Cracking should not lead to spoil appearance. So for members that are visible, the calculated maximum crack width should not exceed 0.30 mm. Also, cracking should not lead to steel corrosion, so for members in aggressive environment the calculated maximum crack width should not lead to a loss of the performance of the structure.

BS 8110-97 provisions are based on Beeby [12] empirical equations,

$$\text{Design surface crack width } W_d = \frac{3 * a_{cr} * \varepsilon_m}{1 + 2\left(\frac{a_{cr} - c_{\min}}{h-x}\right)} \tag{10}$$

where a_{cr} = distance from the point considered to the surface of the nearest longitudinal bar; ε_m = average strain at the level where the cracking is being considered; $c_{\min}$ = minimum cover to the tension steel; h = overall depth of the member; x = depth of neutral axis.

For cracked section, the value of ε_m is expressed as:

$$\varepsilon_m = \varepsilon_1 - \frac{b(h-x)(a'-x)}{3E_s A_s(d-x)} \tag{11}$$

where ε_1 = strain at the level considered, calculated ignoring the stiffening effect of the concrete in the tension zone, b = width of the section at the centroid of the tension steel, a' = distance from the compression face to the point at which the crack width is being calculated.

According to BS 8110-1997 [23], in assessing the strains, the modulus of elasticity of the concrete should be taken as half the instantaneous values.

3. Some significant formulas for crack width given by researchers

3.1. Gergely and Lutz

Gergely and Lutz [8] used test results from Hognestad [24], Kaar and Mattock [25], Kaar and Hognestad [26], Clark [27], and Rusch and Rehm [28], to conclude their equation for the calculation of crack widths at the tension surface.

As stated by Frosch [10], the maximum concrete cover; d_c used in the tests analyzed by Gergely and Lutz [8] was 84 mm and only three specimens of 612 observations had clear covers greater than 64 mm.

The original equation developed by Gergely and Lutz [8] is as follows:

$$W_s = 0.011\sqrt[3]{(c + \phi/2)A_o}(f_s - 34.45) * 10^{-3} \tag{12}$$

where W_s = the most probable maximum crack width at level of steel, mm; f_s = the reinforcement steel stress, N/mm^2; A_o = the area of concrete symmetric with reinforcing steel divided by number of bars, mm^2.

To obtain the maximum crack width at the extreme tensioned fiber, Eq. (12) is multiplied by a factor $\beta = \frac{\mathbf{h-x}}{\mathbf{d-x}}$.

3.2. Oh and Kang

Oh and Kang [9] proposed a formulation for predicting the maximum crack width; $W_{\max}$ and average crack spacing; S in reinforced concrete flexural members. They tested five reinforced concrete beams with design variables including the concrete cover, diameter of steel bars, reinforcement ratios, spacing of steel bars and steel stress. Based on an energy approach, Oh and Kang concluded a new definition of the effective area of concrete in tension and suggested the following equation for the calculation of the maximum crack width:

$$W_{\max} = \phi a_0(\varepsilon_s - 0.0002)R \tag{13}$$

In which,

$$a_0 = 159\left(\frac{t_b}{h-x}\right)^{4.5} + 2.83\left(\frac{A_0}{A_{s1}}\right)^{1/3} \tag{14}$$

$$R = (h-x)/(d-x) \tag{15}$$

where A_0 is the effective area of tensioned concrete around each reinforcing bar; $A_o = A_e/n_b$ and $A_e = b\,h_1$, h_1 is the depth of equivalent area,

$$h_1 = \frac{(h-x)^3}{3(d-x)^2} \tag{16}$$

$$S_m = \phi\left(c_0 + \frac{0.236^* 10^{-6}}{\varepsilon_s^2}\right) \tag{17}$$

In which,

$$c_0 = 25.7\left(\frac{t_b}{h-x}\right)^{4.5} + 1.66\left(\frac{A_o}{A_{s1}}\right)^{1/3} \tag{18}$$

3.3. Frosch

Frosch [10] stated that Eq. (7) proposed by ACI-95 [17] is valid for a relatively narrow range of concrete covers (i.e. up to 63 mm). The use of thicker concrete covers is increasing because research and experience have indicated that the use of thicker covers can increase durability. Therefore, Frosch [10] developed the following simple, theoretically-derived equation to predict crack widths that could be used regardless the actual concrete cover.

$$W_{\max} = 2\frac{f_s}{E_s} d^* \beta \tag{19}$$

where d^* is the controlling cover distance and is taken the greater of $\left\{\begin{array}{l}\sqrt{d_c^2 + d_s^2}\\ \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}\end{array}\right\}$, $\beta = \frac{h-x}{d-x}$ and d_s is the side cover.

4. Comparison between some code equations and other available equations for predicting the crack width of some theoretical models

In this section, the above mentioned building codes and some equations given by researchers are applied to five reinforced concrete sections having different values of reinforcement ratio, bar distribution, and bars grade.

Fig. 1 shows the layout of the five models. The Models 1, 2, and 3 have the same reinforcement (strength; f_y, type: deformed bars, number and diameter), section width; b, concrete strength; f_{cu}. These models varied in section depth; d and consequently have variable reinforcement ratios; μ. Values of μ were: 0.565%, 0.87%, and 1.21% for the three model respectively. Models 2 and 4 have identical properties (same steel reinforcement, same section dimensions) and they varied in bar surface condition and yield stress of steel. Plain mild steel bars were used for Model 4, while high strength deformed bars were used for Model 2. Models 2 and 5 have the same section dimensions, approximately, same area of tension steel but varied in bar distribution (number and diameter). The values of the crack width were calculated at the same steel stress level

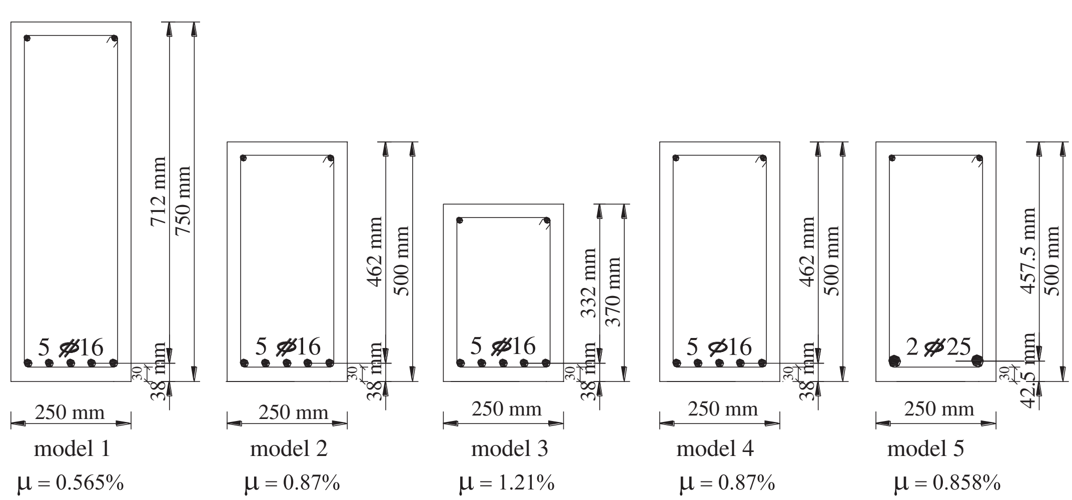

Figure 1 Dimensions and reinforcement of the studied models.

Table 1 Values of the crack width for the studied models.

Equation	Crack width (mm)				
	Model 1	Model 2	Model 3	Model 4	Model 5
Eurocode2	0.127	0.127	0.127	0.107	0.16
ECP-2007	0.107	0.129	0.137	0.123	0.170
ACI 318-95	0.124	0.129	0.136	0.119	0.191
BS 8110-97	0.079	0.087	0.090	0.056	0.162
Gergely and Lutz [8]	0.102	0.107	0.113	0.0682	0.158
Frosch [10]	0.101	0.106	0.111	0.0742	0.231
Oh and Kang [9]	0.147	0.135	0.131	0.0845	0.218

(f_s = 200 N/mm^2 for models 1, 2, 3 and 5 and f_s = 140 N/mm^2 for model 4). Concrete strength was considered as f_{cu} = 30 N/mm^2 and the clear cover = 30 mm.

Table 1 shows the values of crack widths calculated for the studied models for both of code equations and equations given by Gergely and Lutz [8], Frosch [10] and Oh and Kang [9]. The following was observed concerning the results given in the Table 1:

i. Generally, values of crack width predicted by the code equations showed a large scatter among the different code equations and those obtained by previous researchers. Values of crack width, predicted using BS 8110-97, were less than those predicted by other building codes or by equations developed by researchers, especially for Model 4, reinforced with plain mild steel bars. Values of the crack width predicted by the Egyptian code were very close to those predicted by ACI 318-95 and Eurocode2 equations except for model 1 with low reinforcement ratio; μ. The Egyptian code equation predicts values of crack width for sections with low reinforcement ratio smaller than those predicted by ACI or Eurocode2. Also, the table indicates that the equation of Oh and Kang [9] overestimated the values of the crack widths in the case of low steel ratio (Model 1), however ECP-2007 overestimated the value of the crack width in the case of plain bars of model 4.
ii. Comparing the results of the crack widths calculated for the models 1,2 and 3 indicated that as the percentage of the flexural steel ratio, μ, increased the calculated values of the crack width increased. This is given by code equations of ECP-2007, ACI-318-95 and other equations given by Gergely and Lutz [8] and Frosch [10]. With the increase of μ, the concrete contribution in tension decreases and the mean steel strain increases, and then the crack width consequently increases. However, according to Eurocode2, the increase of μ does not affect the crack width. On contrary, the equation of Oh and Kang [9] is found to give low values of crack width with the increase of the flexural steel ratio, μ.
iii. The effect of using different numbers of steel bars with equal area on the crack width is investigated by comparing the results of models 2 and 5. It is well established that the bar distribution is an important factor affecting crack width. With well choice of bar arrangement (larger number, smaller diameter), better bond between concrete and steel occurs and thus reducing the crack spacing. The reduction in crack width due to better bar distribution ranged from 26% to 118%, the lowest value was obtained using the Eurocode2 equation and the highest value was obtained according to Frosch [10] equation. The equation developed by Frosch [10] indicates that crack spacing (and consequently crack width) depends mainly on the distance between reinforcement bars.
iv. The effect of bar surface condition (plain or deformed) on crack width was obtained by comparing the results of models 2 and 4. The use of plain mild steel bars (with low steel stress at service loads) results values of crack width less by about 5–56% compared with model reinforced with the same area of high strength deformed bars. The lowest value was obtained using the ECP-2007 equation and the highest value was obtained by Oh and Kang equation. The effect of bar surface deformation on the calculation of crack width not only affects the crack spacing, but also affects the mean strain. When the bond between the concrete and the steel increased, more tension force is transferred to the concrete between cracks. With the increase in the concrete contribution in tension, less slip between concrete and steel occurs, hence less value of total elongation between them ($\varepsilon_{sm} - \varepsilon_{cm}$); and consequently resulting in less crack width. However, the effect of bar surface deformation is not considered in code formulas except the Egyptian code and Eurocode2.

5. Application of some code equations to some available experimental data

To asses the accuracy of code equations and the most common formulas for predicting crack width in reinforced concrete members, a comparison is carried out with the experimental results reported by Makhlouf and Malhas [11]. The comparison includes two factors affecting the crack width such as the steel stress and concrete cover.

5.1. Makhlouf and Malhas

Makhlouf and Malhas [11] reported results of tests on 11 beams reinforced with high strength deformed bars. The maximum crack width was recorded at the reinforcement level. The specimens consisted of two groups. Group ‘‘A’’ consisted of eight beams four of them were with 22 mm concrete clear cover and the other four were with 52 mm concrete clear cover. Group ‘‘A’’ mainly intended to investigate the effect of

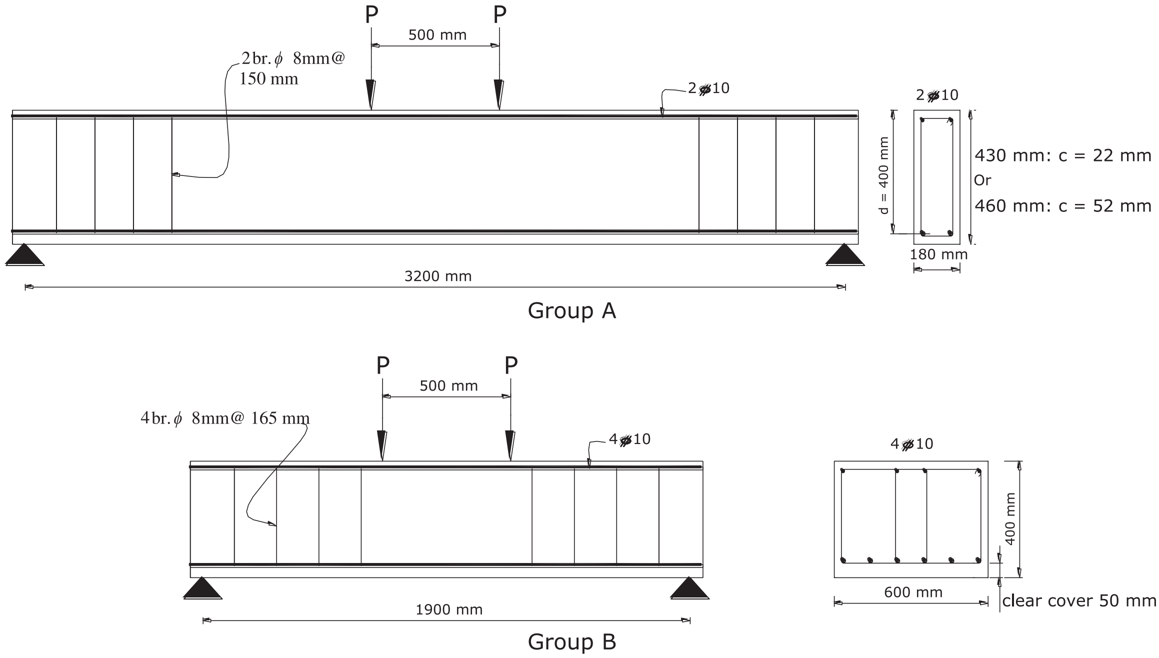

Figure 2 Beam specimens tested by Makhlouf and Malhas [11].

Table 2 Properties of specimens tested by Makhlouf and Malhas [11].

	Specimen no.	*b* (mm)	*h* (mm)	*d* (mm)	Reinforcement steel	Reinforcement ratio; μ (%)	fc′ (N/mm^2)	fy (N/mm^2)	*c* (mm)
Group “A”	GA11	180	430	400	2Ø12	0.31	34	425	22
	GA12	180	430	400	2Ø16	0.56	34	425	22
	GA13	180	430	400	2Ø20	0.87	34	425	22
	GAM	180	430	400	2Ø25	1.36	34	425	22
	GA21	180	460	400	2Ø12	0.31	34	425	52
	GA22	180	460	400	2Ø16	0.56	34	425	52
	GA23	180	460	400	2Ø20	0.87	34	425	52
	GA24	180	460	400	2Ø25	1.36	34	425	52
Group “B”	Gl	600	400	338	4Ø14 + 2020	0.62	40	430	50
	G2	600	400	336	6Ø20	0.94	40	430	50
	G3	600	400	336	9Ø20	1.40	40	430	50

doubling the concrete clear cover on the crack width. Group “B” consisted of 3 wide beams of 600 mm width, 400 mm depth and clear concrete cover = 50 mm. The experimental specimens were tested under a load level of 80–110% of service load. Group “B” aimed to assess the magnitude of crack widths in full-size beams under different levels of steel stress.

Fig. 2 shows the dimensions and loading system of the tested beams while Table 2 gives properties of all specimens tested by Makhlouf and Malhas [11] for both Group “A” and Group “B”.

Measurements were taken on both sides of the beam specimens and at 10 mm above the lower edge. Readings were taken at predetermined load levels that corresponded to a specific safety factor relative to the ultimate failure load.

5.1.1. Effect of steel stress, f_s

The main and the most important factor affecting the crack width is the steel strain (stress) which is directly proportional with the crack width. Makhlouf and Malhas [11] measured the crack width of the specimens of Group “B” at different levels of steel stress, as given in Table 3. Figs. 3–5 display the relationship between the crack width; W_{max}, as calculated using codes’ equations, and the steel stress; $f_{s,}$ for the group “B” specimens: G1, G2, and G3 respectively. A summary of the results obtained from code’s equations is given in Table 3. From the Figures and Table the following are observed:

i. The Egyptian code gives underestimated values of crack width for sections reinforced with low reinforcement ratios (μ = 0.62%), especially at low levels of steel stress

Table 3 Application of codes' equations to beams tested by Makhlouf and Malhas [11] – Group B.

	μ (%)	f_s (N/mm^2)	$\varepsilon_s = f_s/E_s \times 10^{-4}$	Egyptian code 2007[a]			Eurocode2-2001[b]			BS 8110-97		ACI 318–95	Oh-Kang 1987	Measured crack width by Makhlouf and Malhas experiments
				f_{scr2} (N/mm^2)	$\varepsilon_{sm} \times 10^{-4}$	W_k (mm)	$\varepsilon_m \times 10^{-4}$	$\varepsilon_m \times 10^{-4}$	W_k (mm)	$\varepsilon_m \times 10^{-4}$	W_d (mm)	W_{max} (mm)	W_{max} (mm)	$W_{experiments}$, (mm)
G1	0.62	134	6.7	173.7	–	–	−1.8	4.0	0.15	3.6	0.07	0.17	0.12	0.08
		161	8.1	173.7	0.54	0.02	−0.5	4.8	0.18	4.9	0.09	0.20	0.15	0.11
		200	10.0	173.7	4.0	0.12	1.2	6.0	0.23	6.9	0.13	0.25	0.20	0.19
		250	12.5	173.7	7.7	0.23	4.0	7.5	0.29	9.4	0.17	0.32	0.26	0.31
G2	0.94	154	7.7	117.7	4.1	0.11	1.6	4.6	0.16	5.7	0.10	0.20	0.15	0.09
		184	9.2	117.7	6.2	0.16	3.2	5.5	0.19	7.2	0.13	0.24	0.19	0.15
		230	11.5	117.7	9.1	0.24	5.5	6.9	0.24	9.5	0.17	0.30	0.26	0.22
		288	14.4	117.7	12.5	0.32	8.3	8.6	0.30	12.4	0.22	0.38	0.33	0.27
G3	1.40	151	7.6	80.2	5.9	0.12	3.3	4.5	0.13	6.3	0.10	0.18	0.13	0.14
		182	9.1	80.2	7.7	0.15	4.8	5.5	0.16	7.8	0.12	0.21	0.17	0.18
		227	11.4	80.2	10.2	0.20	7.1	6.8	0.20	10.1	0.16	0.27	0.22	0.26
		284	14.2	80.2	13.3	0.27	9.9	9.6	0.28	12.9	0.21	0.33	0.29	0.34

[a] Tensile strength of concrete $f_{ctr} = 0.6\sqrt{f_{cu}} = 4.12$ N/mm^2, – uncracked section according to the Egyptian code.
[b] Tensile strength of concrete $f_{ctm} = 0.3(f_c')^{\frac{2}{3}} = 3.51$ N/mm^2), ε_m^*/s_m according to Eq. (2); $s_m\ \varepsilon_m \geqslant 0.6\frac{f_s}{E_s}$.

($f_s = 0.30\,f_y$ and $f_s = 0.37\,f_y$). According to the Egyptian code, at steel stress $f_s = 0.30\,f_y$ (specimen G1), the section is considered uncracked. However, the values of crack width as proposed by the Egyptian code correlated well with the experimental values, at all levels of steel stress, for sections having percentage of reinforcement ($\mu = 0.94\%$). On the other hand, for sections having higher percentage of reinforcement ($\mu = 1.4\%$) the Egyptian code calculated crack width is lower than the crack width was measured experimentally. That may imply that a maximum reinforcement ratio for crack control should be determined.

ii. The underestimated values of crack width did not appear in Eurocode2 predictions at low levels of steel stress, since Eurocode2 limits the value of ε_m not to be less than $0.6\,f_s/E_s$, as appears from Table 3.
iii. Table 3 gives the values of steel strain ε_s together with the values of the average strain (ε_{sm} or ε_m). The results, obtained from both the Egyptian code and BS 8110, indicate that with the increase of reinforcement ratio μ, the estimated concrete contribution in tension (tension stiffening) decreased. In general, the limitation of average strain introduced by Eurocode2 predicts realistic values of crack width than those predicted by the Egyptian code. In addition, with increase of the steel stress, the contribution of tensioned concrete decreased.
iv. Generally, the ACI 318-95 equation greatly overestimated the values of the crack width, except at high level of steel stress and at high values of reinforcement ratio ($\mu = 1.4\%$). It should be noted that ACI 318-05 switched from the procedure of calculating the crack width and adopts a simplified equation for the maximum bar spacing. For values of steel stress 134–284 N/mm^2 (which are the levels of stress at which crack width were measured experimentally), equation 9 of ACI 318-05 yields maximum permitted bar spacing of 625–250 mm, for concrete clear cover = 50 mm.
v. The values of crack width predicted using BS 8110-97 equations were smaller than the experimental values at high levels of steel stress. As given in Table 3, high values of mean strain ε_m were proposed using BS 8110-97 and this indicates small contribution of tensioned concrete.
vi. The Oh-Kang [9] formula (Eq. (13)) correlated well with the experimental values of crack width for most specimens used in the comparison.

5.1.2. Effect of concrete cover, (c)

The value of concrete cover may be considered as the second important factor that affecting the crack width, but its efficiency is considered in different ways in the building codes. Makhlouf and Malhas [11] reported results of tests on eight beams reinforced with high strength deformed bars as given by Fig. 2 and Table 2, Group "A". The maximum crack width was recorded at the reinforcement level. Four of the beams were with 22 mm concrete clear cover and the other four were with 52 mm concrete clear cover. The aim of these tests was to investigate the effect of increasing the concrete clear cover on the crack width. The results showed that a 16% increase in the crack width was obtained as a result of increasing the concrete cover.

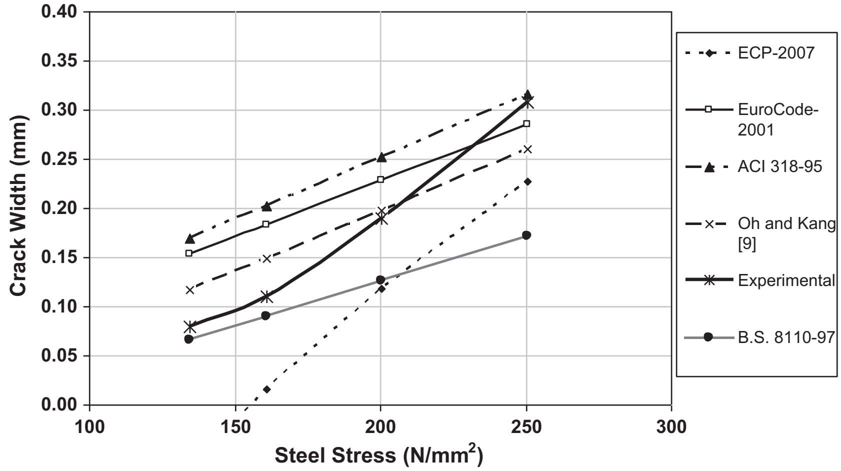

Figure 3 Crack width versus steel stress for G1 (μ = 0.62%) Makhlouf and Malhas [11].

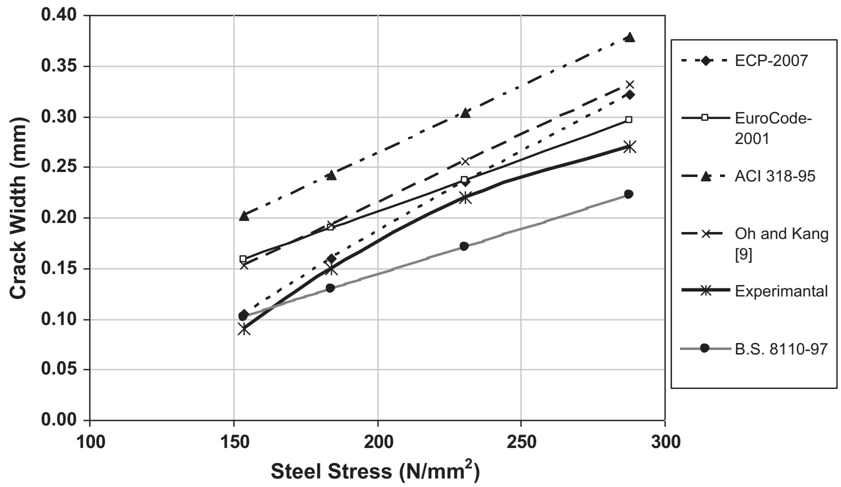

Figure 4 Crack width versus steel stress for G2 (μ = 0.94%) Makhlouf and Malhas [11].

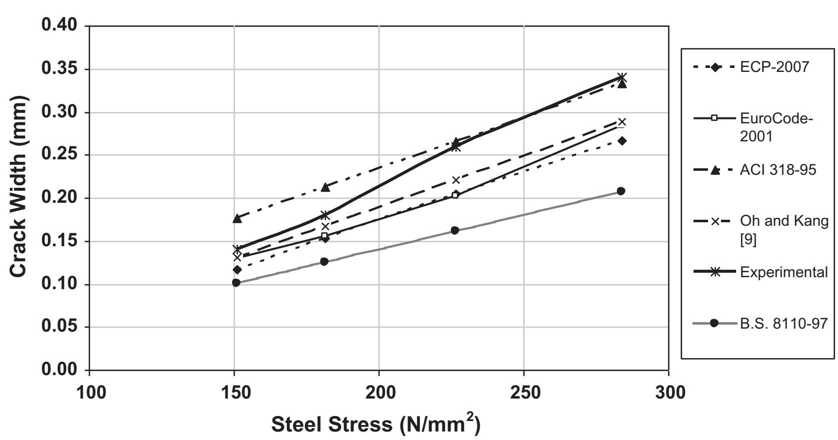

Figure 5 Crack width versus steel stress for G3 (μ = 1.40%) Makhlouf and Malhas [11].

Table 4 gives and compares the values of the crack width calculated using the equations proposed by the building codes for the beams tested by Makhlouf and Malhas [11] and also, the equation developed by Oh and Kang [9]. According to Oh and Kang [9], an increase of the crack width by about 20% is obtained with increase of the concrete cover from

Table 4 Application of codes' equations to beams tested by Makhlouf and Malhas [11] – Group A.

	Egyptian code 2007[a]			Eurocode2-2001[b]			BS 8110-97	ACI 318-95	Oh and Kang [9]
	f_{scr2} (N/mm^2)	$\varepsilon_{sm} \times 10^{-4}$	W_k (mm)	$\varepsilon_m \times 10^{-4}$	$\varepsilon_m^* \times 10^{-4}$	W_k (mm)	W_d (mm)	$W_{\max}$ (mm)	$W_{\max}$ (mm)
GA11	251.2	2.9	0.06	6.2	7.7	0.16	0.12	0.17	0.19
GA21	287.44	–	–	0.5	7.7	0.33	0.14	0.29	0.23
			–			**110.0%**	**20.0%**	**72.4%**	**17.9%**
GA12	144.5	9.5	0.17	8.6	8.6	0.15	0.14	**0.17**	0.21
GA22	165.36	8.5	0.23	5.5	7.7	0.27	0.19	0.29	0.25
			35.5%			**84.5%**	**32.1%**	**73.3%**	**19.2%**
GA13	94.4	11.4	0.18	9.8	9.8	0.14	0.15	**0.17**	0.22
GA23	108.1	10.9	0.25	7.8	7.8	0.26	0.20	0.30	0.27
			40.6%			**79.1%**	**38.5%**	**74.3%**	**20.6%**
GA14	61.9	12.1	0.17	10.5	10.5	0.13	0.14	**0.17**	0.24
GA24	70.9	12.0	0.24	9.2	9.2	0.27	0.20	0.30	0.29
			38.6%			**107.8%**	**45.4%**	**75.6%**	**22.4%**
Average increase in crack width			*38.3%*			*95.4%*	*34.0%*	*73.9%*	***20.0***

$f_s = 255.0$ N/mm^2, $\varepsilon_s = f_s/E_s = 12.75 \times 10^{-4}$, – Uncracked section according to the Egyptian code. Bold values refer to the percentage of the increase of the crack width value in each two specimens, however italic values refer to the average increase in the crack width for all specimens in each column of the table.

22 mm to 52 mm. This amount of increase is very close to the value obtained experimentally by Makhlouf and Malhas [11]. However, such increase in the values of crack width as predicted by the building codes was greatly overestimated as compared to the experimental results. The maximum percentage of increase was recorded by Eurocode2 followed by ACI 318-95 while the Egyptian code 2007 and BS 8110-97 give reasonable predictions.

6. Conclusions

From the results obtained from the comparison of several building codes' equations and from the theoretical work presented in this paper through studying the main variables affecting crack width, such as the concrete cover, steel stress, reinforcement ratio, bar surface, and reinforcement arrangement, the following conclusions may be drawn:

1. Values of crack width predicted by the code equations showed a large scatter among the different code equations and those obtained by other researchers. The Egyptian code equation predicted values very close to those predicted by ACI 318-95 and Eurocode2 equations except sections with low reinforcement ratio. For this case, the Egyptian code equation predicts values of crack width smaller than those predicted by ACI or Eurocode2.
2. Values of crack width, predicted using BS 8110-97, were less than those predicted by other building codes or by equations developed by researchers, especially for sections reinforced with plain mild steel bars. It should be noted that BS 8110 gives rules for bar detailing to ensure control of cracking rather than giving detailed calculations for crack width and crack spacing.
3. With the increase of reinforcement ratio μ, the concrete contribution in tension decreases, the mean steel strain increases, consequently the crack width increases. However, for crack control, it is suggested to limit the reinforcement ratio instead of limiting the steel stress.
4. The reinforcement detailing (i.e. the bars distribution) is an important factor affecting crack width. With the well choice of bar arrangement (larger number, smaller diameter), better bond between concrete and steel occurs resulting in a reduction in the crack spacing. In addition to BS-8110 recommendations for crack control, ACI 318-05 switched for the procedure of calculating the crack width and adopts simplified equation for the maximum bar spacing.
5. The effect of bar surface deformation on the calculation of crack width not only affects the crack spacing, but also affects the mean strain. When the bond between the concrete and the steel increased, more tension force is transferred to the concrete between cracks. With the increase in the concrete contribution in tension, less slip between concrete and steel occurs, hence less value of total elongation between them; (i.e. $\varepsilon_{sm} - \varepsilon_{cm}$); and consequently resulting in less crack width. However, the effect of bar surface deformation is not considered in most codes' formulas except the Egyptian code and Eurocode2.
6. Most equations proposed by the building codes overestimate the effect of the concrete cover on the calculated values of crack width when compared with the experimental results. However, Oh and Kang [9] formula, (Eq. (13), correlated well with the experimental values of crack width for most specimens used in the comparison.
7. The limitation of the average strain ε_m not to be less than $0.6\,f_s/E_s$ introduced by Eurocode2 results in realistic values of crack width when compared with experimental results than those predicted by the Egyptian code at low levels of steel stress for sections reinforced with low reinforcement ratios.
8. Comparison of building codes against experimental results revealed that the Egyptian code gives underestimated values of crack width for members reinforced with low reinforcement ratios especially at low level of steel stresses. The ECP 203-2007 adopted the use of mean strain similar to the equation given by Leonhardt [7]. The Egyptian code equation expresses the tension stiffening as a function of the

steel stress. For the calculation of the steel stress just after the first cracking, the Egyptian code assumes that the force resisted by concrete in tension is completely neglected immediately after the occurrence of the first crack. This assumption produces a very large value of steel stress just after cracking (f_{scr2}) for members having low reinforcement ratios especially, at low levels of steel stress. Values of f_{scr2} higher than f_s indicate uncracked sections. It should be noted that by definition f_{scr2} should be less than f_s. Therefore, for sections with low percentage of reinforcement, it is recommended, when using Eq. (5) of the ECP 203-2007, to limit the value f_{scr2} to f_s (the steel stress under working load conditions).

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