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Boundary states in IIA plane wave background

Yeonjung Kim [a], Jaemo Park [b]

[a] *Department of Physics, KAIST, Taejon 305-701, South Korea*
[b] *Department of Physics, POSTECH, Pohang 790-784, South Korea*

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Abstract

We work out boundary states for type IIA string theory on a plane wave background. By directly utilizing the channel duality, the induced conditions from the open string boundary conditions are imposed on the boundary states. The resulting boundary states correctly reproduce the partition functions of the open string theory for DpDp and DpD$\bar{p}$ cases where Dp branes are half BPS brane if located at the origin of the plane wave background.

1. Introduction

Recently there have been great interests on the string theory on the plane wave background [1]. Initially Metsaev worked out type IIB string theory on the plane wave background in the lightcone gauge [2,3]. Subsequently the type IIB string theory on the plane wave background attracted great interests in relation to the correspondence with $N = 4$ supersymmetric Yang–Mills theories [4].

In a series of paper [5–9], simple type IIA string theory on the plane wave background have been studied. The background for the string theory is obtained by compactifying the 11-dimensional plane wave background on a circle and taking the small radius limit. The resulting string theory has many nice features. It admits light cone gauge where the string theory spectrum is that of the free massive theory as happens in type IIB theory. Furthermore the worldsheet enjoys $(4,4)$ worldsheet supersymmetry. The structure of supersymmetry is simpler in the sense that the supersymmetry commutes with the Hamiltonian so that all members of the same supermultiplet has the same mass. The various $1/2$ BPS D-brane states were analyzed in the lightcone gauge, which gives consistent results with the BPS branes in the matrix model. Subsequently, covariant analysis was carried out for the D-brane spectrums, which agrees with the previous results [10].

In this Letter, we initiated the study of boundary states of type IIA string theories. Partly we were motivated by the works in type IIB side. In [11], the boundary states for type IIB theories were worked out and were shown to exhibit interesting channel duality between open string and closed string. In [12,13], more general cases were studied. In their developments, the lightcone supersymmetries, kinematical as well as dynamical, have played a

E-mail addresses: geni@muon.kaist.ac.kr (Y. Kim), jaemo@physics.postech.ac.kr (J. Park).

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crucial role in the construction of the boundary states. However in the construction of the boundary states in the more general conformal field theories [16,17], the supersymmetries are not important. And many of the theories considered do not have the spacetime supersymmetries. Rather, by directly utilizing the channel duality, one can first construct the boundary states, and then figure out the open string spectrums after the modular transformation and vice versa. In the simplest example of the flat space, starting from the open string boundary conditions, one can obtain the conditions to be imposed on the boundary states by simply interchanging the role of σ and τ of the worldsheet coordinates [14,15].

Indeed Michishita worked out type IIB boundary states along this line of idea and obtained the consistent results with the previous works whenever the comparison is available [19]. Also this approach was adopted in [20]. Here we adopt the same philosophy to obtain the boundary states in type IIA string theory on the plane wave background. The resulting boundary states give the correct open string partition function after the modular transformation. In this Letter we just work out the boundary states for DpDp and DpD$\bar{p}$ case where Dp branes are half BPS at the origin of the plane wave. Obviously there are more general cases to be studied. The detailed explorations of the boundary states of the IIA theory will appear elsewhere [24]. An interesting fact for these Dp branes is that they do not have the dynamical supersymmetries away from the origin of the plane wave background [10]. Similar phenomenon occurs in type IIB cases as well [21,22]. As we finish the draft, we are aware of the work [18] where type IIA boundary states are constructed following the approach of [11,12].

2. Free string theory and D-branes in IIA plane wave background

We closely follow the notation of [7] to describe the type IIA string theory on the plane wave background. The background is given by

$$ds^2 = -2\,dX^+\,dX^- - A\big(x^I\big)\big(dX^+\big)^2 + \sum_{i=1}^{8} dX^i\,dX^i, \tag{2.1}$$

$$F_{+123} = \mu, \qquad F_{+4} = \frac{\mu}{3} \tag{2.2}$$

and

$$A\big(x^I\big) = \sum_{i=1}^{4} \frac{\mu^2}{9}\big(X^i\big)^2 + \sum_{i'=5}^{8} \frac{\mu^2}{36}\big(X^{i'}\big)^2, \tag{2.3}$$

where we define $X^\pm = \frac{1}{\sqrt{2}}(X^0 \pm X^9)$. Throughout this Letter, we use the convention that unprimed coordinates denote $1,2,3,4$ directions while primed coordinates denote $5,6,7,8$ directions. Also unprimed quantities are associated with $1,2,3,4$ directions and primed ones are related to $5,6,7,8$ directions. For the worldsheet coordinates, we use $\partial_\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma)$. The worldsheet action for the closed string is given by

$$\begin{aligned} S_{LC} = -\frac{1}{4\pi\alpha'}\int d^2\sigma\,\Bigg(&-\partial_\mu X^+\partial^\mu X^- + \frac{1}{2}\partial_\mu X^i\partial^\mu X^i + \frac{m^2}{9}\sum_{i=1}^{4} X^i X^i + \frac{m^2}{36}\sum_{i'=1}^{4} X^{i'} X^{i'} \\ &+ \sum_{b=\pm}\big(-i\psi_b^1\partial_+\psi_b^1 - i\psi_b^2\partial_-\psi_b^2\big) + \frac{2im}{3}\psi_+^2\gamma^4\psi_-^1 - \frac{im}{3}\psi_-^2\gamma^4\psi_+^1\Bigg), \end{aligned} \tag{2.4}$$

where $m = \alpha' p^+\mu$ and γ^i are 8×8 matrices satisfying $\{\gamma^i,\gamma^j\} = \delta^{ij}$. The sign of subscript $\psi_\pm^A$ denotes the eigenvalue of γ^{1234} while the superscript $A = 1,2$ denotes the eigenvalue of γ^9. The theory of interest contains two supermultiplets $(X^i, \psi_-^1, \psi_+^2)$ and $(X^{i'}, \psi_+^1, \psi_-^2)$ of $(4,4)$ worldsheet supersymmetry with the masses $\frac{m}{3}$ and

$\frac{m}{6}$, respectively. The fermions of the first supermultiplet has γ^{12349} eigenvalue of 1 while those of the second has the eigenvalue of -1.

The mode expansion for the bosonic coordinates is given by

$$X^i = i\sqrt{\frac{\alpha'}{2}}\sqrt{\frac{1}{\omega_0}}\left(a_0^i e^{-i\omega_0\tau} - a_0^{i\dagger} e^{i\omega_0\tau}\right) + i\sqrt{\frac{\alpha'}{2}}\sum_{n=1}^{\infty}\frac{1}{\sqrt{\omega_n}}\left\{e^{-i\omega_n\tau}\left(a_n^i e^{in\sigma} + \tilde{a}_n^i e^{-in\sigma}\right) - e^{i\omega_n\tau}\left(a_n^{i\dagger} e^{-in\sigma} + \tilde{a}_n^{i\dagger} e^{in\sigma}\right)\right\}, \tag{2.5}$$

where we have $\omega_n = \sqrt{(\frac{m}{3})^2 + n^2}$ with $n \geqslant 0$ and

$$a_0^i \equiv \frac{\sqrt{\frac{\alpha'}{\omega_0}}\, p^i - i\sqrt{\frac{\omega_0}{\alpha'}}\, x^i}{\sqrt{2}}, \qquad a_0^{i\dagger} \equiv \frac{\sqrt{\frac{\alpha'}{\omega_0}}\, p^i + i\sqrt{\frac{\omega_0}{\alpha'}}\, x^i}{\sqrt{2}}. \tag{2.6}$$

The commutation relation is given by

$$\left[a_n^i, a_m^{j\dagger}\right] = \delta^{ij}\delta_{nm}, \quad n, m \geqslant 0, \qquad \left[\tilde{a}_n^i, \tilde{a}_m^{j\dagger}\right] = \delta^{ij}\delta_{nm}, \quad n, m > 0. \tag{2.7}$$

And the mode expansion for the primed coordinates is given by

$$X^{i'} = i\sqrt{\frac{\alpha'}{2}}\sqrt{\frac{1}{\omega'_0}}\left(a_0^{i'} e^{-i\omega'_0\tau} - a_0^{i'\dagger} e^{i\omega'_0\tau}\right) + i\sqrt{\frac{\alpha'}{2}}\sum_{n=1}^{\infty}\frac{1}{\sqrt{\omega'_n}}\left\{e^{-i\omega'_n\tau}\left(a_n^{i'} e^{in\sigma} + \tilde{a}_n^{i'} e^{-in\sigma}\right) - e^{i\omega'_n\tau}\left(a_n^{i'\dagger} e^{-in\sigma} + \tilde{a}_n^{i'\dagger} e^{in\sigma}\right)\right\}, \tag{2.8}$$

where we have $\omega'_n = \sqrt{(\frac{m}{6})^2 + n^2}$ with $n \geqslant 0$ and

$$a_0^{i'} \equiv \frac{\sqrt{\frac{\alpha'}{\omega'_0}}\, p^{i'} - i\sqrt{\frac{\omega_0}{\alpha'}}\, x^{i'}}{\sqrt{2}}, \qquad a_0^{i\dagger'} \equiv \frac{\sqrt{\frac{\alpha'}{\omega'_0}}\, p^{i'} + i\sqrt{\frac{\omega'_0}{\alpha'}}\, x^{i'}}{\sqrt{2}}. \tag{2.9}$$

The commutation relation is given by

$$\left[a_n^{i'}, a_m^{j'\dagger}\right] = \delta^{i'j'}\delta_{nm}, \quad n, m \geqslant 0, \qquad \left[\tilde{a}_n^{i'}, \tilde{a}_m^{j'\dagger}\right] = \delta^{i'j'}\delta_{nm}, \quad n, m > 0. \tag{2.10}$$

The bosonic part of the lightcone Hamiltonian H_{bc} is

$$H_{bc} = \frac{1}{2\pi\alpha' p^+}\int_0^{2\pi} d\sigma_0 \left(\frac{1}{2}P^i P^i + \frac{1}{2}\partial_\sigma X^i \partial_\sigma X^i + \frac{1}{2}\left(\frac{m}{3}\right)^2 X^i X^i + \frac{1}{2}P^{i'} P^{i'} + \frac{1}{2}\partial_\sigma X^{i'} \partial_\sigma X^{i'} + \frac{1}{2}\left(\frac{m}{6}\right)^2 X^{i'} X^{i'}\right) \tag{2.11}$$

with $P^i = \partial_\tau X^i$ and $P^{i'} = \partial_\tau X^{i'}$. In terms of the oscillators

$$p^+ H_{bc} = \omega_0 a_0^{i\dagger} a_0^i + \omega'_0 a_0^{\prime i\dagger} a_0^{i'} + \sum_{n=1}^{\infty}\omega_n\left(a_n^{i\dagger} a_n^i + \tilde{a}_n^{i\dagger}\tilde{a}_n^i\right) + \omega'_n\left(a_n^{i\dagger'} a_n^{i'} + \tilde{a}_n^{i\dagger'}\tilde{a}_n^{i'}\right) + e_{bc0}, \tag{2.12}$$

where e_{bc0} is the zero-point energy contribution. Throughout the Letter, repeated indices i and i' are assumed to be summed over $1, 2, 3, 4$ and $5, 6, 7, 8$, respectively. The fermionic mode expansion is given by

$$\psi_-^1 = \sqrt{\frac{\alpha'}{2}}\left(\chi e^{-i\omega_0\tau} + \chi^\dagger e^{i\omega_0\tau}\right) + \sum_{n=1}^{\infty}\sqrt{\alpha'}\,c_n\left\{e^{-i\omega_n\tau}\left(\tilde{\psi}_n e^{in\sigma} - i\frac{\omega_n - n}{\omega_0}\gamma^4\psi_n e^{-in\sigma}\right) + e^{i\omega_n\tau}\left(\tilde{\psi}_n^\dagger e^{-in\sigma} + i\frac{\omega_n - n}{\omega_0}\gamma^4\psi_n^\dagger e^{in\sigma}\right)\right\}, \tag{2.13}$$

where $c_n = 1/\sqrt{1 + (\frac{\omega_n - n}{\omega_0})^2}$ and

$$\chi = \frac{\tilde{\psi}_0 - i\gamma^4\psi_0}{\sqrt{2}}, \qquad \chi^\dagger = \frac{\tilde{\psi}_0 + i\gamma^4\psi_0}{\sqrt{2}} \tag{2.14}$$

with

$$\psi_+^2 = \sqrt{\frac{\alpha'}{2}}\left(i\gamma^4\chi e^{-i\omega_0\tau} - i\gamma^4\chi^\dagger e^{i\omega_0\tau}\right) + \sum_{n=1}^{\infty}\sqrt{\alpha'}\,c_n\left\{e^{-i\omega_n\tau}\left(\psi_n e^{-in\sigma} + i\frac{\omega_n - n}{\omega_0}\gamma^4\tilde{\psi}_n e^{in\sigma}\right) + e^{i\omega_n\tau}\left(\psi_n^\dagger e^{in\sigma} - i\frac{\omega_n - n}{\omega_0}\gamma^4\tilde{\psi}_n^\dagger e^{-in\sigma}\right)\right\}. \tag{2.15}$$

The anticommutation relations are

$$\left\{\psi_n^a, \psi_m^{b\dagger}\right\} = \delta^{ab}\delta_{nm}, \quad n, m > 0, \qquad \left\{\tilde{\psi}_n^a, \tilde{\psi}_m^{b\dagger}\right\} = \delta^{ab}\delta_{nm}, \quad n, m > 0, \qquad \left\{\chi^a, \chi^{b\dagger}\right\} = \delta^{ab}, \tag{2.16}$$

where a, b range from 1 to 4. The mode expansion of the superpartners for $X^{i'}$ is

$$\psi_+^1 = \sqrt{\frac{\alpha'}{2}}\left(\chi' e^{-i\omega_0'\tau} + \chi'^\dagger e^{i\omega_0'\tau}\right) + \sum_{n=1}^{\infty}\sqrt{\alpha'}\,c_n'\left\{e^{-i\omega_n'\tau}\left(\tilde{\psi}_n' e^{in\sigma} + i\frac{\omega_n' - n}{\omega_0'}\gamma^4\psi_n' e^{-in\sigma}\right) + e^{i\omega_n'\tau}\left(\tilde{\psi}_n'^\dagger e^{-in\sigma} - i\frac{\omega_n' - n}{\omega_0'}\gamma^4\psi_n'^\dagger e^{in\sigma}\right)\right\}, \tag{2.17}$$

where $c_n' = 1/\sqrt{1 + (\frac{\omega_n' - n}{\omega_0'})^2}$ and

$$\chi' = \frac{\tilde{\psi}_0' + i\gamma^4\psi_0'}{\sqrt{2}}, \qquad \chi'^\dagger = \frac{\tilde{\psi}_0' - i\gamma^4\psi_0'}{\sqrt{2}} \tag{2.18}$$

with

$$\psi_-^2 = \sqrt{\frac{\alpha'}{2}}\left(-i\gamma^4\chi' e^{-i\omega_0'\tau} + i\gamma^4\chi'^\dagger e^{i\omega_0'\tau}\right) + \sum_{n=1}^{\infty}\sqrt{\alpha'}\,c_n'\left\{e^{-i\omega_n'\tau}\left(\psi_n' e^{-in\sigma} - i\frac{\omega_n' - n}{\omega_0'}\gamma^4\tilde{\psi}_n' e^{in\sigma}\right) + e^{i\omega_n'\tau}\left(\psi_n'^\dagger e^{in\sigma} + i\frac{\omega_n' - n}{\omega_0'}\gamma^4\tilde{\psi}_n'^\dagger e^{-in\sigma}\right)\right\}. \tag{2.19}$$

The anticommutation relation is given by

$$\{\psi_n'^a, \psi_m'^{b\dagger}\} = \delta^{ab}\delta_{nm}, \quad n,m>0, \qquad \{\tilde{\psi}_n'^a, \tilde{\psi}_m'^{b\dagger}\} = \delta^{ab}\delta_{nm}, \quad n,m>0, \qquad \{\chi'^a, \chi'^{b\dagger}\} = \delta^{ab}. \tag{2.20}$$

The fermionic contribution to the Hamiltonian is

$$p^+ H_{fc} = \omega_0 \chi^\dagger \chi + \omega_0' \chi'^\dagger \chi' + \sum_{n=1}^{\infty} \omega_n \left(\psi_n^\dagger \psi_n + \tilde{\psi}_n^\dagger \tilde{\psi}_n\right) + \omega_n \left(\psi_n^\dagger \psi_n + \tilde{\psi}_n^\dagger \tilde{\psi}_n\right) + e_{fc0}. \tag{2.21}$$

For the closed string, the zero point energy contribution is zero, i.e., $e_{bc0} + e_{fc0} = 0$.

Now we discuss the Hamiltonian of the open strings and their mode expansions. First we deal with the bosonic part of the Hamiltonian

$$H_b = \frac{1}{2\pi\alpha' p^+} \int_0^\pi d\sigma \left(\frac{1}{2}P^i P^i + \frac{1}{2}\partial_\sigma X^i \partial_\sigma X^i + \omega_0^2 X^i X^i + \frac{1}{2}P^{i'} P^{i'} + \frac{1}{2}\partial_\sigma X^{i'} \partial_\sigma X^{i'} + \omega_0'^2 X^{i'} X^{i'}\right) \tag{2.22}$$

with $P^i = \partial_\tau X^i$ and $P^{i'} = \partial_\tau X^{i'}$. For the Neumann boundary condition,

$$X^{i_N} = i\sqrt{\frac{\alpha'}{\omega_0}}\left(a_0^{i_N} e^{-i\omega_0\tau} - a_0^{i_N\dagger} e^{i\omega_0\tau}\right) + i\sqrt{\frac{\alpha'}{2}} \sum_{n=1}^{\infty} \frac{1}{\sqrt{\omega_n}}\left(a_n^{i_N} e^{-i\omega_n\tau} - a_n^{i_N\dagger} e^{i\omega_n\tau}\right)\left(e^{in\sigma} + e^{-in\sigma}\right), \tag{2.23}$$

with the commutation relation being

$$\left[a_n^{i_N}, a_m^{j_N\dagger}\right] = \delta_{nm}\delta^{i_N j_N}, \quad i_N, j_N \geqslant 0. \tag{2.24}$$

The contribution to the Hamiltonian is

$$p^+ H_{b1} = \sum_{n=1}^{\infty} \omega_n a_n^{i_N} a_n^{i_N\dagger} + e_{b01N}. \tag{2.25}$$

For the Dirichlet boundary conditions with $x^{i_D}(\sigma=0) = x_0^{i_D}$, $x^{i_D}(\sigma=\pi) = x_1^{i_D}$

$$\begin{aligned} X^{i_D} = {} & \frac{x_0^{i_D}(e^{\omega_0\sigma} - e^{-\omega_0\sigma}) - x_0^{i_D}(e^{\omega_0(\sigma-\pi)} - e^{-\omega_0(\sigma-\pi)})}{e^{\pi\omega_0} - e^{-\pi\omega_0}} \\ & + i\sqrt{\frac{\alpha'}{2}} \sum_{n=1}^{\infty} \frac{1}{\sqrt{\omega_n}}\left(a_n^{i_D} e^{-i\omega_n\tau} + a_n^{i_D\dagger} e^{i\omega_n\tau}\right)\left(e^{in\sigma} - e^{-in\sigma}\right) \end{aligned} \tag{2.26}$$

with the commutation relation

$$\left[a_n^{i_D}, a_m^{j_D\dagger}\right] = \delta_{nm}\delta^{i_D j_D}, \quad i_D, j_D > 0. \tag{2.27}$$

The contribution to the Hamiltonian is

$$p^+ H = \frac{\omega_0}{4\pi\alpha'} \frac{(e^{\pi\omega_0} + e^{-\pi\omega_0})((x_1^{i_D})^2 + (x_0^{i_D})^2 - 4x_1^{i_D} x_0^{i_D})}{e^{\pi\omega_0} - e^{-\pi\omega_0}} + \sum_{n=1}^{\infty} \omega_n a_n^{i_D} a_n^{i_D\dagger} + e_{boD}. \tag{2.28}$$

For the $x^{i'}$ directions, we have the similar expressions, but we should replace unprimed quantities by primed quantities, e.g., ω_n by ω_n'. The total bosonic Hamiltonian is given by

$$p^+ H_{bo} = \sum_{n=1}^{\infty}\left(\omega_n a_n^{i_D} a_n^{i_D\dagger} + \omega_n a_n^{i_D'} a_n^{i_D'\dagger} + \omega_n a_n^{i_N} a_n^{i_N\dagger} + \omega_n a_n^{i_N'} a_n^{i_N'\dagger}\right)$$

$$+\frac{\omega_0}{4\pi\alpha'}\frac{(e^{\pi\omega_0}+e^{-\pi\omega_0})((\vec{x}_1^{D})^2+(\vec{x}_0^{D})^2-4\vec{x}_1^{D}\cdot\vec{x}_0^{D})}{e^{\pi\omega_0}-e^{-\pi\omega_0}}$$
$$+\frac{\omega_0'}{4\pi\alpha'}\frac{(e^{\pi\omega_0'}+e^{-\pi\omega_0'})((\vec{x}_1'^{D})^2+(\vec{x}_0'^{D})^2-4\vec{x}_1'^{D}\cdot\vec{x}_0'^{D})}{e^{\pi\omega_0'}-e^{-\pi\omega_0'}}+e_{b0}. \tag{2.29}$$

For the fermion, we impose the boundary conditions

$$\psi^1_\pm\big|_{\sigma=0,\pi}=\eta\Omega\psi^2_\mp\big|_{\sigma=0,\pi}. \tag{2.30}$$

Here $\eta=1$ is DpDp case, which means that open string ends on a Dp brane at one end and ends on another Dp brane at the other end. And $\eta=-1$ denotes DpD$\bar{p}$ case where D$\bar{p}$ means an anti-Dp brane. In this Letter we only deal with DpDp and DpD$\bar{p}$ cases, but certainly more general possibilities exist, which will be explored elsewhere. Let us discuss $\eta=+1$ case first. In order for the Dp brane to have the supersymmetry at the origin of the plane wave background, Ω should satisfy

$$\gamma^4\Omega\gamma^4\Omega=-1,\qquad \{\Omega,\gamma^9\}=0,\qquad \{\Omega,\gamma^{1234}\}=0. \tag{2.31}$$

The actual form of the Ω depends on the dimension of the world volume. D2, D4, D6, D8 branes were shown to have the supersymmetry at the origin and the expressions of Ω are tabulated at [7]. However the detailed form of Ω is not important for the subsequent discussions. One interesting point is that depending on the position, the number of the supersymmetries to be preserved by the D-brane is different. The D-branes away from the origin have no dynamical supersymmetry as shown in the covariant analysis [10]. From the conditions above, one obtain the conditions for the open string modes

$$\tilde{\psi}_n=\Omega\psi_n,\qquad \tilde{\psi}_n'=\Omega\psi_n', \tag{2.32}$$

for all n. To obtain the open string mode expansion one should replace $\tilde{\psi}_n$, $\tilde{\psi}_n'$ by the above conditions in the closed string expressions (2.13), (2.15), (2.17) and (2.19). The fermionic contribution to the Hamiltonian is

$$p^+H_{fo}=\sum_{n=1}^{\infty}\left(\omega_n\psi_n^\dagger\psi_n+\omega_n\psi_n'^\dagger\psi_n'\right)+\left(\frac{i}{2}\omega_0\psi_0\gamma^4\Omega\psi_0-\frac{i}{2}\omega_0'\psi_0'\gamma^4\Omega\psi_0'\right)+e_{f0}. \tag{2.33}$$

For $\eta=1$, the zero point energy contribution for Dp brane is given by $e_{b0}+e_{f0}=\frac{1}{2}\omega_0 n_N+\frac{1}{2}\omega_0' n_N'$ with $n_N+n_N'=p-1$ where n_N is the number of Neumann directions among $1,2,3,4$ directions and n_N' is the number of Neumann directions among $5,6,7,8$ directions.

For the DpD$\bar{p}$, we have $\eta=-1$ and the fermion modes have half-integer modings in worldsheet coordinates σ and the energy eigenvalues are given by $\bar{\omega}_n\equiv\sqrt{(\frac{m}{3})^2+(n-\frac{1}{2})^2}$, $\bar{\omega}_n'\equiv\sqrt{(\frac{m}{6})^2+(n-\frac{1}{2})^2}$ and

$$p^+H_f=\sum_{n=1}^{\infty}\left(\bar{\omega}_n\psi^\dagger_{n-\frac{1}{2}}\psi_{n-\frac{1}{2}}+\bar{\omega}_n'\psi'^\dagger_{n-\frac{1}{2}}\psi'_{n-\frac{1}{2}}\right)+e'_{f0}, \tag{2.34}$$

where $\{\psi^a_{n-\frac{1}{2}},\psi^{b\dagger}_{n-\frac{1}{2}}\}=\delta^{ab}_{nm}$ and the similar relation holds for ψ's. The value of zero point energy $e_{b0}+e'_{f0}$ will be given shortly after introducing suitable quantities.

Now one can evaluate the cylinder amplitude for the open string

$$Z_O=\int_0^\infty\frac{dt}{t}\int\alpha'\,dp^+\,dp^-\,\mathrm{Tr}\,e^{-2\pi t(-2\alpha' p^+p^-+p^+H)}, \tag{2.35}$$

where $H = H_{bo} + H_{fo}$ of (2.29) and (2.33). For the evaluation, it is convenient to define

$$f_1^{(m)}(q) = q^{-\Delta_m}\left(1-q^m\right)^{1/2}\prod_{n=1}^{\infty}\left(1-q^{\sqrt{n^2+m^2}}\right), \qquad f_2^{(m)}(q) = q^{-\Delta_m}\left(1+q^m\right)^{1/2}\prod_{n=1}^{\infty}\left(1+q^{\sqrt{n^2+m^2}}\right),$$
$$f_3^{(m)}(q) = q^{-\Delta'_m}\prod_{n=1}^{\infty}\left(1+q^{\sqrt{(n-\frac{1}{2})^2+m^2}}\right), \qquad f_4^{(m)}(q) = q^{-\Delta'_m}\prod_{n=1}^{\infty}\left(1-q^{\sqrt{(n-\frac{1}{2})^2+m^2}}\right), \tag{2.36}$$

where $q = e^{-2\pi t}$ and Δ_m, Δ'_m are defined as

$$\Delta_m = -\frac{1}{(2\pi)^2}\sum_{p=1}^{\infty}\int_0^{\infty} e^{-p^2 s} e^{-\frac{\pi^2 m^2}{s}}, \qquad \Delta'_m = -\frac{1}{(2\pi)^2}\sum_{p=1}^{\infty}(-1)^p\int_0^{\infty} e^{-p^2 s} e^{-\frac{\pi^2 m^2}{s}}. \tag{2.37}$$

The modular transformation property is proved to be [11]

$$f_1^{(m)}(q) = f_1^{(\hat{m})}(\tilde{q}), \qquad f_2^{(m)}(q) = f_4^{(\hat{m})}(\tilde{q}), \qquad f_3^{(m)}(q) = f_3^{(\hat{m})}(\tilde{q}), \tag{2.38}$$

where $\hat{m} = mt$ and $\tilde{q} = e^{-\frac{2\pi}{t}}$. With this expression, the cylinder amplitude is easily evaluated following [11]. The result is

$$\begin{aligned} Z_O = \int_0^{\infty} \frac{dt}{2t^2}(2\sinh\pi t\omega_0)^{2-n_N}(2\sinh\pi t\omega'_0)^{2-n'_N}\, &\frac{f_A^{\omega_0}(q)^4 f_A^{\omega'_0}(q)^4}{f_1^{\omega_0}(q)^4 f_1^{\omega'_0}(q)^4} \\ \times \exp\left[-2\pi t\left(\frac{\omega_0}{4\pi\alpha'}\frac{(e^{\pi\omega_0}+e^{-\pi\omega_0})((\vec{x}_1^D)^2+(\vec{x}_0^D)^2) - 4\vec{x}_1^D\cdot\vec{x}_0^D}{e^{\pi\omega_0}-e^{-\pi\omega_0}}\right)\right] \\ \times \exp\left[-2\pi t\left(\frac{\omega'_0}{4\pi\alpha'}\frac{(e^{\pi\omega'_0}+e^{-\pi\omega'_0})((\vec{x}_1'^D)^2+(\vec{x}_0'^D)^2) - 4\vec{x}_1'^D\cdot\vec{x}_0'^D}{e^{\pi\omega'_0}-e^{-\pi\omega'_0}}\right)\right], \end{aligned} \tag{2.39}$$

where $A = 1$ for DpDp and $A = 4$ for DpD$\bar{p}$. The last two lines of (2.39) are coming from the Dirichlet directions. Using the result of [11], one can see that zero energy contribution for DpD$\bar{p}$ is $\frac{n_N-2}{2}\omega_0 + 4\Delta_{\omega_0} - 4\Delta'_{\omega_0} + \frac{n_{N'}-2}{2}\omega'_0 + 4\Delta_{\omega'_0} - 4\Delta'_{\omega'_0}$. See also [23]. After the modular transformation, with $t = \frac{1}{2\ell}$, $\hat{\omega}_0 = \omega_0 t$, we have

$$\begin{aligned} Z_{C_2} = \int_0^{\infty} d\ell\,(2\sinh\pi\hat{\omega}_0)^{2-n_N}(2\sinh\pi\hat{\omega}'_0)^{2-n'_N}\, &\frac{f_B^{\hat{\omega}_0}(\tilde{q})^4 f_B^{\hat{\omega}'_0}(\tilde{q})^4}{f_1^{\hat{\omega}_0}(\tilde{q})^4 f_1^{\hat{\omega}'_0}(\tilde{q})^4} \\ \times \exp\left[-\left(\frac{\hat{\omega}_0}{2\alpha'}\frac{(e^{2\ell\hat{\omega}_0\pi}+e^{-2\ell\hat{\omega}_0\pi})((\vec{x}_1^D)^2+(\vec{x}_0^D)^2) - 4\vec{x}_1^D\cdot\vec{x}_0^D}{e^{2\ell\hat{\omega}_0\pi}-e^{-2\ell\hat{\omega}_0\pi}}\right)\right] \\ \times \exp\left[-\left(\frac{\hat{\omega}'_0}{2\alpha'}\frac{(e^{2\ell\hat{\omega}'_0\pi}+e^{-2\ell\hat{\omega}'_0\pi})((\vec{x}_1'^D)^2+(\vec{x}_0'^D)^2) - 4\vec{x}_1'^D\cdot\vec{x}_0'^D}{e^{2\ell\hat{\omega}'_0\pi}-e^{-2\ell\hat{\omega}'_0\pi}}\right)\right] \end{aligned} \tag{2.40}$$

with $\tilde{q} = e^{-4\pi\ell}$ where $B = 1$ for DpDp and $B = 2$ for DpD$\bar{p}$. This will be compared with the overlap of the boundary states in the next section.

3. Boundary state construction

As explained in [11], if we have the usual lightcone gauge in the open string we have to take the nonstandard lightcone gauge in the closed string channel where the role of X^+ and p^+ are reversed. In this case,

$$X^+ H_c = p^+ H_{bc} + p^+ H_{fc}, \tag{3.1}$$

where $p^+ H_{bc}$ and $p^+ H_{fc}$ are given by (2.12) and (2.21) respectively with replacing m by $\hat{m} \equiv mt$. This change of the mass parameter is due to the conformal transformation needed going from the open string channel to the closed string channel [11]. If we have a channel duality between closed strings and open strings we can obtain the condition for the boundary states from the boundary conditions for the open strings by interchanging the role of σ and τ coordinates of the world sheets.

For the Neumann boundary conditions for a bosonic coordinate X, $\partial_\sigma X = 0$ at $\sigma = 0$, the corresponding condition for the boundary states is $\partial_\tau X = 0$ at $\tau = 0$. This gives the condition

$$a_n + \tilde{a}_n^\dagger = 0, \qquad \tilde{a}_n + a_n^\dagger = 0, \qquad a_0 + a_0^\dagger = 0 \tag{3.2}$$

whose solution is

$$\exp\left(-\sum_{n\geqslant 1} a_n^\dagger \tilde{a}_n^\dagger - \frac{1}{2}\left(a_0^\dagger\right)^2\right)|0\rangle. \tag{3.3}$$

Here $|0\rangle$ is the vacuum which is annihilated by $a_n, \tilde{a}_n, a_0$. Its CPT conjugate state is given by

$$\langle 0|\exp\left(-\sum_{n\geqslant 1} a_n \tilde{a}_n - \frac{1}{2}a_0^2\right). \tag{3.4}$$

For the Dirichlet conditions, we impose $\partial_\sigma X = 0$ at $\tau = 0$ with $x^i = x_0^i$. We have

$$a_n - \tilde{a}_n^\dagger = 0, \qquad \tilde{a}_n - a_n^\dagger = 0 \quad (n \geqslant 1), \qquad a_0 - a_0^\dagger = -i\sqrt{\frac{2\omega_0}{\alpha'}}\, x_0 \tag{3.5}$$

whose solution is

$$\exp\left(\frac{1}{2}\left(a_0^\dagger - i\sqrt{\frac{2\omega_0}{\alpha'}}\, x_0\right)^2 + \sum_{n\geqslant 1} a_n^\dagger \tilde{a}_n^\dagger\right)|0\rangle. \tag{3.6}$$

For the evaluation of the overlap of the boundary conditions we need

$$\begin{aligned}
&\langle 0|\exp\left(\frac{1}{2}\left(a_0 + i\sqrt{\frac{2\omega_0}{\alpha'}}\, x_1\right)^2\right) e^{-2\pi\ell\omega_0 a_0^\dagger a_0} \exp\left(\frac{1}{2}\left(a_0^\dagger - i\sqrt{\frac{2\omega_0}{\alpha'}}\, x_0\right)^2\right)|0\rangle \\
&\quad = \left(1 - e^{-4\pi\ell\omega_0}\right)^{-1/2} \exp\left(-\frac{\omega_0}{\alpha'}\frac{x_0^2 + x_1^2 - 2x_0 x_1 e^{-2\pi\ell\omega_0}}{1 - e^{-4\pi\ell\omega_0}}\right) \\
&\quad = \frac{\exp\left(-\frac{\omega_0}{2\alpha'}(x_0^2 + x_1^2)\right)}{(1 - e^{-4\pi\ell\omega_0})^{1/2}} \exp\left(-\frac{\omega_0}{2\alpha'}\frac{(e^{2\pi\ell\omega_0} + e^{-2\pi\ell\omega_0})(x_0^2 + x_1^2) - 4x_0 x_1}{e^{2\pi\ell\omega_0} - e^{-2\pi\ell\omega_0}}\right).
\end{aligned} \tag{3.7}$$

Thus the total bosonic boundary state is given by

$$\exp\left(-\sum_{n=1}^{\infty} a_n^{i_N\dagger}\tilde{a}_n^{i_N\dagger} - \frac{1}{2}\left(a_0^{i_N\dagger}\right)^2 - a_n^{i'_N\dagger}\tilde{a}_n^{i'_N\dagger} - \frac{1}{2}\left(a_0^{i'_N\dagger}\right)^2\right),$$

$$\exp\left(\frac{1}{2}\left(a_0^{i_D\dagger} - i\sqrt{\frac{2\omega_0}{\alpha'}}\,x_0^{i_D}\right)^2 + \sum_{n=1}^{\infty} a_n^{i_D\dagger}\tilde{a}_n^{i_D\dagger}\right),$$

$$\exp\left(\frac{1}{2}\left(a_0^{i'_D\dagger} - i\sqrt{\frac{2\omega_0}{\alpha'}}\,x_0^{i'_D}\right)^2 + \sum_{n=1}^{\infty} a_n^{i'_D\dagger}\tilde{a}_n^{i'_D\dagger}\right)|0\rangle, \tag{3.8}$$

where the bosonic vacuum $|0\rangle$ is annihilated by all lowering operators associated with the raising operators appearing in the expression.

Now the boundary condition for the fermions at the open string channel

$$\psi^1_\pm\big|_{\sigma=0} = \eta\psi^2_\mp\big|_{\sigma=0} \tag{3.9}$$

is translated into the conditions at the closed string channel

$$\psi^1_\pm\big|_{\tau=0} = i\eta\psi^2_\mp\big|_{\tau=0}. \tag{3.10}$$

From $\psi^1_- = i\eta\psi^2_+$ at $\tau = 0$, we obtain

$$\chi^\dagger = -\eta\Omega\gamma^4\chi, \qquad \chi = \eta\Omega\gamma^4\chi^\dagger \tag{3.11}$$

which are equivalent if we use $\gamma^4\Omega\gamma^4\Omega = \Omega\gamma^4\Omega\gamma^4 = -1$, which can be proved by using the fact that Ω and γ^4 are either commute or anticommute. For nonzero modes we have

$$\tilde{\psi}_n = i\eta\Omega\psi_n^\dagger, \qquad \psi_n = -i\eta\Omega^T\tilde{\psi}_n^\dagger \tag{3.12}$$

which lead to the same boundary state. From the above conditions we obtain

$$\exp\left(\frac{1}{2}\eta\chi^\dagger\Omega\gamma^4\chi^\dagger + i\eta\sum_{n=1}^{\infty}\tilde{\psi}_n^\dagger\Omega\psi_n^\dagger\right)|0\rangle, \tag{3.13}$$

where the fermionic vacuum state $|0\rangle$ is annihilated by ψ_n, $\tilde{\psi}_n$ and χ. From $\psi^1_+ = i\eta\psi^2_-$ at $\tau = 0$ gives

$$\chi' = -\eta\Omega\gamma^4\chi'^\dagger, \qquad \chi'^\dagger = \eta\Omega\gamma^4\chi' \tag{3.14}$$

and

$$\tilde{\psi}'_n = i\eta\Omega\psi_n'^\dagger, \qquad \psi'_n = -i\eta\Omega^T\tilde{\psi}_n'^\dagger. \tag{3.15}$$

The total fermionic boundary state is given by

$$\exp\left(\frac{1}{2}\eta\chi^\dagger\Omega\gamma^4\chi^\dagger - \frac{1}{2}\eta\chi'^\dagger\Omega\gamma^4\chi'^\dagger + i\eta\sum_{n=1}^{\infty}\tilde{\psi}_n^\dagger\Omega\psi_n^\dagger + i\eta\sum_{n=1}^{\infty}\tilde{\psi}_n'^\dagger\Omega\psi_n'^\dagger\right)|0\rangle, \tag{3.16}$$

where the fermionic vacuum $|0\rangle$ is annihilated by all of the corresponding lowering operators for the raising operators appeared in the expressions. The total boundary states are the product of the bosonic boundary state and the fermionic one. Boundary states at other $\tau = \tau_0$ are generated by the closed string Hamiltonian acting on boundary states at $\tau = 0$, i.e., $|B, \tau = \tau_0\rangle = e^{iX^+H_c\tau_0}|B, \tau = 0\rangle$. From now on all boundary states are assumed to be those at $\tau = 0$.

Let $|B_0, \eta\rangle$ be the boundary states corresponding to the D-branes located at x_0 with η appearing in the definition of the fermion boundary states and $|B_1, \eta'\rangle$ be an analogous states with different values of x_1 and η'. Let $\langle B_1, \eta'|$

be the CPT conjugate states for $|B_1,\eta'\rangle$. The evaluation of the overlap of the boundary state leads to

$$
\begin{aligned}
Z_C &= \int_0^\infty d\ell\,\langle B_1,\eta'|e^{-2\pi\ell X^+ H_C}|B_2,\eta\rangle \\
&= \int d\ell\,\frac{(1-\eta\eta' e^{-4\pi\ell\omega_0})^2(1-\eta\eta'\prod_{n=1}^\infty e^{-4\pi\ell\omega_n})^4(1-\eta\eta'\prod_{n=1}^\infty e^{-4\pi\ell\omega'_n})^4}{(1-e^{-4\pi\ell\omega_0})^2(1-\prod_{n=1}^\infty e^{-4\pi\ell\omega_n})^4(1-\prod_{n=1}^\infty e^{-4\pi\ell\omega'_n})^4} \\
&\quad\times \exp\left(-\frac{\hat\omega_0}{2\alpha'}\left(\left(\vec{x}_0^{\,D}\right)^2+\left(\vec{x}_1^{\,D}\right)^2\right)\right)\exp\left(-\frac{\hat\omega_0}{2\alpha'}\frac{(e^{2\ell\hat\omega_0\pi}+e^{-2\ell\hat\omega_0\pi})((\vec{x}_1^{\,D})^2+(\vec{x}_0^{\,D})^2)-4\vec{x}_1^{\,D}\cdot\vec{x}_0^{\,D}}{e^{2\ell\hat\omega_0\pi}-e^{-2\ell\hat\omega_0\pi}}\right) \\
&\quad\times \exp\left(-\frac{\hat\omega'_0}{2\alpha'}\left(\left(\vec{x}_0^{\,\prime D}\right)^2+\left(\vec{x}_1^{\,\prime D}\right)^2\right)\right)\exp\left(-\frac{\hat\omega'_0}{2\alpha'}\frac{(e^{2\ell\hat\omega'_0\pi}+e^{-2\ell\hat\omega'_0\pi})((\vec{x}_1^{\,\prime D})^2+(\vec{x}_0^{\,\prime D})^2)-4\vec{x}_1^{\,\prime D}\cdot\vec{x}_0^{\,\prime D}}{e^{2\ell\hat\omega'_0\pi}-e^{-2\ell\hat\omega'_0\pi}}\right).
\end{aligned}
$$

For the DpDp boundary condition we have $\eta\eta'=1$ and DpD$\bar{p}$ boundary condition we have $\eta\eta'=-1$. In the process of the calculation we crucially use the fact $\gamma^4\Omega\gamma^4\Omega=-1$. Since Ω commutes with γ^{12349}, two supermultiplets can be separately dealt with. One can check that the above result is coincident with the open string results up to overall normalization factor which can be absorbed into the normalization factor of the boundary state. This provides a consistency check for the boundary state construction carried out here.

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