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Error Propagation and Uncertainty Evaluation for Automatic Control-A Neglected part of Engineering Education

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Abstract

Advancement in science and technology excels control system into a sophisticated level of automation; success of which is hidden in precise measurement. Accuracy of a system is measured in terms of performance uncertainty, the study of which is almost disregarded in engineering and science education though the knowledge of uncertainty measurement is essential to get a high-quality control in automation. Guide to the Expression of Uncertainty in Measurement (GUM) which is recognized universally, provides guidelines for uncertainty measurement in detail. Because of underpinned mathematical expressions of GUM, it is difficult to understand and follow the same without having long term involvement in the similar field. This paper is an effort to explain uncertainty in comparatively simple way to give an idea about the error propagation in a measurement and the calculation of associated uncertainty.

Keywords: uncertainty; automation; measurement; GUM; type B.

1. INTRODUCTION

From household comfort to international security – automation is everywhere. To reduce human labor, assist mankind, increase accuracy automation is a perfect solution. Statistical model, algorithm, artificial intelligence etc are used to implement automation. Among these all statistical model for accurate measurement is the base of modern age automation industries. Work starts form measuring some values and moves forward with numbers of calculation of those values to evaluate the final value. And the final value determines the controlling factor for the system or device. The measure of initial values thus requires high accuracy. To validate the reading obtained from any calculation or measurement, an accurate measurement result traceable to world standard is required; so that it can be accepted globally. Accuracy of measurement can not be 100% due to unavoidable internal and environmental effects. So, measurement result is specified with %of uncertainty when high accuracy measurements are required. Measurement result without uncertainty is incomplete and scientifically wrong. This uncertainty value basically indicates a range in which the actual value must lay. Uncertainty introduced because of variation

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in parameters during measurement. An understanding of the measurement setup along with strong mathematical knowledge is must but not enough for accuracy evaluation. One must be able to extract those process variables which are effecting the parameter under measurement and the person should have a clear idea about their flow through the setup.

Uncertainty measurement is thus a very basic but highly important feature of scientific research work. Unfortunately, in science and technical courses uncertainty or accuracy measurement is completely neglected even in graduate and undergraduate level. Only in few departments some basic definitions are taught. Even error is described in those syllabuses considering the old conceptions; dividing in two groups- systematic errors and random errors. Recent trend in measurement technique follows a completely different way to estimate errors. The standard is defined in Guide to the Expression of Uncertainty in Measurement (GUM) discussed in detail later in this paper.

Improvement in automation technology is the blessing of high accuracy low rage measurement. Length started from millimeter goes down to micrometer and now-a-days it comes down to nanometer scale. Lower dimension measurements require more careful evolution of uncertainty. The lower the dimension means the tougher the measurement and thus only clear understanding of uncertainty results in high precision measurements. At the moment if we consider the world's scenario, people are mostly working in interdisciplinary fields with micro or nanoscale value. Numbers of research centers all over the world are involved in doing different major interdisciplinary projects. They are publishing their works in different journals, getting patents and depending on published works advanced works are planned. Without having a standard representation no measurement result can be accepted by others. Till now many scientists and researchers depend on others to validate their result because of lack of measurement knowledge. It delays the work and increases the cost. Thus person having knowledge of measurement with sufficiently good accuracy is in high demand. Young brain should be trained properly to fulfill the requirement.

2. BACKGROUND

Previously there was no standard procedure defined to follow to do measurement, calculate uncertainty and to report it. Errors for each measurement were then specified in two different terms- systematic and random error. Laboratories in different part of the world had their own defined methodologies of measurement and with measurement result they attached the detail of it. To compare the result of two different laboratories the measurement methodologies need to be compared first. It made the system clumsy, complex and time consuming.

In 1993, an international standard is first proposed to calculate and report uncertainty to overcome the inconveniences associated with it. The International Organization for Standardization together with six international organizations[a] which are mainly working on measurement and standard, published GUM in 1993 and revised in 1995. This GUM is now recognized and followed by all international organizations.

To realize GUM theory, as uncertainty is explained in it completely mathematically, a good understanding of mathematics, statistics and engineering is required. To implement the knowledge to calculate uncertainty during research, indentifying all the sources of errors and classifying them is the first requirement and most tough job. Continuous involvement in this field with higher level of research work helps one to work with it. A comparative simplified document on uncertainty measurement following GUM is published by National Measurement Laboratory (part of NMI).

The new international standard has no concept of random or systematic error. Here all error sources are classified into two categories type A and type B[1]. These types of uncertainty not at all resemble those conventional classifications.

3. CALCULATION PROCEDURE

To do uncertainty measurement, first step is to form a mathematical model of the setup; because mathematical model offers a virtual visibility to the flow of process value through different process parameters. For a large setup this model becomes complex and hence then that model is split down for simplicity. Parameters contributing in measurement uncertainty are indentified from the equations of mathematical model of setup. The nature of contribution of different parameters and the dominating factors are also realized. In a system all entity may not have significant contribution to the final uncertainty. The effects due to those components can be neglected.
If the final reading of uncertainty goes outside acceptability limit, the dominant uncertainty contributing components are then examined and modification in setup is done accordingly to reduce uncertainty in measurement. Mathematical model of a physical setup thus smoothens the calculation procedure, reduces time consumption and labor cost.

3.1. Type A uncertainty

Statistical analysis of series of observations is used to evaluate type A uncertainty. From a large number of readings, taken repeatedly under same conditions, standard deviation of the measurement is calculated. Standard deviation offers an idea about the deviation of measured value from expected value. Mathematically the steps to calculate standard deviation and type A uncertainty are described as follows:

1. Measurement mean

$$x_m = \frac{1}{n}\sum_{k=1}^{n} x_k \qquad (1)$$

Where x_m denotes output whose value is estimated from n independent observations x_k

2. Standard deviation:

$$\sigma = \sqrt{\frac{\sum_{k=1}^{n}(x_k - x_m)^2}{n-1}} \qquad (2)$$

3. Type A uncertainty

$$u_A = \frac{\sigma}{\sqrt{n}} \qquad (3)$$

Type A estimate is reliable only when 'n' is sufficiently large.

3.2. Type B Uncertainty

Type B uncertainty occurs due to the presence of different random as well as systematic error contributing factors. Sources of uncertainty that are local to the measurement process but not satisfactorily allowing a statistical analysis require type B evaluations. So the calculation of type B uncertainty components is not based on a statistical analysis. The value of type B uncertainty comes from a higher-level calibration laboratory or process, and its value is usually reported as expanded uncertainty, U. In the evolution of type B it is considered that random errors cannot be corrected and systematic error can, theoretically at least, be corrected or eliminated from the result. The uncertainty of each contributing element is here considered separately to calculate overall of total uncertainty which in turn generates U and is reported with measurement value.

3.3. Degree of Freedom (DOF)

Uncertainty of measurement can be presented using graph like histogram[2]. Large number of measurement data is taken to obtain that graphical shape. The independent x-axis presents number of standard deviation and y-axis represents probability density with respect to x-axis. The final curve is called uncertainty distribution of measurement. It can be of any shape. This curve provides information about uncertainty contribution of the component.

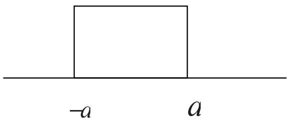

Figure 1: rectangular distribution of uncertainty

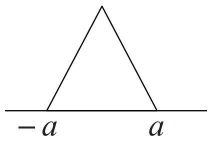

Figure2: triangular distribution of uncertainty

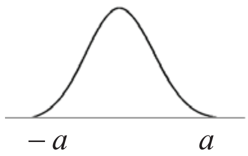

Figure 3: Gaussian distribution of uncertainty

Most common shapes are rectangular (uniform), triangular and Gaussian (normal). The contribution due to rectangular shape is $\frac{a}{\sqrt{3}}$ when $\pm a$ the limit of shape is shown in figure1. Similarly for triangular distribution it is $\frac{a}{\sqrt{6}}$ and for Gaussian it is $\frac{a}{3}$.

The shape of uncertainty distribution is affected by the number of measurements taken to get the shape. The lower the number of measurement less is the knowledge of distribution; the higher the number, the more the information about distribution population. This number of measurement is reflected in degree of freedom (DOF), k which is numerical value without unit. In case of type A uncertainty calculation, mathematically DOF is represented as follows

$$v = N - M$$

Where, N is number of measurements and M is number of quantities calculated.

Say, for an example, to measure the length of a rod data are taken 8 times. Then the average value of these is calculated to get the uncertainty. So number of calculated value used for uncertainty evaluation is 1.

Therefore, DOF $v = 8 - 1 = 7$

For type B uncertainty, value of DOF is considered as infinity when upper and lower limits for a distribution curve are known. But if these values include any uncertainty then a less number of DOF is assigned. Then

$$v \approx \frac{1}{2}\left(\frac{\partial u}{u}\right)^{-2} \qquad (4)$$

where $\frac{\partial u}{u}$ is the relative uncertainty in uncertainty.

It is seen that, the higher the DOF, more reliable the value of uncertainty means higher the accuracy of final result.

The effective degree of freedom for final result is obtained by Welch-Saitterthwaite formula:

$$v_{eff} = \frac{u_c^{\,4}}{\sum_{i=0}^{N}\left[\frac{c_i^{\,4}.u_i^{\,4}}{v_i}\right]} \qquad (5)$$

where c_i represents the sensitivity co-efficient and u_c represents combined uncertainty which is described later in this article.

Sensitivity coefficient of uncertainty offers the information about the stability of final result with respect to the variation of one or more uncertainty components. The lower the value of sensitivity co-efficient refers better performance of the system or device. Desire value of sensitivity co-efficient is always less than unity. Sensitivity co-efficient can be determined mathematically from the model expression taking partial derivative of the same with respect to the uncertainty component.

$$c_i = \partial f / \partial x_i \qquad (6)$$

where f is the model function and x_i is i^{th} input estimate.

The sensitivity co-efficient can also be obtained experimentally. The change in output is observed due to the change in input of a specific contributing element over a specified range and the % change is calculated as $c_i = \partial y / \partial u(x_i)$ when the change in input was over a range $[\pm \partial u(x_i)/2]$. This is the most appropriate way of sensitivity of uncertainty evolution.

3.4. Uncertainty Representation

Uncertainty of all components is then combined together to get the final type B uncertainty of the measurement. If any two or more uncertainty components are correlated then the calculation of uncertainties becomes complex. That correlation is derived using basic probability theory. When all the contributing components are independent with respect to each other and are related by an approximately linear relationship the final type B uncertainty is calculated by

$$u_B = \sqrt{\sum_{i=1}^{N}(c_i.u_i)^2} \qquad (7)$$

Combined standard uncertainty is done taking positive square root of individual square of type A and type B uncertainty[3].

$$u_c = \sqrt{u_A^2 + u_B^2} \quad \text{.......} (8)$$

Normally in report of measurement, expanded uncertainty U is described. The calculation of U includes coverage factor k value of which is determined from student-t table. In this table the value is given against different DOF and confidence level. Higher the confidence level denotes better the measurement.

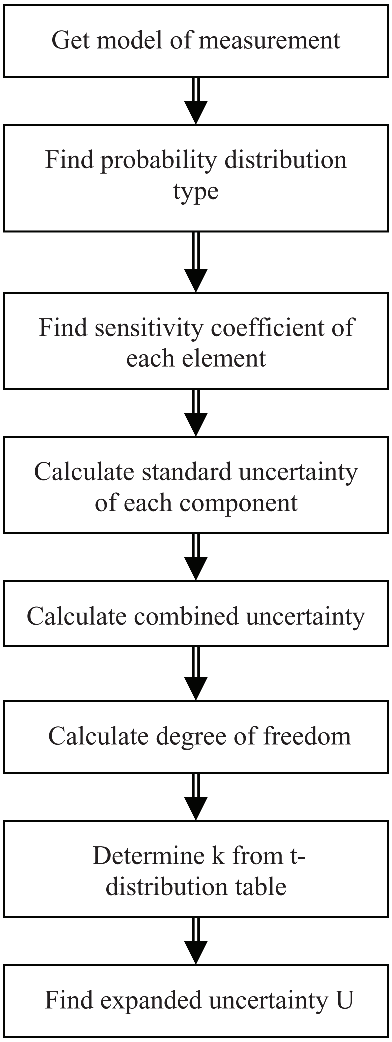

FIGURE 4: FLOWCHART FOR EXPANDED UNCERTAINTY CALCULATION

$$U = k \times u_c \quad \text{.......} (9)$$

If k is not reported, then a conservative way of proceeding is to assume k = 2.

4. DISCUSSION

The standard way of representing uncertainty in final result of measurement is in percentage (%). Sometimes it is also presented in unit which must be same as of the subject under test. Final result of a measurement can be expressed mathematically as

Result= measured value ± % uncertainty
Or
Result= measured value ± uncertainty

The % uncertainty can be evaluated easily and directly form relative uncertainty; but calculation of uncertainty in specific unit is a bit intricate as it includes some more calculation and computation. Main complexity occurs when the dimension of contributing factors is not same as the subject's dimension. The unit change then done by sequential calculation of mathematical formulas which are derived from the flow of error contributed by that particular factor of which the unit needs to be changed. It introduces complexity in tabulation.

No measurement can be guaranteed as perfect. The uncertainty expression ensures measurement of a good quality only. It is very important to report any measurement result with uncertainty which is hardly done in practice till today. Uncertainty measurement using GUM defined method ensures the results traceability to international standards. Traceability is an important property of a test result, particularly if it is to be used for legal or regulatory purposes.

Understanding and implementing uncertainty measurement is not easy as GUM is fortified with an extensive mathematical basis. The paper is an attempt to describe the error propagation and uncertainty calculation method in a simpler way mainly for the graduate/undergraduate engineering students.

5. ISO GUM DEFINITIONS

Standard uncertainty, u

Uncertainty of the result of a measurement expressed as a standard deviation.

Type A evaluation (of uncertainty)

Method of evaluation of uncertainty by the statistical analysis of series of observations

Type B evaluation (of uncertainty)

Method of evaluation of uncertainty by means other than the statistical analysis of series of observations

Combined standard uncertainty, u

standard uncertainty of the result of a measurement when that result is obtained from the values of a number of other quantities, equal to the positive square root of a sum of terms, the terms being the variances or covariances of these other quantities weighted according to how the measurement result varies with changes in these quantities

Coverage factor, k

Numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded Uncertainty.

Expanded uncertainty, U

It defines an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand.

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