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Three-loop Higgs self-coupling beta-function in the Standard Model with complex Yukawa matrices

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Abstract

Three-loop renormalization group equations for the Higgs self-coupling and Higgs mass parameter are recalculated in the case of complex Yukawa matrices which encompass the general flavor structure of the Standard Model. In addition, the anomalous dimensions for both the quantum Higgs field and its vacuum expectation value are presented in the $\overline{\text{MS}}$-scheme. A numerical study of the latter quantities is carried out for a certain set of initial parameters.

Keywords: Standard Model; Renormalization group

The discovery of the Higgs boson [1,2] confirms the fact that the Standard Model turns out to be a perfect model describing physics at the electroweak scale. In spite of all attempts to find something beyond the SM, no stringent evidences of new particles were found.

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Recent analyses [3–6] based on three-loop renormalization group equations [7–9] demonstrated that the SM can be extrapolated up to very high scales without the necessity to introduce additional degrees of freedom.

Unfortunately, current experimental uncertainty in the strong coupling constant and the top quark mass do not allow us to make an accurate prediction whether the SM vacuum is stable only up to $\mathcal{O}(10^{10})$ GeV or up to the Plank scale. It is not surprising that in the above-mentioned studies focused on vacuum stability the flavor structure of the SM was neglected.

In this work, we extend our recent results on Higgs potential parameters to the case of general Yukawa matrices. This kind of result can be important not only in precise studies of vacuum stability, but also in an analysis of different flavor patterns (see, e.g., a review [10]), which can again originate from some New Physics.

The corresponding two-loop expressions [11] can be deduced from the general results of Refs. [12–15]. The three-loop gauge-coupling beta-functions with the full flavor structure were calculated for the first time in Ref. [16] and confirmed later by our group [17]. It should be noted that the expressions presented in this paper *cannot* be obtained from the known results [9,18] in the SM with only one fermion family coupled to the Higgs boson. This is due to the fact that the simple fermion-loop counting and naive generalization of the substitution rules from Refs. [16,17] are not sufficient to distinguish certain Yukawa-matrix traces, which can appear in the final results for the considered quantities (see below). As a consequence, a direct evaluation of Feynman diagrams with explicit flavor indices is required.

For this kind of calculation the Feynman rules for `DIANA` [19], which were used in our previous studies, were appropriately rewritten and a simple routine dealing with explicit flavor indices was developed. In order to validate our codes, we also recalculated the results for the gauge coupling beta-functions, thus confirming the expressions given in Refs. [16,17].

The calculation is carried out in an almost automatic way with the help of the infra-red rearrangement (IRR) [20] procedure implemented in our codes. We start with the Lagrangian of the unbroken SM with the full flavor structure given in our previous paper [17]. For the reader's convenience we present here the terms describing the fermion–Higgs interactions and the Higgs field self-interaction

$$\mathcal{L}_{\text{Yukawa}} = -\big(Y_u^{ij}\big(Q_i^L\Phi^c\big)u_j^R + Y_d^{ij}\big(Q_i^L\Phi\big)d_j^R + Y_l^{ij}\big(L_i^L\Phi\big)l_j^R + \text{h.c.}\big), \tag{1}$$

$$\mathcal{L}_{\text{H}} = (D_\mu\Phi)^\dagger(D_\mu\Phi) - V_H(\Phi), \tag{2}$$

$$V_H(\Phi) = m^2\Phi^\dagger\Phi + \lambda\big(\Phi^\dagger\Phi\big)^2, \qquad \Phi^\dagger\Phi = \frac{h^2+\chi^2}{2} + \phi^+\phi^-. \tag{3}$$

Here λ and $Y_{u,d,l}$ denote the Higgs quartic and Yukawa matrices, respectively. The left-handed quark and lepton SU(2) doublets, Q_i^L, and L_i^L, carry flavor indices $i = 1, 2, 3$. The same is true for the SU(2) singlets corresponding to the right-handed SM fermions u_i^R, d_i^R, and l_i^R. The Higgs doublet Φ with hypercharge $Y_W = 1$ is decomposed in terms of the component fields:

$$\Phi = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(h + i\chi) \end{pmatrix}, \qquad \Phi^c = i\sigma^2\Phi^\dagger = \begin{pmatrix} \frac{1}{\sqrt{2}}(h - i\chi) \\ -\phi^- \end{pmatrix}. \tag{4}$$

The charge-conjugated Higgs doublet Φ^c has $Y_W = -1$ and enters into the Yukawa interactions of the right-handed up-type quarks. We neglect the Higgs mass parameter in the Lagrangian since the corresponding anomalous dimension can be found from the $\overline{\text{MS}}$-renormalization constant of the $|\Phi|^2$ operator (see, e.g., [18,21]).

The utilized IRR prescription consists of the introduction of an auxiliary mass parameter M in every propagator and the subsequent expansion in external momenta

$$\frac{1}{(q+p)^2} \to \frac{1}{q^2-M^2}\left[1+\sum_{j=1}^{k}(-1)^j\left(\frac{2qp+p^2}{q^2-M^2}\right)^j\right] \tag{5}$$

up to a sufficient order k until the resulting term leads to a finite expression.[1] In Eq. (5) q and p are linear combinations of internal and external momenta, respectively. The fully massive vacuum integrals obtained via above-mentioned procedure can be easily evaluated by means of the `MATAD` package [22] or `BAMBA` code developed by V.N. Velizhanin. The price to pay for the absence of spurious IR divergencies in the IRR procedure is the necessity to introduce additive mass counter-terms to cancel spurious UV divergent contributions to the "masses" of gauge and scalar bosons. Only after renormalization is carried out one can safely put $M=0$. It turns out that this kind of prescription is equivalent to the "exact" propagator decomposition of Refs. [23,24], in which one can find further details on the approach. It is worth mentioning that the inclusion of the mentioned counter-terms is mandatory to preserve the transversality of the gauge-boson self-energies, which is a consequence of gauge invariance. In this work we recalculated all needed two-loop counter-terms, for both the SM parameters and the auxiliary boson masses.

In order to find the renormalization constants for λ we consider symmetric four-point Green functions with external Higgs particles h. A special script which takes into account the permutation symmetry of external lines, allows us to substantially reduce the number of calculated three-loop diagrams (from about 8 million to about 600 thousand). It is worth mentioning that the number of diagrams, which has to be evaluated, can be further reduced with the help of the `graph_state` library [25] (by about 200 thousand in the considered case). The latter allows one to find isomorphic Feynman diagrams by using the generalization of graph labeling and ordering algorithm[2] proposed in [26] (Nickel index).

As in our previous paper [18] the anomalous dimension of the Higgs mass parameter m^2 is inferred from a certain set of Feynman diagrams contributing to the scalar four-point Green function with two neutral and two charged external Higgs bosons. In all diagrams from this set both lines associated with external charged particles are connected to a single quartic vertex that mimics the insertion of the $|\Phi|^2$ operator.

From the corresponding renormalization constants Z_{hhhh} and $Z_{hh[\phi^+\phi^-]}$ we obtain ($\hat{\lambda} \equiv \lambda/(16\pi^2)$)

$$Z_{\hat{\lambda}} = \frac{Z_{hhhh}}{Z_h^2}, \qquad Z_{m^2} = \frac{Z_{hh[\phi^+\phi^-]}}{Z_h}, \tag{6}$$

where Z_h is nothing else but the renormalization constant for the Higgs propagator,[3] and $Z_{\hat{\lambda}}$, Z_{m^2} enter into the relations between the bare parameters $\hat{\lambda}_{\text{Bare}}$, m^2_{Bare} and the corresponding renormalized ones

$$\hat{\lambda}_{\text{Bare}}\mu^{-2\epsilon} = Z_{\hat{\lambda}}\hat{\lambda} = \hat{\lambda} + \sum_{l=1}^{\infty}\sum_{n=1}^{l}\frac{c_{\hat{\lambda}}^{(l,n)}}{\epsilon^n}, \tag{7}$$

$$m^2_{\text{Bare}} = Z_{m^2}m^2 = m^2\left(1+\sum_{l=1}^{\infty}\sum_{n=1}^{l}\frac{c_{m^2}^{(l,n)}}{\epsilon^n}\right). \tag{8}$$

[1] One should take into account the divergent contributions from the product of divergent factors originating from counter-terms and finite Feynman integrals.

[2] The generalization also takes into account fields on internal lines.

[3] Due to unbroken SU(2) invariance all the fields from the Higgs doublet have the same renormalization constant $Z_h^{1/2}$.

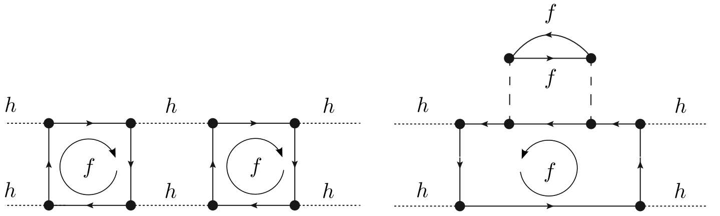

Fig. 1. Two example Feynman diagrams contributing to the four-point vertex of the neutral Higgs bosons h, which cannot be separated by the trick given in Refs. [16,17]. The left graph give rises to the product $\mathcal{Y}_{ff}\mathcal{Y}_{ff}$, while the right one produces $\mathcal{Y}_{f}\mathcal{Y}_{fff}$, $f = u, d, l$.

Here μ is the $\overline{\mathrm{MS}}$ renormalization scale, $\epsilon = (4 - D)/2$ is the parameter of dimensional regularization, and $c^{(l,n)}_{\lambda,m^2}$ denotes the l-loop contribution to the coefficient of $1/\epsilon^n$ in the considered renormalization constants.

The required renormalization group coefficients are extracted from the single pole in ϵ with the help of the following formulae:

$$\beta_{\hat{\lambda}} = \frac{d\hat{\lambda}(\mu,\epsilon)}{d\ln\mu^2}\bigg|_{\epsilon=0} = \sum_{l=1}^{\infty} l \cdot c^{(l,1)}_{\hat{\lambda}}, \qquad \gamma_{m^2} = \frac{d\ln m^2(\mu,\epsilon)}{d\ln\mu^2}\bigg|_{\epsilon=0} = \sum_{l=1}^{\infty} l \cdot c^{(l,1)}_{m^2}. \tag{9}$$

The explicit expressions[4] for $\beta_{\hat{\lambda}}$ and γ_{m^2} can be found in ancillary files of the arXiv version of the paper. The results depend on traces of different combinations of the Yukawa matrices which we list here ($f = u, d, l$) for convenience

$$\begin{aligned}
&\mathcal{Y}_f \equiv \frac{\mathrm{tr}\, Y_f Y_f^\dagger}{16\pi^2}, \qquad \mathcal{Y}_{ff} \equiv \frac{\mathrm{tr}\, Y_f Y_f^\dagger Y_f Y_f^\dagger}{(16\pi^2)^2},\\
&\mathcal{Y}_{fff} \equiv \frac{\mathrm{tr}\, Y_f Y_f^\dagger Y_f Y_f^\dagger Y_f Y_f^\dagger}{(16\pi^2)^3}, \qquad \mathcal{Y}_{ffff} \equiv \frac{\mathrm{tr}\, Y_f Y_f^\dagger Y_f Y_f^\dagger Y_f Y_f^\dagger Y_f Y_f^\dagger}{(16\pi^2)^4},\\
&\mathcal{Y}_{ud} = \frac{\mathrm{tr}\, Y_u Y_u^\dagger Y_d Y_d^\dagger}{(16\pi^2)^2}, \qquad \mathcal{Y}_{udd} = \frac{\mathrm{tr}\, Y_u Y_u^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger}{(16\pi^2)^3},\\
&\mathcal{Y}_{uud} = \frac{\mathrm{tr}\, Y_u Y_u^\dagger Y_u Y_u^\dagger Y_d Y_d^\dagger}{(16\pi^2)^3}, \qquad \mathcal{Y}_{uuud} = \frac{\mathrm{tr}\, Y_u Y_u^\dagger Y_u Y_u^\dagger Y_u Y_u^\dagger Y_d Y_d^\dagger}{(16\pi^2)^4},\\
&\mathcal{Y}_{uudd} = \frac{\mathrm{tr}\, Y_u Y_u^\dagger Y_u Y_u^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger}{(16\pi^2)^4}, \qquad \mathcal{Y}_{udud} = \frac{\mathrm{tr}\, Y_u Y_u^\dagger Y_d Y_d^\dagger Y_u Y_u^\dagger Y_d Y_d^\dagger}{(16\pi^2)^4},\\
&\mathcal{Y}_{uddd} = \frac{\mathrm{tr}\, Y_u Y_u^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger}{(16\pi^2)^4}.
\end{aligned} \tag{10}$$

In these expressions the product $Y_f Y_f^\dagger$ corresponds to the propagation of the right-handed fermion f. Since there is no right-handed flavor-changing current coupled to a SM gauge field, the expressions of the form $Y_{f'} Y_f^\dagger$ with $f \neq f'$ do not appear in the results.

A comment on the necessity to introduce explicit flavor indices is in order. From Eq. (10) one can immediately deduce that the traces $\mathcal{Y}_{uudd}$ and $\mathcal{Y}_{udud}$ cannot be distinguished by the trick

[4] It is worth mentioning that for the color algebra the FORM package COLOR [27] was utilized.

used in Refs. [16,17], since both these expressions give rise to the same factor $y_u^4 y_d^4$ with y_t and y_b being the Yukawa couplings of top and bottom quarks, respectively. Moreover, in the case of two fermion traces involving Yukawa interactions the situation is even worse. For example, the contribution to the four-point vertex originating from the diagrams presented in Fig. 1 cannot be separated by the above-mentioned method.

If one neglects the mixing between generations together with the Yukawa couplings of the first two fermion families, one obtains the known expressions [8]. The one- and two-loop contributions given in Refs. [11,13] can be reproduced by means of identification of Y_u, Y_d and Y_l with $\mathbf{H}^+$, $\mathbf{F}_\mathbf{d}^+$ and $\mathbf{F}_\mathbf{L}^+$, respectively. In addition, the Higgs self-coupling should be rescaled $\lambda \to \lambda/2$.

To save space, we do not show the results for $\hat{\lambda}$ and m^2 themselves but present here an interesting combination of these quantities which can be associated with the three-loop anomalous dimension γ_v of the "tree-level" vacuum expectation value defined by the expression (see, e.g., Ref. [28]).

$$v(\mu) = \sqrt{-\frac{m^2(\mu)}{\lambda(\mu)}}. \tag{11}$$

From Eq. (11) one can deduce that

$$\gamma_v = \frac{1}{2}\left(\gamma_{m^2} - \frac{\beta_{\hat{\lambda}}}{\hat{\lambda}}\right) = \gamma_v^{(1)} + \gamma_v^{(2)} + \gamma_v^{(3)} + \cdots, \tag{12}$$

with $\gamma_v^{(l)}$ being the l-loop contribution given by the following expressions

$$\begin{aligned}
\gamma_v^{(1)} = {} & \frac{1}{\hat{\lambda}}\left(\frac{3(\mathcal{Y}_{dd} + \mathcal{Y}_{uu})}{2} + \frac{\mathcal{Y}_{ll}}{2}\right) - \frac{9}{80}\frac{a_1 a_2}{\hat{\lambda}} - \frac{27}{800}\frac{a_1^2}{\hat{\lambda}} - \frac{9}{32}\frac{a_2^2}{\hat{\lambda}} \\
& - 3\left(\hat{\lambda} + \frac{\mathcal{Y}_d + \mathcal{Y}_u}{2}\right) - \frac{\mathcal{Y}_l}{2} + \frac{9a_1}{40} + \frac{9a_2}{8},
\end{aligned} \tag{13}$$

$$\begin{aligned}
\gamma_v^{(2)} = {} & \frac{1}{\hat{\lambda}}\left(\frac{3}{2}(\mathcal{Y}_{udd} + \mathcal{Y}_{uud}) - \frac{15(\mathcal{Y}_{ddd} + \mathcal{Y}_{uuu})}{2} - \frac{5\mathcal{Y}_{lll}}{2}\right) - 10a_s(\mathcal{Y}_d + \mathcal{Y}_u) \\
& - \frac{a_1 a_2}{\hat{\lambda}}\left(\frac{27\mathcal{Y}_d}{40} + \frac{33\mathcal{Y}_l}{40} + \frac{63\mathcal{Y}_u}{40}\right) - a_1\left(\frac{9\hat{\lambda}}{5} + \frac{5\mathcal{Y}_d}{16} + \frac{15\mathcal{Y}_l}{16} + \frac{17\mathcal{Y}_u}{16}\right) \\
& + \frac{a_1^2}{\hat{\lambda}}\left(-\frac{9\mathcal{Y}_d}{80} + \frac{9\mathcal{Y}_l}{16} + \frac{171\mathcal{Y}_u}{400}\right) + \frac{a_1}{\hat{\lambda}}\left(-\frac{\mathcal{Y}_{dd}}{5} + \frac{3\mathcal{Y}_{ll}}{5} + \frac{2\mathcal{Y}_{uu}}{5}\right) + \frac{21\mathcal{Y}_{ud}}{4} \\
& - a_2\left(9\hat{\lambda} + \frac{45(\mathcal{Y}_d + \mathcal{Y}_u)}{16} + \frac{15\mathcal{Y}_l}{16}\right) + \frac{a_2^2}{\hat{\lambda}}\left(\frac{9(\mathcal{Y}_d + \mathcal{Y}_u)}{16} + \frac{3\mathcal{Y}_l}{16}\right) - \frac{7\mathcal{Y}_{ll}}{8} \\
& + \frac{a_1^2 a_2}{\hat{\lambda}}\left(\frac{n_G}{5} + \frac{717}{1600}\right) + \frac{a_1 a_2^2}{\hat{\lambda}}\left(\frac{n_G}{5} + \frac{97}{320}\right) + \frac{a_1^3}{\hat{\lambda}}\left(\frac{3n_G}{25} + \frac{531}{8000}\right) \\
& + a_1^2\left(-\frac{n_G}{4} - \frac{903}{1600}\right) + \frac{a_2^3}{\hat{\lambda}}\left(n_G - \frac{497}{64}\right) + a_2^2\left(\frac{241}{64} - \frac{5n_G}{4}\right) + 63\hat{\lambda}^2 \\
& + \left(\frac{8a_s}{\hat{\lambda}} - \frac{21}{8}\right)(\mathcal{Y}_{dd} + \mathcal{Y}_{uu}) - \frac{189 a_1 a_2}{160} + 18\hat{\lambda}(\mathcal{Y}_d + \mathcal{Y}_u) + 6\hat{\lambda}\mathcal{Y}_l,
\end{aligned} \tag{14}$$

$$
\begin{aligned}
\gamma_v^{(3)} = {} & \frac{a_1^3 a_2}{\hat\lambda}\left(\frac{n_G^2}{9} + n_G\left(\frac{18\,001}{24\,000} - \frac{183\zeta_3}{250}\right) - \frac{81\zeta_3}{320} + \frac{29\,779}{64\,000}\right) \\
& + \frac{a_1^2 a_2^2}{\hat\lambda}\left(\frac{n_G^2}{9} - n_G\left(\frac{63\zeta_3}{50} + \frac{149}{3600}\right) - \frac{7857\zeta_3}{3200} + \frac{64\,693}{19\,200}\right) \\
& + \frac{a_1^4}{\hat\lambda}\left(\frac{n_G^2}{10} + n_G\left(\frac{12\,441}{16\,000} - \frac{171\zeta_3}{250}\right) - \frac{8019\zeta_3}{160\,000} + \frac{12\,321}{256\,000}\right) \\
& + \frac{a_2^4}{\hat\lambda}\left(\frac{5n_G^2}{6} + n_G\left(\frac{45\zeta_3}{2} + \frac{14\,749}{384}\right) + \frac{2781\zeta_3}{256} - \frac{982\,291}{6144}\right) \\
& + \frac{a_1 a_s}{\hat\lambda}\left(-\frac{68\mathcal{Y}_{dd}\zeta_3}{5} + \frac{641\mathcal{Y}_{dd}}{60} + \frac{28\mathcal{Y}_{uu}\zeta_3}{5} - \frac{931\mathcal{Y}_{uu}}{60}\right) - \frac{6}{\hat\lambda}\mathcal{Y}_{ll}\mathcal{Y}_{ud} \\
& + a_1^3\left(-\frac{7n_G^2}{18} + n_G\left(\frac{57\zeta_3}{50} - \frac{1523}{600}\right) - \frac{1863\zeta_3}{4000} - \frac{9323}{4000}\right) + \frac{9a_1\mathcal{Y}_l^2}{40} \\
& + \frac{a_2 a_s}{\hat\lambda}\left(-12\zeta_3(\mathcal{Y}_{dd} + \mathcal{Y}_{uu}) + \frac{31(\mathcal{Y}_{dd} + \mathcal{Y}_{uu})}{4} - 48\mathcal{Y}_{ud}\zeta_3 + 4\mathcal{Y}_{ud}\right) \\
& + a_2^3\left(-\frac{35n_G^2}{18} - n_G\left(\frac{45\zeta_3}{2} + \frac{4163}{144}\right) - \frac{3807\zeta_3}{32} + \frac{53\,563}{1152}\right) \\
& + \frac{a_1 a_2 a_s}{\hat\lambda}\left(\frac{54}{5}\zeta_3(\mathcal{Y}_d + \mathcal{Y}_u) - \frac{699\mathcal{Y}_d}{40} - \frac{747\mathcal{Y}_u}{40}\right) - 90a_2\hat\lambda^2\zeta_3 \\
& + \frac{a_1^2 a_s}{\hat\lambda}\left(\frac{81}{25}\zeta_3(\mathcal{Y}_d + \mathcal{Y}_u) - \frac{2049\mathcal{Y}_d}{400} - \frac{1761\mathcal{Y}_u}{400}\right) + 324\hat\lambda\mathcal{Y}_{ud}\zeta_3 \\
& + a_1 a_s\left(-6\mathcal{Y}_d\zeta_3 + \frac{991\mathcal{Y}_d}{120} - \frac{102\mathcal{Y}_u\zeta_3}{5} + \frac{2419\mathcal{Y}_u}{120}\right) + 162\hat\lambda\mathcal{Y}_d\mathcal{Y}_u \\
& + \frac{a_1 a_2^3}{\hat\lambda}\left(\frac{n_G^2}{9} + \frac{8341n_G}{2880} + \frac{243\zeta_3}{64} + \frac{54\,053}{11\,520}\right) + \frac{a_s}{\hat\lambda}\mathcal{Y}_{uud} + \frac{a_s}{\hat\lambda}\mathcal{Y}_{udd} \\
& + a_1^2 a_2\left(n_G\left(\frac{27\zeta_3}{50} - \frac{243}{80}\right) - \frac{1809\zeta_3}{800} - \frac{25\,767}{1600}\right) + \frac{9a_2\mathcal{Y}_l^2}{8} - \frac{41a_s\mathcal{Y}_{ud}}{2} \\
& + \frac{a_1 a_2^2}{\hat\lambda}n_G\left(\frac{3\mathcal{Y}_l}{20} - 3\left(\frac{\mathcal{Y}_d}{4} + \frac{\mathcal{Y}_u}{20}\right)\right) + \frac{a_1^2 a_2}{\hat\lambda}n_G\left(\frac{3\mathcal{Y}_l}{20} - 3\left(\frac{\mathcal{Y}_d}{4} + \frac{\mathcal{Y}_u}{20}\right)\right) \\
& + a_1 a_2^2\left(n_G\left(\frac{9\zeta_3}{10} - \frac{27}{8}\right) + \frac{279\zeta_3}{160} - \frac{3849}{128}\right) - \frac{21\mathcal{Y}_l\mathcal{Y}_{ud}}{4} - 90\hat\lambda\mathcal{Y}_{ll}\zeta_3 \\
& + a_1^2 n_G\left(\frac{129\hat\lambda}{20} + \frac{31\mathcal{Y}_d}{80} + \frac{117\mathcal{Y}_l}{80} + \frac{127\mathcal{Y}_u}{80}\right) + 24a_s\mathcal{Y}_{ud}\zeta_3 + \frac{3145\mathcal{Y}_{lll}}{32} \\
& + a_s^2\left(16n_G(\mathcal{Y}_d + \mathcal{Y}_u) + 12\zeta_3(\mathcal{Y}_d + \mathcal{Y}_u) - \frac{455(\mathcal{Y}_d + \mathcal{Y}_u)}{3}\right) \\
& - \frac{a_1^3}{\hat\lambda}n_G\left(\frac{57\mathcal{Y}_d}{100} + \frac{99\mathcal{Y}_l}{100} + \frac{129\mathcal{Y}_u}{100}\right) - 1008\hat\lambda^3\zeta_3 + 9\hat\lambda\mathcal{Y}_l^2 - 28\mathcal{Y}_l\mathcal{Y}_{ll} \\
& + \frac{a_2^2}{\hat\lambda}n_G\left(\frac{39(\mathcal{Y}_{dd} + \mathcal{Y}_{uu})}{8} + \frac{13\mathcal{Y}_{ll}}{8} + 6\mathcal{Y}_{ud}\right) + a_1^2 n_G + \frac{a_1^2}{\hat\lambda} + a_1 a_2^2
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{a_1^2}{\hat{\lambda}}n_G\left(\frac{83\mathcal{Y}_{dd}}{40}-\frac{39\mathcal{Y}_{ll}}{40}+\frac{23\mathcal{Y}_{uu}}{40}\right)+a_1^2a_s+a_1^2a_2+a_1n_G\\
&+a_2^2n_G\left(\frac{129\hat{\lambda}}{4}+\frac{63(\mathcal{Y}_d+\mathcal{Y}_u)}{16}+\frac{21\mathcal{Y}_l}{16}\right)+\frac{a_1}{\hat{\lambda}}+a_1a_s-1281\hat{\lambda}^3\\
&-\frac{27a_2^2}{\hat{\lambda}}\left(\frac{1}{32}\left(\mathcal{Y}_d^2+\mathcal{Y}_u^2\right)+\frac{\mathcal{Y}_d\mathcal{Y}_u}{16}\right)-\frac{20a_s^2}{\hat{\lambda}}n_G(\mathcal{Y}_{dd}+\mathcal{Y}_{uu})\\
&+\frac{a_2^2a_s}{\hat{\lambda}}\left(27\zeta_3(\mathcal{Y}_d+\mathcal{Y}_u)-\frac{651(\mathcal{Y}_d+\mathcal{Y}_u)}{16}\right)+\frac{407}{160}\frac{a_1^2}{\hat{\lambda}}\mathcal{Y}_{dd}\zeta_3\\
&+\frac{297}{16\hat{\lambda}}(\mathcal{Y}_d\mathcal{Y}_{lll}+\mathcal{Y}_{ddd}\mathcal{Y}_l+\mathcal{Y}_l\mathcal{Y}_{uuu}+\mathcal{Y}_{lll}\mathcal{Y}_u)+\frac{27a_1^2a_2}{\hat{\lambda}}\left(\frac{\mathcal{Y}_l\zeta_3}{20}+\frac{\mathcal{Y}_u\zeta_3}{50}\right)\\
&-18a_1\left(\hat{\lambda}\zeta_3(\hat{\lambda}+\mathcal{Y}_l)+\frac{\hat{\lambda}\mathcal{Y}_u\zeta_3}{5}\right)+81a_1\left(\frac{1}{40}\left(\mathcal{Y}_d^2+\mathcal{Y}_u^2\right)+\frac{\mathcal{Y}_d\mathcal{Y}_u}{20}\right)\\
&+a_2a_s\left(\frac{489(\mathcal{Y}_d+\mathcal{Y}_u)}{8}-54\zeta_3(\mathcal{Y}_d+\mathcal{Y}_u)\right)+a_2^2a_sn_G\left(18\zeta_3-\frac{135}{8}\right)\\
&-\frac{a_2^3}{\hat{\lambda}}n_G\left(\frac{27(\mathcal{Y}_d+\mathcal{Y}_u)}{4}+\frac{9\mathcal{Y}_l}{4}\right)+81a_2\left(\frac{1}{8}\left(\mathcal{Y}_d^2+\mathcal{Y}_u^2\right)+\frac{\mathcal{Y}_d\mathcal{Y}_u}{4}\right)\\
&+\frac{a_1^3a_s}{\hat{\lambda}}n_G\left(\frac{1683}{2000}-\frac{99\zeta_3}{125}\right)+81\left(\hat{\lambda}\left(\mathcal{Y}_d{}^2+\mathcal{Y}_u^2\right)+\frac{\mathcal{Y}_{lll}\zeta_3}{2}\right)\\
&-84(\mathcal{Y}_d\mathcal{Y}_{ll}+\mathcal{Y}_{dd}\mathcal{Y}_l+\mathcal{Y}_l\mathcal{Y}_{uu}+\mathcal{Y}_{ll}\mathcal{Y}_u)+\frac{a_2^3a_s}{\hat{\lambda}}n_G\left(\frac{153}{16}-9\zeta_3\right)\\
&+a_1^2a_sn_G\left(\frac{66\zeta_3}{25}-\frac{99}{40}\right)+\frac{18}{\hat{\lambda}}\zeta_3(\mathcal{Y}_{dddd}+\mathcal{Y}_{uddd}+\mathcal{Y}_{uuud}+\mathcal{Y}_{uuuu})\\
&+\frac{891}{16\hat{\lambda}}(\mathcal{Y}_d+\mathcal{Y}_u)(\mathcal{Y}_{ddd}+\mathcal{Y}_{uuu})-\frac{135}{16\hat{\lambda}}(\mathcal{Y}_d+\mathcal{Y}_u)(\mathcal{Y}_{udd}+\mathcal{Y}_{uud})\\
&+\frac{819}{32}\frac{a_2^2}{\hat{\lambda}}\zeta_3(\mathcal{Y}_{dd}+\mathcal{Y}_{uu})+\frac{27}{2}\frac{a_2}{\hat{\lambda}}\zeta_3(\mathcal{Y}_{ddd}+\mathcal{Y}_{uuu})-\frac{9}{16}\frac{a_2^2}{\hat{\lambda}}\mathcal{Y}_l(\mathcal{Y}_d+\mathcal{Y}_u)\\
&-\frac{297}{8}\frac{a_2^3}{\hat{\lambda}}\zeta_3(\mathcal{Y}_d+\mathcal{Y}_u)-\frac{81}{100}\frac{a_1^2a_2}{\hat{\lambda}}\mathcal{Y}_d\zeta_3+\frac{2229}{80}\frac{a_1a_2}{\hat{\lambda}}\mathcal{Y}_{uu}\zeta_3+\frac{3}{8}\frac{a_1a_2}{\hat{\lambda}}\mathcal{Y}_d\mathcal{Y}_l\\
&+\frac{1143}{80}\frac{a_1a_2}{\hat{\lambda}}\mathcal{Y}_{ll}\zeta_3+\frac{933}{80}\frac{a_1a_2}{\hat{\lambda}}\mathcal{Y}_{dd}\zeta_3-\frac{81}{40}\frac{a_1a_2^2}{\hat{\lambda}}\mathcal{Y}_u\zeta_3-\frac{27}{10}\frac{a_1a_2^2}{\hat{\lambda}}\mathcal{Y}_d\zeta_3\\
&-\frac{93}{20}\frac{a_1a_2}{\hat{\lambda}}\mathcal{Y}_{ud}\zeta_3+\frac{9}{20}\frac{a_1a_2^2}{\hat{\lambda}}\mathcal{Y}_l\zeta_3-\frac{87}{40}\frac{a_1a_2}{\hat{\lambda}}\mathcal{Y}_l\mathcal{Y}_u+\frac{63}{40}\frac{a_1a_2}{\hat{\lambda}}\mathcal{Y}_d\mathcal{Y}_u\\
&-\frac{70\,563}{12\,800}\frac{a_1^2a_2}{\hat{\lambda}}\mathcal{Y}_u-\frac{59\,913}{12\,800}\frac{a_1^2a_2}{\hat{\lambda}}\mathcal{Y}_l-\frac{39\,627}{12\,800}\frac{a_1^2a_2}{\hat{\lambda}}\mathcal{Y}_d-\frac{48a_s^2}{\hat{\lambda}}\left(\mathcal{Y}_d^2+\mathcal{Y}_u^2\right)\\
&-\frac{13\,437}{256}\frac{a_2^2}{\hat{\lambda}}(\mathcal{Y}_{dd}+\mathcal{Y}_{uu})-\frac{3411}{64}\frac{a_2}{\hat{\lambda}}(\mathcal{Y}_{ddd}+\mathcal{Y}_{uuu})+\frac{17\,217}{512}\frac{a_2^3}{\hat{\lambda}}(\mathcal{Y}_d+\mathcal{Y}_u)\\
&+a_1^2\left(\frac{561}{400}-\frac{33\zeta_3}{25}\right)-\frac{477}{64}\frac{a_2}{\hat{\lambda}}(\mathcal{Y}_{udd}+\mathcal{Y}_{uud})-\frac{12\,537}{2560}\frac{a_1a_2^2}{\hat{\lambda}}\mathcal{Y}_d
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{36}{\hat{\lambda}}\big(\mathcal{Y}_{ud}^2+\mathcal{Y}_{uudd}\zeta_3\big)-\frac{9309}{2560}\frac{a_1a_2^2}{\hat{\lambda}}\mathcal{Y}_u-\frac{5499}{2560}\frac{a_1a_2^2}{\hat{\lambda}}\mathcal{Y}_l\\
&+\frac{457}{3}\frac{a_s^2}{\hat{\lambda}}(\mathcal{Y}_{dd}+\mathcal{Y}_{uu})-\frac{45}{16}\frac{}{\hat{\lambda}}\mathcal{Y}_l(\mathcal{Y}_{udd}+\mathcal{Y}_{uud})+\frac{6}{\hat{\lambda}}\big(\mathcal{Y}_{ll}{}^2+\mathcal{Y}_{llll}\zeta_3\big)\\
&-\frac{120a_s}{\hat{\lambda}}\zeta_3(\mathcal{Y}_{ddd}+\mathcal{Y}_{uuu})-\frac{2957}{800}\frac{a_1^2}{\hat{\lambda}}\mathcal{Y}_{uu}\zeta_3-\frac{2103}{400}\frac{a_1^2}{\hat{\lambda}}\mathcal{Y}_l\mathcal{Y}_u\\
&+\frac{24a_s}{\hat{\lambda}}\zeta_3(\mathcal{Y}_{udd}+\mathcal{Y}_{uud})-\frac{16a_s^2}{\hat{\lambda}}\zeta_3(\mathcal{Y}_{dd}+\mathcal{Y}_{uu})+\frac{2591}{640}\frac{a_1a_2}{\hat{\lambda}}\mathcal{Y}_{dd}\\
&+\frac{1871}{640}\frac{a_1a_2}{\hat{\lambda}}\mathcal{Y}_{uu}-\frac{123}{400}\frac{a_1^2}{\hat{\lambda}}\mathcal{Y}_d\mathcal{Y}_l+\frac{549}{640}\frac{a_1a_2}{\hat{\lambda}}\mathcal{Y}_{ll}+\frac{51}{400}\frac{a_1^2}{\hat{\lambda}}\mathcal{Y}_d\mathcal{Y}_u\\
&+\frac{273}{32}\frac{a_2^2}{\hat{\lambda}}\mathcal{Y}_{ll}\zeta_3-\frac{135}{32}\frac{a_1^2}{\hat{\lambda}}\mathcal{Y}_{ll}\zeta_3+a_1\left(\frac{153}{80}-\frac{9\zeta_3}{5}\right)-\frac{63}{80}\frac{a_1a_2}{\hat{\lambda}}\mathcal{Y}_l^2\\
&-\frac{99}{80}\frac{a_1a_2}{\hat{\lambda}}\mathcal{Y}_u^2+\frac{81}{80}\frac{a_1a_2}{\hat{\lambda}}\mathcal{Y}_d^2+\frac{81}{200}\frac{a_1^3}{\hat{\lambda}}\mathcal{Y}_l\zeta_3-\frac{711}{16}a_2\hat{\lambda}(\mathcal{Y}_d+\mathcal{Y}_u)\\
&+\frac{459}{8}a_2^2\zeta_3(\mathcal{Y}_d+\mathcal{Y}_u)-\frac{351}{2}a_2\zeta_3(\mathcal{Y}_{dd}+\mathcal{Y}_{uu})-\frac{301}{64}\frac{a_1a_2}{\hat{\lambda}}\mathcal{Y}_{ud}\\
&-\frac{27}{200}\frac{a_1^3}{\hat{\lambda}}\mathcal{Y}_d\zeta_3+\frac{27}{100}\frac{a_1^3}{\hat{\lambda}}\mathcal{Y}_u\zeta_3-\frac{117}{4}\frac{a_2^2}{\hat{\lambda}}\mathcal{Y}_{ud}\zeta_3-\frac{99}{10}\frac{a_1}{\hat{\lambda}}\mathcal{Y}_{lll}\zeta_3+\frac{9}{50}\frac{a_1^2}{\hat{\lambda}}\mathcal{Y}_{ud}\zeta_3\\
&-\frac{51}{10}\frac{a_1}{\hat{\lambda}}\mathcal{Y}_{uuu}\zeta_3+\frac{27}{20}a_1\mathcal{Y}_l(\mathcal{Y}_d+\mathcal{Y}_u)-288a_s\hat{\lambda}\zeta_3(\mathcal{Y}_d+\mathcal{Y}_u)-\frac{99}{8}\frac{a_2^3}{\hat{\lambda}}\mathcal{Y}_l\zeta_3\\
&+\frac{42}{5}\frac{a_1}{\hat{\lambda}}\mathcal{Y}_{uud}\zeta_3-\frac{39}{5}\frac{a_1}{\hat{\lambda}}\mathcal{Y}_{udd}\zeta_3+\frac{27}{4}a_2\mathcal{Y}_l(\mathcal{Y}_d+\mathcal{Y}_u)+\frac{15}{2}\frac{a_1}{\hat{\lambda}}\mathcal{Y}_{ddd}\zeta_3\\
&+54a_2\hat{\lambda}\zeta_3(\mathcal{Y}_d+\mathcal{Y}_u)+\frac{9}{2}\frac{a_2}{\hat{\lambda}}\mathcal{Y}_{lll}\zeta_3-\frac{711}{20}a_1a_2\mathcal{Y}_u\zeta_3-\frac{243}{10}a_1a_2\mathcal{Y}_l\zeta_3\\
&+\frac{297}{5}a_1a_2\hat{\lambda}\zeta_3+\frac{249}{16\hat{\lambda}}(\mathcal{Y}_{uddd}+\mathcal{Y}_{uuud})-\frac{20\,681}{19\,200}\frac{a_1^2}{\hat{\lambda}}\mathcal{Y}_{dd}-\frac{18}{5}a_1a_2\mathcal{Y}_d\zeta_3\\
&-\frac{128\,829}{64\,000}\frac{a_1^3}{\hat{\lambda}}\mathcal{Y}_u-\frac{106\,083}{64\,000}\frac{a_1^3}{\hat{\lambda}}\mathcal{Y}_l-\frac{36\,129}{64\,000}\frac{a_1^3}{\hat{\lambda}}\mathcal{Y}_d-\frac{11\,269}{3840}\frac{a_1^2}{\hat{\lambda}}\mathcal{Y}_{uu}\\
&+\frac{54}{\hat{\lambda}}\big(\mathcal{Y}_{dd}^2+\mathcal{Y}_{uu}^2\big)-\frac{6669}{1280}\frac{a_1^2}{\hat{\lambda}}\mathcal{Y}_{ll}+\frac{5973}{3200}\frac{a_1^2}{\hat{\lambda}}\mathcal{Y}_{ud}-\frac{39}{4\hat{\lambda}}(\mathcal{Y}_{dddd}+\mathcal{Y}_{uuuu})\\
&-\frac{2241}{800}\frac{a_1^2}{\hat{\lambda}}\mathcal{Y}_l^2-\frac{1857}{800}\frac{a_1^2}{\hat{\lambda}}\mathcal{Y}_u^2+\frac{7101}{256}a_2^2(\mathcal{Y}_d+\mathcal{Y}_u)-\frac{4479}{256}\frac{a_2^2}{\hat{\lambda}}\mathcal{Y}_{ll}\\
&+\frac{231}{800}\frac{a_1^2}{\hat{\lambda}}\mathcal{Y}_d^2-252(\mathcal{Y}_d+\mathcal{Y}_u)(\mathcal{Y}_{dd}+\mathcal{Y}_{uu})+\frac{19a_s}{\hat{\lambda}}(\mathcal{Y}_{ddd}+\mathcal{Y}_{uuu})\\
&+\frac{6453}{32}a_2(\mathcal{Y}_{dd}+\mathcal{Y}_{uu})+\frac{5739}{512}\frac{a_2^3}{\hat{\lambda}}\mathcal{Y}_l+\frac{567}{128}\frac{a_2^2}{\hat{\lambda}}\mathcal{Y}_{ud}-\frac{5111}{320}\frac{a_1}{\hat{\lambda}}\mathcal{Y}_{ddd}\\
&-\frac{3467}{320}\frac{a_1}{\hat{\lambda}}\mathcal{Y}_{uuu}+\frac{2299}{320}\frac{a_1}{\hat{\lambda}}\mathcal{Y}_{udd}-\frac{1337}{320}\frac{a_1}{\hat{\lambda}}\mathcal{Y}_{uud}+\frac{36}{\hat{\lambda}}\mathcal{Y}_{ll}(\mathcal{Y}_{dd}+\mathcal{Y}_{uu})\\
&-\frac{18}{\hat{\lambda}}\mathcal{Y}_{ud}(\mathcal{Y}_{dd}+\mathcal{Y}_{uu})-\frac{3735}{8}\hat{\lambda}(\mathcal{Y}_{dd}+\mathcal{Y}_{uu})+\frac{243}{2}\zeta_3(\mathcal{Y}_{ddd}+\mathcal{Y}_{uuu})
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1449}{4}\hat{\lambda}^2(\mathcal{Y}_d+\mathcal{Y}_u)-\frac{1343}{4}a_s(\mathcal{Y}_{dd}+\mathcal{Y}_{uu})-\frac{1137}{64}\frac{a_2}{\hat{\lambda}}\mathcal{Y}_{lll}-\frac{3}{32}\frac{a_2^2}{\hat{\lambda}}\mathcal{Y}_l^2\\
&+\frac{29\,223a_1a_2\mathcal{Y}_u}{640}+\frac{2247}{200}a_1^2\mathcal{Y}_u\zeta_3+\frac{2079}{200}a_1^2\mathcal{Y}_l\zeta_3-270\hat{\lambda}\zeta_3(\mathcal{Y}_{dd}+\mathcal{Y}_{uu})\\
&+\frac{15\,633a_1a_2\mathcal{Y}_l}{640}+\frac{15\,459a_1a_2\mathcal{Y}_d}{640}+\frac{10\,881a_1a_2\hat{\lambda}}{160}+468a_s\zeta_3(\mathcal{Y}_{dd}+\mathcal{Y}_{uu})\\
&+\frac{99}{16\hat{\lambda}}\mathcal{Y}_l\mathcal{Y}_{lll}-\frac{81}{64}\frac{a_1}{\hat{\lambda}}\mathcal{Y}_{lll}+\frac{177}{200}a_1^2\mathcal{Y}_d\zeta_3-\frac{96a_s^2}{\hat{\lambda}}\mathcal{Y}_d\mathcal{Y}_u+306a_s\hat{\lambda}(\mathcal{Y}_d+\mathcal{Y}_u)\\
&-\frac{63}{4}\mathcal{Y}_{ud}(\mathcal{Y}_d+\mathcal{Y}_u)+\frac{567}{50}a_1^2\hat{\lambda}\zeta_3+\frac{1599a_1\hat{\lambda}\mathcal{Y}_l}{80}-\frac{117}{2}a_2\mathcal{Y}_{ll}\zeta_3\\
&-\frac{1323}{80}a_1\hat{\lambda}\mathcal{Y}_d+54\hat{\lambda}\mathcal{Y}_l(\mathcal{Y}_d+\mathcal{Y}_u)-\frac{531}{10}a_1\mathcal{Y}_{dd}\zeta_3+\frac{513a_1\hat{\lambda}\mathcal{Y}_u}{80}\\
&+\frac{351}{2}a_2^2\hat{\lambda}\zeta_3+\frac{243a_1\mathcal{Y}_{ll}\zeta_3}{10}-\frac{237}{16}a_2\hat{\lambda}\mathcal{Y}_l+\frac{153}{8}a_2^2\mathcal{Y}_l\zeta_3-\frac{81}{2}a_2\mathcal{Y}_{ud}\zeta_3\\
&+\frac{3a_1\mathcal{Y}_{ud}\zeta_3}{10}-\frac{27}{2}a_1\mathcal{Y}_{uu}\zeta_3+18a_2\hat{\lambda}\mathcal{Y}_l\zeta_3+18a_1\hat{\lambda}\mathcal{Y}_d\zeta_3\\
&+\frac{175\,399a_1^2\mathcal{Y}_d}{19\,200}+\frac{101\,791a_1^2\mathcal{Y}_u}{19\,200}+\frac{9435(\mathcal{Y}_{ddd}+\mathcal{Y}_{uuu})}{32}+\frac{43\,011a_1^2\hat{\lambda}}{1600}\\
&-\frac{183}{32}(\mathcal{Y}_{udd}+\mathcal{Y}_{uud})-\frac{369a_1^2\mathcal{Y}_l}{6400}+\frac{2367a_2^2\mathcal{Y}_l}{256}-54\zeta_3(\mathcal{Y}_{udd}+\mathcal{Y}_{uud})\\
&+\frac{7949a_1\mathcal{Y}_{dd}}{160}-\frac{375}{8\hat{\lambda}}\mathcal{Y}_{uudd}+\frac{3531a_2^2\hat{\lambda}}{64}+\frac{3353a_1\mathcal{Y}_{uu}}{160}+\frac{108}{\hat{\lambda}}\mathcal{Y}_{dd}\mathcal{Y}_{uu}\\
&+\frac{411a_1\hat{\lambda}^2}{10}-\frac{33}{4\hat{\lambda}}\mathcal{Y}_{udud}+\frac{2151a_2\mathcal{Y}_{ll}}{32}-\frac{13}{4\hat{\lambda}}\mathcal{Y}_{llll}+\frac{495a_2\mathcal{Y}_{ud}}{16}-\frac{483\hat{\lambda}^2\mathcal{Y}_l}{4}\\
&-\frac{441a_1\mathcal{Y}_{ll}}{32}+\frac{411a_2\hat{\lambda}^2}{2}-\frac{1245\hat{\lambda}\mathcal{Y}_{ll}}{8}+\frac{121a_1\mathcal{Y}_{ud}}{16}-\frac{1197\hat{\lambda}\mathcal{Y}_{ud}}{4},
\end{aligned}
\tag{15}
$$

where the following notation was used for the gauge couplings:

$$a_i=\left(\frac{5}{3}\frac{g_1^2}{16\pi^2},\frac{g_2^2}{16\pi^2},\frac{g_s^2}{16\pi^2}\right). \tag{16}$$

Let us also mention that the results for $\beta_{\hat{\lambda}}$, γ_{m^2} and γ_v are independent of gauge-fixing parameters and the corresponding renormalization constants satisfy the so-called pole equations [29]. This serves as a crucial test of the correctness of the calculated three-loop contributions.

It is worth pointing that the expressions (13)–(15) do not coincide with the anomalous dimension[5] of the Higgs doublet

$$\gamma_\Phi=-\frac{1}{2}\frac{d\ln Z_h}{d\ln\mu^2}. \tag{17}$$

The latter, if taken in the Landau gauge (see, e.g., Refs. [30,31] for details), corresponds the anomalous dimension of VEV obtained via minimization of the effective potential [32–34].

[5] The corresponding expression can also be found in ancillary files of the arXiv version of the paper.

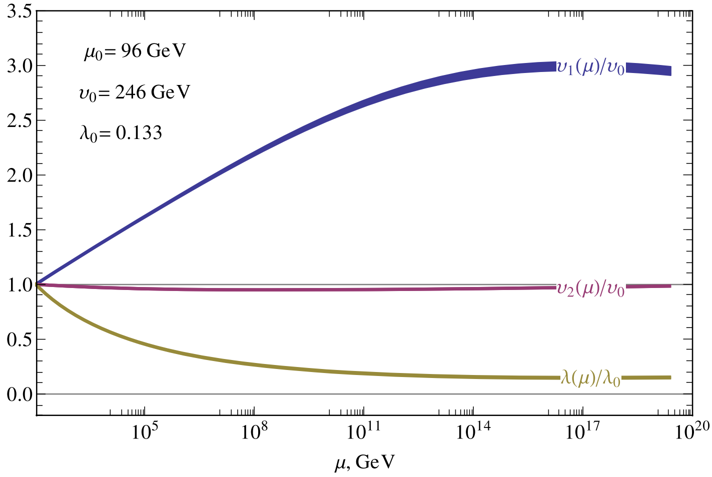

Fig. 2. The scale dependence of $v_1(\mu) \equiv \sqrt{m^2(\mu)/\lambda(\mu)}$ and $v_2(\mu)$, which minimizes the SM effective potential. The width of the curves corresponds to the difference between two- and three-loop running. All parameters are normalized by their initial values at the scale μ_0 and we assume that $v_1(\mu_0) = v_2(\mu_0) = v_0$.

In Fig. 2 one can see an example of the VEV running driven by two different anomalous dimensions: $v_1(\mu)$ by γ_v from Eqs. (13)–(15) and $v_2(\mu)$ by γ_Φ in the Landau gauge. The initial scale is chosen to be $\mu_0 \simeq 96$ GeV [35] at which one expects the threshold corrections for $v_1(\mu)$ to be small. The boundary values for the couplings are also taken from Ref. [36] and we made the assumption that $v_2(\mu_0) = v_1(\mu_0) = v_0 \simeq 246$ GeV. For convenience, we divide all the running quantities in Fig. 2 by their boundary values. It is clear that $v_1(\mu)$ increases significantly with μ, while the scale dependence of $v_2(\mu)$ is rather smooth. This is due to the fact that the anomalous dimension of $v_1(\mu)$ is correlated with $-\beta_\lambda/\lambda$, and at μ_0 we have a large positive contribution $-\beta_\lambda(\mu_0)/\lambda_0 \simeq 0.08$ to γ_v. In Fig. 3 the scale dependence of $\lambda(\mu)$ and $\beta_\lambda(\mu)$ is presented. In addition, we plot the second derivate $\ddot{\lambda}(\mu)$, which can be of some interest in scenarios with $\lambda = \beta_\lambda = 0$ at some scale. From Fig. 3 one can see that for a chosen set of initial parameters [36] the beta-function β_λ reaches zero at 10^{17} GeV, while λ and $\ddot{\lambda}$ are still positive at this scale.

It is fair to mention that different implementation [6] of threshold corrections, which relate the $\overline{\text{MS}}$ parameters to some measured quantities, leads to a different boundary value of the top Yukawa coupling. The latter drives λ to negative values at the scales of order 10^{10} GeV rendering the SM vacuum unstable. Since, in our opinion, both procedures,[6] if implemented consistently, should render the same values for dimensionless couplings, this discrepancy requires further investigation.

To conclude, by explicit calculation we extended our results presented in Ref. [18] to the case of complex Yukawa matrices. We also provided the anomalous dimensions γ_Φ and γ_v of the Higgs doublet and the running vacuum expectation value (VEV) defined as $v \equiv \sqrt{m^2/\lambda}$, respectively. In addition, the scale dependence of the considered quantities are studied numerically.

[6] The essential difference in matching procedures of Refs. [36] and [6] stems from the way one treats the so-called tadpole contributions [37] or, in other words, whether $v_1(\mu)$ or $v_2(\mu)$ is used in the relations between $\overline{\text{MS}}$-running masses and couplings.

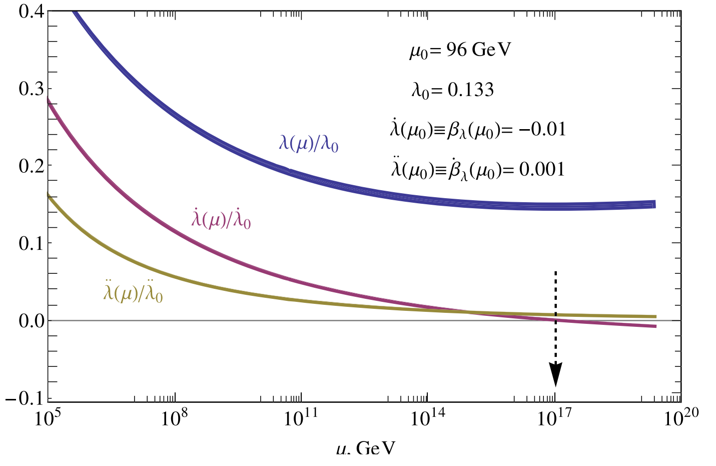

Fig. 3. The running of the Higgs self-coupling is given together with the scale dependence of its first (β_λ) and second derivatives. The width of the curves corresponds to the difference between two- and three-loop running. All parameters are normalized by their initial values at the scale μ_0. The arrow points to the scale at which $\beta_\lambda = 0$.

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