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Kähler corrections for the volume modulus of flux compactifications

G. von Gersdorff [a], A. Hebecker [b]

[a] *Department of Physics and Astronomy, Johns Hopkins University, 3400 N. Charles Street, Baltimore, MD 21218, USA*
[b] *Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16 und 19, D-69120 Heidelberg, Germany*

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Abstract

No-scale models arise in many compactifications of string theory and supergravity, the most prominent recent example being type IIB flux compactifications. Focussing on the case where the no-scale field is a single unstabilized volume modulus (radion), we analyse the general form of supergravity loop corrections that affect the no-scale structure of the Kähler potential. These corrections contribute to the 4d scalar potential of the radion in a way that is similar to the Casimir effect. We discuss the interplay of this loop effect with string-theoretic α' corrections and its possible role in the stabilization of the radion.

In flux compactifications of type IIB supergravity, all complex structure moduli and the dilaton are generically fixed by the non-trivial superpotential induced by the 3-form field strength [1,2]. However, this superpotential is independent of the Kähler moduli. Even if supersymmetry is broken by the non-zero vacuum expectation value of the superpotential W, one of the flat directions associated with the Kähler moduli survives. The resulting 4d model is of no-scale type and the no-scale field T is the Kähler modulus related to an overall rescaling of the compact volume. Perturbative corrections generically renormalize the Kähler potential, destroy the no-scale structure and lift the flat directions. We will be interested in loop corrections to the no-scale Kähler potential of the volume modulus T (the radion). In the large-volume limit, such corrections should be calculable within the low-energy effective field theory. They are potentially relevant for the stabilization of the radion and the uplifting to a metastable de Sitter vacuum [3,4].

To understand the supergravity 1-loop corrections, we first focus on a situation where $W = 0$ and supersymmetry is unbroken. We consider the corrections to the radion kinetic term and to the Einstein–Hilbert term of the 4d effective theory. Before Weyl rescaling, these corrections are independent of the 10d Planck

E-mail addresses: gero@pha.jhu.edu (G. von Gersdorff), a.hebecker@thphys.uni-heidelberg.de (A. Hebecker).

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mass and their form can therefore be inferred from dimensional arguments. It is then straightforward to derive the corresponding Kähler corrections, which are of the form $1/(T+\bar{T})^2$, with the compact volume scaling as $V \sim (\mathrm{Re}\,T)^{3/2}$. After a small non-zero W has been introduced as a perturbation, they induce a potentially important contribution to the radion scalar potential.

For the purpose of our technical discussion, we first adopt a slightly more general perspective. Consider a d-dimensional supergravity theory, compactified to 4d on a k-dimensional manifold ($d = 4 + k$) for which its total volume V corresponds to a flat direction. We write the metric as

$$ds^2 = g_{\mu\nu}\,dx^\mu\,dx^\nu + R(x)^2\tilde{g}_{mn}\,dy^m\,dy^n, \tag{1}$$

where Greek and Latin indices run over $0,\dots,3$ and $5,\dots,d$, respectively, and the decomposition $g_{mn} = R^2\tilde{g}_{mn}$ is defined in such a way that the volume of the compact space measured with the metric $\tilde{g}_{mn}$ is 1. The physical volume is $V = R^k$. In spite of its various interesting physical effects [2], we neglect for simplicity the possible warp factor, i.e., we assume that $g_{\mu\nu}$ does not depend on y. This is justified in the large volume limit.

Assuming that the fundamental d-dimensional Einstein–Hilbert term has coefficient $M^{d-2}/2$, the 4d action reads

$$S = \int d^4x\,\sqrt{g}(MR)^k\frac{M^2}{2} \times \left[\mathcal{R} + k(k-1)(\partial\ln R)^2 + \cdots\right], \tag{2}$$

where $\mathcal{R}$ is the 4d curvature scalar. A possible dilaton dependence of the coefficient M^{d-2} has not been made manifest since we assume that the dilaton (as well as other moduli) are stabilized at a high scale.

We now set $M = 1$ and perform a Weyl rescaling

$$g_{\mu\nu} \to R^{-k}g_{\mu\nu}, \tag{3}$$

which takes us to the Einstein frame action

$$S = \int d^4x\,\sqrt{g}\left[\frac{1}{2}\mathcal{R} - \frac{k(k+2)}{4}(\partial\ln R)^2 + \cdots\right]. \tag{4}$$

Note that the reason for our very explicit derivation of this familiar action is the importance of the intermediate form, Eq. (2), for the subsequent discussion of quantum corrections.

We further assume that the effective 4d theory is an $N = 1$ no-scale model [5] where the flat direction R is described by a no-scale field T with $\mathrm{Re}\,T = R^\alpha$. By comparing Eq. (4) with the kinetic term derived from the standard no-scale Kähler potential $K = -3\ln(T+\bar{T})$, we find

$$\alpha = \sqrt{k(k+2)/3}. \tag{5}$$

Although our analysis is general, we will primarily focus on two cases:

- $d = 5$ ($k = 1$) compactifications of minimal 5d supergravity on S^1/Z_2 with supersymmetry broken by the Scherk–Schwarz mechanism [6]. In this case, the no-scale field is given by $T = R + iA_5$, A_5 being the fifth component of the graviphoton. This is in agreement with the value $\alpha = 1$ implied by Eq. (5). The constant superpotential characteristic of the no-scale model is proportional to the Scherk–Schwarz parameter. This simple and familiar example will provide us with a useful consistency check for our results.[1]
- $d = 10$ ($k = 6$) flux compactifications of type IIB string theory, where the internal compact space is a Calabi–Yau orientifold. These indeed result in a no-scale model with no-scale field $T = R^4 + ib$ [2], where $b = \mathrm{Im}\,T$ stems from the dimensional reduction of the RR four form. Note that, again, the relation between $\mathrm{Re}\,T$ and R is correctly given by the exponent of Eq. (5).

Perturbative corrections ΔK to the Kähler potential generically destroy the no-scale structure. After supersymmetry is broken by the addition of a constant superpotential W (which we consider to be a parametrically small effect), this Kähler correction generates a non-trivial potential for the volume modulus. A common approach in field-theoretic model building is to calculate this potential (i.e., the Casimir energy) and, if required, to infer the corresponding Kähler correction (see, e.g., [8] and, in particular, [9]).

Here, we instead consider the Kähler correction directly in the model with $W = 0$, i.e., before SUSY breaking. We identify the structure of ΔK from the

[1] For the relation of 5d Scherk–Schwarz breaking to 4d no-scale models see Ref. [7].

corrections to the kinetic term of the field R and to the 4d Einstein term. These corrections are most easily understood in the 4d action before Weyl rescaling, Eq. (2).

The tree-level action is invariant under shifts in $\operatorname{Im} T$ and we can expect the 1-loop correction to respect this symmetry. The finite corrections, to be added to the action of Eq. (2), then take the form

$$\Delta S = \int d^4x \sqrt{g}\big[F(R)\mathcal{R} + G(R)(\partial R)^2\big], \tag{6}$$

i.e., there is no explicit dependence on $\operatorname{Im}(T)$.

The form of the functions F and G follows from dimensional arguments. At 1-loop, the corrections arise simply from the propagation of d-dimensional free fields in the compact space. Alternatively, one may say that they arise from a summation of a Kaluza–Klein tower of 4d fields with mass splitting $\sim 1/R$. In any case, the d-dimensional Planck mass M does not enter these corrections and the only scale known to these corrections is the compactification radius R. Thus, from the requirement of a dimensionless 4d action, we have $F(R) \sim 1/R^2$ and $G(R) \sim 1/R^4$, i.e., the corrections are

$$R^{-2}\mathcal{R} \quad \text{and} \quad R^{-4}(\partial R)^2. \tag{7}$$

In the above, we have pretended that the field-theoretic one-loop corrections are finite. If they are not, a UV cutoff scale (say the string scale α') enters the result. However, such cutoff-dependent contributions can always be absorbed in a local d-dimensional action (including higher-dimension operators). The leading operators relevant for us are those of Eq. (2) (a d-dimensional cosmological constant is not generated in supersymmetric theories). Subdominant terms may be important. For example, the α' corrections of [10] (see also [11]) considered recently in this context [12] are of this type. Our present result of Eq. (7) is limited to those corrections which cannot be viewed as the dimensional reductions of d-dimensional local operators.

The terms in Eq. (7) correspond to the operators before Weyl rescaling. Going to the Einstein frame,[2] we find that both operators give corrections to the kinetic term

$$\big(R^{-(k+4)} + \cdots\big)(\partial R)^2, \tag{8}$$

where '$\cdots$' stand for terms which are suppressed by inverse powers of R in the limit $R \to \infty$. Rewriting Eq. (8) in terms of T, we see that we need a ΔK which induces a kinetic term

$$(T + \bar{T})^{-\frac{k+2}{\alpha} - 2}\, \partial T \cdot \partial \bar{T}. \tag{9}$$

We conclude that

$$\Delta K \sim \frac{1}{(T + \bar{T})^c} \quad \text{with} \quad c = \frac{k+2}{\alpha} = \sqrt{\frac{3(k+2)}{k}}. \tag{10}$$

We now have all the necessary information to calculate the form of the one-loop potential for R that arises if a non-zero W is included. Using the standard supergravity formula for the scalar potential we find the Einstein-frame result

$$V^{\mathrm{E}}_{\mathrm{Casimir}}(R) \sim |W|^2 (T + \bar{T})^{-(c+3)}. \tag{11}$$

The numerical prefactor, which we have suppressed in the above expression, includes a term $c(c-1)$. This vanishes for $c = 0$ and $c = 1$, i.e., in the two cases where ΔK preserves the no-scale structure (at least in the large-R limit).

Returning to the frame used in Eqs. (2) and (6) (which we will refer to as the Brans–Dicke frame) by undoing the Weyl rescaling Eq. (3), we find

$$V^{\mathrm{BD}}_{\mathrm{Casimir}}(R) \sim |W|^2 R^{-3\alpha + k - 2}. \tag{12}$$

In the example of the 5d compactification on S^1 or S^1/Z_2 with Scherk–Schwarz SUSY breaking, this gives the well-known 1-loop potential $V(R) \sim |W|^2 R^{-4}$. Since the Scherk–Schwarz parameter is dimensionless, this correction has to behave as one would expect in a massless non-SUSY field theory on dimensional grounds. Indeed, the R^{-4} behaviour is the familiar scaling of the Casimir energy, which ensures that the 4d potential has mass dimension 4.

In the case of 10d flux compactifications, which is our primary interest in this Letter, we obtain a correction

$$V^{\mathrm{BD}}_{\mathrm{Casimir}}(R) \sim |W|^2 R^{-8}. \tag{13}$$

This has to be compared with the perturbative string-theoretic (α') corrections [10,12] recently considered

[2] Note that the Weyl rescaling has to be modified in the presence of the first operator in Eq. (7).

in this context, which scale as

$$V_{\alpha'}^{\mathrm{BD}}(R) \sim |W|^2 R^{-6}. \tag{14}$$

Even though our Casimir correction is subdominant, it is clearly less so than non-perturbative corrections to the superpotential, which are expected to be exponentially suppressed at large volume.

Thus, if a (meta-)stable minimum at large volume can be found in the combined potential

$$V^{\mathrm{BD}}(R) \sim |W|^2 \big(c_{\alpha'} R^{-6} + c_{\mathrm{Casimir}} R^{-8}\big), \tag{15}$$

one may hope that this result will survive a more detailed analysis. Naively, such a minimum appears at

$$R_{\min}^2 = \frac{4|c_{\mathrm{Casimir}}|}{3|c_{\alpha'}|}, \tag{16}$$

whenever $c_{\alpha'} < 0$ and $c_{\mathrm{Casimir}} > 0$. However, as long as this value cannot be made parametrically large, there is clearly no reason to neglect higher-order and non-perturbative corrections.

A similar situation has recently been discussed in the 5d field-theoretic context, where the interplay of 1- and 2-loop Casimir energy effects was used to stabilize a 5d model in a controlled way [13] (for earlier related ideas see, e.g., [14]). The key there was the possibility of finding a class of models with a hierarchy between the coefficients of the two leading terms in the $1/R$ expansion.

The obvious parameter that could create such a hierarchy in the present context is the value of α'. To see this in more detail, we write the type IIB supergravity action not in terms of α' and the dilaton, but rather in terms of α' and the 10d Planck mass M. Then the tree-level part depends only on M while the α' correction (and therefore the coefficient $c_{\alpha'}$) involve an explicit factor α'^3. Since c_{Casimir} depends only on the tree-level supergravity action, we conclude that

$$(R_{\min} M)^2 \sim 1/\big(M^6 \alpha'^3\big), \tag{17}$$

which will be large at small α'. Unfortunately, this corresponds to the strong coupling regime of string theory. By the S self duality of the type IIB theory, the small-α' regime has a dual description with Regge slope $\tilde{\alpha}' \sim \alpha'^{-1}$. We expect $\tilde{\alpha}'$ corrections to arise in this theory, implying that the coefficient of the R^{-6} term can never be made small.

If an explicit calculation of c_{Casimir}, along the lines of [15], were available in a sufficiently large class of models (with known $c_{\alpha'}$), one could attempt to isolate geometries where the above minimum occurs accidentally at large R. Work in this direction is under way [16]. In the absence of a detailed study based on such explicit results, we can make the following proposal for how a large value of $R_{\min}$ may arise: Recall that $c_{\alpha'}$ is proportional to the Euler number χ of the underlying manifold. We can now think of a topologically complicated space, where the two Hodge numbers $h^{1,1}$ and $h^{2,1}$ are large while $\chi = 2(h^{1,1} - h^{2,1})$ is small. In this limit one might expect that, because of the large number of light fields (and the presumably large number of corresponding Kaluza–Klein towers), the coefficient c_{Casimir} will be large. Thus, stabilization at large $R_{\min}$ should naturally occur.

To summarize, we have derived the parametrical form of 1-loop supergravity Kähler corrections to the volume modulus of type IIB flux compactifications. We have found the leading finite correction to be of the form $\Delta K \sim 1/(T + \bar{T})^2$ with $\mathrm{Re}\, T \sim R^4 \sim V^{2/3}$. In the presence of a non-zero vacuum value of the superpotential W, this gives rise to a scalar potential of the form $|W|^2/R^8$, which is subdominant relative to the potential contribution $|W|^2/R^6$ induced by α' corrections. We note that our correction, which resembles the Casimir energy effect discussed extensively in field-theoretic models, is dominant for manifold with vanishing Euler number. Furthermore, for specific compact spaces, this Casimir correction may combine with the α' correction to ensure volume stabilization at large R. We expect the Casimir correction discussed in this Letter to be relevant for a wide class of models and stabilization mechanisms.

Note added in proof

After submission of this Letter, a closely related string calculation appeared [17].

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