Generalized dilaton-Maxwell cosmic string and wall solutions

John R. Morris

Physics Department, Indiana University Northwest, 3400 Broadway, Gary, IN 46408, USA

Abstract

The class of static solutions found by Gibbons and Wells for dilaton-electrodynamics in flat spacetime, which describe nontopological strings and walls that trap magnetic flux, is extended to a class of dynamical solutions supporting arbitrarily large, nondissipative traveling waves, using techniques previously applied to global and local topological defects. These solutions can then be used in conjunction with S-duality to obtain more general solitonic solutions for various axidilaton-Maxwell theories. As an example, a set of dynamical solutions is found for axion, dilaton, and Maxwell fields in low energy heterotic string theory using the $SL(2, \mathbb{R})$ invariance of the equations of motion.
© 2006 Elsevier B.V. Open access under CC BY license.

PACS: 11.27.+d; 98.80.Cq; 04.50.+h

Keywords: Dilaton-electrodynamics; Cosmic strings; Traveling waves

1. Introduction

The common forms of cosmic string and domain wall solutions that have been studied so extensively in the past [1] involve discrete or continuous symmetries that are broken by the vacuum states. For instance, a simple Z_2 domain wall described by a single real scalar field φ arises from an action

$$S_W = \int d^4x \, \sqrt{-g} \left[\frac{1}{2} (\partial_\mu \varphi)^2 - \frac{1}{4} \lambda (\varphi^2 - \eta^2)^2 \right] \tag{1}$$

and a local $U(1)$ cosmic string described by the complex scalar field ϕ and gauge field A_μ can be obtained from an Abelian-Higgs action

$$S_{CS} = \int d^4x \, \sqrt{-g} \left[\frac{1}{2} (|D_\mu \phi|^2) - \frac{1}{4} \lambda (|\phi|^2 - \eta^2)^2 - \frac{1}{4} F_{\mu\nu}^2 \right] \tag{2}$$

with $D_\mu = \partial_\mu + ieA_\mu$. The action for a global string can be obtained by setting $A_\mu = 0$. In the solutions admitted by these actions the scalar self coupling λ and the symmetry breaking parameter η determine the thickness of the wall or string. For

the local $U(1)$ string the magnetic flux is trapped inside the string core where $\phi \to 0$ and the gauge field becomes massless. Whether or not closed form analytic solutions are known for these solitonic objects, it has been shown [2] that they can support traveling wave solutions of arbitrary amplitude that are nondissipative, i.e., they do not result in the radiation of scalar or gauge particles. Such radiation must come from nontraveling wave solutions [3]. The method used in Ref. [2] to show the existence of nondissipative traveling waves relies on the definition of new coordinates X and Y that are related to the Cartesian coordinates x and y by $X = x - \psi(t, z)$, $Y = y - \chi(t, z)$, where $\psi(t, z)$ and $\chi(t, z)$ are traveling wave solutions satisfying $(\partial_t^2 - \partial_z^2)\xi(t, z) = 0$, $\partial^\mu \xi \partial_\mu \xi = 0$, where $\xi = \psi, \chi$ and ψ and χ are both functions of either $t - z$ or $t + z$. Then the dynamical solutions, for instance $\phi(t, x, y, z)$ and $A_\mu(t, x, y, z)$ for cosmic strings, are obtained from the functions $\Phi(X, Y)$ and $\mathcal{A}_\mu(X, Y)$, where $\Phi(x, y)$ and $\mathcal{A}_\mu(x, y)$ are static solutions for a string stretching along the z axis.

More specifically, for a static domain wall located in the y–z plane, the center of the wall is at $x = 0$. The replacement $x \to X = x - \psi(t \pm z)$, $\Phi(x) \to \Phi(X) = \phi(x, z, t)$ gives a solution describing a traveling wave in the wall, with the wall's center described by the surface $X = x - \psi(t \pm z) = 0$. In a

similar manner, for a string stretching along the z axis, the replacements $x \to X = x - \psi(t \pm z)$, $y \to Y = y - \chi(t \pm z)$ describe a string with center at $X = 0$, $Y = 0$, and the dynamical solutions are given by [2,4] $\phi(x, y, z, t) = \Phi(X, Y)$ and $A_\mu(x, y, z, t) = (\partial X^M/\partial x^\mu)\mathcal{A}_M(X, Y, z, t)$, where $X^M = (t, X, Y, z)$. Extended methods were also developed to obtain the metric for gravitating solitons with traveling waves [4,5].

A 4d dilaton coupling to strings and walls (in the Einstein conformal frame) can be included in an action of the form

$$S' = \int d^4 x \sqrt{-g} \left\{ \frac{1}{2}(\partial_\mu \phi)^2 - U(\phi) + e^{-\alpha\phi}\mathcal{L} \right\} \tag{3}$$

where ϕ is a 4d dilaton field (which may be a modulus field associated with a compactified extra dimension) and the Lagrangian $\mathcal{L}$ is built from the scalar and gauge fields describing the wall or string [6]. (The Einstein–Hilbert contribution to the action has been dropped for the case of a flat spacetime.) The dilaton field ϕ is a real valued scalar field, but the fields giving rise to strings or walls are the fields in $\mathcal{L}$. Again, the strings or walls develop because of a broken symmetry.

2. Dilaton-Maxwell theory

The string and wall solutions found by Gibbons and Wells [7] are quite different from the types of topological soliton solutions discussed above. The Gibbons–Wells solutions are exact, analytical solutions of pure dilaton-Maxwell theory. The action for dilaton-electrodynamics in a flat spacetime is

$$S = \int d^4 x \left(\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{4}e^{-2\tilde{\kappa}\phi}F_{\mu\nu}F^{\mu\nu} \right) \tag{4}$$

where $\tilde{\kappa}$ is a dimensioned parameter, not to be confused with a Newtonian gravitation constant, and $g_{\mu\nu} = \eta_{\mu\nu} = (+, -, -, -)$. The action in (4) contains a real scalar field ϕ which couples to the electromagnetic fields nonminimally. There is no spontaneous symmetry breaking for ϕ as is the case for the ordinary string and wall fields in the actions described in (1)–(3).

For future convenience we list the electromagnetic field tensors

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix},$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix} \tag{5}$$

with $E_x = E_1$, $B_x = B_1$, etc. being the physical fields and $F_{ij} = F^{ij}$. The equations of motion that follow from (4), along with the Bianchi identity, are

$$\Box\phi + \frac{1}{2}\tilde{\kappa}e^{-2\tilde{\kappa}\phi}F_{\mu\nu}F^{\mu\nu} = 0, \tag{6}$$

$$\nabla_\mu\left(e^{-2\tilde{\kappa}\phi}F^{\mu\nu}\right) = 0, \qquad \nabla_\mu \tilde{F}^{\mu\nu} = 0, \tag{7}$$

where the dual tensor is $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$. The set of equations in (7) is just the set of Maxwell equations

$$\nabla \cdot \mathbf{D} = 0, \qquad \nabla \times \mathbf{H} - \dot{\mathbf{D}} = 0,$$

$$\nabla \cdot \mathbf{B} = 0, \qquad \nabla \times \mathbf{E} + \dot{\mathbf{B}} = 0 \tag{8}$$

with $\mathbf{D} = \epsilon\mathbf{E}$ and $\mathbf{B} = \mu\mathbf{H}$ and effective dielectric and permeability functions given by $\epsilon = \mu^{-1}$, where

$$\mu = \epsilon^{-1} = e^{2\tilde{\kappa}\phi} \tag{9}$$

and the index of refraction is $\sqrt{\epsilon\mu} = 1$. We can then rewrite (6) as

$$\nabla^2\phi - \partial_t^2\phi = -\tilde{\kappa}e^{-2\tilde{\kappa}\phi}\left(\mathbf{B}^2 - \mathbf{E}^2\right)$$
$$= -\tilde{\kappa}e^{2\tilde{\kappa}\phi}\left(\mathbf{H}^2 - \mathbf{D}^2\right). \tag{10}$$

3. Static dilaton string and wall solutions

The static z-independent solutions discovered by Gibbons and Wells are obtained by assuming an ansatz where $\phi = \Phi(x, y)$, $\mathbf{E} = 0$, $\mathbf{H} = (0, 0, \mathcal{H}) = \text{const}$, and $\mathbf{B} = (0, 0, \mathcal{B}) = \mu\mathbf{H}$. I.e., we assume that there is a region of space where $H_k = \delta_{k3}\mathcal{H}$ is constant, so that

$$\mathcal{B}(x, y) = \mu(x, y)\mathcal{H} = e^{2\tilde{\kappa}\Phi(x,y)}\mathcal{H}. \tag{11}$$

The Maxwell equations in (8) are then trivially satisfied, and the dilaton equation (10) reduces to

$$\left(\partial_x^2 + \partial_y^2\right)\Phi = -\tilde{\kappa}\mathcal{H}^2 e^{2\tilde{\kappa}\Phi}. \tag{12}$$

Eq. (12) is the Euclidean Liouville equation [8] (see also, for example, Refs. [9,10]). Its solution [7] can be written in terms of $\zeta = x + iy$ as

$$\mu(\zeta) = e^{2\tilde{\kappa}\Phi(\zeta)} = \frac{4}{\tilde{\kappa}^2\mathcal{H}^2}\frac{|f'(\zeta)|^2}{(1 + |f(\zeta)|^2)^2}, \tag{13}$$

where f is a holomorphic function of ζ and $f'(\zeta) = df(\zeta)/d\zeta$. As pointed out in Ref. [7], the functions f and $1/f$ give the same solution Φ.

The interesting feature of the class of solutions given by (11) and (13) is that there is a trapping of magnetic flux in the solution cores where Φ, μ, and $\mathcal{B}$ all become large. Gibbons and Wells present two specific examples of solutions, one for a cylindrically symmetric cosmic string, and one for a planar wall.

A cylindrically symmetric solution describing a cosmic string lying along the z axis is given by $f = a\zeta$ with a being a constant. The solution for Φ is then

$$\mu = e^{2\tilde{\kappa}\Phi} = \frac{4a^2}{\tilde{\kappa}^2\mathcal{H}^2}\frac{1}{(1 + a^2\rho^2)^2}, \tag{14}$$

where $\rho^2 = |\zeta|^2 = x^2 + y^2$. The total magnetic flux

$$\Phi_{\text{mag}} = \int \mathbf{B} \cdot \hat{n}\, dA = \pi\mathcal{H}\int_0^\infty \mu(\rho)\, d\left(\rho^2\right) = \frac{4\pi}{\tilde{\kappa}^2\mathcal{H}} \tag{15}$$

associated with this solution is finite with the flux being concentrated near the core. The parameter a is arbitrary and the

total flux does not depend on it. (Around 80% of the total flux is trapped within a region where $\rho a \lesssim 2.5$.) We also have that $\mu, \mathcal{B} \to 0$ as $\rho \to \infty$. The string solution has a logarithmically divergent energy per unit length, resembling that of a global string rather than a local one.

For a wall-like solution Gibbons and Wells consider a solution generated by $f(\zeta) = \exp(\zeta)$, giving

$$\mu = e^{2\tilde{\kappa}\Phi} = \frac{1}{\tilde{\kappa}^2 \mathcal{H}^2} \frac{1}{\cosh^2 x}. \tag{16}$$

This describes a wall-like solution with an amount of flux per unit length of $\frac{2}{\tilde{\kappa}^2 \mathcal{H}}$. Inside the wall $\mu, \mathcal{B}$ are large, and asymptotically $\mu, \mathcal{B} \to 0$ as $x \to \pm\infty$.

4. Solutions with traveling waves

We use the solution generating technique developed by Vachaspati and Vachaspati [2] to describe traveling waves on strings and walls. We want to find solutions $\phi(x, y, z, t)$, $F_{\mu\nu}(x, y, z, t)$ to the equations of motion (8)–(10). We start with static solutions described by (11)–(13) and make replacements $x \to X$, $y \to Y$; that is, we make a coordinate transformation from the coordinates $x^\mu = (t, x, y, z)$ to the coordinates X^M, where we define the new coordinates

$$X^M = \left(X^0, X^1, X^2, X^3\right) = (t, X, Y, z),$$
$$X = x - \psi(t, z), \qquad Y = y - \chi(t, z),$$
$$X^0 = x^0 = t, \qquad X^3 = x^3 = z. \tag{17}$$

The functions ψ and χ must be traveling waves that satisfy $(\partial_t^2 - \partial_z^2)\xi = 0$ and $\partial_\mu \xi \partial^\mu \xi = 0$, where $\xi = \psi, \chi$ so that both ψ and χ are functions of either $t - z$ or $t + z$. We then take the dilaton field to be given by

$$\phi\left(x^\mu\right) = \phi(x, y, z, t) = \Phi(X, Y). \tag{18}$$

Since $(\partial_t^2 - \partial_z^2)\Phi(X, Y) = 0$, we have $\Phi(X, Y)$ satisfying an equation of motion having the same form as in (12) (with the same constant $\mathcal{H}$) with the replacements $x \to X$ and $y \to Y$. Of course, we must also determine the new $\mathbf{E}$ and $\mathbf{B}$ fields for the new coordinate system so that we get the same right-hand side of (10) with $e^{2\tilde{\kappa}\phi}(\mathbf{H}^2 - \mathbf{D}^2) = e^{2\tilde{\kappa}\Phi(X,Y)}\mathcal{H}^2$. This is easily done by considering the tensor transformation of the field tensor $\mathcal{F}_{MN}(X^M)$, where $\mathcal{F}_{MN}$ is composed of the electric and magnetic fields

$$\mathcal{E}_k = 0, \qquad \mathcal{B}_k(X, Y) = \delta_{k3}\mathcal{B}(X, Y) = \delta_{k3}e^{2\tilde{\kappa}\Phi(X,Y)}\mathcal{H}. \tag{19}$$

That is, we use the static solutions with $x, y \to X, Y$. We then have

$$F_{\mu\nu}\left(x^\alpha\right) = \frac{\partial X^M}{\partial x^\mu} \frac{\partial X^N}{\partial x^\nu} \mathcal{F}_{MN}\left(X^A\right). \tag{20}$$

Using (5), (19) and (20) give the dynamical electric and magnetic fields $\vec{E}(x, y, z, t)$ and $\vec{B}(x, y, z, t)$ with components

$$E_1 = E_x = -\dot{\chi}\mathcal{B}, \qquad B_1 = B_x = \psi'\mathcal{B},$$
$$E_2 = E_y = \dot{\psi}\mathcal{B}, \qquad B_2 = B_y = \chi'\mathcal{B},$$
$$E_3 = E_z = (\psi'\dot{\chi} - \dot{\psi}\chi')\mathcal{B} = 0, \qquad B_3 = B_z = \mathcal{B}, \tag{21}$$

where $\dot{\psi} = \partial_0 \psi$, $\psi' = \partial_z \psi$, etc. We also have for the fields $\mathbf{D}(x^\mu)$ and $\mathbf{H}(x^\mu)$

$$\mathbf{D} = \epsilon\mathbf{E} = e^{-2\tilde{\kappa}\Phi(X,Y)}\mathbf{E}, \tag{22a}$$
$$\mathbf{B} = \mu\mathbf{H} = e^{2\tilde{\kappa}\Phi(X,Y)}\mathbf{H}. \tag{22b}$$

From $\mathbf{H} = \mu^{-1}\mathbf{B}$ and $B_3 = \mathcal{B} = e^{2\tilde{\kappa}\Phi(X,Y)}\mathcal{H}$, we find $H_3 = H_z = \mathcal{H}$ and $e^{2\tilde{\kappa}\phi}(\mathbf{H}^2 - \mathbf{D}^2) = e^{2\tilde{\kappa}\Phi(X,Y)}\mathcal{H}^2$.

Using the facts that $\partial_x = \partial_X$, $\partial_y = \partial_Y$, one can verify that the solutions given by (18) and (21) are x, y, z, t dependent solutions to the equations of motion (8)–(10). Note that we *do not* make a tensor transformation on the metric, but rather we take $g_{\mu\nu}(x^\mu) = g_{MN}(X^M)$, i.e., we take $\eta_{\mu\nu} = \eta_{MN}$ in order to maintain the *form invariance* of the equations of motion, as opposed to a covariance under a general coordinate transformation. The solutions $\phi(x^\mu)$, $F_{\mu\nu}(x^\mu)$ describe traveling waves of arbitrary size propagating in the $\pm z$ direction through otherwise static string or wall solutions that trap magnetic flux. The center of a wall is now located at $X = x - \psi(t, z) = 0$, and the center of a string at $X = x - \psi(t, z) = 0$, $Y = y - \chi(t, z) = 0$, and the entrapped region of magnetic flux in the core of the solution moves along with the dilaton field. Furthermore, since the nonstatic field $\Phi(X, Y)$ has the same functional form as the static field $\Phi(x, y)$ and the electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$ are proportional to $\mathcal{B}(X, Y)$, and $\mathcal{B}(X, Y)$ has the same functional form as the static field $\mathcal{B}(x, y)$, then if the static solutions drop off to zero away from the solution core, then the same must hold for the nonstatic solutions. We therefore conclude that the dynamical solutions are nondissipative and do not radiate energy in the form of either dilatons or photons.

To make this more explicit, we note that the scalar dilaton field can be written in terms of a traveling wave part $\Phi(X, Y)$ plus a radiative part $\delta\phi(x, y, z, t)$, i.e., $\phi(x, y, z, t) = \Phi(X, Y) + \delta\phi(x, y, z, t)$, as done for the case of scalar radiation from oscillating domain walls in Ref. [3]. Since $\delta\phi = 0$ for the case of traveling waves, these solutions generate no dilatonic radiation in the form of particles escaping to an infinite distance from the soliton core. We conclude that scalar dilaton radiation can come only from solitons that support nontraveling waves (e.g., standing waves). Concerning electromagnetic radiation, we note that the motion of the soliton core will be associated with a locally time dependent magnetic flux and induced electric field, so that there is, in general, a locally nonvanishing Poynting vector. From the fields in (21) and (22) we find

$$\vec{S} = \vec{E} \times \vec{H} = \epsilon(X, Y)(\vec{E} \times \vec{B})$$
$$= \mathcal{H}\mathcal{B}(X, Y)\left(\dot{\psi}, \dot{\chi}, -(\dot{\psi}\psi' + \dot{\chi}\chi')\right). \tag{23}$$

The equation of motion for the dilaton can be written in the form

$$\nabla^2 \Phi = -\tilde{\kappa}\mathcal{H}\mathcal{B}. \tag{24}$$

For a single localized soliton with $\nabla\Phi \to 0$ asymptotically, we must have that $\mathcal{B} \to 0$ at asymptotic distances, which implies that $\vec{S} \to 0$ at asymptotic distances from the solitonic core. To have $\int \vec{S} \cdot \hat{n}\, dA \to 0$ through a surface asymptotically distant from the soliton, we require that $\mathcal{B} \to 0$ faster

than the inverse distance, asymptotically. For example, if asymptotically $|\nabla \Phi| \sim \rho^{-n}$, $n > 0$, for a string-like solution, then $|\nabla^2 \Phi| \sim \rho^{-(n+1)} \sim \mathcal{B}$. It follows that $S_\rho = \mathcal{H}\mathcal{B}(X,Y)(\frac{x}{\rho}\dot{\psi} + \frac{y}{\rho}\dot{\chi}) \sim \mathcal{B} \to 0$ faster than ρ^{-1}, so that no electromagnetic energy flows radially in or out through a cylindrical surface at $\rho \to \infty$. A similar argument can be made for a wall-like solution. However, electromagnetic energy does flow in and out from the soliton core *locally* as the soliton wiggles around. Also there is an energy flow in the z direction, within the string or wall. Also, multisoliton solutions (e.g., a network of strings [7]) exist, and each soliton is assumed to have the same type of radiative behavior as a single isolated soliton.

Finally, we can note that the gauge field $A_\mu(x^\mu)$ for the dynamical solutions can be related to the gauge field $\mathcal{A}_M(X^M)$ for the static solutions, with $x^\mu \to X^M$, by the tensor transformation [4]

$$A_\mu(x^\sigma) = (\partial_\mu X^M)\mathcal{A}_M(X^S), \tag{25}$$

where X^M is given by (17) and $A_\mu(x)$ and $\mathcal{A}_M(X)$ generate the field strengths $F_{\mu\nu}(x)$ and $\mathcal{F}_{MN}(X)$, respectively. From $F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$ and $\mathcal{F}_{MN}(X) = \partial_M \mathcal{A}_N(X) - \partial_N \mathcal{A}_M(X)$, (25) yields (20), the field tensor for the dynamical solutions.

5. Extensions using S-duality

The equations of motion given by (6) and (7) exhibit an S-duality, i.e., they are invariant under the duality transformations

$$\phi \to -\phi, \qquad F_{\mu\nu} \to e^{-2\tilde{\kappa}\phi}\tilde{F}_{\mu\nu}, \tag{26}$$

where $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$. This symmetry could be used to generate new solutions. For example, if the duality transformations are applied to the static solutions of (11) and (13), we have $\epsilon \to \mu$, $\mu \to \epsilon$, $\mathbf{E} \to \mathbf{H}$, $\mathbf{B} \to -\mathbf{D}$, $\mathbf{D} \to \mathbf{B}$, and $\mathbf{H} \to -\mathbf{E}$. Consequently we have the new solutions

$$\mathcal{D}(x,y) = \epsilon(x,y)\mathcal{E} = e^{-2\tilde{\kappa}\Phi(x,y)}\mathcal{E}, \tag{27a}$$

$$\epsilon(\zeta) = e^{-2\tilde{\kappa}\Phi(\zeta)} = \frac{4}{\tilde{\kappa}^2\mathcal{E}^2}\frac{|f'(\zeta)|^2}{(1+|f(\zeta)|^2)^2} \tag{27b}$$

describing strings or walls in a region of constant electric field $\mathbf{E} = (0,0,\mathcal{E})$ that entrap the displacement field $\mathcal{D}$, as pointed out in [7].

This method can also be used in effective field theories with more fields and other dual symmetries, such as $SL(2,\mathbb{R})$. An example is provided in [11] where the authors consider an effective 4d low energy heterotic string theory containing electromagnetic, dilaton, and axion fields. Starting with a solution with zero axion, nontrivial new solutions with a nonzero axion can be generated from the $SL(2,\mathbb{R})$ symmetry. This technique could be applied to other theories exhibiting $SL(2,\mathbb{R})$ duality as well, such as supergravities [12] and nonlinear electrodynamics coupled to an axion and dilaton [13,14], and to low energy string theories with generalized couplings of dilaton and modulus fields to gauge fields, along with generalized duality transformations [15]. Here, we follow the example in Ref. [11] by taking the dynamic Gibbons–Wells type of solutions obtained

in the last section, along with a vanishing axion field, and using these solutions as input to generate, via the $SL(2,\mathbb{R})$ dual symmetry, new dynamical solutions for an effective low energy heterotic string theory that possesses a nonzero axion field (in the flat space limit). Using the units of Ref. [11], we write the action (in a flat spacetime) for a truncated version of the effective 4d low energy heterotic string theory [11] as[1]

$$S = \int d^4x \left[\frac{1}{2}(\partial\phi)^2 + \frac{1}{12}e^{-2\phi}H_{\mu\nu\rho}H^{\mu\nu\rho} - e^{-\phi}F_{\mu\nu}F^{\mu\nu} \right], \tag{28}$$

where ϕ is the dilaton, $H_{\mu\nu\rho}$ is an antisymmetric rank three tensor field, and the axion field Ψ is defined through

$$H^{\mu\nu\rho} = -e^{2\phi}\epsilon^{\mu\nu\rho\sigma}\partial_\sigma\Psi. \tag{29}$$

The equations of motion are

$$\nabla_\mu(e^{-\phi}F^{\mu\nu} + \Psi\tilde{F}^{\mu\nu}) = 0, \tag{30a}$$

$$\nabla_\mu\tilde{F}^{\mu\nu} = 0, \tag{30b}$$

$$\Box\phi - e^{-\phi}F_{\mu\nu}F^{\mu\nu} - e^{2\phi}\partial_\mu\Psi\partial^\mu\Psi = 0, \tag{30c}$$

$$\Box\Psi + 2\partial_\mu\phi\partial^\mu\Psi + e^{-2\phi}F_{\mu\nu}\tilde{F}^{\mu\nu} = 0. \tag{30d}$$

Upon setting $\Psi = 0$, $\mathbf{E} = 0$, $B_k = \delta_{3k}\mathcal{B}$, these equations reduce to the form of those in (6) and (7), the solutions to which are given by (18) and (20). The equations of motion in (30) are invariant under the duality transformations [11]

$$\lambda \to \lambda' = \frac{a\lambda + b}{c\lambda + d},$$
$$F_{\mu\nu} \to F'_{\mu\nu} = (c\Psi + d)F_{\mu\nu} - ce^{-\phi}\tilde{F}_{\mu\nu},$$
$$ad - bc = 1, \tag{31}$$

where $\lambda = \Psi + ie^{-\phi}$. Using the solutions in (18) and (20) along with $\Psi = 0$ as input, (31) yields a new solution set given by

$$\Psi' = \frac{ace^{-2\phi} + bd}{c^2e^{-2\phi} + d^2}, \qquad e^{-\phi'} = \frac{e^{-\phi}}{c^2e^{-2\phi} + d^2},$$
$$F'_{\mu\nu} = dF_{\mu\nu} - ce^{-\phi}\tilde{F}_{\mu\nu} \tag{32}$$

with the various components of $F_{\mu\nu}$ being given by (21). This represents a generalization of the class of solutions of the Gibbons–Wells type describing wiggly strings and walls, with nonvanishing axion fields, for the effective low energy heterotic string theory described by (28). These types of solutions may be of physical interest, for example, in providing a natural solution to the monopole problem [7].

6. Brief summary

To summarize, we have found dynamical solutions of the flat-space dilaton-Maxwell theory that describe nontopological solitons entrapping magnetic flux and supporting nondissipative traveling waves. These solutions, given by Eqs. (18)–(22), depend on a holomorphic function $f(\mathbf{Z})$, where $\mathbf{Z} = X + iY$,

---

[1] We use a different (mostly negative) metric.

and

$$\mu(\mathbf{Z}) = e^{2\tilde{\kappa}\Phi(\mathbf{Z})} = \frac{4}{\tilde{\kappa}^2 \mathcal{H}^2} \frac{|f'(\mathbf{Z})|^2}{(1 + |f(\mathbf{Z})|^2)^2}. \tag{33}$$

These z independent solitons can represent codimension 1 or 2 objects, or a network of such objects, depending upon the choice for the function f. These static or dynamic solutions, along with their dilaton-Maxwell duals (see, e.g., (27)), can be further used as seed solutions in axidilaton-Maxwell theories with the use of S-duality. An example has been provided for low energy string theory with toroidal compactification, where the dilaton-Maxwell solutions are used along with a vanishing axion field to generate a new class of $SL(2, \mathbb{R})$ dual solutions describing different static or dynamic solitonic flux-trapping configurations with nonvanishing axion field (Eq. (32)). The types of nontopological solitons discussed here differ from the usual topological solitons not only from a mathematical standpoint, but also have different physical characteristics, and can arise naturally in classes of axidilaton-Maxwell theories, including low energy string theories.

References

[1] See, for example, A. Vilenkin, E.P.S. Shellard, Cosmic Strings and Other Topological Defects, Cambridge Univ. Press, 1994.
[2] Vachaspati, T. Vachaspati, Phys. Lett. B 238 (1990) 41.
[3] T. Vachaspati, A.E. Everett, A. Vilenkin, Phys. Rev. D 30 (1984) 2046.
[4] A. Economou, D. Harari, M. Sakellariadou, Phys. Rev. D 45 (1992) 433.
[5] D. Garfinkle, T. Vachaspati, Phys. Rev. D 42 (1990) 1960.
[6] See, for example, R. Gregory, C. Santos, Phys. Rev. D 56 (1997) 1194.
[7] G.W. Gibbons, C.G. Wells, Class. Quantum Grav. 11 (1994) 2499.
[8] J. Liouville, J. Math. Pures Appl. 18 (1853) 71.
[9] E. D'Hoker, R. Jackiw, Phys. Rev. D 26 (1982) 3517.
[10] E. Witten, Phys. Rev. Lett. 38 (1977) 121.
[11] P. Das, P. Jain, S. Mukherji, Int. J. Mod. Phys. A 16 (2001) 4011.
[12] E. Lozano-Tellechea, T. Ortin, Nucl. Phys. B 569 (2000) 435.
[13] G.W. Gibbons, D.A. Rasheed, Nucl. Phys. B 454 (1995) 185.
[14] G.W. Gibbons, D.A. Rasheed, Phys. Lett. B 365 (1996) 46.
[15] M. Cvetic, A.A. Tseytlin, Nucl. Phys. B 416 (1994) 137.