The 5th Indonesia International Conference on Innovation, Entrepreneurship, and Small Business (IICIES 2013)

A Mathematical Model of the Profit-Loss Sharing (PLS) Scheme

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Abstract
In financial world, lending money is a part of investment where the borrower get an opportunity to start up or enlarge their business, and as in the fair world, the lender should also take profits on the borrower’s investment. In Islamic finance, the profit (and later also the loss) of an investment is coming from the profit-loss sharing concept. We proposed a mathematical model of micro-credit scheme based on this concept. The model is implemented onto real data taken from some low income Indonesian traders and the simulated one. We deliberately choose these respondents due to they are traditionally targeted by usurers (or rentenir in Indonesian) who lend money with high interest rate and penalties so, in many cases, it will makes them broke instead of having their business improve. The important issues in the model are how to generate the simulated data representing the real data, determination of required parameters and to find an optimal portion of profit share which is quite fair for the lender and the borrower. The model shows that the borrower will benefit from the scheme even though sometimes she has losses in particular days, and the lender will get good return as an investment return.

1. Introduction

Shariah, the code of law derived from the Quran and from the teachings and example of Mohammad SAW, ensures its protection from any exploitation through unjust and unfair dealings. It prohibits Riba (interest), Gharar (lack of information disclosure), Qimar (gambling) and Mysur (games of chance involving deception) [1,2]. Two prohibitions of Riba and Gharar are significantly important if we do an economic activity such as providing small loans, or micro-credit. Riba in lending money is prohibited in Islamic economy system (See QS.ar-Rum : 39 etc) because it frequently leads to exploitation through unjust and unfair dealings. This is also prohibited by other religions, such as Jew and Christian (See Exodus 22:25, Deuteronomy 23: 19, Luke 19:22-23 etc). The micro-credit loan is often considered one of the most effective and flexible strategies in the fight against global poverty.
In facts, however, this loan does not achieve its goal to help impoverished people to become self employed. One of the reasons is that the participant fails to pay his/her debt because of some losses in his/her economic activity and later cannot pay the accumulated interest (Riba). In many cases, his/her collateral will be taken by the lender so that the participant might be even poorer than before. To circumvent this practice of exploitation of impoverished people, Shariah gives a better concept than taking interest as a reward when someone gives his/her idle money to be borrowed by other people who need a cash. One thing should be remembered that the money borrowed is strictly for economic activity not for consumable goods. The Profit-Loss Sharing (PLS) concept gives reward to the lender with inconstant base depending on how much the borrower has got profit or loss from his/her economic activity. This concept, again, is excellent in theory but not perfect in practice. Because a privilege to alter paying debt is available, a criminal person can always claim loss despite of the real condition happening. Indeed this person is practicing Gharar which is also prohibited in the Shariah. The pursuit of profit-loss sharing concept needs thorough procedures and systems. In sequence, the lender and the borrower are well-prepared to involve in this Islamic economic activity.

Nowadays banks based on shariah system are rapidly growing worldwide, with strong growth in Saudi Arabia, Malaysia and Kuwait. This growth might be resulted from the fact that Muslims represent 21.01% of the world's population and their assets are about USD 250 billion - 1 trillion to invest [5,6] growing at 15% annually. It is still in a query whether the PLS concept is the main attractor to the potential costumers of the Islamic banking and it gives real benefit comparing to the conventional bank practices. The study at Islamic banks in Malaysia [7] shows that on the Islamic bank financing strictly PLS based is rarely provided and the loans are charged closely to conventional interest rate. A small study in Indonesia deduces the same situation [8]. It concludes that the rapid growth in Islamic banking is largely driven by rising amount of the Islamic law compliants worldwide rather than by the advantages of the PLS concept.

In this paper we propose a mathematical model of micro-credit based on PLS concept which is implemented on low income traders at a local traditional market. The capital of this micro-credit is IDR 1 million which is about USD 115.4 which is a significant amount of money for these traders. Notice that in 2010 Indonesian population still living in poverty, which is under USD 1.25 and 2 (Purchasing power parity) a day are respectively 18.1% and 46.1% [9,10]. The traders are all women, as suggested in [3], which are a promising agent to against poverty.

Some parameters should be defined to determine the boundaries in order to fulfill the condition that we avoid exploitation through unjust and unfair dealings but it is still good investment. It should be emphasized here that we clearly distinguish between investments and charity. If the initial intention of this loan is as charity loan without the need to pay back, so this model can not be implemented. First the proposed model of debt repayment from the borrower to the investor containing PLS scheme will be explained in Section 2. In Section 3, we also model the loan particularly provided by traditional usurers (or rentenir in Bahasa Indonesia) who lend money with high interest rate and penalties, so it provides parameters needed for the next section. The model with PLS concept should give benefit to the lender who play role as an investor to the micro-credit system but yet it should consider the borrower’s condition. We define the optimality problems to be solved in Section 4, and the implementation results and conclusions are in Section 5.
2. Model of PLS scheme

Implementing PLS into the micro-credit model needs some assumption to be fulfilled first, which are following.

- The loan is given to borrowers who already have trading activities, so this is not the initial capital of their shops. Therefore the average daily profit of the traders before joining this credit scheme can be recorded.
- The borrowers should be trustworthy in reporting their daily profits.
- The loan is spent to improve their trading activity not at personal consumable goods.

In this model, the lender gives a capital loan $A$ to be borrowed as much as IDR 1 million. The borrower should hand over back money daily consisting of the basic installment for paying the lending money and some portion of profit sharing. The period of this credit is 52 days. It should be emphasized that the lending process in one type of investment allowing by Islamic Law as long as there is a profit-loss sharing between the borrower and investor. This type of investment is called as $Musyarakah$[14, 15].

The total amount of payment $S_t(p)$ at day-$t$ is written as below:

$$S_t(p) = I_t + B_t(p) + C_t, \quad t = 1, 2, \ldots, T. \quad (1)$$

Here for this case, $T$ is 52 days. $I_t$ is the basic installment on day-$t$ for paying the principal. The sharing of profit is determined in variable $B_t(p)$ which contains the profit sharing portion $p$ where its optimal value is the important question to be answered later. The sharing of loss reflects on the values of three variables. When the borrower suffers a loss (which will be defined later), she is exempt from paying $B_t(p)$ and $I_t$ on that day, but she still has to pay only the basic installment $I_t^b$ later which is accumulated in the debt $H_t$. Remember that the term debt for $H_t$ is due to late payment of the basic installment. There is no a penalty to this late payment. When the borrower is capable to pay, the debt and other amount from previous debts (if any) will be paid as a payable debt $C_t$.

Now we calculate a profit sharing when the borrower gains good profit. The calculation of the profit sharing with portion $p > 0$ is formulated in the below equation.

$$B_t(p) = \begin{cases} p(w_t - I_t - C_t), & (w_t - I_t - C_t) > 0, \\ 0, & (w_t - I_t - C_t) \leq 0. \end{cases} \quad (2)$$

Here $w_t$ is the net profit on day-$t$ if the value is positive or a money loss if the value is negative. We determine a good profit by $(w_t - I_t - C_t) > 0$. Having deducted from the basic installment $I_t$ and the payment of debt $C_t$, the excessive profit will be shared to the investor in a portion of $p$. Now choosing the optimal value of $p$ is the question to be answered in this paper, which will be explained in Section 4. The borrower will have a customize value $p$ depending on her trading revenue. The other problem to be solved is to determine some parameters stating the fair treatment between the lender and borrower that
could bring the good causes of this model. Therefore we need to model the usurer scheme in the next section.

3. **Usurer Scheme and its implementation**

The purpose of the model of usurer scheme, where the loan is paid back with high interest rate and penalties so it could makes the borrower’s condition is worst, is to set an upper bound of the value of required parameters where the PLS model should not cross.

A usurer gives a loan IDR 1 million with an interest rate 30%, so that the payback amount is IDR 1.3 million that should be paid in 52 days. The borrower should pay an administrative cost IDR 100,000 on the first day of the borrowing period so the borrower only receives cash IDR 900,000 instead of IDR 1 million. In the next 52 days, the borrower should hand over IDR 25,000 per day, which is calculated from the amount of the payback divided by 52. This money is called installment \( I_t \). There is a penalty IDR 1,000 per day if the borrower fails to pay fully this installment. The total amount of money \( S_t \) on day-\( t \) received by the lender can be written as the equation below:

\[
S_t = I_t + C_t, \quad t = 1, 2, \ldots, 52.
\]  

(3)

Note that variables \( S_t \), \( I_t \), and \( C_t \) appear in this model have similar definition with the variables from the model in section 2 but they are significantly different. Here the value of installment \( I_t \) contains interest. The illustration of this scheme on a borrower data can be found in table 1.

Now we analyze the scheme by calculating parameters indicating the gains for the usurer and the borrower respectively. The values of these parameters are obtained by implementing the scheme on a simulated data based on real data taken from the incomes of a small group of 4 low income traders [11, 12]. To determine a parameter for the benefit for the usurer, we calculate the rate of return of the usurer scheme, called \( r_u \), using the cash-flow analysis [13]. The rate of return is the value \( r_u \) which satisfies the following the present value equation:

\[
1,000,000 = 100,000 + \frac{S_1}{(1 + r_u)} + \frac{S_2}{(1 + r_u)^2} + \cdots + \frac{S_{52}}{(1 + r_u)^{52}}
\]  

(4)

Table 1: An illustration of the one week payment for usurer scheme

<table>
<thead>
<tr>
<th>Day</th>
<th>Profit (_t)</th>
<th>( I_t )</th>
<th>( C_t )</th>
<th>( H_t )</th>
<th>Pty</th>
<th>( S_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35,000</td>
<td>25,000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>25,000</td>
</tr>
<tr>
<td>2</td>
<td>-75,000</td>
<td>0</td>
<td>0</td>
<td>25,000</td>
<td>1,000</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>90,000</td>
<td>25,000</td>
<td>26,000</td>
<td>0</td>
<td>0</td>
<td>51,000</td>
</tr>
<tr>
<td>4</td>
<td>25,000</td>
<td>25,000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>25,000</td>
</tr>
<tr>
<td>5</td>
<td>10,000</td>
<td>0</td>
<td>0</td>
<td>25,000</td>
<td>1,000</td>
<td>0</td>
</tr>
</tbody>
</table>
If we use the real data, we obtain from the simulation, the values of $r_u$ are from 44.73% – 47.01% per month which are higher than the original interest 30%. They are in fact 375.73 – 394.88 % per year, which are far much higher from the credit rates of Indonesian Banks which are around 9 - 12% per year. Due to this result, we use the values of $r_u$ in the determination of upper bound of the rate of return $r_j$ for the PLS model. It is desirable that the value of $r_j$ is much more less than $r_u$. The making of this bound is explained in the next section.

The second parameter to be determined is to measure how much the borrower will gain at the end of the loan period. The parameter $s_{ur}$ is the comparison between the total sum of the net profit after the deduction due to daily payment and the total sum of the net profit without deduction. The total sum with deduction is calculated regarding the borrower has paid all her debt. If there is an accumulated debt left at the end of the loan period or $H_{S_2} \neq 0$ then this should be deducted from the total sum. With assumption that the collected money is always put in the bank with interest rate compounded daily, the total sum is calculated using future value (FV) formulae with annual BI rate $r_{bi} = 7\%$. Here BI stands for Bank Indonesia (Central Bank of Indonesia).

Implementing to the real data, values of $s_{ur}$ varies according to the borrower’s initial asset. One borrower’s value is 0.90 whoever the average of her daily profits is about five times higher than others. Other borrowers have values about 0.47 – 0.58 meaning that the borrowers can bring back home the money only less than 58% of their total profits during 52 days. It is said the usurer takes about half of their profit by lending her in fact only IDR 900,000 for 52 days. This justifies the statement that credits from usurer for micro-trader cannot help improving low-level economic of the trader even make it worst. This result also will be used as the lower bound for the related parameter $s_s$ for PLS model.

4. An optimization problem in determining $p$

Now we implement the Model of PLS concept into the real and simulated data. A problem occurs when we want to find the best value of $p$, the portion of profit sharing in function $B_i(p)$. This can be obtained from the solution of a multi-objective optimization problem which maximizes the gains for both the lender and the borrower at once.

<table>
<thead>
<tr>
<th>Day</th>
<th>Profit $P^*_n$</th>
<th>$I_n$</th>
<th>$C_n$</th>
<th>$H_n$</th>
<th>$B_n$</th>
<th>$S_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35,000</td>
<td>16,666</td>
<td>0</td>
<td>0</td>
<td>180</td>
<td>16,846</td>
</tr>
<tr>
<td>2</td>
<td>-75,000</td>
<td>0</td>
<td>0</td>
<td>16,666</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>90,000</td>
<td>16,666</td>
<td>16,666</td>
<td>0</td>
<td>555</td>
<td>33,889</td>
</tr>
<tr>
<td>4</td>
<td>25,000</td>
<td>16,666</td>
<td>0</td>
<td>0</td>
<td>82</td>
<td>16,748</td>
</tr>
</tbody>
</table>
In determining the optimization problems, we need to find the criteria whenever a value of $p$ is optimal for each borrower.

- $H_T = 0$, meaning that at the final on day-T, the total debts has to be able to paid off.
- The rate of the PLS model’s return $r_s$ is bounded that is $r_{bi} < r_s < r_u$, meaning that the return for the investor should be less than the return gained from the usurer scheme and greater than the rate of return on saving the money in the conventional bank. The latter also means that this investment should be more profitable than only save money in the bank.
- Find the value of portion $s_{sr}$, which measures how much the borrower will gain at the end of the loan period, such that $s_{sr} > s_{ur}$ meaning that the PLS’s gain is greater than the borrower’s gain from the usurer scheme.

Now the model is implemented into the real data illustrated on Table 2. The capital to be lent for 60 days to a trader is IDR 1,000,000, and the basic installment $I_b = 1,000,000 \div 60 \approx 16,666$.

Table 3 shows the results from the implementation of this model to data of the group of 4 low income traders, as in the usurer scheme.

<table>
<thead>
<tr>
<th>No</th>
<th>$p$</th>
<th>$r_s$</th>
<th>$s_{sr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0098</td>
<td>0.0066</td>
<td>0.9198</td>
</tr>
<tr>
<td>2</td>
<td>0.0965</td>
<td>0.0067</td>
<td>0.5498</td>
</tr>
<tr>
<td>3</td>
<td>0.1019</td>
<td>0.0077</td>
<td>0.6407</td>
</tr>
<tr>
<td>4</td>
<td>0.0851</td>
<td>0.0059</td>
<td>0.6330</td>
</tr>
</tbody>
</table>

The rate of return for the PLS investment is around 17.7% – 23.1% per month which is still a profitable investment for the investor. Values of $s_{sr}$ vary between 54.98% – 91.98%, which means that the borrowers can bring back home the money much higher than of the usurer scheme. Similar pattern to the usurer scheme, the variety of the latter values depends on the asset owned by the trader. The average of daily profits of trader 1 is IDR 252.433 which is the higher average among traders. The profit share for the investor for lending IDR 1,000,000 is the smallest ($p=0.98\%$) meaning that the loan might have small effect on the trading activity. The amount of the daily average profits from the smallest to the biggest is IDR 44.933 (trader 2), 54.083 (trader 4), and 58.167 (trader 3), which resembles the order of $s_{sr}$, which are 55% (2), 63% (4), and 64% (3). However, the smallest to the biggest values of $p$ and $r_s$ belong to trader 3, 2 and 4. Discrepancies of the order of these parameters from of the previous parameters are still an open problem.
5. Conclusion

We conclude the micro-credit of PLS concept, given to this small group of low income traders, gives flexibility and more benefits for the lender and it is still a good investment to the investor. Due to the flexibility of the model, there is no doubt that we can have this conclusion for larger number of real data.

References


M. Yunus, Banker to the Poor: Micro-Lending and the Battle against World Poverty, 1999, PublicAffairs.


