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A Bounded Velocity Postulate and Special Relativity

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Although the theory of special relativity follows in a straightforward manner from the assumption that the speed of light is invariant as measured by observers having uniform velocities with respect to one another [2], it would be interesting to know that the theory also follows from the assumption that there is some number l such that no velocity exceeds l. The article, "Einstein — A Natural Completion of Newton" [1] outlines a candidate for such a derivation based on the following conjecture.

Conjecture. Suppose l is a number and suppose f is a real valued function with domain the set of ordered number pairs (v_a, v_b) with v_a and v_b having opposite algebraic signs. Then the following two statements are equivalent:

1. (a) if $|v_a| < l$ and $|v_b| < l$ then $|v_a - v_b|/f(v_a, v_b) < l$.
(b) $[(v_a - v_b)/f(v_a, v_b)] \to 0$ as $v_a \to 0$ and $v_b \to 0$,
(c) $[|v_a - v_b|/f(v_a, v_b)] \to l$ as $|v_a| \to l$ for each number v_b,
(d) $[|v_a - v_b|/f(v_a, v_b)] \to l$ as $|v_b| \to l$ for each number v_a,
2. $f(v_a, v_b) = 1 - v_a v_b / l^2$.

However, this conjecture is not true. Each function of the following form satisfies statement 1, a, b, c, and d above, but only in case $N = 1$ does statement 2 hold.

$$f(v_a, v_b) = N\left(1 + \frac{|v_a v_b|}{l^2}\right) - (N-1)\left(\frac{|v_a| + |v_b|}{l}\right), \qquad N > 0. \tag{1}$$

Let $|v_a| \equiv l - \epsilon_a$ and $|v_b| \equiv l - \epsilon_b$, with $\epsilon_a > 0$ and $\epsilon_b > 0$ and note that

$$|v_a - v_b|/f(v_a, v_b) = \frac{l^2[l - \epsilon_a + l - \epsilon_b]}{Nl^2 + N(l - \epsilon_a)(l - \epsilon_b) - l(N-1)[l - \epsilon_a + l - \epsilon_b]}$$

$$= \frac{2l^3 - l^2(\epsilon_a + \epsilon_b)}{2l^2 - l(\epsilon_a + \epsilon_b) + N(\epsilon_a \epsilon_b)}. \tag{2}$$

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It is apparant from (1) that statement 1(b) is true. It is apparent from (2) that statement 1(a) is true since

$$l[2l^2 - l(\epsilon_a + \epsilon_b) + N(\epsilon_a \epsilon_b)] > 2l^3 - l^2(\epsilon_a + \epsilon_b).$$

Equation (2) also implies statements 1(c) and 1(d).

The following is a stronger condition than statement 1(b) and would be an appropriate condition to include in statement 1 of the conjecture

1 (b′) $f(v_a, v_b) \to 1$ as $v_a \to 0$ and $v_b \to 0$.

To obtain a function f which satisfies 1(a), (b′), (c), and (d), but which does not satisfy statement 2, the N in (1) could be defined as a function of (v_a, v_b) so that

$$N(v_a, v_b) = \frac{K + |v_a||v_b|}{K} \tag{3}$$

with K a positive number.

References

1. A. Ramakrishnan, Einstein—A natural completion of Newton, *J. Math. Anal. Appl.* **42** (1973), 377–380.
2. M. Steib, Extraction of relativistic concepts, *Amer. J. Phys.* **44** (1976), 60–62.