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Physics Letters B 632 (2006) 532–536

PHYSICS LETTERS B

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A note on particles and scalar fields in higher-dimensional nutty spacetimes

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Received 7 September 2005; accepted 28 October 2005

Available online 9 November 2005

Editor: M. Cvetič

Abstract

In this note, we study the integrability of geodesic flow in the background of a very general class of spacetimes with NUT-charge(s) in higher dimensions. This broad set encompasses multiply NUT-charged solutions, electrically and magnetically charged solutions, solutions with a cosmological constant, and time-dependant bubble-like solutions. We also derive first-order equations of motion for particles in these backgrounds. Separability turns out to be possible due to the existence of non-trivial irreducible Killing tensors. Finally, we also examine the Klein–Gordon equation for a scalar field in these spacetimes and demonstrate complete separability.

PACS: 04.50.+h; 98.80.Cq

1. Introduction

Taub–NUT solutions arise in a very wide variety of situations in both string theory and general relativity. NUT-charged spacetimes, in general, are studied for their unusual properties which typically provide rather unique counterexamples to many notions in Einstein gravity. They are also widely studied in the context of issues of chronology protection in the AdS/CFT correspondence. Understanding the nature of geodesics in these backgrounds, as well as scalar field propagation, could prove to be very interesting in further exploration of these spacetimes.

There is a strong need to understand explicitly the structure of geodesics in the background of black holes in anti-de Sitter space in the context of string theory and the AdS/CFT correspondence. This is due to the recent work in exploring black hole singularity structure using geodesics and correlators in the dual CFT on the boundary [1–6]. Black holes with charge are particularly interesting for this type of analysis since the charges are reinterpreted as the R-charges of the dual theory. The class of solutions dealt with in this Letter also include black holes that carry both NUT and electric charges in various dimensions, and could prove very interesting in this sort of analysis.

In this Letter we explore a very general metric describing a wide variety of spacetimes with NUT-charge(s). In addition further metrics can also be obtained from these through various analytic continuations (which does not affect separability as demonstrated for these class of metrics). As such, the study of separability in this set of spacetimes encompasses the cases of both singly and multiply NUT-charged solutions, electrically and magnetically charged solutions with NUT parameter(s), solutions with a cosmological constant and NUT parameters(s), and time-dependant bubble-like NUT-charged solutions. Many of these describe very interesting gravitational instantons. Some of these solutions include static backgrounds, while others are time-dependant and provide very interesting backgrounds for studying both string theory and general relativity. Some of these solutions, especially the bubble-like ones, are particularly interesting in the context of string theory as they arise in the context of topology changing processes, e.g. they show up as possible end states for Hawking evaporation, and they show up in transitions of black strings in closed string tachyon condensation.

We study the separability of the Hamilton–Jacobi equation in these spacetimes, which can be used to describe the motion of classical massive and massless particles (including photons). We use this explicit separation to obtain first-order equations of

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doi:10.1016/j.physletb.2005.10.076

motion for both massive and massless particles in these backgrounds. The equations are obtained in a form that could be used for numerical study, and also in the study of black hole singularity structure using geodesic probes and the AdS/CFT correspondence. We also study the Klein–Gordon equation describing the propagation of a massive scalar field in these spacetimes. Separation again turns out to be possible with the usual multiplicative ansatz.

Separation is possible for both equations in these metrics due to the existence of non-trivial second-order Killing tensors. The Killing tensors, in each case, provides an additional integral of motion necessary for complete integrability.

There has been a lot of work recently dealing with geodesics and integrability in black hole backgrounds in higher dimensions both with and without the presence of a cosmological constant [7–16]. Of particular note in the context of this Letter are [12,14] which deal with black holes with NUT parameters in some special cases. This work extends, and generalizes, some of the results obtained in these papers.

2. Overview of the metrics

The class of metrics dealt with in this Letter, and their generalizations obtained via analytic continuations, have been constructed and analyzed in [17–22], as well as some references contained therein. We will very briefly describe the metrics, and some of the various types of spacetimes that can be obtained from them. As mentioned earlier, separability for all the metrics is addressed by dealing with the class we do here, since analytic continuations do not affect separability of either the Hamilton–Jacobi or Klein–Gordon equation (though they do affect the physical interpretations of the various variables and their associated conserved quantities).

The general spacetimes we study are described by the metrics

$$ds^2 = -F(r)\left[dt + \sum_{i=1}^{p} 2N_i f_i(\theta_i)\, d\phi_i\right]^2 + \frac{dr^2}{F(r)} + \sum_{i=1}^{p}\left(r^2+N_i^2\right)\left(d\theta_i^2 + g_i^2(\theta_i)\, d\phi_i^2\right). \tag{2.1}$$

A very general class of metrics in even dimensions where the (ϕ_i, θ_j) sector has the form $M_1 \times M_2 \times \cdots \times M_p$, with each M_i a two-dimensional space of constant curvature δ_i. In this case the functions are given by

$$\begin{aligned}
\delta_i = 1:&\quad f_i(\theta_i) = -\cos\theta_i, &\quad g_i^2(\theta_i) = \sin^2\theta_i,\\
\delta_i = 0:&\quad f_i(\theta_i) = -\theta_i, &\quad g_i^2(\theta_i) = 1,\\
\delta_i = -1:&\quad f_i(\theta_i) = -\cosh\theta_i, &\quad g_i^2(\theta_i) = \sinh^2\theta_i,
\end{aligned} \tag{2.2}$$

and an expression for $F(r)$ can be found in [21] along with a detailed description. Generalizations to include electric charge are obtained by suitably modifying $F(r)$, and can be found in [20,22]. Metrics describing "bubbles of nothing" also fall under this class and can be found in [19]. Examples of NUT-charged spacetimes in cosmological backgrounds also fall in this framework and can be found in [19].

For the purposes of analyzing separability, some odd-dimensional NUT-charged spacetimes also fall under this category. For instance in five dimensions (i.e. $p = 2$) a NUT-charged spacetime is obtained by taking $g_2(\theta_2) = 0$ and $N_2 = 0$, i.e. a metric of the form

$$ds^2 = -F(r)(dt - 2N_1\cosh\theta_1\, d\phi_1)^2 + \frac{dr^2}{F(r)} + \left(r^2+N_1^2\right)\left(d\theta_1^2 + \sinh^2\theta_1\, d\phi_1^2\right) + r^2\, d\theta_2^2. \tag{2.3}$$

This describes a spacetime in an AdS background; similar dS and flat background spacetimes can be obtained by following the prescriptions in (2.2) while maintaining $g_2(\theta_2) = 0$ and $N_2 = 0$. Generalizations to higher odd-dimensional spacetimes are obvious.

Various twists of these spacetimes can also be obtained through analytic continuations. For instance, using the prescriptions $t \to i\theta$, $\theta \to it$, we can obtain time-dependant bubbles. In five dimensions in an AdS background, some examples obtained via this prescription, and a few other suitable obvious variable redefinitions are

$$\begin{aligned}
ds^2 &= F(r)(d\theta_1 + 2N_1\cos t\, d\phi)^2 + \frac{dr^2}{F(r)} + \left(r^2+N_1^2\right)\left(-dt^2 + \sin^2 t\, d\phi^2\right) + r^2\, d\theta_2^2,\\
ds^2 &= F(r)(d\theta_1 + 2N_1\sinh\phi\, dt)^2 + \frac{dr^2}{F(r)} + \left(r^2+N_1^2\right)\left(d\phi^2 - \cosh^2\phi\, dt^2\right) + r^2\, d\theta_2^2,\\
ds^2 &= F(r)(d\theta_1 + 2N_1\cosh\phi\, dt)^2 + \frac{dr^2}{F(r)} + \left(r^2+N_1^2\right)\left(d\phi^2 - \sinh^2\phi\, dt^2\right) + r^2\, d\theta_2^2,\\
ds^2 &= F(r)(d\theta_1 + 2N_1 e^{\phi}\, dt)^2 + \frac{dr^2}{F(r)} + \left(r^2+N_1^2\right)\left(d\phi^2 - e^{2\phi}\, dt^2\right) + r^2\, d\theta_2^2.
\end{aligned} \tag{2.4}$$

For future use, we give the determinant of the metric (2.1)

$$g = -\prod_{i=1}^{p}\left(r^2+N_i^2\right)^2 g_i^2(\theta_i). \tag{2.5}$$

The components of the inverse metric are

$$\begin{aligned}
g^{tt} &= \sum_{i=1}^{p}\frac{4N_i^2 f_i^2(\theta_i)}{(r^2+N_i^2)g_i^2(\theta_i)} - \frac{1}{F(r)},\\
g^{t\phi_i} &= -\frac{2N_i f_i(\theta_i)}{g_i^2(\theta_i)(r^2+N_i^2)},\\
g^{\phi_i\phi_j} &= \frac{\delta_{ij}}{(r^2+N_i^2)g_i^2(\theta_i)},\\
g^{rr} &= F(r),\\
g^{\theta_i\theta_j} &= \frac{\delta_{ij}}{r^2+N_i^2}.
\end{aligned} \tag{2.6}$$

These formulae are somewhat tedious to derive, but can be proved using a few Maple calculations, and then using mathematical induction [23].

3. The Hamilton–Jacobi equation and separability

The Hamilton–Jacobi equation in a curved background is given by

$$-\frac{\partial S}{\partial\lambda}=H=\frac{1}{2}g^{\mu\nu}\frac{\partial S}{\partial x^\mu}\frac{\partial S}{\partial x^\nu}, \tag{3.1}$$

where S is the action associated with the particle and λ is some affine parameter along the worldline of the particle. Note that this treatment also accommodates the case of massless particles, where the trajectory cannot be parametrized by proper time.

3.1. Separability

We can attempt a separation of coordinates as follows. Let

$$S=\frac{1}{2}m^2\lambda-Et+\sum_{i=1}^{p}L_{\phi_i}\phi_i+\sum_{i=1}^{p}S_{\theta_i}(\theta_i)+S_r(r). \tag{3.2}$$

t and ϕ_i are cyclic coordinates, so their conjugate momenta are conserved. The conserved quantity associated with time translation is the energy E, and those with rotation in the ϕ_i are the corresponding angular momenta L_{ϕ_i}. Then, using the components of the inverse metric (2.6), the Hamilton–Jacobi equation (3.1) is written to be

$$\begin{aligned}-m^2&=\sum_{i=1}^{p}\frac{4N_i^2f_i^2(\theta_i)}{(r^2+N_i^2)g_i^2(\theta_i)}E^2-\frac{E^2}{F(r)}\\&\quad-\sum_{i=1}^{p}\frac{4N_if_i(\theta_i)}{(r^2+N_i^2)g_i^2(\theta_i)}(L_{\phi_i})(-E)\\&\quad+\sum_{i=1}^{p}\frac{1}{(r^2+N_i^2)g_i^2(\theta_i)}L_{\phi_i}^2\\&\quad+F(r)\left[\frac{dS_r(r)}{dr}\right]^2+\sum_{i=1}^{p}\frac{1}{r^2+N_i^2}\left[\frac{dS_{\theta_i}(\theta_i)}{d\theta_i}\right]^2.\end{aligned} \tag{3.3}$$

After some manipulation, we can recursively separate out the equation into

$$\begin{aligned}-m^2&=-\frac{E^2}{F(r)}+F(r)\left[\frac{dS_r(r)}{dr}\right]^2+\sum_{i=1}^{p}\frac{K_i}{r^2+N_i^2},\\K_i&=\left[\frac{dS_{\theta_i}(\theta_i)}{d\theta_i}\right]^2+\left[\frac{L_{\phi_i}+2N_if_i(\theta_i)E}{g_i(\theta_i)}\right]^2.\end{aligned} \tag{3.4}$$

For future reference we will use the notation $K=\sum_{i=1}^{p}K_i$. Also note that for the metrics obtained through analytic continuations discussed earlier, the issue of separability is clearly not affected. However, for an analytic continuation of the form $t\to i\theta$, $\theta\to it$, we need to replace $E\to -iL_\theta$, and the energy is no longer conserved as we have a time dependant background. However, now the angular momentum L_θ associated to θ is conserved. Similar substitutions need to be made for any other analytic continuations or variable redefinitions used to define the new metrics.

3.2. The equations of motion

To derive the equations of motion, we will write the separated action S from the Hamilton–Jacobi equation in the following form:

$$\begin{aligned}S&=\frac{1}{2}m^2\lambda-Et+\sum_{i=1}^{p}L_{\phi_i}\phi_i+\int^{r}\sqrt{\mathcal{R}(r')}\,dr'\\&\quad+\sum_{i=1}^{p}\int^{\theta_i}\sqrt{\Theta_i(\theta_i')}\,d\theta_i',\end{aligned} \tag{3.5}$$

where

$$\begin{aligned}F(r)\mathcal{R}(r)&=-\sum_{i=1}^{p}\frac{K_i}{r^2+N_i^2}+\frac{E^2}{F(r)}-m^2,\\\Theta_i(\theta_i)&=K_i-\left[\frac{L_{\phi_i}+2N_if_i(\theta_i)E}{g_i(\theta_i)}\right]^2.\end{aligned} \tag{3.6}$$

To obtain the equations of motion, we differentiate S with respect to the parameters m^2, K_i, E, L_{ϕ_i} and set these derivatives to equal other constants of motion. However, we can set all these new constants of motion to zero (following from freedom in choice of origin for the corresponding coordinates, or alternatively by changing the constants of integration). Following this procedure, we get the following equations of motion:

$$\begin{aligned}&\frac{\partial S}{\partial m^2}=0\Rightarrow\lambda=\int\frac{dr}{F(r)\sqrt{\mathcal{R}(r)}},\\&\frac{\partial S}{\partial K_i}=0\Rightarrow\int\frac{d\theta_i}{\sqrt{\Theta_i}}=\int\frac{1}{(r^2+N_i^2)}\frac{dr}{F(r)\sqrt{\mathcal{R}(r)}},\\&\frac{\partial S}{\partial L_{\phi_i}}=0\Rightarrow\phi_i=\int\frac{L_{\phi_i}+2N_if_i(\theta_i)E}{g_i^2(\theta_i)}\frac{d\theta_i}{\sqrt{\Theta_i(\theta_i)}},\\&\frac{\partial S}{\partial E}=0\\&\quad\Rightarrow t=\int\frac{E}{F^2(r)}\frac{dr}{\sqrt{\mathcal{R}(r)}}\\&\qquad-\sum_{i=1}^{p}\int\frac{2N_iL_{\phi_i}f_i(\theta_i)+4N_i^2f_i^2(\theta_i)E}{g_i^2(\theta_i)}\frac{d\theta_i}{\sqrt{\Theta_i(\theta_i)}}.\end{aligned} \tag{3.7}$$

It is often more convenient to rewrite these in the form of first-order differential equations obtained from (3.7) by direct differentiation with respect to the affine parameter:

$$\begin{aligned}\frac{dr}{d\lambda}&=F(r)\sqrt{\mathcal{R}(r)},\\\frac{d\theta_i}{d\lambda}&=\frac{\sqrt{\Theta_i(\theta_i)}}{r^2+N_i^2},\\\frac{d\phi_i}{d\lambda}&=\frac{L_{\phi_i}+2N_if_i(\theta_i)E}{g_i^2(\theta_i)(r^2+N_i^2)},\\\frac{dt}{d\lambda}&=\frac{E}{F(r)}-\sum_{i=1}^{p}\frac{2N_iL_{\phi_i}f_i(\theta_i)+4N_i^2f_i^2(\theta_i)E}{g_i^2(\theta_i)(r^2+N_i^2)}.\end{aligned} \tag{3.8}$$

4. Dynamical symmetry

The general class of metrics discussed here are stationary and "axisymmetric"; i.e., $\partial/\partial t$ and $\partial/\partial\phi_i$ are Killing vectors and have associated conserved quantities, $-E$ and L_{ϕ_i}. In general, if ξ is a Killing vector, then $\xi^\mu p_\mu$ is a conserved quantity, where p is the momentum of the particle. Note that this quantity is first order in the momenta.

The additional constants of motion K_i which allowed for complete integrability of the equations of motion is not related to a Killing vector from a cyclic coordinate. These constants are, rather, derived from irreducible second-order Killing tensors in which permit the complete separation of equations. Killing tensors are not symmetries on configuration space, and cannot be derived from a Noether procedure, and are rather, symmetries on phase space. They obey a generalization of the Killing equation for Killing vectors (which do generate symmetries in configuration space by the Noether procedure) given by

$$\mathcal{K}_{(\mu\nu;\rho)} = 0, \tag{4.1}$$

where $\mathcal{K}$ is any second-order Killing tensor, and the parentheses indicate complete symmetrization of all indices.

The Killing tensors can be obtained from the expressions for the separation constants K_i in each case. If the particle has momentum p, then the Killing tensor $\mathcal{K}_{\mu\nu}$ is related to the constant K via

$$K = \mathcal{K}^{\mu\nu} p_\mu p_\nu = \mathcal{K}^{\mu\nu} \frac{\partial S}{\partial x^\mu}\frac{\partial S}{\partial x^\nu}. \tag{4.2}$$

We can use the expression for the K_i in terms of the θ_i equations.

For the Taub–NUT metrics analyzed above, the expression for K_i from (3.4) is

$$K_i = \left[\frac{dS_{\theta_i}(\theta_i)}{d\theta_i}\right]^2 + \left[\frac{L_{\phi_i} + 2N_i f_i(\theta_i)E}{g_i(\theta_i)}\right]^2. \tag{4.3}$$

Thus, from (4.2) we can easily read

$$\mathcal{K}_i = \partial_{\theta_i}\otimes\partial_{\theta_i} + \frac{1}{g_i(\theta_i)^2}\big[\partial_{\phi_i}\otimes\partial_{\phi_i} + 4N_i^2 f_i^2(\theta_i)\partial_t\otimes\partial_t - 2N_i f_i(\theta_i)\,\mathrm{sym}(\partial_{\phi_i}\otimes\partial_t)\big]. \tag{4.4}$$

We can easily check using Maple [23], that the Killings tensors do satisfy the Killing equation.

Note that if any of the NUT parameters N_k were zero, then the corresponding Killing tensor $\mathcal{K}_k$ would simply be the usual Killing tensor of the underlying two-dimensional space M_k (which is a reducible one in the case of a homogenous constant curvature space M_k as is the case for many situations here). In general, however, a non-zero NUT parameter N_k provides a non-trivial coupling between the (r, ϕ_i, θ_i) sectors, and the existence of the Killing vectors ∂_{ϕ_i} and ∂_t along is not enough to ensure complete separability. It is the existence of these non-trivial irreducible Killing tensors $\mathcal{K}_i$ that provides the addition separation constants K_i necessary for complete separation of each space M_i from another space M_j, as well as separation of the angular sectors completely from the radial sector. These tensors are irreducible since they are not simply linear combinations of tensor products of Killing vectors of the spacetime.

5. The scalar field equation

Consider a scalar field Ψ in a gravitational background with the action

$$S[\Psi] = -\frac{1}{2}\int d^D x\,\sqrt{-g}\big((\nabla\Psi)^2 + \alpha R\Psi^2 + m^2\Psi^2\big), \tag{5.1}$$

where we have included a curvature dependent coupling. However, in these (anti)-de Sitter and flat backgrounds with charges, R is constant (proportional to the cosmological constant Λ). As a result we can trade off the curvature coupling for a different mass term. So it is sufficient to study the massive Klein–Gordon equation in this background. We will simply set $\alpha = 0$ in the following. Variation of the action leads to the Klein–Gordon equation

$$\frac{1}{\sqrt{-g}}\partial_\mu\big(\sqrt{-g}g^{\mu\nu}\partial_\nu\Psi\big) = m^2\Psi. \tag{5.2}$$

Using the explicit expressions for the components of the inverse metric (2.6) and the determinant (2.5), the Klein–Gordon equation for a massive scalar field in this spacetime can be written as

$$\begin{aligned} m^2\Psi = {} & \left[\sum_{i=1}^{p}\frac{4N_i^2 f_i^2(\theta_i)}{(r^2+N_i^2)g_i^2(\theta_i)} - \frac{1}{F(r)}\right]\partial_t^2\Psi \\ & - \sum_{i=1}^{p}\frac{4N_i f_i(\theta_i)}{g_i^2(\theta_i)(r^2+N_i^2)}\partial^2_{t\phi_i}\Psi \\ & + \sum_{i=1}^{p}\frac{1}{(r^2+N_i^2)g_i^2(\theta_i)}\partial^2_{\phi_i}\Psi \\ & + \frac{1}{\prod_{i=1}^{p}(r^2+N_i^2)}\frac{\partial}{\partial r}\left[\prod_{i=1}^{p}(r^2+N_i^2)F(r)\frac{\partial\Psi}{\partial r}\right] \\ & + \sum_{i=1}^{p}\frac{1}{(r^2+N_i^2)g_i(\theta_i)}\frac{\partial}{\partial\theta_i}\left[g_i(\theta_i)\frac{\partial\Psi}{\partial\theta_i}\right]. \end{aligned} \tag{5.3}$$

We assume the usual multiplicative ansatz for the separation of the Klein–Gordon equation

$$\Psi = \Phi_r(r)e^{-iEt}e^{i\sum_{i=1}^{p}L_{\phi_i}\phi_i}\prod_{i=1}^{p}\Phi_{\theta_i}(\theta_i). \tag{5.4}$$

Then we can easily completely separate the Klein–Gordon equation as

$$\begin{aligned} K_i = {} & \frac{1}{g_i(\theta_i)\Phi_{\theta_i}(\theta_i)}\frac{d}{d\theta_i}\left[g_i(\theta_i)\frac{d\Phi_{\theta_i}(\theta_i)}{d\theta_i}\right] \\ & - \left[\frac{L_{\phi_i}+2N_i f_i(\theta_i)E}{g_i(\theta_i)}\right]^2, \end{aligned}$$

$$-m^2 = \frac{1}{\prod_{i=1}^{p}(r^2+N_i^2)} \frac{d}{dr}\left[\prod_{i=1}^{p}(r^2+N_i^2) F(r) \frac{d\Phi_r(r)}{dr}\right] + \frac{E^2}{F(r)} + \sum_{i=1}^{p} \frac{K_i}{r^2+N_i^2}, \tag{5.5}$$

where the K_i are again separation constants. At this point we have completely separated out the Klein–Gordon equation for a massive scalar field in these spacetimes.

We note the role of the Killing tensors in the separation terms of the Klein–Gordon equations in these spacetimes. In fact, the complete integrability of geodesic flow of the metrics via the Hamilton–Jacobi equation can be viewed as the classical limit of the statement that the Klein–Gordon equation in these metrics also completely separates.

6. Conclusions

We studied the complete integrability properties of the Hamilton–Jacobi and the Klein–Gordon equations in the background of a very general class of Taub–NUT metrics in higher dimensions, which include the cases of both singly and multiply NUT-charged solutions, electrically and magnetically charged solutions with NUT parameter(s), solutions with a cosmological constant and NUT parameters(s), and time-dependant bubble-like NUT-charged solutions, and other very interesting gravitational instantons. Complete separation of both the Hamilton–Jacobi and Klein–Gordon equations in these backgrounds is demonstrated. This is due to the enlarged dynamical symmetry of the spacetime. We construct the Killing tensors in these spacetimes which explicitly permit complete separation. We also derive first-order equations of motion for classical particles in these backgrounds. It should be emphasized that these complete integrability properties are a fairly non-trivial consequence of the specific form of the metrics, and generalize several such remarkable properties for other previously known metrics.

Further work in this direction could include the study of higher-spin field equations in these backgrounds, which is of great interest, particularly in the context of string theory. Explicit numerical study of the equations of motion for specific values of the black hole parameters could lead to interesting results. The geodesic equations presented can also readily be used in the study of black hole singularity structure in an AdS background using the AdS/CFT correspondence.

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