Double Diffusive Convection in an Inclined Parallelogrammic Porous Enclosure

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Abstract

This paper reports the numerical investigation of double diffusive natural convection in an inclined parallelogrammic porous enclosure. The two opposing side walls of the enclosure are maintained at uniform, but different temperatures and concentrations, while the top and bottom walls are kept adiabatic and impermeable. Using the Darcy model, the governing equations are solved by a stable and implicit finite difference method. From the numerical simulations, it has been found that the inclination angle of the enclosure has significant influence on convective flow, heat and mass transfer characteristics.

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1. Introduction

Natural convection in finite enclosures continues to be an active area of research during past few decades, due to its wide range of applications. Among finite enclosures, the parallelogrammic enclosure present high potential to be used in heat and mass transfer applications, as they can act as heat and mass transfer promoters, being usually referred to as heat and mass transfer diodes. Many of the recent works reveal that the concept of natural convection in a parallelogrammic enclosure is of universal importance [1-3].

The phenomenon of double diffusive natural convection in fluid saturated porous media has been motivated by its wide range of applications in many engineering and scientific fields. A comprehensive review concerning double diffusive natural convection in a fluid saturated porous media is presented in detail by Neild and Bejan [4] in their monograph. Mamou et al. [5] studied natural convective flow inside a rectangular enclosure with heat and mass

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flux boundary conditions on the vertical walls and adiabatic conditions on the horizontal walls. Double diffusive natural convection in square and rectangular enclosures is of considerable interest in many engineering applications. Using Darcy model, Trevisan and Bejan [6] performed a combined analytical and numerical investigation of heat and mass transfer in a square enclosure. Other important studies on double diffusive convection in this geometry are by Goyeau et al. [7] and Bennacer et al. [8]. Later, double diffusive natural convection in an inclined rectangular porous enclosure has been numerically studied by Farhany and Turan [9]. They found that the average Nusselt and Sherwood numbers decreases with the aspect ratio, but increases with the angle of inclination. Recently, the effect of discrete heat and solute source on double diffusive convection in a vertical annulus has been numerically examined by Sankar et al. [10]. Most of the existing studies in the literature on double diffusive convection in finite enclosures are mainly concentrated in rectangular and annular enclosures. However, due to the presence of sloping walls, the heat and mass transfer behaviour in a parallelogrammic porous and non-porous enclosure is more interesting compared to the rectangular cavity [8, 9]. It has been shown that the flow structure, thermal and concentration fields have undergone drastic change in a parallelogrammic enclosure with the Rayleigh number, the inclination angle and the aspect ratio of the enclosure. After a meticulous survey of the existing literature, we found that less attention has been paid to double diffusive convection and the corresponding heat and mass transfer rates in a parallelogrammic enclosure. In particular, to the best of our knowledge, the effect of inclination of parallelogrammic enclosure has not been examined on the double diffusive natural convection. Hence, the objective of the present numerical study is to analyze the heat and mass transfer performance in an inclined parallelogrammic enclosure filled with a fluid saturated porous medium.

2. Mathematical formulation

The physical configuration considered in the present study is shown in Fig. 1. The model consists of a two dimensional parallelogrammic enclosure of height $H$ and width $L$. Let $\alpha$ be the angle of inclination of the enclosure with respect to the horizontal plane. The vertical walls are maintained at different temperatures ($T_h$ & $T_c$) and concentrations ($S_h$ & $S_c$) such that, $T_h > T_c$ and $S_h > S_c$, while the horizontal walls are assumed to be adiabatic and impermeable. The fluid is assumed to be incompressible and comply with the Boussinesq approximation. Also, the effects of inertial force and viscous dissipation are assumed to be negligible. Further, to transform the parallelogrammic enclosure to a square enclosure, the transformation $X = x - y \tan \phi$, $Y = y$ has been used. Using Darcy’s law, the dimensionless governing equations of momentum, energy and the concentration in terms of transformed coordinates ($\xi, \eta$) are:

\[
\frac{\partial^2 \psi}{\partial \xi^2} - 2 \frac{\sin \phi}{A} \frac{\partial^2 \psi}{\partial \xi \partial \eta} + \frac{1}{A^2} \frac{\partial^2 \psi}{\partial \eta^2} = ReT \cos \phi \left\{ \frac{\sin \alpha}{A} \left[ \frac{\partial \theta}{\partial \eta} + N \frac{\partial C}{\partial \eta} \right] - \cos (\phi - \alpha) \left[ \frac{\partial \theta}{\partial \xi} + N \frac{\partial C}{\partial \xi} \right] \right\}
\]

Fig: 1 Physical configuration and coordinate system.
\[
\frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial \psi}{\partial \eta} \frac{\partial \theta}{\partial \xi} - \frac{\partial \psi}{\partial \eta} \frac{\partial \theta}{\partial \eta} = A \cos \phi \left( \frac{\partial^2 \theta}{A \partial \xi^2} - 2 \frac{\sin \phi}{A} \frac{\partial^2 \theta}{\partial \xi \partial \eta} + \frac{1}{A^2} \frac{\partial^2 \theta}{\partial \eta^2} \right)
\]

(2)

\[
\frac{\partial C}{\sigma \partial \xi} + \frac{\partial \psi}{\partial \eta} \frac{\partial C}{\partial \xi} - \frac{\partial \psi}{\partial \eta} \frac{\partial C}{\partial \eta} = \frac{A}{\cos \phi} \frac{1}{Le} \left( \frac{\partial^2 C}{A \partial \xi^2} - 2 \frac{\sin \phi}{A} \frac{\partial^2 C}{\partial \xi \partial \eta} + \frac{1}{A^2} \frac{\partial^2 C}{\partial \eta^2} \right)
\]

(3)

The following dimensionless variables are used in the above equations:

\[
\xi = X / L, \eta = Y / H \cos \phi, \tau = \alpha_m / \sigma LH \cos \phi, \psi = \psi^* / \alpha_m, \theta = (T - T_e) / \Delta T, C = (S - S_e) / \Delta S,
\]

where \(T_e = (T_h + T_c) / 2, \Delta T = T_h - T_c, S_e = (S_h + S_c) / 2\) and \(\Delta S = S_h - S_c\).

In the above equations, \(\psi\) is the stream function, \(A\) is the aspect ratio, \(Ra_T\) is the thermal Rayleigh number, \(N\) is the buoyancy ratio and \(Le\) is the Lewis number, and are defined by \(A = \frac{H}{L}, Ra_T = \frac{gK \beta_T \Delta T \eta}{\nu \alpha_m}, N = \frac{\beta_c \Delta S}{\beta_T \Delta T}, Le = \frac{\alpha_m}{D}\).

The relevant dimensionless boundary conditions are:

\[
\psi = 0, \quad \theta = \frac{1}{2}, \quad \eta = \frac{1}{2} \quad \text{at} \quad \xi = 0; \quad \psi = 0, \quad \theta = \frac{1}{2}, \quad \eta = -\frac{1}{2} \quad \text{at} \quad \xi = 1 \quad \text{and} \quad \psi = 0, \quad \frac{\partial \theta}{\partial \eta} = \frac{\partial C}{\partial \eta} = 0 \quad \text{at} \quad \eta = 0, \quad A
\]

The overall heat and mass transfer performances at the walls are defined in terms of the average Nusselt and Sherwood numbers and are given below:

\[
Nu = -\int_0^1 \frac{1}{\cos \phi} \left( \frac{\sin \phi}{A} \frac{\partial \theta}{\partial \eta} - \frac{\partial \theta}{\partial \xi} \right) d\eta \quad \text{and} \quad Sh = -\int_0^1 \frac{1}{\cos \phi} \left( \frac{\sin \phi}{A} \frac{\partial C}{\partial \eta} - \frac{\partial C}{\partial \xi} \right) d\eta.
\]

3. Numerical method and validation

The governing equations (1) - (3) along with the boundary conditions are solved by the alternative direction implicit method (ADI) and successive line over relaxation method (SLOR). The time derivative is approximated by forward difference, whereas the convection and diffusion terms are discretized using a second order central difference. The resulting finite difference equations are in the form of tri diagonal matrix, which can be efficiently solved by Thomas algorithm. The steady state results are obtained as an asymptotic limit to the transient solutions. In the present study, uniform grids have been used in the \(\xi - \eta\) plane of the computational domain, and a proper grid independence study has been carried out by varying the grids from a coarse grid size of \(51 \times 51\) to a finer grid size of \(201 \times 201\). After thorough investigation on the grid independence tests, all simulations are performed with \(101 \times 101\) grids for \(A = 1\). For brevity, the details of the grid independence tests are not presented in the paper. Further, we have developed an in-house FORTRAN code for the present model and have been successfully validated against various benchmark solutions. For this, we have obtained simulations of double diffusive convection in a square porous enclosure \((\alpha = 0, \phi = 0)\) and obtained the average Nusselt and Sherwood numbers. Table 1 provides comparison of average Nusselt and Sherwood numbers from the present results against three benchmark results in a square porous enclosure. A careful observation of the Table 1 reveals that the agreement between our study and the existing studies is quite acceptable. Also we found an excellent agreement between the present streamlines and isotherms, and those of Baytas and Pop [1] in a non-inclined parallelogrammic enclosure \((\alpha = 0)\), which has not been provided here for the sake of brevity.

4. Results and discussion

The objective of this study is to explore the influence of inclination angle of a parallelogrammic porous enclosure on the double diffusive convective flow, heat and mass transfer characteristics. In the present study, we fix \(A = 1\) and \(Le = 2\), while other governing parameters are varied over wide ranges. The Darcy-Rayleigh number \(Ra\) and the buoyancy ratio \(N\) are varied in the ranges of \(50 \leq Ra \leq 1000\) and \(-5 \leq N \leq 5\). Also, the inclination angle of the
parallelogrammic cavity ($\alpha$), tilt angle of the sloping walls ($\phi$) are varied in the ranges of $-60^\circ \leq \alpha \leq 60^\circ$ and $-60^\circ \leq \phi \leq 60^\circ$. The detailed numerical simulations are performed for wide range of physical and geometrical parameters and the results are presented in the form of streamlines, isotherms, isoconcentrations, average Nusselt and Sherwood numbers.

### Table 1. Comparison of average Nusselt and Sherwood numbers for $Le = 10$, $N = 0$, $A = 1.0$, $\alpha = 0$, $\phi = 0$.

<table>
<thead>
<tr>
<th>Ra = 100</th>
<th>Ra = 200</th>
<th>Ra = 400</th>
<th>Ra = 1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nu</td>
<td>Sh</td>
<td>Nu</td>
<td>Sh</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Trevisan and Bejan [6]</td>
<td>3.27</td>
<td>15.61</td>
<td>5.61</td>
</tr>
<tr>
<td>Farhan and Turan[9]</td>
<td>3.12</td>
<td>14.06</td>
<td>5.01</td>
</tr>
<tr>
<td>Present work</td>
<td>3.11</td>
<td>13.26</td>
<td>4.96</td>
</tr>
</tbody>
</table>

### 4.1. Effect of Buoyancy ratio

Figure 2 illustrates the effect of buoyancy ratio on the streamlines, isotherms and isoconcentrations. To show the true effects of buoyancy ratio, we have chosen three different values of buoyancy ratios and fixed the values of $Ra = 500$, $\alpha = \phi = 30^\circ$ and $Le = 2$. For negative value of $N$, the thermal and solutal buoyancies have opposing nature. For $N=-5$, due to opposing nature of two buoyant forces, the flow circulation rate is lowest and leads to the formation of two eddies in the middle of the enclosure. Also, when the buoyancy forces are opposing each other, the thermal buoyancy drives the flow in upwards, while the flow direction is downwards for solutal buoyancy. However, the solutal buoyancy dominates over thermal buoyancy even though two driving forces are opposite. This tendency is also manifested in the plots of isotherms and isoconcentrations, where flow is directed downwards. Further, the maximum stream function value is positive and hence a counter clockwise circulation flow pattern exists in the enclosure. In the case of no solute transfer ($N = 0$), the double diffusive convective flow, heat and mass transfer rates are mainly driven by thermal buoyancy force, and the influence of solute buoyancy can be neglected. Also, the streamline contours reveal a clockwise unicellular flow driven by the thermal buoyancy force, which is similar to the thermal convection. Interestingly, as observed in the case of opposing buoyancy forces ($N = -5$), the thermal and solutal contours are moving in upper direction. The isotherm and isoconcentration contours reveal the existence of thermal and solutal stratification in the enclosure. For $N=+5$, the thermal and solutal buoyancy forces are acting in the same direction, which is termed as aiding double diffusive convective flow. Due to the combined effect of both buoyancy forces, the flow rate has been marginally increases and leads to the formation of a single large cell, stretched diagonally along the corners. In the case of aiding flow, the thermal and solutal buoyancy effects augment each other, and as a result, the isotherms and isoconcentrations reveal a strengthened stratification. Also, the formation of thermal and solutal boundary layers visibly exists for the aiding flow compared to other two cases.
The combined effect of Darcy-Rayleigh number and buoyancy ratio on the average Nusselt and Sherwood numbers in the ranges of $-5 \leq N \leq +5$ and $50 \leq Ra \leq 500$ is presented in Fig. 3. The other parameters are kept fixed at $\phi = 30$, $\alpha = 30$ and $Le = 2$. An overview of the figure reveals that the heat and mass transfer rates are increasing functions of $Ra$ and $N$. At all values of $Ra$, it can be observed that the average Nusselt and Sherwood numbers increases when the buoyancy ratio increases ($N > -1$) or buoyancy ratio decreases ($N < -1$). From the results, we found that both the average Nusselt and Sherwood numbers acquired the minimum values, when $N = -1$. Further, it is observed that both $Nu$ and $Sh$ attain their maximum value when the buoyancy ratio is maximum ($N = 5$). In particular, the mass transfer rate (average Sherwood number) for both aiding and opposing buoyancy ratios is higher compared to the heat transfer rate (average Nusselt number). In general, as the magnitude of buoyancy ratio increases, the solutal buoyancy force enhances and dominates over the thermal buoyancy for opposing as well as aiding flows, and this fact contributes to the overall increase in mass transfer rates compared to heat transfer rates.

### 4.2. Effect of tilt angle ($\phi$)

The parallelogrammic enclosure is usually referred to as heat and mass transfer diode, since this geometry can act as heat and mass transfer inhibitors or promoters. As mentioned earlier, the tilt angle ($\phi$) of the sloping walls of a parallelogrammic enclosure plays a key role to control the convective flow and the corresponding heat and mass transfer characteristics. From practical point of view, the positive values of $\phi$ act as heat transfer promoters and negative values cause thermal insulation effect. Figure 3 displays the contour plots of streamlines for different values of $\phi$ by fixing $Ra = 500$, $\alpha = 30$, $Le = 2$ and $N = 1$. A careful observation of the streamline patterns reveals the distinct effect of tilt angle on the flow fields. By observing the extreme value of stream function, it can be concluded that the flow circulation rate is lower for negative tilt angle ($\phi < 0$), but higher flow rates are observed for $\phi \geq 0$. A
similar effect has also been found on thermal and solutal distributions. Further, the streamlines, isotherms and isoconcentrations corroborate horizontal symmetry for all tilt angles.

Fig: 4 Effect of tilt angle (ϕ) on streamlines (left), isotherms (middle) and isoconcentration (right) for A=1, Ra=500, α = 30°, Le =2 and N =1. (Top) ϕ = -45°, T_{\text{max}}=18.4, (centre) ϕ = 0°, T_{\text{max}}=22.2 and (bottom) ϕ = 45°, T_{\text{max}}=22.1.

The influence of tilt angle (ϕ) and Darcy-Rayleigh number (Ra) on the heat and mass transfer rates are demonstrated in Fig. 5 for wide range of Ra and ϕ by fixing other parameters. An overview of the results clearly showing that the heat and mass transfer rates increases as ϕ increases from -60° to 15° and decreases as ϕ varies from 15° to 60°. Another important observation of the results is that the average Nusselt and Sherwood numbers increases with an increase in Darcy-Rayleigh number. Among the parameter ranges considered in our study, we found that the maximum heat and mass transfer takes place for the combination of Ra = 1000 and ϕ = 15°, whereas minimum heat and mass transfer rates occur for Ra = 100 and ϕ = -60°.

Fig: 5 Effect of tilt angle (ϕ) and Darcy-Rayleigh number (Ra) on the average Nusselt and Sherwood numbers for α=30°, A=1, Le=2.0, N=1.0.

4.3. Effect of inclination angle of the enclosure (α)

The influence of inclination angle on the streamlines is reported in Fig. 6. The values of thermal Darcy–Rayleigh number, aspect ratio, tilt angle, Lewis number and buoyancy ratio are, respectively, fixed at Ra = 500, A = 1, ϕ = 30°, Le=2.0 and N = 1, while three different inclination angles (α = -45°, 0° and 45°) are considered. The influence of inclination angle of the enclosure on the flow pattern is visibly apparent from Fig. 6. When α = -45°, the streamlines reveal a lower flow circulation rate and formation of two weak eddies at the centre of the enclosure. Also, the isotherms and isoconcentrations are parallel to the sloping walls, indicating conduction and diffusion is the
major mode and heat and mass transfer respectively. However, as the inclination angle increases, the flow rate is enhanced in many orders of magnitude and two weak eddies merge to form a single stronger elliptical eddy in the middle of the enclosure. The isotherms and isoconcentrations also reveal a strong variation, indicating the convective strength in the enclosure. As the value of $\alpha$ is increased further ($\alpha = 45^\circ$), the intensity of the flow increases and the direction of the main cell is changed in the enclosure. The isothermal and isoconcentration contours exhibit stratification of temperature and concentration.

![Fig: 6 Effect of inclination angle ($\alpha$) on the streamlines (left), isotherms (middle) and isoconcentration (right) for $Ra=500$, $\phi=30$, $A=1$, $Le=2.0$, $N=1.0$. (Top) $\alpha = -45^\circ$, $|\psi_{max}|=3.0$, (centre) $\alpha = 0$, $|\psi_{max}|=15.2$ and (bottom) $\alpha = 45^\circ$, $|\psi_{max}|=24.9$.]

The heat and mass transfer rates are important quantitative measures of the problem and are investigated in Fig. 7 for three Darcy–Rayleigh numbers ($Ra$) and wide range of inclination angles ($\alpha$). In general, the Darcy–Rayleigh number characterizes the influence of external forces on the convective motion driven by the combined buoyancies. Hence, it can be seen from Fig. 7 that the heat and mass transfer rates increase with $Ra$. However, a careful observation of Fig. 7 reveals that the heat and mass transfer rates are increases with $\alpha$ up to $45^\circ$ and then decreases. Therefore, the optimal location for maximum heat and mass transfer not only depends on $Ra$, but also strongly depends on the inclination angle of the enclosure. 

![Fig: 7 Effect of inclination angle and Darcy-Rayleigh number on the average Nusselt and Sherwood numbers for $\phi=30^\circ$, $A=1$, $Le=2.0$, $N=1.0$.]
5. Conclusion

In this study, the influence two angles ($\alpha$ and $\phi$) on double diffusive natural convection in an inclined porous parallelogrammic enclosure has been investigated using the Darcy model. Comparisons with previously published work under the limiting cases of the problem are performed and are found to be in good agreement. The results are presented in the form of streamlines, isotherms, isoconcentrations, average Nusselt and Sherwood numbers for wide range of governing physical and geometrical parameters. What follows is a brief summary of the major results obtained from the present investigations.

The flow pattern, thermal and solutal fields, heat and mass transfer rates inside the enclosure strongly depends on the Darcy-Rayleigh number and the inclination angles. Increasing the Rayleigh number and inclination angle always lead to an increase in the heat and mass transfer performance of the enclosure. In addition, heat and mass transfer parameters reveal different behaviour when the buoyancy ratio varies between -5 to +5. The average Nusselt and Sherwood numbers increases as $Ra$ increases with respect to $N$. Further, depending on the need of the applications, it is possible to control the flow circulation, heat and mass transfer rates through the Darcy-Rayleigh number, tilt and inclination angles. Hence, it has been concluded that the heat and mass transfer rates can be either enhanced or suppressed with a proper combination of $Ra$, $\alpha$ and $\phi$. The results of this study clearly confirm the strong prospective of parallelogrammic enclosure filled with fluid saturated porous media for heat and mass transfer applications. In particular, the diode effect is one of the main characteristic of parallelogrammic enclosure and this can be used with great advantage for the inhibition or promotion of the heat and mass transfer processes.

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