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Time-like pion electromagnetic form factors in k_T factorization with the next-to-leading-order twist-3 contribution

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ABSTRACT

We calculate the time-like pion electromagnetic form factor in the k_T factorization formalism with the inclusion of the next-to-leading-order (NLO) corrections of the leading-twist and sub-leading-twist contributions. It's found that the total NLO correction can enhance (reduce) the magnitude (strong phase) of the leading order form factor by 20%–30% ($< 15°$) in the considered invariant mass squared $q^2 > 5$ GeV^2, and the NLO twist-3 correction plays the key role to narrow the gap between the pQCD predictions and the measured values for the time-like pion electromagnetic form factor.

1. Introduction

As a very important physical observable which may help us to understand the hadrons' structure and the transition from the perturbative to the non-perturbative region, the pion meson electromagnetic (EM) form factor has been the hot subject of numerous experimental and theoretical investigations for a long time.

During past four decades, the pion EM form factors [1] have been measured frequently by many groups [2–8]. The space-like pion EM form factor was firstly measured by Harvard & Cornell collaboration in the range $0.15 \le Q^2 \le 10$ GeV^2 with the electro-production processes in 1970s [2], and then measured by DESY collaboration at the fixed point ($Q^2 = 0.35, 0.70$ GeV^2) in the similar processes almost at the same time [3]. In the new century, this space-like form factor was measured separately by Jefferson Lab F_π Collaboration in the region $0.60 \le Q^2 \le 1.60$ and at the fixed point $Q^2 = 2.45$ GeV^2 [4]. For the time-like pion EM form factor, Cyclotron Laboratory reported their result at the point $q^2 = 0.176$ GeV^2 in the electro-production process [5], then NOVOSIBIRSK collaboration and ORSAY collaboration measured this form factor independently in the region $0.64 \le q^2 \le 1.40$ GeV^2 [6] and $1.35 \le q^2 \le 2.38$ GeV^2 [7] through the e^+e^- annihilation process respectively. Recently, CLEO Collaboration also reported their precision measurements of this form factor at the relatively large q^2 ($q^2 = 9.6, 13.48$ GeV^2) [8]. A comprehensive summary of experimental measurements for the time-like pion EM form factor can be found in Ref. [9].

On the theory side, pion EM form factors also attracted much attentions. The space-like one was studied at different energy regions in QCD by using the different approaches. For example, it was investigated in high and moderate energy region in Ref. [10] and Refs. [11,12] respectively, while it's asymptotic behavior at the extremely large q^2 was studied in Ref. [13]. In Refs. [14–16], the space-like pion form factor was studied carefully in the perturbative QCD theory, and it was also studied in the sum rules formalism [17]. For the time-like pion EM form factor [18], it's high q^2 behavior was evaluated at $q^2 = M^2_{J/\Psi}$, which was found to be two times larger than the space-like one [19]. The Sudakov effect for the time-like form factor was discussed in Refs. [20,21] and the asymptotic behavior of the integration singularity for the time-like form factor is the same as that for the space-like one. The conformal symmetry was also used to analyze the time-like form factor [22] and it is shown explicitly that the time-like form factor, which was obtained by the analytic continuation of the space-like one, agree well with the dispersion relation. What's more, the light-cone QCD investigation [23,24] confirmed recently that the effects of the power suppressed sub-leading twist' and the genuine soft QCD correction' contributions turn out to be dominant at low- and moderate-energies.

With removing the end-point singularities by the Sudakov factors [25,26], the k_T factorization theorem [27] is successful to deal with the exclusive processes with a large momentum transfer [28]. In the k_T factorization theorem, the space-like pion EM form factor was re-examined with the inclusion of the Sudakov

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suppression [29]. Three-parton contribution to pion EM form factor in k_T factorization was also investigated in Refs. [30] and it's found that such contribution is rather small in size and therefore can be dropped safely.

After completing the NLO calculations for the space-like pion EM form factor at leading twist (twist-2) [31], the authors also studied the NLO twist-2 time-like pion EM form factor [32] and found that the NLO twist-2 correction to the twist-2 leading order (LO) magnitude (strong phase) is lower than 25% (10°) at the large invariant mass squared $q^2 > 30$ GeV^2. In Refs. [33,34], the authors calculated the sub-leading twist's (twist-3) contribution from pion meson distribution amplitudes (DAs) to the exclusive $B \to \pi$ transition form factors and the space-like pion EM form factor, and they found that this power-suppressed contribution is large in the low and moderate q^2 regions. In this paper, therefore, we will evaluate the NLO twist-3 contribution to the time-like pion EM form factor with the calculation for the NLO correction to the space-like pion EM form factor at twist-3 level [34].

This paper is organized as follows. In Section 2, we give the LO analysis for the time-like pion EM form factor. In Section 3, the NLO twist-3 corrections to the time-like form factor will be calculated from the space-like one by analytical continuation. Section 4 contains the numerical analysis of the NLO effects, and the conclusion will also be given in this section.

2. Leading order analysis

In this section we will present the LO factorization formula for the time-like pion EM form factor and evaluate the contributions from the two-parton twist-2 and twist-3 pion meson DAs. The LO quark diagram for the relative time-like and space-like pion EM form factor corresponding to the process $\gamma^\star \to \pi\pi\,(\pi\gamma^\star \to \pi)$ are illustrated in Fig. 1(a) and 1(b), respectively.

One should note that the kinetics for the time-like pion EM form factor are different from the space-like ones, because both two mesons are outgoing in Fig. 1(a), while one meson is incoming and the other is outgoing in Fig. 1(b). In the light-cone coordinates, the momenta p_1 and p_2 in Fig. 1(a) are parameterized as

$$p_1 = (p_1^+, 0, \mathbf{0_T}), \quad p_2 = (0, p_2^-, \mathbf{0_T}); \quad p_1^+ = p_2^- = \frac{Q}{\sqrt{2}}, \tag{1}$$

$$k_1 = (x_1 p_1^+, 0, \mathbf{k}_{1T}), \quad k_2 = (0, x_2 p_2^-, \mathbf{k}_{2T}),$$
$$q^2 = Q^2 = (p_1 + p_2)^2, \tag{2}$$

with q^2 being the invariant mass squared of the intermediate virtual photon, k_1 (k_2) is the momentum carried by the valence quark (anti-quark) of meson M_1 (M_2) with the momentum fraction x_1 (x_2) denoting the strength of the quark (anti-quark) to form the corresponding meson. Then the time-like (space-like) pion EM

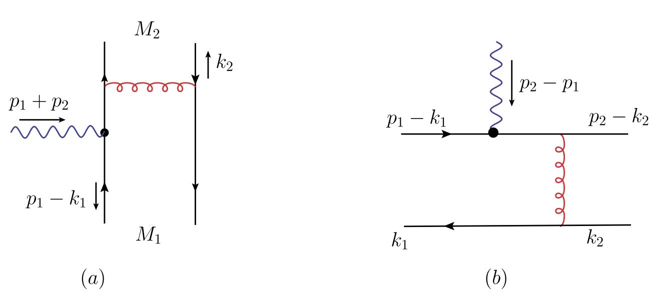

Fig. 1. The LO diagrams for the time-like (a) and space-like (b) pion electromagnetic form factor, with • here representing the electromagnetic vertex.

form factor G_π (F_π) can be specified through the following matrix elements [24]:

$$e(p_1 - p_2)_\mu G_\pi(q^2)$$
$$= < \pi^\pm(p_2)\pi^\mp(p_1)\,|\,J_\mu^{EM}(p_1+p_2)\,|\,0 >, \tag{3}$$
$$e(p_1 + p_2)_\mu F_\pi(Q^2)$$
$$= < \pi^\pm(p_2)\,|\,J_\mu^{EM}(p_1-p_2)\,|\,\pi^\pm(p_1) >, \tag{4}$$

where J_μ^{EM} is the EM current. The space-like momentum transfers in Eq. (4) is $Q^2 = -q^2 = -(p_1 - p_2)^2$, which is different from the time-like one as described in Eq. (2).

From Fig. 1(a) and Fig. 1(b), one can write down the LO time-like and space-like hard kernels

$$H_a^{(0)}(x_i, \mathbf{k_{iT}}, Q^2) = \frac{-ie_q 32\pi\alpha_s C_F N_C Q^2}{(p_2+k_1)^2(k_2+k_1)^2} \cdot \Big\{ x_1 p_{1\mu} \phi^A(x_1)\phi^A(x_2)$$
$$- 2r_0^2\Big[(p_{2\mu} + x_1 p_{1\mu})\phi^P(x_1)\phi^P(x_2)$$
$$- (p_{2\mu} - x_1 p_{1\mu})\phi^T(x_1)\phi^P(x_2)\Big]\Big\}, \tag{5}$$

$$H_b^{(0)}(x_i, \mathbf{k_{iT}}, Q^2) = \frac{ie_q 32\pi\alpha_s C_F N_C Q^2}{(p_2-k_1)^2(k_2-k_1)^2} \cdot \Big\{ x_1 p_{1\mu} \phi^A(x_1)\phi^A(x_2)$$
$$+ 2r_0^2\Big[(p_{2\mu} - x_1 p_{1\mu})\phi^P(x_1)\phi^P(x_2)$$
$$- (p_{2\mu} + x_1 p_{1\mu})\phi^T(x_1)\phi^P(x_2)\Big]\Big\}, \tag{6}$$

where $\phi^A(x_1)$ and $\phi^{P,T}(x_1)$ ($\phi^A(x_2)$ and $\phi^{P,T}(x_2)$) represent the twist-2 and twist-3 pion meson DAs for the corresponding meson with the momentum p_1 (p_2), the chiral parameter is defined as $r_0^2 = m_0^2/Q^2$ with the chiral mass $m_0 = 1.74$ GeV. By comparing Eq. (5) with Eq. (6), we can find that the LO time-like hard kernel has the similar structure with the space-like one, the only difference is the direction of the valence quark momentum k_1, which will flow into the internal propagators. Then we can obtain the LO time-like hard kernel from the space-like one by direct replacement $-\not{k}_1 \to \not{k}_1$ for the internal propagators, which implied that the time-like form factor can also be obtained from the space-like one by analytical continuation from $-Q^2$ to Q^2 in the invariant mass squared q^2 space. This is the basic idea being used to calculate the NLO time-like pion EM form factor in this paper.

The LO time-like and space-like pion EM form factor can be obtained by combining Eqs. (3), (5) and Eqs. (4), (6) respectively, and they are written as,

$$Q^2 G^{(0)}(x_i, Q^2, \mathbf{k}_{iT})$$
$$= \frac{128\pi Q^4 \cdot \alpha_s(\mu)}{(p_2+k_1)^2(k_1+k_2)^2} \int_0^1 dx_1 dx_2 \int_0^\infty \frac{d^2\mathbf{k}_{1T}}{2\pi}\frac{d^2\mathbf{k}_{2T}}{2\pi}$$
$$\cdot \Big\{ -x_1\phi^A(x_1)\phi^A(x_2) + 2r_0^2\Big[(1-x_1)\phi^P(x_1)\phi^P(x_2)$$
$$+ (1+x_1)\phi^T(x_1)\phi^P(x_2)\Big]\Big\}, \tag{7}$$

$$Q^2 F^{(0)}(x_i, Q^2, \mathbf{k}_{iT})$$
$$= \frac{128\pi Q^4 \cdot \alpha_s(\mu)}{(p_2-k_1)^2(k_1-k_2)^2} \int_0^1 dx_1 dx_2 \int_0^\infty \frac{d^2\mathbf{k}_{1T}}{2\pi}\frac{d^2\mathbf{k}_{2T}}{2\pi}$$
$$\cdot \Big\{ x_1\phi^A(x_1)\phi^A(x_2) + 2r_0^2\Big[(1-x_1)\phi^P(x_1)\phi^P(x_2)$$
$$- (1+x_1)\phi^P(x_1)\phi^T(x_2)\Big]\Big\}. \tag{8}$$

The relation $\phi^T(x) = \partial\phi^A(x)/(6\partial_x)$ has been considered in the process to derive out Eq. (7).

For the time-like case, the denominator in Eq. (7) is expanded as,

$$(p_2+k_1)^2(k_1+k_2)^2 = \left(x_1 Q^2 - \mathbf{k}_{1T}^2 + i\epsilon\right)\left(x_1x_2Q^2 - |\mathbf{k}_{1T}+\mathbf{k}_{2T}|^2 + i\epsilon\right), \quad (9)$$

and then the internal gluon/quark may go on mass shell, which will generate an image part in the hard kernel according to the principle-value prescription:

$$\frac{1}{k_T^2-\beta-i\epsilon} = \Pr\left(\frac{1}{k_T^2-\beta}\right) + i\pi\cdot\delta(k_T^2-\beta). \quad (10)$$

But in the space-like case, no image part would appear because the internal gluon/quark can't go on mass shell. The denominator in Eq. (6) is expanded as,

$$(p_2-k_1)^2(k_1-k_2)^2 = (x_1Q^2+\mathbf{k}_{1T}^2)(x_1x_2Q^2+|\mathbf{k}_{1T}+\mathbf{k}_{2T}|^2). \quad (11)$$

Now we consider the end-point behaviors of the LO form factors. We here examine first the end-point behavior in Eqs. (7), (8) by using the asymptotic pion meson DAs [34] in Eqs. (12) for the elaboration.

$$\phi_\pi^A(x) = 6f_\pi x(1-x), \quad \phi_\pi^P(x) = f_\pi, \quad \phi_\pi^T(x) = f_\pi(1-2x). \quad (12)$$

Then the end-point behavior of the integrands in Eqs. (7), (8) can be expressed roughly as

$$Q^2G^{(0)}(x_i, Q^2, \mathbf{k}_{iT}) \propto \frac{-9x_1x_1x_2(1-x_1)(1-x_2)+r_\pi^2(1-x_1-x_1^2)}{(p_2+k_1)^2(k_1+k_2)^2}, \quad (13)$$

$$Q^2F^{(0)}(x_i, Q^2, \mathbf{k}_{iT}) \propto \frac{9x_1x_1x_2(1-x_1)(1-x_2)+r_\pi^2x_1^2}{x_1x_1x_2Q^4}, \quad (14)$$

where the first (second) term in Eqs. (13), (14) describes the contributions from the twist-2 (twist-3) DAs. In the expansions of Eq. (14), the transverse momentum contributions in the internal propagators were absorbed into the effective momentum fraction x_i. From the expressions in Eqs. (7)–(14), one can see the following points:

(i) The twist-2 contribution to the LO pion EM form factor, no matter for the time-like case or the space-like one, has no end-point singularity because of the cancelation of them between the denominator and numerator. The twist-3 contribution, however, will generate the end-point singularities, although they are power-suppressed by r_π^2 in the large momentum transfers region. The LO twist-3 space-like form factor is behaved as $1/x_1$, for example, the twist-3 DAs would give the dominate contribution in the small and intermediate momentum transfers region.

(ii) Since the Sudakov factor from threshold resummation [26] can suppress effectively the end-point singularity from the twist-3 contribution, a rough estimate shows that the major contribution to the LO space-like form factor in Eq. (14) comes from the region of $x_1 \sim 0.1$ and $x_2 \sim 0.5$. Then the twits-2 contribution to the LO space-like form factor will become as large as that from twist-3 contribution at the point $Q^2 \sim 7.4$ GeV^2, which has been confirmed by the numerical result in Ref. [34].

(iii) The second term in Eq. (13) is proportional to $1-x_1-x_1^2$, which is much larger than the second term in Eq. (14) since it is proportioned to $x_1^2 \sim 10^{-2}$. The end-point singularity for the time-like form factor induced by the twist-3 DAs, consequently, is much higher than that for the space-like one. The twist-3 contribution to the time-like form factor is then much larger than the twist-2 contribution in the low and moderate q^2 region. Simple estimation shows that these two kinds of contributions may become similar in size in the high $Q^2 \sim 300$ GeV^2 region.

By making the Fourier transformation for function $Q^2G^{(0)}(x_i, Q^2, \mathbf{k}_{iT})$ in Eq. (7) from the transversal momentum space ($\mathbf{k}_{iT}$) to the conjugate-parameter space ($\mathbf{b}_i$), we obtain the standard double-b convolution LO time-like pion EM form factor [24,32,35]:

$$\begin{aligned} Q^2G_{II}^{(0)} = &\int_0^1 dx_1dx_2\int_0^\infty b_1db_1b_2db_2 128\pi Q^4\cdot\alpha_s(\mu) \\ &\cdot\exp[-S(x_i;b_i;Q;\mu)] \\ &\cdot\{-x_1\phi^A(x_1)\phi^A(x_2) + 2r_0^2\big[(1-x_1)\phi^P(x_1)\phi^P(x_2) \\ &+(1+x_2)\phi^T(x_1)\phi^P(x_2)\big]\cdot S_t(x_i)\}\cdot K_0(i\sqrt{x_1x_2}Qb_2) \\ &\cdot\big[K_0(\sqrt{x_1}Qb_1)I_0(\sqrt{x_1}Qb_2)\theta(b_1-b_2)+(b_1\leftrightarrow b_2)\big], \end{aligned} \quad (15)$$

where the Sudakov exponent $S = S(x_1,b_2;M_B;\mu) + S(x_2,b_2;M_B;\mu)$ is the k_T resummation factor, the Sudakov factor $S_t(x_i) = S_t(x_1)\cdot S_t(x_2)$ refers to the threshold resummation factor, K_0 and I_0 are the Bessel functions:

$$K_0(iz) = \frac{i\pi}{2}H_0^{(1)}(iz); \quad H_0^{(1)}(iz) = H_0^{(1)}(z) = J_0(z)+iN_0(Z); \quad I_0(z) = J_0(z). \quad (16)$$

Since the k_T factorization theorem applies to processes dominated by small x contribution, so the NLO correction to the space-like pion EM form factor [31,34] has been calculated with the hierarchy $x_1Q^2, x_2Q^2 \gg x_1x_2Q^2, \mathbf{k}_T^2$ for convenience. Since there is no end-point singularity for the LO pion form factor from the twist-2 DAs, we can ignore the transverse momenta for the internal quark propagator safely for the twist-2 contribution as elaborated in Ref. [32], then the denominator for the first term in Eq. (7) is reduced to

$$(p_2+k_1)^2(k_1+k_2)^2 = x_1Q^2(x_1x_2Q^2 - |\mathbf{k}_{1T}+\mathbf{k}_{2T}|^2 + i\epsilon). \quad (17)$$

The LO time-like pion EM form factor from the twist-2 DAs can be written in a single-b convolution formula as follows:

$$\begin{aligned} Q^2G_{T2,I}^{(0)} = &\int_0^1 dx_1dx_2\int_0^\infty b_1db_1b_2db_2 128\pi Q^4\cdot\alpha_s(\mu) \\ &\cdot\exp[-S(x_i;b_i;Q;\mu)] \\ &\cdot\Big\{-x_1\phi^A(x_1)\phi^A(x_2)\Big\}\cdot K_0(i\sqrt{x_1x_2}Qb_2). \end{aligned} \quad (18)$$

In Ref. [32], the authors confirmed that the numerical results in the standard double-b convolution as in Eq. (15) are approximately equal to the values of the single-b convolution as shown in Eq. (18), which furthermore showed that the major source of the strong phase is the internal gluon propagator for the twist-2 contribution.

The LO time-like form factor from the twist-3 DAs, however, has a high power end-point singularity, the single-b approximation is therefore not valid for the twist-3 DAs's contribution. So in the next section we have to calculate the NLO twist-3 hard kernel in time-like form factor by using the double-b convolution method.

3. NLO correction to the twist-3 time-like pion EM form factors

The LO analysis in the last section show that the time-like hard kernel can be obtained from the space-like one by the simple space transfer: $-Q^2 \to Q^2$. Because of the Lorentz invariant QCD theory, it's believed that this analytical continuation should be hold at NLO.

In k_T factorization theorem, the NLO hard kernel for pion EM form factor is derived by taking the difference of the NLO ($\mathcal{O}(\alpha_s^2)$) quark diagrams and the convolutions of the LO ($\mathcal{O}(\alpha_s)$) hard kernel with the NLO ($\mathcal{O}(\alpha_s)$) effective diagrams for meson wave functions. For the space-like pion EM form factors as described explicitly in Refs. [31,34], the ultraviolet divergences are just absorbed into the renormalized coupling constant $\alpha_s(\mu)$ with the massless pion meson, the infrared divergences in the soft region are canceled by themselves in the quark diagrams, and the infrared divergences in the collinear region for the quark diagrams can be absorbed into the high order non-perturbative meson wave functions.

For the time-like form factor, the NLO twist-2 hard kernel has been calculated in Ref. [32] and then the only unknown NLO correction at present is the one from the twist-3 DAs. With the NLO twist-3 space-like hard kernels calculated in Ref. [34], we can obtain the NLO twist-3 time-like hard kernel by the analytical continuation $-Q^2 \to Q^2$. For this purpose, we firstly define two types of LO twist-3 time-like hard kernels $H^{(0)}_{T3,1}$ ($H^{(0)}_{T3,2}$) proportioned to the Lorentz structure $p_{1\mu}$ ($p_{2\mu}$) from Eq. (5):

$$H^{(0)}_{T3,1}(x_i, \mathbf{k_{iT}}, Q^2) = \frac{ie_q 32\pi\alpha_s C_F N_C Q^2}{(p_2+k_1)^2(k_2+k_1)^2} \cdot 2r_0^2 x_1 p_{1\mu}\left[\phi^P(x_1)+\phi^T(x_1)\right]\phi^P(x_2), \quad (19)$$

$$H^{(0)}_{T3,2}(x_i, \mathbf{k_{iT}}, Q^2) = \frac{ie_q 32\pi\alpha_s C_F N_C Q^2}{(p_2+k_1)^2(k_2+k_1)^2} \cdot 2r_0^2 p_{2\mu}\left[\phi^P(x_1)-\phi^T(x_1)\right]\phi^P(x_2). \quad (20)$$

By substituting $Q^2+i\epsilon$ for the momentum transfers of the virtual photon, and $x_1x_2Q^2-(\mathbf{k}_{1T}+\mathbf{k}_{1T})^2+i\epsilon\,(x_1Q^2-\mathbf{k}_{1T}^2+i\epsilon)$ for the internal gluon (quark), we can obtain the NLO twist-3 hard kernels for the time-like $\pi^+\pi^-$ production process from the NLO twist-3 space-like one [34]. The NLO twist-3 time-like hard kernels can then be written as the form of

$$\begin{aligned} &H^{(1)}_{T3,1}(x_i, \mathbf{k_{iT}}, Q^2, \mu, \mu_f) \\ &= h_{T3,1}(x_i, \mathbf{k}_{iT}, Q, \mu, \mu_f)\cdot H^{(0)}_{T3,1}(x_i, \mathbf{k_{iT}}, Q^2) \quad (21)\\ &H^{(1)}_{T3,2}(x_i, \mathbf{k_{iT}}, Q^2, \mu, \mu_f) \\ &= h_{T3,2}(x_i, \mathbf{k}_{iT}, Q, \mu, \mu_f)\cdot H^{(0)}_{T3,2}(x_i, \mathbf{k_{iT}}, Q^2). \quad (22)\end{aligned}$$

By setting the renormalized and factorized scales both at the internal hard scale $\mu=\mu_f=t$, and using the follow relations,

$$\begin{aligned} &\ln(-Q^2-i\epsilon) = \ln(Q^2) - i\pi, \\ &\ln(\mathbf{k}_{1T}^2 - x_1Q^2+i\epsilon) = \ln(\mathbf{k}_{1T}^2 - x_1Q^2) + i\pi\Theta(\mathbf{k}_{1T}^2 - x_1Q^2) \\ &\ln(\mathbf{k}_T^2 - x_1x_2Q^2+i\epsilon) \\ &= \ln(\mathbf{k}_T^2 - x_1x_2Q^2) + i\pi\Theta(\mathbf{k}_T^2 - x_1x_2Q^2), \quad (23)\end{aligned}$$

the relevant correction functions $h_{T3,1}$, $h_{T3,2}$ in Eqs. (21), (22) can be written as,

$$\begin{aligned} &h_{T3,1}(x_i, \mathbf{k}_{iT}, Q, t) \\ &= \frac{\alpha_s C_F}{4\pi}\left[\frac{9}{4}\ln\left(\frac{t^2}{Q^2}\right) - \frac{53}{16}\ln\delta'_{12} - \frac{23}{16}\ln x'_1 - \frac{1}{8}\ln^2 x_2 \right.\\ &\left. - \frac{9}{8}\ln x_2 - \frac{137\pi^2}{96} + \frac{337}{64} + i\pi\frac{5}{2}\right], \quad (24)\\ &h_{T3,2}(x_i, \mathbf{k}_{iT}, Q, t) \\ &= \frac{\alpha_s C_F}{4\pi}\left[\frac{9}{4}\ln\left(\frac{t^2}{Q^2}\right) - 4\ln\delta'_{12} - \frac{1}{2}\ln^2 x'_1 + 2\ln x_2 \right.\\ &\left. - \frac{15\pi^2}{24} + \frac{\ln 2}{4} + \frac{11}{2} + i\pi\left(\frac{7}{4}+\ln x'_1\right)\right], \quad (25)\end{aligned}$$

where $\ln\delta'_{12} \equiv \ln((\mathbf{k}_{1T}+\mathbf{k}_{2T})^2 - x_1x_2Q^2+i\epsilon) - \ln Q^2$ and $\ln x'_1 \equiv \ln(\mathbf{k}_{1T}^2 - x_1Q^2+i\epsilon)$.

We can then obtain the NLO twist-3 time-like correction functions $h_{T3,1}$, $h_{T3,2}$ in the parameter space $\mathbf{b}_i$ by the Fourier transformation from the transverse momentum space $\mathbf{k}_{iT}$ to $\mathbf{b}_i$ space. The correction functions in $\mathbf{b}$ space takes the form of

$$\begin{aligned} &h_{T3,1}(x_i, b_i, Q, t) \\ &= \frac{\alpha_s C_F}{4\pi}\left[\frac{9}{4}\ln\left(\frac{t^2}{Q^2}\right) - \frac{53}{32}\ln\left(\frac{4x_1x_2}{Q^2b_2^2}\right) - \frac{23}{32}\ln\left(\frac{4x_1}{Q^2b_1^2}\right)\right.\\ &\left. - \frac{1}{8}\ln^2 x_2 - \frac{9}{8}\ln x_2 - \frac{137\pi^2}{96} + \frac{19}{4}\gamma_E + \frac{337}{64} + i\pi\frac{39}{8}\right], \quad (26)\\ &h_{T3,2}(x_i, b_i, Q, t) \\ &= \frac{\alpha_s C_F}{4\pi}\left[\frac{9}{4}\ln\left(\frac{t^2}{Q^2}\right) - 2\ln\left(\frac{4x_1x_2}{Q^2b_2^2}\right) - \frac{1}{8}\ln^2\left(\frac{4x_1}{Q^2b_1^2}\right)\right.\\ &+ \left(\frac{\gamma_E}{2} + \frac{3}{4}i\pi\right)\ln\left(\frac{4x_1}{Q^2b_1^2}\right) + 2\ln x_2 - \frac{\pi^2}{4} - \frac{\gamma_E^2}{2} + 4\gamma_E \\ &\left. + \frac{\ln 2}{4} + \frac{11}{2} + i\pi\left(\frac{15}{4} - \frac{3}{2}\gamma_E\right)\right], \quad (27)\end{aligned}$$

where γ_E is the Euler constant.

With the NLO twist-3 correction function in Eqs. (26), (27) and the NLO twist-2 correction function in Ref. [32], we can obtain the NLO time-like pion EM form factor in k_T factorization formula as the form of

$$\begin{aligned} Q^2G^{(1)}_{\rm II} &= 128\pi Q^4\cdot\alpha_s(\mu)\cdot\int_0^1 dx_1dx_2\int_0^\infty b_1db_1b_2db_2 \\ &\cdot\exp[-S(x_i;b_i;Q;\mu)]\cdot\Big\{-x_1\phi^A(x_1)\phi^A(x_2)\cdot h_{T2} \\ &+2r_0^2\Big[\Big(\phi^P(x_1)+\phi^T(x_1)\Big)\phi^P(x_2)\cdot h_{T3,2} \\ &+x_1\Big(\phi^T(x_1)-\phi^P(x_1)\Big)\phi^P(x_2)\cdot h_{T3,1}\Big]\cdot S_t(x_i)\Big\} \\ &\cdot K_0(i\sqrt{x_1x_2}Qb_2) \\ &\cdot\left[K_0(\sqrt{x_1}Qb_1)I_0(\sqrt{x_1}Qb_2)\theta(b_1-b_2)+(b_1\leftrightarrow b_2)\right], \quad (28)\end{aligned}$$

where the NLO twist-2 correction function h_{T2} derived from sing-b formula is expressed as the following form [32],

$$
\begin{aligned}
h_{T2}(x_i, b_2, Q, t) = \frac{\alpha_s C_F}{4\pi} \Big\{ & -\frac{3}{4}\ln\left(\frac{t^2}{Q^2}\right) - \frac{1}{4}\ln^2\left(\frac{4x_1x_2}{Q^2 b_2^2}\right) \\
& - \frac{17}{4}\ln^2 x_1 + \frac{27}{8}\ln x_1 \ln x_2 \\
& + \left(\frac{17}{8}\ln x_1 + \frac{23}{16} + \gamma_E + i\frac{\pi}{2}\right)\ln\left(\frac{4x_1x_2}{Q^2 b_2^2}\right) \\
& - \left(\frac{13}{8} + \frac{17\gamma_E}{4} - i\frac{17\pi}{8}\right)\ln x_1 + \frac{31}{16}\ln x_2 \\
& - \frac{\pi^2}{2} + (1-2\gamma_E)\pi + \frac{\ln 2}{2} + \frac{53}{4} - \frac{23\gamma_E}{8} \\
& - \gamma_E^2 + i\pi\left(\frac{171}{16} + \gamma_E\right) \Big\}. \qquad (29)
\end{aligned}
$$

4. Numerical results and discussions

In this section we present the numerical results for the time-like pion EM form factor induced by the distribution amplitudes with different twists at LO and NLO level. Non-asymptotic pion meson DAs as given in Eq. (30) with the inclusion of the high order effects are adopted in our numerical calculation.

$$
\phi_\pi^A(x) = \frac{3 f_\pi}{\sqrt{6}} x(1-x)\left[1 + a_2^\pi C_2^{\frac{3}{2}}(u) + a_4^\pi C_4^{\frac{3}{2}}(u)\right],
$$

$$
\begin{aligned}
\phi_\pi^P(x) &= \frac{f_\pi}{2\sqrt{6}}\Big[1 + \left(30\eta_3 - \frac{5}{2}\rho_\pi^2\right)C_2^{\frac{1}{2}}(u) \\
&\quad - 3\left(\eta_3\omega_3 + \frac{9}{20}\rho_\pi^2\left(1 + 6a_2^\pi\right)\right)C_4^{\frac{1}{2}}(u)\Big], \\
\phi_\pi^T(x) &= \frac{f_\pi}{2\sqrt{6}}(1-2x)\Big[1 + 6\left(5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho_\pi^2 - \frac{3}{5}\rho_\pi^2 a_2^\pi\right) \\
&\quad \cdot \left(1 - 10x + 10x^2\right)\Big], \qquad (30)
\end{aligned}
$$

where the Gegenbauer moments a_i^π, the parameters η_3, ω_3 and ρ_π are adopted from Refs. [36–39]:

$$
\begin{aligned}
& a_2^\pi = 0.25, \quad a_4^\pi = -0.015, \quad \rho_\pi = m_\pi/m_0, \\
& \eta_3 = 0.015, \quad \omega_3 = -3.0, \qquad (31)
\end{aligned}
$$

with $f_\pi = 0.13$ GeV, $m_\pi = 0.13$ GeV, $m_0 = 1.74$ GeV.

The LO and NLO pQCD predictions for the magnitude and strong phase of the time-like pion EM form factor from twist-2 and twist-3 DAs are illustrated in Fig. 2 and Fig. 3 respectively. By summing up the different twists' contributions, the total pQCD prediction for these physical quantities are shown in Fig. 4. From Figs. 2, 3 and 4, one can see the following points:

(1) For the LO form factor induced by the twist-2 DAs, the single-b convolution formula is a good approximation for the region of $q^2 > 30$ GeV^2 because the single-b convolution result is close

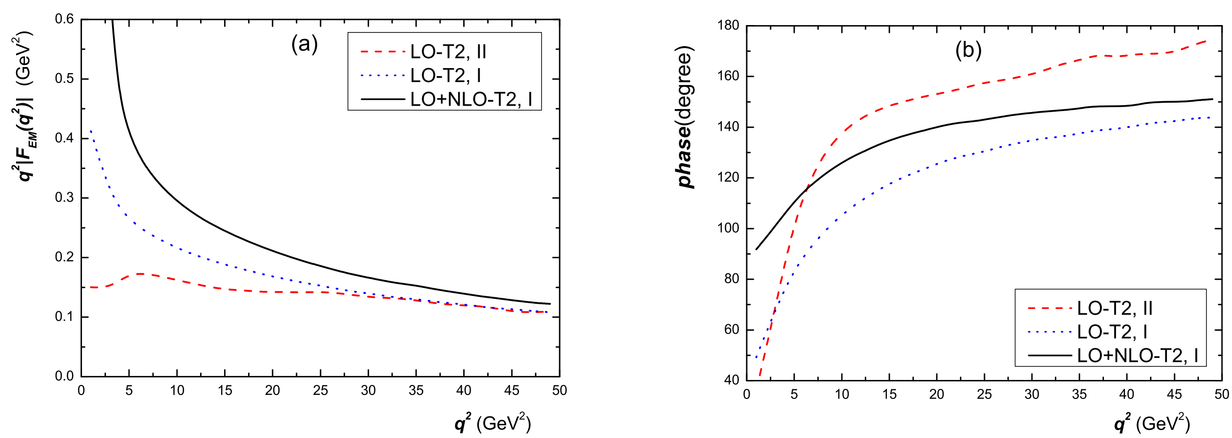

Fig. 2. The pQCD predictions for the magnitude and strong phase of the time-like pion EM form factors induced by the twist-2 DAs ϕ^A. The Rome symbol "II" ("I") refers to the form factors calculated in double-b (single-b) convolution formula as described in Eq. (15) (Eq. (18)).

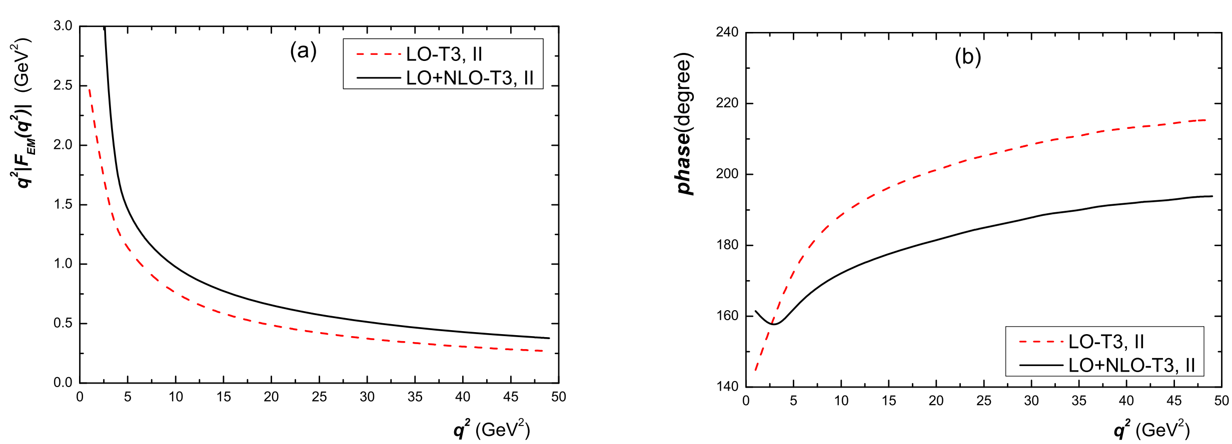

Fig. 3. The pQCD predictions for the magnitude and strong phase of the time-like pion EM form factors induced by the twist-3 DAs $\phi^{P,T}$. The Rome symbol "II" refers to the form factors calculated in double-b convolution formula as described in Eq. (15).

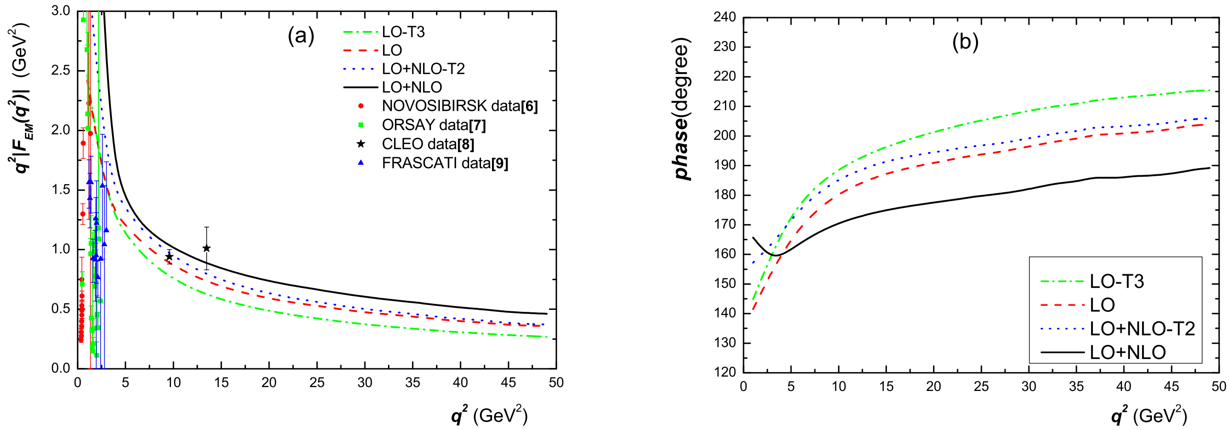

Fig. 4. The pQCD predictions for the magnitude and strong phase of the time-like pion EM form factor as described in Eqs. (15), (28) at the LO and NLO level. As a comparison, those currently available measured values [6–9] for fixed q^2 are also shown in Fig. 4(a).

to the standard double-b convolution result in this q^2 region. Of course, this approximation can be understood by the fact that the internal gluon propagator carry almost all the strong phase for this twist-2 case with no end-point singularity.

(2) The NLO twist-2 correction to the magnitude (strong phase) of the LO twist-2's contribution is smaller than 25% ($10°$) in the region of $q^2 > 30$ GeV^2. The NLO twist-3 correction to the magnitude (strong phase) of the LO twist-3's contribution is smaller than 35% ($20°$) in the region of $q^2 > 5$ GeV^2.

(3) At the LO level, because of the high power singularity, the twist-3 contribution is much larger than the twist-2 part in our considered region of $1 < q^2 < 49$ GeV^2. So the obvious NLO twist-3 correction can enhance the LO pQCD prediction and therefore improve the agreement with the data, especially in the region of $q^2 > 5$ GeV^2. The NLO correction with the inclusion of both twist-2 and twist-3 contributions can enhance (reduce) the magnitude (strong phase) of the LO one by 20%–30% ($< 15°$) in the region of $q^2 > 5$ GeV^2. The NLO pQCD prediction for time-like form factor therefore become well consistent with the CLEO data in the region of $5 < q^2 < 15$ GeV^2, as shown explicitly by the solid curve in Fig. 4(a).

(4) We also consider the second moment of the pion DAs with the recent NLO pQCD fitting [40] ($a_2 = 0.005$) and the precise lattice calculation (say $a_2 = 0.136$) [41] in our pQCD calculation, and find that usage of these new second moments don't leads to any large modifications to the pQCD predictions.

Our numerical result at LO is a little smaller than the one in Ref. [32], since we here used the different input DAs and the different choice of the QCD scale λ_{QCD}. In Ref. [32], λ_{QCD} is chosen at the fixed value 0.2 GeV. In this paper, however, the QCD scale is varying in the transition process according to the internal hard scale, and λ_{QCD} is numerically around 0.25 GeV here.

In this paper, we firstly gave a brief review for the LO time-like and space-like pion EM form factor evaluated in the k_T factorization theorem, and then calculated the NLO twist-3 correction to the LO time-like pion EM form factor by making the analytic continuation of the NLO twist-3 space-like correction for the corresponding space-like form factor. And finally we made the numerical calculations for the time-like pion EM form factor with the inclusion of the NLO twist-2 and twist-3 corrections.

From the analytical analysis and the numerical results for the LO and NLO pQCD predictions for the time-like pion EM form factor, we found that:

(i) The LO twist-3 contribution is much larger than the twist-2 contribution since the high power end-point singularity.

(ii) The NLO twist-2 correction to the LO twist-2 contribution for the magnitude (phase) is less than 25% ($10°$) in the region of $q^2 > 30$ GeV^2. The NLO twist-3 correction to the LO twist-3 contribution for the magnitude (phase) of the LO form factor is less than 35% ($10°$) in the region of $q^2 > 5$ GeV^2.

(iii) The total NLO correction with the inclusion of both the twist-2 and twist-3 contributions can enhance (reduce) the magnitude (phase) of the LO form factor by 20%–30% ($< 15°$) in the region of $q^2 > 5$ GeV^2, and consequently the NLO pQCD prediction for the pion EM form factor under consideration become well consistent with the CLEO data.

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