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# Feature extraction based DCT on dynamic signature verification

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## KEYWORDS

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**Abstract** Signature verification is one of the most accepted biometric techniques, because a signature is a part of everyday life, although less accurate than biometric techniques such as using the iris. In this field, much attention has been paid to features, because a verification system should be able to overcome problems such as forgeries, insensitivity to intra-personal variability and sensitivity to inter-personal variability. In this paper, we present a simple and efficient approach to on-line signature verification, based on a discrete cosine transform, which has been applied to 44 time signals, such as position, velocity, pressure and angle of pen. Experiments are carried out on two benchmark databases, SVC2004 and SUSIG. The forward feature selection algorithm is used to search for the best performing feature subsets. The proposed system is tested with different classifiers, with skilled forgery, and equal error rates were 3.61%, 2.04% and 1.49% for SVC2004 Task1&2, Task2 and SUSIG databases, respectively.

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## 1. Introduction

Biometric verification techniques require a user to present identifying information based on an unchangeable personal feature. This may be a physical characteristic, such as a fingerprint or an iris, or it may be characteristic behavior, such as a signature or voice [1]. Signatures have been considered a typical form of authentication in our society for hundreds of years. Signature verification is the most natural and friendly approach in personal authentication for many biometric-based verification systems.

A signature is a simple, concrete expression of the unique variations in human hand geometry. The way a person signs his or her name is known to be characteristic of that individual. Signatures are learnt and acquired over a period of time rather than being a physiological characteristic, and are influenced by the physical and emotional conditions of a subject.

A signature verification system must be able to detect forgeries, and, at the same time, reduce rejection of genuine

signatures. Analysis of signatures based on the method used to capture the signatures is divided into two main categories; off-line and on-line. In off-line verification, the signature patterns are signed on paper, and then scanned by plate-form scanners. On-line signature patterns possess more information than off-line patterns. There are not only static geometrical shapes but also dynamic writing information, such as speed, acceleration, and pressure, etc. On-line signature verification methods have proved to be more accurate than off-line methods [2].

Signatures are subject to intra-personal variations. Hence, a signature verification system is feasible only if the system is insensitive to intra-personal variability, but sensitive to inter-personal variability [3]. Even when insensitive to intra-personal variations, the system must possess the discriminating power to foil skillful forgers.

Significant research has been conducted in feature extraction and selection for the application of on-line signature verification [4–8]. All these features may be important for some problems, but for a given task, only a small subset of features is relevant. In addition to a reduction in storage requirements and computational cost, these may also lead to an improvement in general performance. On the other hand, selection of a feature subset requires a multicriterion optimization function, e.g. the number of features and accuracy of classification.

Many different on-line signature verification algorithms have been proposed by research groups around the world, and some commercial products are also available. In many publications, signatures are classified with neural networks,

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such as multilayer perceptrons, time-delay neural networks, or self-organizing maps [9–11], the hidden Markov model (HMM) or other models with probabilistic backgrounds [8,12,13], dynamic time warping (DTW) or similar techniques based on dynamic programming [14,15]. Other classifier paradigms, such as the support vector machine [16], fuzzy logic [17], the statistical model [18] and combinations of them [19,20], are investigated.

In this work, we present a simple and effective approach for an on-line signature verification system. First, the Discrete Cosine Transform (DCT) is performed on the time signals of the signature, and then DCT coefficients create a feature vector. The advantage of using the DCT is the ability to compactly represent an on-line signature using a fixed number of coefficients, which leads to fast matching algorithms. More importantly, the fixed-length is better suited, or even necessary, in certain applications related to information theory and biometric systems. Finally, several classifiers are adopted for the classification task.

The remainder of the paper is organized as follows: A brief review of features in the general area of on-line signature verification and related works are given in Section 2. In Section 3, the proposed system is described and followed by preprocessing in Section 4. Section 5 explains the features used in the system proposed. Section 6 reports experimental procedures and the results of the different features' effects on the performance of the signature verification system. Finally, conclusions are drawn in Section 7.

## 2. Previous work on features

Approaches to on-line signature verification are generally classified into two groups; those based on global or parametric-based approaches and those often referred to as local or function-based approaches [5]. In parametric approaches, a set of parameters is selected to describe a signature pattern, and the parameters of the reference and test signatures are compared to decide if the signature is genuine. On the other hand, function-based approaches represent a signature pattern as a function of time, and compare the characteristics of the signatures locally on a point-to-point or segment-to-segment basis [21].

In the parametric approach, signatures can be described in a compact form, so the enrollment data size is typically very small and constant. In the literature, several hundred parameters have been proposed for signature verification. Some are obtained from time signals of the signature, and are specifically devoted to on-line signature verification. The average, the root mean square, and maximum and minimum values are generally derived from the position, displacement, speed and acceleration time functions representative of a signature [22–24]. Other parameters are determined as coefficients obtained from mathematical transforms. Transforms of Fourier [25], discrete cosine [26] and wavelet [19] have been proposed for on-line signature verification. Other typical parameters for on-line signature verification describe the signature apposition process as total signature time duration, pen-down time ratio, and number of pen-lifts (pen-down, pen-up) etc. [23,24].

More importantly, this approach is expected to be more stable against variations in local regions, which are common in signatures. On the other hand, the parametric features are robust to noise. The parametric approach has advantages in terms of algorithmic simplicity, computational speed and storage requirements. However, the main problem in a parametric approach lies in the selection of a subset of features with adequate discriminating power [27]. Hence, for improved performance using a parametric approach, the selection of

personalized features that can overcome the problem of intra-personal and inter-personal variability is a critical factor [28].

The local or functional approach is divided into two categories: local time-based features, which extract features based on the time domain, and local strokes-based features, which extract features based on writing strokes. Typical signature functions include horizontal and vertical components of position, velocity, acceleration, pressure and force, all against time. Velocity is generally considered to be more informative than position and acceleration for dynamic signature verification. Pressure and force functions have also been frequently used, and specific devices have been developed to capture them directly during the signing process. Another way of characterizing a signature is through analysis of the “stroke”, which is, for example, the pen-down, pen-up movement of the pen on the digitizer.

These approaches retain more information of the signing process than the parametric approaches [3]. On the other hand, local features provide rich descriptions of writing shapes, and are powerful for discriminating writers. The main difficulty in this approach is how to reliably find the correspondence between segments.

From a theoretical viewpoint, the personalized feature subset should be constituted in respect of its discrimination power. However, from a practical viewpoint, it is unrealistic to assume the availability of all possible skilful forgeries of signatures to verify the relative discriminating power of a specific feature. In conventional signature verification systems, the personalized feature subset is selected just according to how small the standard deviation of a feature is across a sample of genuine training signatures [24]. This is not reliable, as a feature with a small standard deviation may not necessarily be a good candidate feature. On the contrary, it can even be damaging to verification, as it may be a feature that can be imitated with ease [17].

In signal analysis or the pattern recognition field, it is a common practice to transform the original signal into another form to investigate a certain property more effectively. For instance, the well-known FFT transforms the signal into the frequency domain revealing many useful characteristics, in respect to signal frequency, that were ambiguous in the original signal [29]. Likewise, a proper transform can be an effective tool for the analysis of dynamic characteristics in time series patterns.

There are two algorithms which are used in computing the dissimilarity of two signatures which depend on the types of features used. For global features, a Euclidean distance, for instance, would be used. This is because the number of features extracted is equal. For local features, the commonly used algorithms would be DTW and HMM.

An on-line signature verification system based on local information and a one-class classifier, i.e. the Linear Programming Descriptor classifier (LPD), was presented by Nanni and Lumini [26]. The authors investigated and described how the information was extracted as time functions of various dynamic properties of the signatures, and then the discrete 1-D Wavelet Transform (WT) was performed on these features. The DCT was used to reduce the approximation coefficients vector obtained by WT to a feature vector of a given dimension. Moreover, the LPD classifier is trained using DCT coefficients. The experimental results using all 5000 signatures from 100 subjects of the SUBCORPUS-100 MCYT bimodal biometric database were presented, yielding performance improvement for both random and skilled forgeries, and obtained an Equal Error Rate (EER) of 5.2% in skilled forgeries.

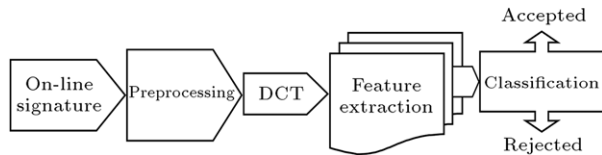


Figure 1: Block diagram of the proposed signature verification system.

Fabregas and Faundez-Zanuy have presented a new system for on-line signature verification based on DCT feature extraction with discriminability feature selection [30]. They performed a complete set of simulations with the largest available online signature database, MCVT, which consists of 330 people with genuine and skilled forgeries performed by five other different users. The main contribution of this work is the management of FTE situations by means of a new proposal, called intelligent enrolment, which consists of consistency checking in order to automatically reject low quality samples. This strategy enhances the performance of the system to 22%, when 8% of the users are left out. In this situation, 8% of the people cannot be enrolled in the system and must be verified by other biometrics or human abilities. They achieved a 5.26% minimum Detection Cost Function (DCF) for skilled forgeries.

### 3. System overview

Figure 1 shows the main modules of the signature verification system.

The system performance is evaluated using the databases of the SVC2004 [31] and SUSIG [32]. The SVC2004 database provided two different signature databases, namely, Task1 and Task2. Each signature is represented as a sequence of points, which contains the  $X$  and  $Y$  coordinates, the time stamp and pen status (pen-up or pen-down). In Task2, additional information, like azimuth, altitude and pressure, is available. Each database contains of 40 sets of signatures; 20 genuine signatures from one signer and 20 skilled forgeries from at least four other signers.

The SUSIG database consists of two parts, namely, visual and blind sub-corpora. SUSIG consists of the signatures of 110 signers. Visual sub-corpus was collected using an Interlink Electronics' ePad-ink tablet signature tablet, with a built-in LCD screen providing visual feedback. For each subject, there are 20 genuine and 10 forgery signatures. Genuine signatures were collected in two different sessions. The blind sub-corpus was collected using the Wacom Graphire2 pressure sensitive tablet, without visual feedback. For each subject, there are 10 genuine and 10 forged signatures. Genuine signatures were collected in a single session. The signature data consists of  $X$  and  $Y$  coordinates, time stamp, pressure level and a pen-up or -down indicator. In this paper, we use visual sub-corpus.

The altitude is the angle between the pen and the surface. The azimuth denotes the clockwise rotation of the pen around the  $X$  axis. Figure 2 describes the parameters.

In signature verification, forgeries are often classified into the following three types:

1. Random forgery: where the forger has either no knowledge about the original signature and uses his/her own signature instead of the signature supposed to be tested.
2. Simple forgery: where the forger does not make any effort to simulate a genuine signature but has access to the name of the author.
3. Skilled forgery: where the forger can see the genuine signature, has time to practice imitations and tries to simulate a

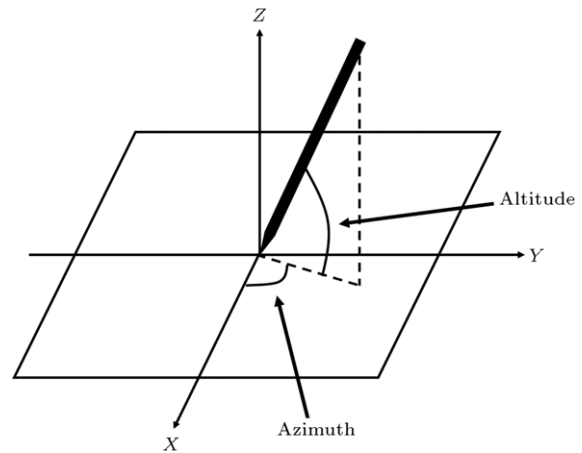


Figure 2: Azimuth and altitude angles of the pen with respect to the plane of the tablet.

genuine signature as closely as possible to the original, although it is not professional.

On the contrary, forgers generally have difficulty in imitating dynamic characteristics at the same time. Thus, dynamics information still preserves its discriminative ability [1]. Signature dynamics is considered a biometric feature for its dynamics information arises from the involuntary behavior of the author. Even in an off-line case where no dynamics information is available, it is known that the key to discriminating between individual signatures includes peculiarities caused by involuntary motion rather than overall shape characteristics [1].

### 4. Preprocessing

Preprocessing of on-line signatures is commonly done to remove variations that are thought to be unrelated to the verification performance. Smoothing and rotation are among the most common preprocessing steps.

Tablets are involved in capturing signatures which may have lower resolution. Extracting local features from jagged signature trajectories, and then using them for verification, may lead to poor system performance. To solve this problem, we employed cubic splines for smoothing purposes, due to their nice mathematical properties. After smoothing the signatures, the isolated strokes in a word are joined together to form one single stroke. The polar coordinates,  $(r, \theta)$ , are used to remove the rotation of the signature. Also, we displaced the origin of coordinates to the first point of the signature.

### 5. Feature extraction

Feature extraction plays a very important role in on-line signature verification. The databases provide discrete time signals, i.e. the horizontal  $x(t)$  and vertical  $y(t)$  positions of the pen also provide the pressure  $p(t)$ , azimuth angle  $az(t)$  and altitude angle  $al(t)$ , of the pen.

Some signals have to be calculated from derivatives of the basic signal. We use the following estimate for the derivative:

$$D(x_i) = \frac{(x_i - x_{i-1}) + (x_{i+1} - x_i)/2}{2}. \quad (1)$$

This estimate is simply the average of the slope of the line through the point in question and its left neighbor, and the slope of the line through the left and right neighbors. Empirically,

Table 1: List of signals used.

Feature description		Feature description	
1	Coordinate $x(t)$	2	Coordinate $y(t)$
3	Absolute position, $r(t) = \sqrt{x^2(t) + y^2(t)}$	4	Velocity in $x$ , $v_x(t)$
5	Velocity in $y$ , $v_y(t)$	6	Absolute velocity, $v(t) = \sqrt{v_x^2(t) + v_y^2(t)}$
7	Velocity of $r(t)$ , $v_r(t)$	8	Angle of velocity vector, $v = \text{tg}^{-1} \frac{v_y(t)}{v_x(t)}$
9	Sinuous of angle, $\sin(\theta_v) = \frac{v_y(t)}{v(t)}$	10	Cosine of angle, $\cos(\theta_v) = \frac{v_x(t)}{v(t)}$
11	Acceleration in $x$ , $a_x(t)$	12	Acceleration in $y$ , $a_y(t)$
13	Absolute acceleration, $a(t) = \sqrt{a_x^2(t) + a_y^2(t)}$	14	Tangential acceleration, $a_t(t) = \dot{v}(t) = \frac{v_x(t)a_x(t) + v_y(t)a_y(t)}{v(t)}$
15	Centripetal acceleration, $a_c(t) = v(t)\dot{\theta}_v(t) = \frac{v_x(t)a_y(t) - v_y(t)a_x(t)}{v(t)}$	16	Acceleration of $r(t)$ , $a_r(t)$
17	Angle of acceleration vector, $\theta_a = \text{tg}^{-1} \frac{a_y(t)}{a_x(t)}$	18	Sinuous of angle, $\sin(\theta_a) = \frac{a_y(t)}{a(t)}$
19	Cosine of angle, $\cos(\theta_a) = \frac{a_x(t)}{a(t)}$	20	Angle of centripetal acceleration vector, $\beta_a = \text{tg}^{-1} \frac{a_y(t)}{a_x(t)}$
21	Sinuous of angle, $\sin(\beta_a) = \frac{a_c(t)}{a(t)}$	22	Cosine of angle, $\cos(\beta_a) = \frac{a_r(t)}{a(t)}$
23	Jerk in $x$ , $j_x(t)$	24	Jerk in $y$ , $j_y(t)$
25	Absolute jerk, $j(t) = \sqrt{j_x^2(t) + j_y^2(t)}$	26	Tangential jerk, $j_t(t) = \dot{a}(t) = \frac{a_x(t)j_x(t) + a_y(t)j_y(t)}{a(t)}$
27	Centripetal jerk, $j_c(t) = a(t)\dot{\theta}_a(t) = \frac{a_x(t)j_y(t) - a_y(t)j_x(t)}{a(t)}$	28	Jerk of $r(t)$ , $j_r(t)$
29	Angle of jerk vector, $\theta_j = \text{tg}^{-1} \frac{j_y(t)}{j_x(t)}$	30	Sinuous of angle, $\sin(\theta_j) = \frac{j_y(t)}{j(t)}$
31	Cosine of angle, $\cos(\theta_j) = \frac{j_x(t)}{j(t)}$	32	Angle of centripetal jerk vector, $\beta_j = \text{tg}^{-1} \frac{j_y(t)}{j_x(t)}$
33	Sinuous of angle, $\sin(\beta_j) = \frac{j_c(t)}{j(t)}$	34	Cosine of angle, $\cos(\beta_j) = \frac{j_r(t)}{j(t)}$
35	Pressure, $p(t)$	36	Velocity of pressure, $v_p(t)$
37	Acceleration of pressure, $a_p(t)$	38	Azimuth angle, $az(t)$
39	Velocity of azimuth angle, $v_{az}(t)$	40	Acceleration of azimuth angle, $a_{az}(t)$
41	Altitude angle, $al(t)$	42	Velocity of altitude angle, $v_{al}(t)$
43	Acceleration of altitude angle, $a_{al}(t)$	44	Curvature, $c(t) \log \frac{v_x(t)a_y(t)v_y(t)a_x(t)}{v^3(t)}$

this estimate is more robust to outliers than any estimate considering only two data points.

We use 44 signals for classification and evaluation of the signature verification system. The complete set of signals is given in Table 1. After all those 44 signals have been made up, the DCT of each signal was calculated for feature extraction.

### 5.1. Discrete cosine transform

In this paper, we use DCT, and, with the transform, the dynamic characteristics of the original signal are reflected to the transformed patterns. This transform is applied to an on-line signature verification system to evaluate the effectiveness of the approach.

The most common DCT definition of a 1-D sequence of length  $N$  is:

$$X(k) = \alpha(k) \sum_{n=0}^{N-1} x(n) \cos \left[ \frac{\pi(2n+1)k}{2N} \right]. \quad (2)$$

For  $k = 0, 1, 2, \dots, N-1$ . Similarly, the inverse transformation is defined as:

$$x(n) = \sum_{k=0}^{N-1} \alpha(k) X(k) \cos \left[ \frac{\pi(2k+1)n}{2N} \right]. \quad (3)$$

For  $n = 0, 1, 2, \dots, N-1$ . In both Eqs. (1) and (2),  $\alpha(k)$  is defined as:

$$\alpha(k) = \begin{cases} \sqrt{\frac{1}{N}} & k = 0 \\ \sqrt{\frac{2}{N}} & k \neq 0. \end{cases} \quad (4)$$

DCT works better than other well-known techniques for dimensionality reduction. This transform has two important properties, i.e. decorrelation and energy compaction. The DCT provides a good compromise between information packing

ability and computational complexity. The principle advantage of transformation is the removal of redundancy between samples. This leads to uncorrelated transform coefficients, which can be encoded independently. The efficiency of a transformation scheme can be directly gauged by its ability to pack input data into as few coefficients as possible. This allows the quantizer to discard coefficients with relatively small amplitudes without introducing visual distortion in the reconstructed image. A sample signature and its signals, with the resultant energy of DCT coefficients, are shown in Figure 3.

An on-line signature must be an adequate match with the reference signatures of the claimed identity in both shape and dynamic properties, in order to be accepted. Scale invariance is more complicated, due to the additional dimension of time. If a signature is only scaled in space, while keeping the signing duration the same, dividing each coefficient's magnitude by  $X(1)$  achieves scale normalization. However, for the more general case involving both scale and time variations (factor  $\alpha(k)$ ), we use a simple approach, namely, the coefficients size is normalized to a standard deviation of one.

## 6. Experiments and results

In order to evaluate the effectiveness of the feature and system, several experiments were carried out. In signature verification systems, a number of reference signatures (3–10 samples) from each signer to be enrolled are collected which are used to measure the variations within his/her signatures. In this paper, five reference signatures have been chosen randomly from genuine signatures. The sets of training data are defined as six genuine signatures for each signer and eight signatures from skilled forgeries (randomly selected for each signer). The remaining signatures are used for verification. In these experiments, the forgery signatures consist of two types; random forgeries and skilled forgeries. Therefore, in the verification phase, one signature of every other signer has been chosen randomly for the random forgery samples.

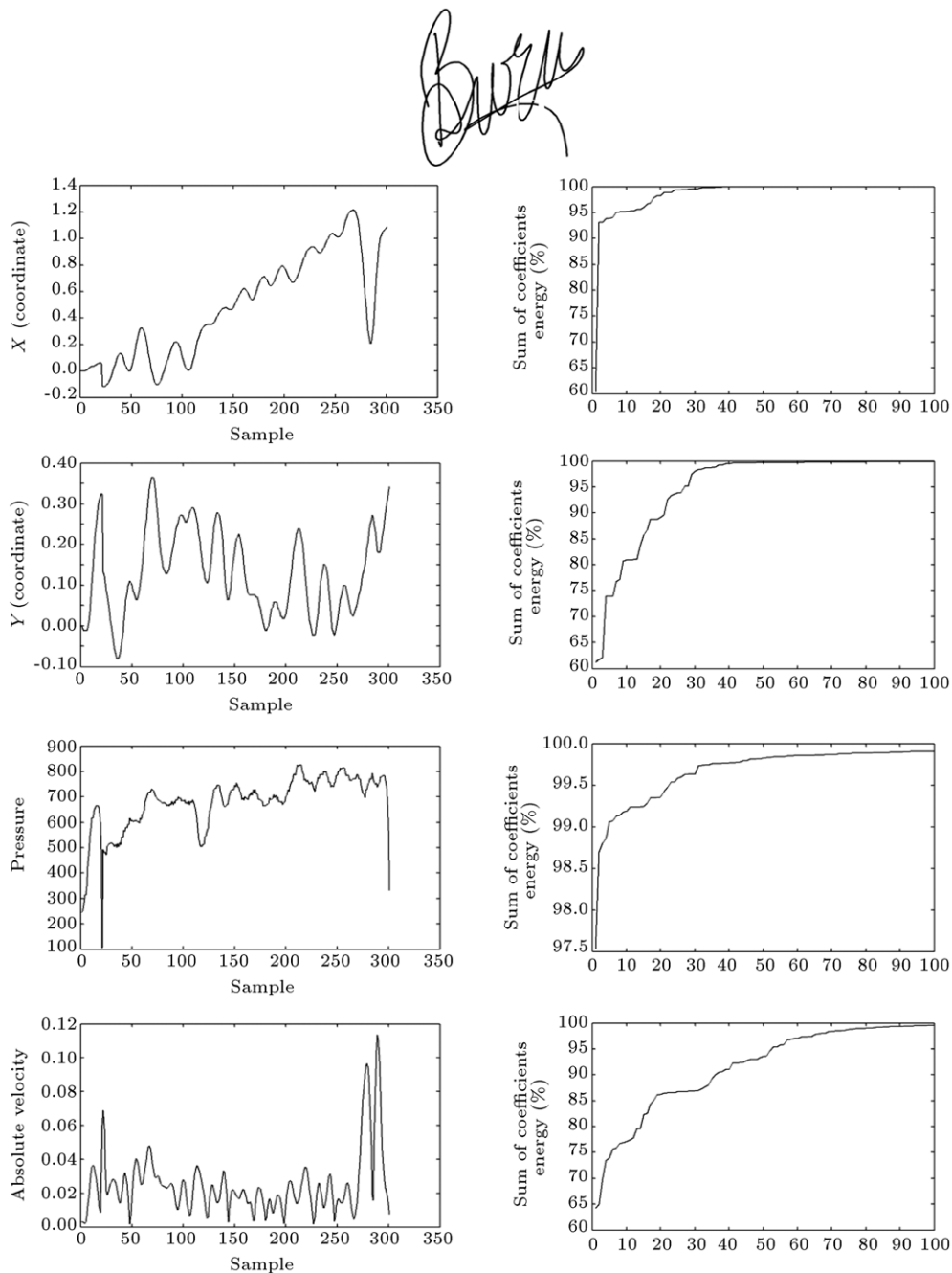


Figure 3: Signature pattern (top). Signals (left) and energy compaction by DCT versus number of coefficients (right).

During the training and verification phase, a subject provides his/her test signature to be compared against the claimed user's reference set signature. In order to match two signatures, the following stages are applied:

1. A basis feature matrix is calculated from the reference signatures of each signer. For this, we calculate the minimum, mean, maximum and sum values for each DCT coefficient in the reference set.
2. The distance between the feature matrix of reference and test signatures computes.
3. The matrix of the feature distance is also normalized. Let the maximum feature distance be  $D_{\max}$  for a feature. Every

feature in the matrix of the feature distance is normalized by Eq. (5) to map to a value between 0 and 1.

$$d_{\text{Normalized}} = e^{-\frac{d}{2D_{\max}}} \quad (5)$$

We utilize all these distances, treating them as features in a two-class classification problem, where the aim is to verify if the signer is the person that he/she claims to be. Finally, we decide the test signature's acceptance with a preset threshold.

We use a Parzen Window Classifier (PWC) for classification of signatures, which makes the recognition performance more stable with respect to the system parameters [33]. The parzen



window density estimate can be used to approximate the probability density of genuine and forgery groups.

To evaluate the experiments, we determined EER and Receiver Operating Characteristic (ROC) curves. The EER is generally adopted as a unique measure for characterizing the performance level of a biometric system and it indicates the provided security level. The EER is the point at which these two error rates, i.e. False Acceptance Rate (FAR) and False Rejection Rate (FRR), cross. FAR represents the probability that a false match occurs, while FRR represents the probability that a false rejection occurs. The ROC curve plots the FAR against the FRR. Basically, ROC depicts the tradeoff between FAR and FRR at various thresholds.

The whole experiment is repeated 20 times to provide better statistical accuracy and then the average values of EER for all 20 trials are calculated. For every trial, the training set is randomly selected.

### 6.1. Features analysis

In order to analyze the discriminative power of features, this experiment was undertaken. Different methods have been used in the literature for analysis of signature features' stability and repeatability [34–37]. Dimauro et al. in their study of local features stability have introduced a warping function that allows  $m$  to  $n$  points to be matched [34]. However, such an approach may not be practical in cases of extreme values of features.

Lei and Govindaraju analyzed the consistency and discriminative power of on-line features using a distance-based measure that is optimized for each feature of study [35]. They view signature verification as a one category classification problem, and use the distances between features to distinguish them rather than the feature values themselves.

Guest, for example, uses the Coefficient Of Variance (COV) and the analysis of variance (ANOVA) on a set of global features of the signature data [36]. However, his approach is not applicable to local features which consist of a time series of data.

In recent studies, the authors compared the capability of several on-line and off-line features in distinguishing between genuine and forged signatures by using two analysis techniques of ANOVA and EER values [37]. This study has shown that basic on-line features, such as velocity, angle along trajectory, pressure and acceleration, are good in discriminating between genuine and skilled forgeries for on-line HMM-based signature verification systems.

In the first series of experiments, each dynamic feature was examined separately and only with skilled forgeries. The process of signature classification was carried out using only one dynamic feature per test. These results are obtained using, first, 10 DCT coefficients per signal.

A comparison of the discriminative ability of each feature can be gauged by observing the EER values in Table 2. This comparison was applied to both normalized and not normalized data. As can be seen from the results, the normalization process significantly improves the classification rate for the SUSIG database, and also for some features seen in the SVC2004 database. The best EER achieved using normalized data was 12.47% for the SVC2004 database, and 3.66% for the SUSIG database. This proves that normalization is not always important and should be incorporated carefully into a signature verification system.

The 10 signals with the lowest error in each group are shown darker. On the basis of the observed verification results

for each signal, it is discovered that position signals ( $X, Y, R$ ) have a low verification error rate in comparison with others. Also, the classification based on the azimuth signal and its derivatives gave better results than the altitude signal. Of course, it should be noted that these results do not indicate the discriminative power of features, and only represents each feature's classification error.

It must be noted that none of the EERs in Table 2 are low enough for real applications because we used only one feature at a time for verification in our experiments. How to combine these features optimally is an open question, and further experiments are necessary to claim the consistency of any given feature.

### 6.2. Feature selection

The analysis of individual features allows prediction of which types of features are likely to be part of an optimal multidimensional feature vector. Nevertheless, the existing relationships or correlation between them may alter this intuitive reasoning, and features that perform well individually may not do so in combination with others. Therefore, we perform a feature selection over the whole set of proposed features to obtain optimal feature subsets.

With a large number of initial features, an exhaustive search of the feature subset space becomes computationally intractable, as an initial set of  $N$  features would result in  $2^N - 1$  possible combinations. Many algorithms exist for reducing this time down to reasonable limits. Therefore, we use feature selection based on the Sequential Forward Feature Selection (SFFS) algorithm, which is applied to random and skilled forgeries. The SFFS algorithm is one of the best performing methods reported [38]. For evaluation of features, the SFFS algorithm computes the sum of estimated Mahalanobis distances. Finally, feature selection is based on selecting an increasing number of ranked features.

We repeated the experiments 20 times with a different number of features on the verification set. First, 10 DCT coefficients are extracted from each signal and each DCT coefficient is supposed to be a feature. The number of features was varied between 10 and 100, in steps of 10. In Figure 4, the evolution of the system EER, according to the size of the optimum feature vector selected by the SFFS algorithm, is depicted. It can be seen that the behavior for different databases is similar in both cases of normalized and not normalized data. Also, the results show that normalization increases the performance of EER for skilled forgery, and decreases the EER for random forgery.

If the evolution of the EER is carefully observed, it can be noticed that plots decrease more steeply until a stable region with almost 60–80 ranked features. Moreover, these results reveal that the verification performance is significantly better for Task2 in SVC2004, as compared with Task1&2. These suggest that pressure, azimuth and altitude information increase the performance of the verification system, and improve stability and the discriminative ability of feature vectors against inter-personal variability.

The best verification phase results are summarized in Table 3, separately, for random and skilled forgeries. As can be seen, for the SUSIG database, random forgery EER performance is lower than skilled forgery tests, but it is not very intuitive. One would expect random forgery results to be much lower, as, after all, these are not even true forgeries but other people's genuine signatures. This is partly due to a significant emphasis on the

Table 2: Average EER (%) versus first 10 DCT coefficients of time signals.

	SVC2004		SUSIG			SVC2004		SUSIG	
	Normalized	Not normalized	Normalized	Not normalized		Normalized	Not normalized	Normalized	Not normalized
$x(t)$	12.47	14.88	4.66	6.34	$j_x(t)$	30.35	29.28	21.76	17.57
$y(t)$	14.37	15.35	5.18	8.87	$j_y(t)$	27.53	24.37	21.44	14.83
$r(t)$	12.60	14.96	4.69	5.78	$j(t)$	20.47	19.26	9.99	8.74
$v_x(t)$	19.84	19.16	12.48	16.11	$j_i(t)$	22.76	20.88	11.94	8.33
$v_y(t)$	18.38	18.35	14.33	14.37	$j_c(t)$	19.98	17.85	20.53	10.15
$v(t)$	17.79	17.73	5.36	11.21	$j_r(t)$	28.51	27.41	23.01	16.65
$v_r(t)$	18.28	16.98	12.29	14.39	$\theta_j$	30.69	28.30	18.98	11.03
$\theta_v$	26.48	25.78	17.23	21.48	$\sin \theta_j$	26.35	26.52	30.82	31.48
$\sin \theta_v$	18.51	18.72	12.66	13.21	$\cos \theta_j$	26.40	26.75	30.50	30.96
$\cos \theta_v$	17.81	18.31	7.89	9.44	$\beta_j$	36.93	34.98	29.74	10.52
$a_x(t)$	21.70	19.44	18.78	11.43	$\sin \beta_j$	21.15	20.62	21.60	22.72
$a_y(t)$	22.07	19.04	20.34	12.03	$\cos \beta_j$	29.98	30.17	32.68	33.42
$a(t)$	19.61	17.68	7.16	7.48	$p(t)$	18.86	15.29	7.26	17.28
$a_c(t)$	21.94	18.51	13.98	6.98	$v_p(t)$	23.67	22.64	33.98	27.71
$a_c(t)$	16.82	18.83	16.74	11.34	$a_p(t)$	37.36	32.57	33.22	28.21
$a_r(t)$	19.19	17.49	20.26	11.42	$az(t)$	18.31	15.47	-	-
$\theta_a$	27.47	26.16	16.45	19.36	$v_{az}(t)$	24.96	22.27	-	-
$\sin \theta_a$	19.28	19.58	18.59	19.93	$a_{az}(t)$	29.16	25.79	-	-
$\cos \theta_a$	21.04	21.83	18.46	18.41	$al(t)$	19.98	20.49	-	-
$\beta_a$	31.84	31.11	26.03	24.16	$v_{al}(t)$	30.08	27.53	-	-
$\sin \beta_a$	17.28	17.84	17.22	18.57	$a_{al}(t)$	31.34	28.52	-	-
$\cos \beta_a$	23.71	24.72	18.67	19.13	$c(t)$	44.12	47.82	41.91	39.79

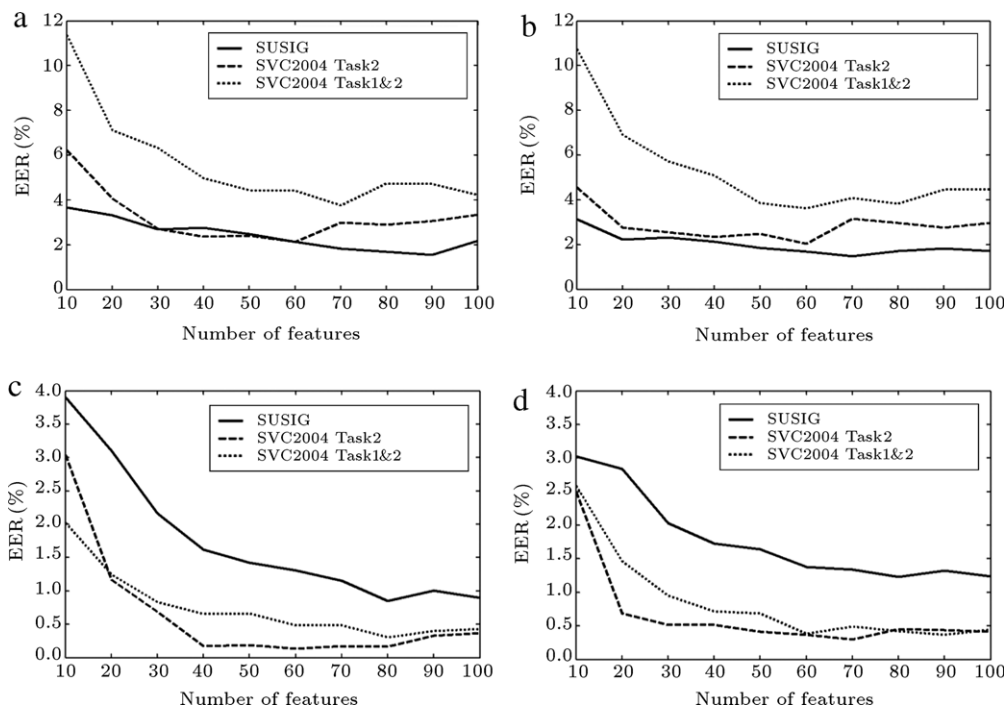


Figure 4: Verification performance in terms of the size of the ranked feature. Left figures for normalized data and right figures for not normalized data. (a) and (b) Skilled forgery. (c) and (d) Random forgery.

correct timing of a signature. Analysis of the random forgery errors has shown that intentional forgeries in the skilled and highly skilled sets are, on average, twice as long in duration compared to genuine signatures.

### 6.3. Comparison with other methods

In this section, we compare the results of the proposed classifier of signature verification with that of other classifiers, i.e. fuzzy  $k$ -nearest neighbor (FKNNC) and support vector machine (SVMC). Parameter  $k$  in FKNNC fixes to five. The kernel

function in SVMC is a linear function that is defined as:

$$K(X_i, X_j) = \text{Sign}(X_i \cdot X_j + 1) \cdot (X_i \cdot X_j + 1), \quad (6)$$

where  $X_i, X_j$  are training samples. The results are shown in Figure 5 for normalized data with 60 ranked features. It is noted from Figure 5 that PWC leads to better EER for skilled and random forgeries in comparison to other classifiers, for all cases. The FKNN classifier is better than the SVM classifier only for SVC2004 Task1&2 databases. With the parzen window classifier and for skilled forgery, we obtained the average of the EER performance of 5.24%, 2.13% and 2.06% for Task1&2, Task2 and SUSIG, respectively.

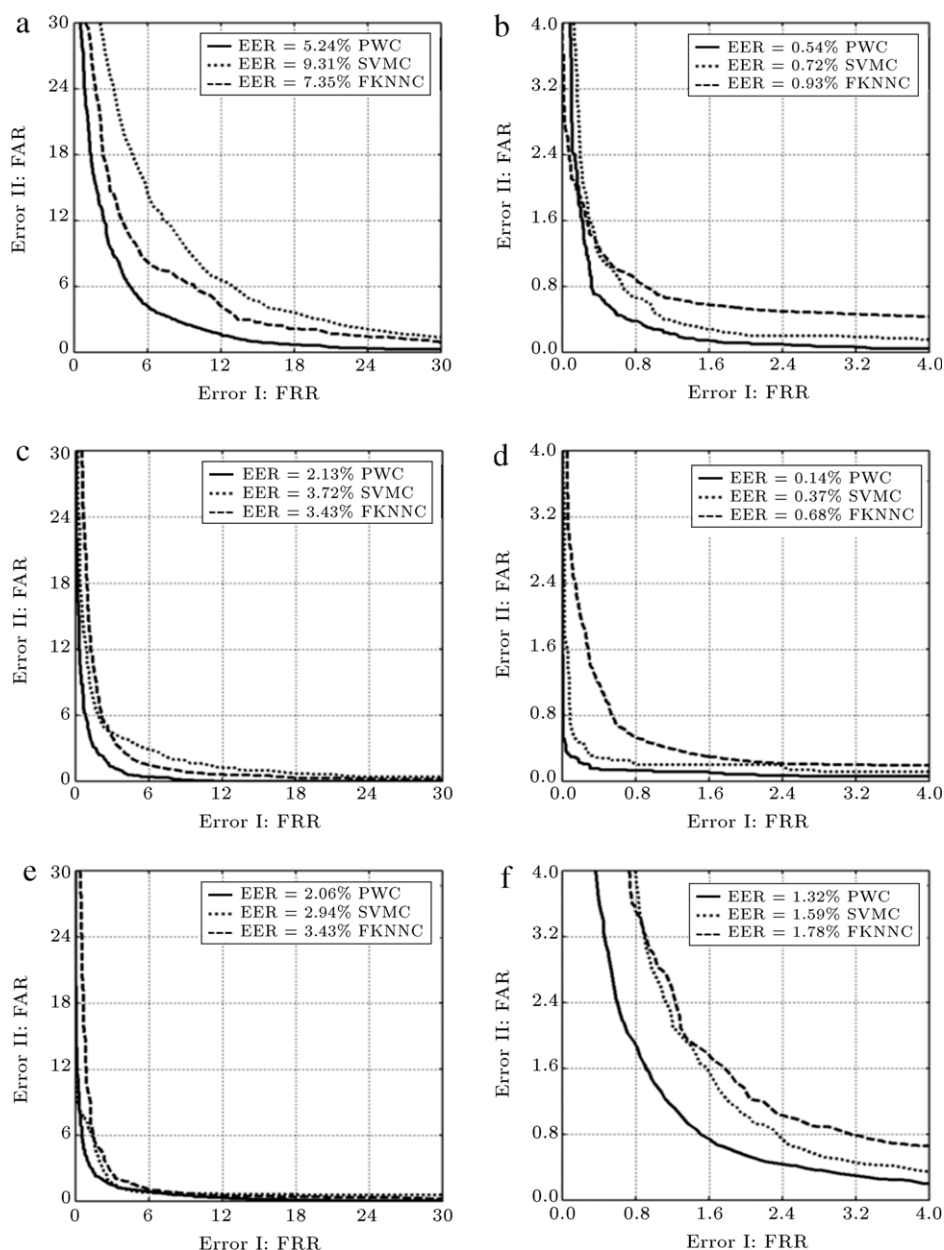


Figure 5: Comparison of average ROC curves with normalized data. Left figures for skilled forgery and right figures for random forgery. (a) and (b) Task1&2. (c) and (d) Task2. (e) and (f) SUSIG.

Table 3: The best verification performance for different databases. Average EER in (%).

	Skilled forgery		Random forgery	
	Normalized	Not normalized	Normalized	Not normalized
<b>Task1&amp;2</b>	3.76	3.61	0.31	0.49
	70 features	60 features	80 features	90 features
<b>Task2</b>	2.13	2.04	0.14	0.37
	60 features	60 features	60 features	60 features
<b>SUSIG</b>	1.54	1.49	0.84	1.23
	90 features	70 features	80 features	80 features

#### 6.4. Comparison with previous works

To illustrate the performance of our proposed method, we compared the results with other approaches. It is difficult to

make a comparison between different signature verification techniques based on different databases. Hence, here, we just compared the performance achieved by some of the suggested signature verification techniques with the same database.

From the results shown in Table 4, it is clear that the proposed technique yields a significant lower EER value than the other signature verification techniques, excluding the best results of the SVC2004 competition. In this competition, we tested more than 15 systems from industry and academia, and found that the best equal error rates are 2.84% and 2.89% for Task1 and Task2, respectively. However, we are sure that the achieved EER value can be further reduced if the set of features included global features as the signature time, the number of pen-ups and so on.

Another advantage of the proposed method by the use of DCT coefficients is the compression of feature data, which



Table 4: The best verification performance for different databases with skilled forgery.

Reference	Database	Method	Feature	EER (%)
SVC2004 competition [39]	SVC2004 Task1&2	DTW	$v_x v_y$	Task1: 2.84 Task2: 2.89
Lei et al. [40]	SVC2004 Task2	DTW+ER2	$x y$	7.2%
Fierrez-Aguilar et al. [41]	SVC2004 Task2	DTW, HMM	All signals and their first order derivatives	DTW: 14.26 HMM: 15.04 Fusion: 10.91
Fierrez-Aguilar et al. [42]	SVC2004 Task2	HMM	$x y t p a z a l$	7.14
Hu and Wang [43]	SVC2004 Task2	Majority classifier	Local: $v_x v_y$ Global: 8 features	Local: 4 Global: 16.38 Fusion: 3.02
Adamski and Saeed [44]	SVC2004 Task2	DTW	$y p a z a l$	7
Yanikoglu and Kholmatov [45]	SUSIG	FFT, DTW	$x y$	$xy$ : 6.20 DTW: 3.30 $xy + DTW$ : 3.03
Khalil el al. [46]	SUSIG	DTW	$v \cos \Theta_v$	3.06
Gruber et al. [47]	SVC2004 Task2	SVM	$x y p a z a l$	SVM-Euclid: 13.84 SVM-DTW: 16.06 SVM-LCSS: 6.84
Proposed method	SVC2004 SUSIG	Parzen window	DCT coefficients	Task1&2: 3.61 Task2: 2.04 SUSIG: 1.49

reduces the elapsed time and storage space for training and verification processes. The time for DTW and HMM methods is very long.

## 7. Conclusion and future work

Although signature verification is not one of the safest biometric solutions, the use of it in business practices is still justified. Moreover, signature verification has a very promising future. One major drawback is that humans are not consistent when signing their signatures. In this paper, a robust signature verification system has been proposed, based on DCT coefficients and the parzen window classifier. Our proposed method can extract basic dynamic features from signature time signals, and compress signature data, while keeping the rough form and basic information of signatures. Especially in the context of skilled forgery, where inter-personal variability in the number of features becomes negligible, an effective analysis of features based on time signals is essential for attainment of a suitable performance.

Ideally we need features that are stable, i.e. do not change very much between different genuine signatures, and which are hard to forge. For attaining this purpose, capturing signals via a tablet digitizer and using the extracted dynamics information has been considered, mainly in the form of simple parameters of DCT coefficients. These features are efficient and experimental results confirm that the proposed method is promising. The summation condition in the verification process guides the system towards an accurate decision.

The extensive experiments conducted show that the proposed method has achieved a considerable reduction in EER. The results show that:

1. The basic on-line features, position signals  $x(t)$ ,  $y(t)$  and  $r(t)$ , are better than other signals in discriminating between genuine signatures and forgeries.
2. On some of the signals, the amplitude normalization of DCT coefficients performed more poorly than for no normalization, for skilled forgeries, probably because it helped to compensate for the forgers' slower movements.

3. Different classifiers will be experimented and compared. Experimental results indicate that the best performance of the proposed method is achieved when the parzen window classifier is applied. In this case, with skilled forgery, the minimum equal error rates are attained as 3.61%, 2.04% and 1.49% for Task1&2, Task2 and SUSIG databases, respectively, which is highly acceptable in signature verification systems.
4. The proposed method is very fast in training, feature extraction, and matching, in comparison with the DTW system.
5. The study also reveals the fact that different databases have contradictory results, probably due to the nationality and language of signers. For some languages, the  $x$ -coordinate typically grows linearly with time, with small oscillations on the linear curve, while the  $y$ -coordinate shows a more oscillatory variation with time.

Future work will focus on having a lower EER by adding more features that are found useful in other studies. Also, we will design a further two stage signature verification system using global and local features.

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