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Physics Letters B 639 (2006) 465–470
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Realistic few-body physics in the $dd \to \alpha\pi^0$ reaction

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Received 13 March 2006; received in revised form 10 April 2006; accepted 20 April 2006

Available online 21 July 2006

Editor: J.-P. Blaizot

Abstract

We use realistic two- and three-nucleon interactions in a hybrid chiral-perturbation-theory calculation of the charge-symmetry-breaking reaction $dd \to \alpha\pi^0$ to show that a cross section of the experimentally measured size can be obtained using LO and NNLO pion-production operators. This result supports the validity of our power counting scheme and demonstrates the necessity of using an accurate treatment of ISI and FSI. It also becomes evident that a full calculation requires the use of consistent chiral nuclear forces to overcome the visible model dependence of our result.

1. For most purposes, hadronic isospin states can be considered as charge symmetric, i.e., invariant under a rotation by 180° around the 2-axis in isospin space. Charge symmetry (CS) is a subset of the general isospin symmetry, charge independence (CI), which requires invariance under any rotation in isospin space. In quantum chromodynamics (QCD), CS implies that dynamics are invariant under the exchange of the up and down quarks [1]. However, since the up and down quarks do have different masses ($m_u \neq m_d$) [2,3], the QCD Lagrangian is not charge symmetric. This symmetry violation is called charge symmetry breaking (CSB). The different electromagnetic interactions of the up and down quarks break CI. Observing the effects of CSB interactions therefore provides a probe of m_u and m_d.

Two exciting recent observations of CSB in experiments involving the production of neutral pions stimulate our attention. Many years of effort led to the observation of CSB in $np \to d\pi^0$ at TRIUMF. The CSB forward–backward asymmetry of the differential cross section was found to be $A_{\text{fb}} = [17.2 \pm 8(\text{stat}) \pm 5.5(\text{sys})] \times 10^{-4}$ [4]. In addition, the final experiment at the IUCF Cooler ring reported a very convincing $dd \to \alpha\pi^0$ signal near threshold ($\sigma = 12.7 \pm 2.2$ pb at $T_d = 228.5$ MeV and 15.1 ± 3.1 pb at 231.8 MeV) [5]. The $dd \to \alpha\pi^0$ reaction violates CS since the deuterons and the α-particle are self-conjugate under the CS operator, with a positive eigenvalue, while the neutral pion wave function changes sign.

The study of CSB π^0 production reactions presents an exciting new opportunity to learn about the influence of quark masses in nuclear physics, and to use effective field theory (EFT) to improve our understanding of how QCD works [6]. This is because chiral symmetry of QCD determines the form of pionic interactions. Electromagnetic CSB is typically of the same order of magnitude as the strong one, and also can be handled using EFT.

The EFT for the Standard Model at momenta comparable to the pion mass, $Q \sim m_\pi$, is chiral perturbation theory (χPT) [7].

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0370-2693
doi:10.1016/j.physletb.2006.04.058

This EFT has been extended [8–13] to momenta relevant to pion production, $Q \sim \sqrt{m_\pi M}$ with M the nucleon mass. (For a review and further references, see Ref. [13].)

EFT with the operators of Ref. [14] was used to correctly predict the sign of the forward–backward asymmetry in $np \to d\pi^0$ [9]. For the $dd \to \alpha\pi^0$ reaction, we surveyed various mechanisms using initial-state plane-wave functions and simplified final-state wave functions [10]. In this simplified model, we found that the formally leading-order (LO) production mechanism is suppressed through symmetries in the wave functions and studied other mechanisms. The contributions from next-to-next-to-leading-order (NNLO) diagrams are too small to account for the observed cross section—a cross section of only 0.9 pb was found. We also included short-range pion emission, which contributes at N^{4}LO through contact vertices whose strengths are a priori unknown. We used resonance saturation, by means of CSB effects in Z-diagrams, as motivated by a successful phenomenological model [15] of the charge-symmetry-conserving (CSC) reaction $pp \to pp\pi^0$. For the simplified wave functions, we then found a cross section of the observed order of magnitude.

Our aim here is to take advantage of recent significant advances in four-body theory [16,17] that allow us to include the effects of deuteron–deuteron interactions in the initial state, and to use bound-state wave functions with realistic two- and three-nucleon interactions. The calculations presented here are hybrid: the pion-production operators are constructed using EFT, but the nuclear interactions used to obtain the wave functions are not. No calculation of this kind can be considered to be completely well-founded unless the operators and wave functions are constructed from the same convergent EFT. However, the interactions do include one-pion exchange, so their long-range behavior is founded in EFT. Moreover, we employ several potentials to gauge the sensitivity of the various production operators to the shorter-ranged parts of the interaction. Indeed, we will show below, that there is a visible model dependence for some of the operators. Ultimately, we will therefore require consistent, chiral nuclear forces for the calculation of the wave functions.

The present study does not include all diagrams appearing at NNLO. A complete analysis demands the inclusion of loop diagrams. Their evaluation requires a careful treatment of divergences as pointed out in Ref. [11], and understood only recently [12]. For technical reasons, photon exchange is so far only considered in the final-state wave function. In this Letter, we concentrate on the important effects of initial- and final-state interactions (ISI and FSI), and anticipate that the use of the present incomplete set of operators should be sufficient to get order-of-magnitude estimates and to demonstrate the technique.

2. We summarize the power counting of the previous study [10], which contains explicit expressions for the operators. At LO, there is only one contribution, represented by Fig. 1a: pion rescattering in which the CSB occurs through the seagull pion-nucleon terms linked to the nucleon-mass splitting. This contribution stems from the chiral transformation properties of the quark operators that generate CSB, which are two: (i) the up-down mass difference, which breaks chiral symmetry as a component of a chiral four-vector; (ii) electromagnetic quark interactions, which break chiral symmetry as components of a chiral anti-symmetric rank-two tensor. In lowest order, there exist two seagull operators involving a nucleon interacting with two pions, one of which is neutral. Their strengths are determined by the quark-mass and electromagnetic contributions to the nucleon mass splitting, δm_N and $\bar{\delta} m_N$, respectively. δm_N is proportional to $\varepsilon(m_u + m_d)$, where $\varepsilon \equiv (m_u - m_d)/(m_u + m_d) \approx 1/3$, while $\bar{\delta} m_N$ is the fine-structure constant α times a typical hadronic mass. The only existing constraint on these two terms is $\delta m_N + \bar{\delta} m_N = M_n - M_p = 1.29$ MeV and the model dependent estimate $\bar{\delta} m_N = -(0.76 \pm 0.3)$ MeV based on the Cottingham sum rule [21]. Verifying the theory requires that the two terms be constrained independently. The most natural reaction to study is πN scattering. Ref. [2] predicted a significant difference between the $\pi^0 p$ and $\pi^0 n$ scattering lengths that is not presently observable, as discussed in Ref. [18]. Effects of these terms in the nuclear potential [19] are relatively small or suffer from other unknowns as in πd scattering [20]. This leaves the investigation of CSB in the two reactions, $np \to d\pi^0$ and $dd \to \alpha\pi^0$, as very promising possibilities. For definiteness, in this Letter we use the central value of the estimate from the Cottingham sum rule. The leading diagram, Fig. 1(a), is $O[\varepsilon m_\pi^2/(f_\pi^3 M Q)]$, where $f_\pi = 92.4$ MeV denotes the pion decay constant and $Q \approx \sqrt{m_\pi M}$ a typical momentum.

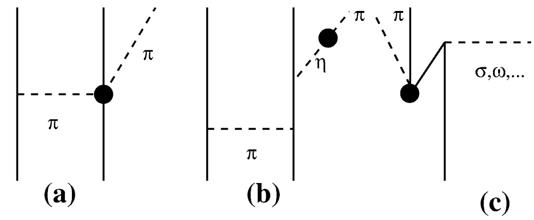

Fig. 1. Diagrams of $np \to d\pi^0$; the solid circle indicates CSB.

We refer to this contribution as “pion exchange”. For completeness, we distinguish the parts proportional to δm_N and $\bar{\delta} m_N$ and denote the contributions by $\mathcal{M}_{\mathrm{PE}} = \mathcal{M}_{\mathrm{PE},\delta m_N} + \mathcal{M}_{\mathrm{PE},\bar{\delta} m_N}$.

There is no NLO contribution. At NNLO, suppressed by $O(m_\pi/M)$, there exists a recoil correction of the LO term (labeled $\mathcal{M}_{\mathrm{rec}} = \mathcal{M}_{\mathrm{rec},\delta m_N} + \mathcal{M}_{\mathrm{rec},\bar{\delta} m_N}$). Its strength is also determined by δm_N and $\bar{\delta} m_N$. Therefore, its contribution allows us to estimate the size of NNLO contributions. The recoil correction to the πNN vertex is linear in the energy of the virtual pion. Such operators were studied in Ref. [22] for the reactions $NN \to NN\pi$, and we use the prescription provided there and applied in Ref. [10]. Demonstrating the validity of this recipe for a four-body environment deserves further study.

At the same order new parameters appear. In particular, a term arises in which a one-body CSB operator ($\propto \beta_1 + \bar{\beta}_3$) is sandwiched between initial- and final-state wave functions, as illustrated in, e.g., Fig. 1(b). We refer to this as the one-body term ($\mathcal{M}_{1b}$). The terms $\beta_1 = O(\epsilon m_\pi^2/M^2)$ and $\bar{\beta}_3 = O(\alpha/\pi)$ arise from, respectively, the quark-mass-difference and electromagnetic contributions to the isospin-violating pion–nucleon coupling. Neither β_1 nor $\bar{\beta}_3$ can be extracted from experiment yet. To allow us to provide numerical results we estimate these terms by modeling [23] β_1 by π-η mixing, see Fig. 1(b),

$$\beta_1 = \bar{g}_\eta \langle \pi^0 | H | \eta \rangle / m_\eta^2, \tag{1}$$

where $\langle \pi^0 | H | \eta \rangle = -4200\ \text{MeV}^2$ is the π–η-mixing matrix element [24], and $\bar{g}_\eta = g_{\eta NN} f_\pi / M$ the η-nucleon coupling constant. An early analysis [25] using one-boson-exchange potentials in NN scattering gave $g_{\eta NN}^2/4\pi = 3.86$ (used in Ref. [9]), but the data show little sensitivity to η exchange and high-accuracy fits can be achieved [26] using $g_{\eta NN}^2/4\pi = 0$. Indeed, the possibility of a vanishing coupling constant had been raised earlier. The detailed analysis of NN total cross sections and $p\bar{p}$ data using dispersion relations [27] found that $g_{\eta NN}^2/4\pi = 0$. This is consistent with extractions from the nucleon pole in the amplitude $\pi N \to \eta N$ that give [28] $0.5 > g_{\eta NN}^2/4\pi \geqslant 0$. Photoproduction reactions on a nucleon [29] (see their Fig. 2) yield the small value $g_{\eta NN}^2/4\pi = 0.1$. To be consistent with our earlier study [10], we use $g_{\eta NN}^2/4\pi = 0.51$ for the results shown below, but also examine the effects of using $g_{\eta NN}^2/4\pi = 0.10$. Both values are roughly consistent with the size expected using power counting arguments [23]. We assume the sign predicted by $SU(3)$ symmetry, as in Ref. [10].

The effects of electromagnetic interactions as well as strong CSB were included in computing the α-particle wave functions, where the former effect is dominant. These interactions generate a small isospin $T = 1$ component of the wave function that enables a non-zero contribution of CSC production operators. To estimate the effects of the admixtures, we calculate the production matrix element using the CSC counterpart of diagram Fig. 1(b) (referred to as $\mathcal{M}_{\text{WF}}$).

A number of other CSB mechanisms enter at N^{3}LO or higher, including additional loop diagrams and short-range interactions. The lowest order where four-nucleon contact interactions start to contribute is N^{4}LO, that is, $O(m_\pi/M)$ below NNLO. To estimate their strength, Ref. [10] evaluated certain tree-level contributions as indicated by Fig. 1(c), which represents the exchange of heavy mesons (σ, ω, ρ) via a Z-graph mechanism, with π–η mixing generating CSB at pion emission ($\mathcal{M}_\sigma$, $\mathcal{M}_\omega$ and $\mathcal{M}_\rho$). Another Z-graph (labeled as $\mathcal{M}_{\rho\omega}$) arises in which the CSB occurs in the heavy-meson exchange via ρ–ω mixing along with strong pion emission at the vertex. The Z-graphs are believed to be important because their inclusion leads to a quantitative description of the total cross section for the reaction $pp \to pp\pi^0$ near threshold [15]. Our present results use the coupling constants and parameters of Ref. [10], see their Table 1. However, in the future it will be necessary to reassess the procedure in light of recent developments concerning the treatment of divergences in EFT loop diagrams [12].

3. The various mechanisms generate pion-production kernels that are sandwiched between final- and initial-state wave functions to provide a transition matrix element $\mathcal{M}$. We restrict our analysis to $T_d = 228.5$ MeV, as the effects of a small change in energy are captured mainly by the change in the phase-space factor. The cross section is related to the matrix elements by

$$\sigma = 4.303\ \text{pb}\ \left|\mathcal{M} \times 10^4\ \text{fm}^2\right|^2. \tag{2}$$

Table 1
Complex $dd \to \alpha\pi^0$ amplitudes at $T_d = 228.5$ MeV in units of $10^{-4}\ \text{fm}^{-2}$. PWA denotes the plane-wave approximation. ISI results also include the initial-state interaction

	PWA		ISI	
	CDB + TM99	AV18 + TM99	CDB + TM99	AV18 + TM99
$\mathcal{M}_{\text{PE},\delta m_N}$	0.35	−0.07	−1.51 + i1.87	−0.76 + i0.74
$\mathcal{M}_{\text{PE},\bar{\delta} m_N}$	0.06	−0.01	−0.28 + i0.35	−0.14 + i0.14
$\mathcal{M}_{\text{PE}}$	0.41	−0.08	−1.79 + i2.22	−0.90 + i0.88
$\mathcal{M}_{\text{rec},\delta m_N}$	0.41	0.34	−0.81 + i0.74	−0.63 + i0.59
$\mathcal{M}_{\text{rec},\bar{\delta} m_N}$	0.08	0.06	−0.15 + i0.14	−0.12 + i0.11
$\mathcal{M}_{\text{rec}}$	0.49	0.40	−0.96 + i0.88	−0.75 + i0.70
$\mathcal{M}_{1b}$	1.76	1.60	−2.51 + i1.84	−1.94 + i1.60
$\mathcal{M}_\sigma$	0.46	0.31	−0.56 + i0.64	−0.32 + i0.42
$\mathcal{M}_\omega$	0.51	0.38	−0.53 + i0.44	−0.35 + i0.34
$\mathcal{M}_\rho$	0.24	0.15	−0.33 + i0.34	−0.18 + i0.19
$\mathcal{M}_{\rho\omega}$	1.17	0.87	−1.32 + i1.51	−0.84 + i1.07
$\mathcal{M}_{\text{WF}}$	−0.15	−0.14	+0.51 − i0.13	+0.41 − i0.14

We present our new results in stages. First we introduce realistic bound-state wave functions, while continuing to use the plane-wave approximation (PWA). The techniques to solve the four-body problem have been presented by Nogga et al. [16]. To be specific, we present results using both the AV18 [30] and CD-Bonn 2000 [26] two-nucleon potentials combined with a properly adjusted Tucson–Melbourne (TM99) [31] three-nucleon force. The combination guarantees that the α-particle binding energy is reproduced with high accuracy. Additional calculations using the Urbana-IX [32] three-nucleon potential resulted in essentially identical results and will be presented elsewhere [33].

Table 1 summarizes our results for the transition amplitudes that add to $\mathcal{M}$, labeled according to the various mechanisms described above. The one-body term is predicted rather model independently. Using these matrix elements for the one-body operator leads to a cross section of 10–13 pb, which is accidentally in good agreement with the experiment. Compared to our toy-model calculation [10], we find an increase of the cross section by a factor of 10, showing that the high-momentum tail of the wave function is important, especially for the one-body term. Using the smaller, but also realistic, coupling $g_{\eta NN}^2/4\pi = 0.10$ would reduce the resulting cross section by a factor of 5. The one-body term is formally subleading. However, the toy-model calculation showed that the pion-exchange term is suppressed due to the symmetry of the α-particle wave function. This result persists for the realistic α-particle wave functions: the amplitude does not vanish exactly, but remains smaller than the one-body term. This term is quite sensitive to the chosen nuclear interaction, pointing to sensitivity to the short-range part of the potential and to the small components of the α-particle wave function.

Since the LO term is suppressed and the one-body term is not well constrained by resonance saturation, it is interesting to look at the pion-recoil term. Its parameters are better determined than β_1, since they are related to the nucleon mass difference and the Cottingham sum rule. Our calculation may therefore give a trustful estimate of the size of the NNLO contributions. The results are a rather model-independent amplitude of approximately 1/3 the size of the one-body term and are in line with the power counting.

In contrast, we find that all the Z-graphs give unexpectedly large contributions, especially the ρ–ω exchange operator. Also, the contributions add constructively, so that their sum tends to overwhelm the one-body term. This model of resonance saturation thus gives results in vast disagreement with the power counting.

CSB effects on the final-state wave functions ($\mathcal{M}_{\mathrm{WF}}$) are smaller than pion recoil and insensitive to the chosen nuclear interaction, indicating that these terms are well constrained using phenomenological interactions.

4. The next step is to present the effects of including the ISI. The correct treatment of this involves the solution of the four-body scattering problem at center-of-mass energies greater than 200 MeV. In spite of the tremendous progress achieved in recent years on the solution of the four-nucleon problem [17,34], advances in obtaining exact solutions are limited to energies below the four-particle breakup threshold. To understand our pion-production reaction it is necessary to go beyond the distortions obtained through an effective optical-model potential fitted to the elastic $\vec{d}d$ scattering data. This is because important pion production occurs in which the deuterons interact, break up, and then emit a pion. At very high energies the use of Glauber approximation is justified, but in the threshold energy regime for pion production the wave length associated with the relative $d+d$ on-shell momentum is close to the size of the deuteron. Therefore we obtain an approximate solution of the Yakubovsky [35] equation for the four-nucleon scattering wave function that is made up of two terms: the first involves the bound-state wave functions of the two deuterons times a plane wave describing the relative motion between them; the second requires the breakup of one of the deuterons followed by the three-body scattering of the $N+d$ system into the three-particle continuum in the presence of the remainder spectator nucleon.

Such an approximation is based on the lowest-order terms in the Neumann series expansion of the four-particle Yakubovsky equation, leading to the following expression for the scattering wave function,

$$\left|\Psi^{\rho_0}\right\rangle \simeq \left|\phi^{\rho_0}\right\rangle + \sum_j \sum_{i\rho} G_0 t_i G_0 U_{ij}^{\rho} \bar{\delta}_{\rho\rho_0} \left|\phi_j^{\rho_0}\right\rangle, \tag{3}$$

where ρ denotes one of the seven two-body partitions, four of $(3)+1$ type and three of $(2)+(2)$ type, and i is a pair interaction that is internal to ρ; j is both internal to ρ and ρ_0. The initial-state wave function component $|\phi_j^{\rho_0}\rangle$ carries the appropriate bound-state wave function components of the target and projectile times a relative plane wave between their respective center of mass. As usual, ρ_0 specifies the two-body entrance channel, $\bar{\delta}_{\rho\rho_0} = 1 - \delta_{\rho\rho_0}$, G_0 is the four-free-particle Green's function and t_i the t-matrix for pair i. If ρ_0 corresponds to a $2+2$ initial state, then ρ can only be a $(3)+1$ two-body partition and U_{ij}^{ρ} is the solution of the three-body Alt, Grassberger and Sandhas (AGS) [36] equation for the three-particles that make up subsystem ρ. The first term in Eq. (3) corresponds to the initial-state wave function; the second term requires the breakup of one of the bound pairs followed by the scattering of either one of the particles from the remaining bound pair, leading to four free particles in the continuum. The particle-pair scattering into the continuum takes place in the presence of the fourth one, and therefore, by energy conservation, the total energy available is the total four-body center-of-mass energy minus the relative kinetic energy of the fourth particle relative to the center of mass of the other three. Thus the four-body scattering wave function we construct contains all orders in the pair interaction but also three-particle correlations in first-order perturbation at all possible energies that are consistent with four-particle energy conservation.

For four identical nucleons Eq. (3) may be written as

$$\left|\Psi_{dd}^{+}\right\rangle = |\phi_{dd}\rangle + \frac{1}{\sqrt{12}}\left(1 + P - P_{34}P + \tilde{P}\right) \times (1 - P_{34})\left|\psi_{dd}^{(12,3)4}\right\rangle, \tag{4}$$

where $P = P_{12}P_{23} + P_{13}P_{23}$ and $\tilde{P} = P_{13}P_{24}$ are permutation operators whose appropriate combination generates the 6 (12) components of the Yakubovsky wave function that are of $2+2$ $(1+3)$ type. The symmetrized $d+d$ initial-state wave function [10] $|\phi_{dd}\rangle$ is given by

$$|\phi_{dd}\rangle = \frac{1}{\sqrt{6}}\left(1 + P - P_{34}P + \tilde{P}\right) \times \xi_d(12)\xi_d(34)\,\mathrm{Exp}(12-34), \tag{5}$$

where $\xi_d(ij)$ is the deuteron wave function for the pair (ij), and $\mathrm{Exp}(12-34)$ represents the relative plane wave between the two deuterons. The second term in Eq. (4) mandates the use of a specific choice of wave-function component for nucleons 1, 2, 3 and 4,

$$\left|\Psi_{dd}^{(12,3)4}\right\rangle = G_0\langle 12,3|U_0(z)\big|(12)3\big\rangle\xi_d(34), \tag{6}$$

where connectivity increases from left to right and $z = E - \frac{4}{3}k^2 + i0$, k being the relative momentum between nucleon 4 and the center of mass of (123). The breakup operator $U_0 = tG_0U$ and the corresponding matrix element $\langle 12,3|U_0(z) \times |(12)3\rangle$ represents the scattering of nucleon 3 from the bound state of (12) leading to three free nucleons in the continuum where nucleons 1 and 2 are last to interact through their respective t-matrix t. The operator U is the AGS three-body scattering operator that satisfies the integral equation

$$U = PG_0^{-1} + PtG_0U, \tag{7}$$

from which one calculates Nd elastic scattering amplitudes. The permutation operator P is the same as used in Eq. (4) and corresponds to the sum of the two cyclic permutations of particles 1, 2 and 3. We extract from Eq. (4) the $^3\mathrm{P}_0$ partial wave in the entrance channel to compute the threshold cross section for $dd \to \alpha\pi^0$.

Details regarding the numerical solution of the scattering problem and evaluation of the pion-production matrix-element will be presented elsewhere [33]. Here, in Table 1, we simply present the results for the transition amplitudes, which acquire imaginary parts due to the presence of the initial-state interaction. Generally, we observe a significant enhancement of all contributions to the amplitude.

It is immediately apparent that the pion-exchange term, which is supposed to be LO, is now of the size of the NNLO terms considered here, namely the pion-recoil and one-body operators. It still shows a sizable model dependence, which could visibly influence our final result for the cross section. A more consistent treatment of nuclear interactions and production operators will be necessary in the future.

The pion-recoil and one-body terms remain relatively model independent. Both, therefore, can serve as an order-of-magnitude estimate of the cross section. Our results for these matrix elements correspond to cross sections between 4.5 and 42 pb. Again, the one-body contribution is larger than the pion-recoil term. Both would come close to each other for the smaller choices of the strength of the one-body term. Our explicit calculation shows that the NNLO contribution provides a strength consistent with the experiment. We stress that the strong enhancement due to initial-state interactions and higher-momentum components of the α-particle wave functions are necessary to find NNLO contributions of the required size.

The ISI also enhances short-range contributions from Z-graphs, but by far-smaller amounts. We still find relative contributions much larger than expected from the power counting and, again, all the contributions add up constructively. One possible explanation would be that there is simply no convergent EFT for the reaction $dd \to \alpha\pi^0$ and the Z-graphs would need to be included as done in this work. However, if this was true the value of the empirical cross section would be the result of subtle cancellations amongst various terms from very different origins, a very unlikely coincidence. The more likely interpretation is that the power counting works, but the Z-graphs simply provide the wrong model to estimate the four-nucleon operators. If this is the case we may even drop them all together from our investigations, since they are of high order.

5. This Letter extends our earlier study of the reaction $dd \to \alpha\pi^0$ by using realistic wave functions for the four-nucleon ISI and FSI. We also provide numerical estimates for some diagrams that lead to a cross section of the right order of magnitude supporting the power counting given the suppression of the LO. In addition, the present results allow us to identify a few issues that deserve further study (in addition to the inclusion of all diagrams up to NNLO).

- Given the dramatic influence of initial-state interactions, it is of paramount importance that new experimental constraints be obtained for the deuteron–deuteron interactions in the energy region close to the pion-production threshold. Besides data on elastic dd scattering, also data on other pion-production reactions with the same initial $^3P_0$ state are needed. The most obvious examples are the CSC reactions $dd \to {}^3\mathrm{H}/\mathrm{He}N\pi$ recently measured at COSY,[2] which are an interesting alternative to elastic dd scattering because only a few partial waves contribute in the entrance channel.
- Another important issue is to better understand the role of the Z-graphs used to estimate the size of four-nucleon operators. As shown above, their size is much larger than indicated by their N^{4}LO power-counting order. Thus, it is necessary to reassess the procedure in light of recent developments in EFT, especially concerning the treatment of divergences in loop diagrams [12]. Given the large number of experimental data, especially for $pp \to pp\pi^0$, much insight can be obtained. For these charge-symmetry conserving reactions, we can investigate the convergence of the chiral expansion explicitly by comparing the contributions of different orders.
- Finally, we found some dependence on the chosen nuclear interaction model. This is probably related to the presumed inconsistency between the chosen nuclear interaction and the production operators. Such an inconsistency can only be resolved by applying nuclear interactions based on chiral perturbation theory [7,37] that need to be extended to higher cutoffs as outlined in Ref. [38].

The central result of this letter is that the inclusion of ISI and FSI enhances the contribution of NNLO diagrams of the production operator enough so that they are able to account for the measured cross section. Together with the insight that the LO is suppressed, this supports the EFT approach to this reaction. However, to study the rate of convergence of the chiral expansion for the charge-symmetry breaking reaction, a calculation to N^{3}LO is necessary. Our current work demonstrates that a careful treatment of the nuclear effects is required for a final analysis.

In view of a planned measurement of the reaction $\vec{d}d \to \alpha\pi^0$ at higher energies at COSY [39]—where p waves will be relevant—and of the experimental determination of A_{fb} [4], we will have an increased database on cross sections, which will allow us to disentangle the various contributions. Although much remains to be done before any precise statements about the values of the parameters δm_N, $\bar{\delta} m_N$ can be made, we are now at threshold of understanding how the light-quark masses makes a difference in nuclear physics.

Acknowledgements

We thank Andy Bacher, Allena Opper, and Ed Stephenson for useful discussions and encouragement, without which this work would not have been completed. This research was partially funded by FCT grant POCTI/37280/FNU/2001 (A.C.F.), NSF grant PHY-0457014 (A.G.), DOE grants DE-FG02-97ER41014 (G.A.M.), DE-FG02-93ER40756 (A.G.) and DE-FG02-04ER41338 (U.v.K.), and a Sloan Research Fellowship (U.v.K.).

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