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Three-point boundary value problems for nonlinear second-order impulsive q -difference equations

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Abstract

The quantum calculus on finite intervals was studied recently by the authors in *Adv. Differ. Equ.* 2013:282, 2013, where the concepts of q_k -derivative and q_k -integral of a function $f : J_k := [t_k, t_{k+1}] \rightarrow \mathbb{R}$ have been introduced. In this paper, we prove existence and uniqueness results for nonlinear second-order impulsive q_k -difference three-point boundary value problems, by using Banach's contraction mapping principle and Krasnoselskii's fixed-point theorem.

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1 Introduction

In this article, we investigate the nonlinear second-order impulsive q_k -difference equation with three-point boundary conditions

$$\begin{cases} D_{q_k}^2 x(t) = f(t, x(t)), & t \in J := [0, T], t \neq t_k, \\ \Delta x(t_k) = I_k(x(t_k)), & k = 1, 2, \dots, m, \\ D_{q_k} x(t_k^+) - D_{q_{k-1}} x(t_k) = I_k^*(x(t_k)), & k = 1, 2, \dots, m, \\ x(0) = 0, & x(T) = x(\eta), \end{cases} \quad (1.1)$$

where $0 = t_0 < t_1 < t_2 < \dots < t_k < \dots < t_m < t_{m+1} = T$, $f : J \times \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function, $I_k, I_k^* \in C(\mathbb{R}, \mathbb{R})$, $\Delta x(t_k) = x(t_k^+) - x(t_k)$ for $k = 1, 2, \dots, m$, $x(t_k^+) = \lim_{h \rightarrow 0} x(t_k + h)$, $\eta \in (t_j, t_{j+1})$ a constant for some $j \in \{0, 1, 2, \dots, m\}$ and $0 < q_k < 1$ for $k = 0, 1, 2, \dots, m$.

The theory of quantum calculus on finite intervals was developed recently by the authors in [1]. In [1] the concepts of q_k -derivative and q_k -integral of a function $f : J_k := [t_k, t_{k+1}] \rightarrow \mathbb{R}$, are defined and their basic properties proved. As applications, existence and uniqueness results for initial value problems for first- and second-order impulsive q_k -difference equations are proved.

The book by Kac and Cheung [2] covers many of the fundamental aspects of the quantum calculus. In recent years, the topic of q -calculus has attracted the attention of several researchers and a variety of new results can be found in the papers [3–15] and the references cited therein.

Impulsive differential equations, that is, differential equations involving an impulse effect, appear as a natural description of observed evolution phenomena of several real-world problems. For some monographs on impulsive differential equations we refer to [16–18].

In the present paper we prove existence and uniqueness results for the impulsive boundary value problem (1.1) by using Banach’s contraction mapping principle and Krasnoselskii’s fixed-point theorem. The rest of this paper is organized as follows: In Section 2 we present the notions of q_k -derivative and q_k -integral on finite intervals and collect their properties. The main results are proved in Section 3, while examples illustrating the results are presented in Section 4.

2 Preliminaries

In this section we present the notions of q_k -derivative and q_k -integral on finite intervals. For a fixed $k \in \mathbb{N} \cup \{0\}$ let $J_k := [t_k, t_{k+1}] \subset \mathbb{R}$ be an interval and $0 < q_k < 1$ be a constant. We define q_k -derivative of a function $f : J_k \rightarrow \mathbb{R}$ at a point $t \in J_k$ as follows.

Definition 2.1 Assume $f : J_k \rightarrow \mathbb{R}$ is a continuous function and let $t \in J_k$. Then the expression

$$D_{q_k}f(t) = \frac{f(t) - f(q_k t + (1 - q_k)t_k)}{(1 - q_k)(t - t_k)}, \quad t \neq t_k, \quad D_{q_k}f(t_k) = \lim_{t \rightarrow t_k} D_{q_k}f(t), \quad (2.1)$$

is called the q_k -derivative of function f at t .

We say that f is q_k -differentiable on J_k provided $D_{q_k}f(t)$ exists for all $t \in J_k$. Note that if $t_k = 0$ and $q_k = q$ in (2.1), then $D_{q_k}f = D_q f$, where D_q is the well-known q -derivative of the function $f(t)$ defined by

$$D_q f(t) = \frac{f(t) - f(qt)}{(1 - q)t}. \quad (2.2)$$

In addition, we should define the higher q_k -derivative of functions.

Definition 2.2 Let $f : J_k \rightarrow \mathbb{R}$ is a continuous function, we call the second-order q_k -derivative $D_{q_k}^2 f$ provided $D_{q_k}f$ is q_k -differentiable on J_k with $D_{q_k}^2 f = D_{q_k}(D_{q_k}f) : J_k \rightarrow \mathbb{R}$. Similarly, we define the higher-order q_k -derivative $D_{q_k}^n : J_k \rightarrow \mathbb{R}$.

The properties of the q_k -derivative are summarized in the following theorem.

Theorem 2.3 Assume $f, g : J_k \rightarrow \mathbb{R}$ are q_k -differentiable on J_k . Then:

- (i) The sum $f + g : J_k \rightarrow \mathbb{R}$ is q_k -differentiable on J_k with

$$D_{q_k}(f(t) + g(t)) = D_{q_k}f(t) + D_{q_k}g(t).$$

- (ii) For any constant α , $\alpha f : J_k \rightarrow \mathbb{R}$ is q_k -differentiable on J_k with

$$D_{q_k}(\alpha f)(t) = \alpha D_{q_k}f(t).$$

(iii) The product $fg : J_k \rightarrow \mathbb{R}$ is q_k -differentiable on J_k with

$$\begin{aligned} D_{q_k}(fg)(t) &= f(t)D_{q_k}g(t) + g(q_k t + (1 - q_k)t_k)D_{q_k}f(t) \\ &= g(t)D_{q_k}f(t) + f(q_k t + (1 - q_k)t_k)D_{q_k}g(t). \end{aligned}$$

(iv) If $g(t)g(q_k t + (1 - q_k)t_k) \neq 0$, then $\frac{f}{g}$ is q_k -differentiable on J_k with

$$D_{q_k}\left(\frac{f}{g}\right)(t) = \frac{g(t)D_{q_k}f(t) - f(t)D_{q_k}g(t)}{g(t)g(q_k t + (1 - q_k)t_k)}.$$

Definition 2.4 Assume $f : J_k \rightarrow \mathbb{R}$ is a continuous function. Then the q_k -integral is defined by

$$\int_{t_k}^t f(s) d_{q_k}s = (1 - q_k)(t - t_k) \sum_{n=0}^{\infty} q_k^n f(q_k^n t + (1 - q_k^n)t_k), \tag{2.3}$$

for $t \in J_k$. Moreover, if $a \in (t_k, t)$ then the definite q_k -integral is defined by

$$\begin{aligned} \int_a^t f(s) d_{q_k}s &= \int_{t_k}^t f(s) d_{q_k}s - \int_{t_k}^a f(s) d_{q_k}s \\ &= (1 - q_k)(t - t_k) \sum_{n=0}^{\infty} q_k^n f(q_k^n t + (1 - q_k^n)t_k) \\ &\quad - (1 - q_k)(a - t_k) \sum_{n=0}^{\infty} q_k^n f(q_k^n a + (1 - q_k^n)t_k). \end{aligned}$$

Note that if $t_k = 0$ and $q_k = q$, then (2.3) reduces to the q -integral of a function $f(t)$, defined by $\int_0^t f(s) d_q s = (1 - q)t \sum_{n=0}^{\infty} q^n f(q^n t)$ for $t \in [0, \infty)$.

Theorem 2.5 For $t \in J_k$, the following formulas hold:

- (i) $D_{q_k} \int_{t_k}^t f(s) d_{q_k}s = f(t)$;
- (ii) $\int_{t_k}^t D_{q_k}f(s) d_{q_k}s = f(t)$;
- (iii) $\int_a^t D_{q_k}f(s) d_{q_k}s = f(t) - f(a)$ for $a \in (t_k, t)$.

3 Main results

Let $J = [0, T]$, $J_0 = [t_0, t_1]$, $J_k = (t_k, t_{k+1}]$ for $k = 1, 2, \dots, m$. Let $PC(J, \mathbb{R}) = \{x : J \rightarrow \mathbb{R} : x(t) \text{ is continuous everywhere except for some } t_k \text{ at which } x(t_k^+) \text{ and } x(t_k^-) \text{ exist and } x(t_k^-) = x(t_k), k = 1, 2, \dots, m\}$. $PC(J, \mathbb{R})$ is a Banach space with the norm $\|x\|_{PC} = \sup\{|x(t)|; t \in J\}$.

Lemma 3.1 The unique solution of problem (1.1) is given by

$$\begin{aligned} x(t) &= -t \sum_{k=1}^j \left(\int_{t_{k-1}}^{t_k} f(s, x(s)) d_{q_{k-1}}s + I_k^*(x(t_k)) \right) \\ &\quad - \frac{t}{T - \eta} \sum_{k=j+1}^m \left(\int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^s f(\sigma, x(\sigma)) d_{q_{k-1}}\sigma d_{q_{k-1}}s + I_k(x(t_k)) \right) \end{aligned}$$

$$\begin{aligned}
 & - \frac{t}{T-\eta} \sum_{k=j+1}^m \left(\int_{t_{k-1}}^{t_k} f(s, x(s)) d_{q_{k-1}} s + I_k^*(x(t_k)) \right) (T - t_k) \\
 & + \frac{t}{T-\eta} \int_{t_j}^{\eta} \int_{t_j}^s f(\sigma, x(\sigma)) d_{q_j} \sigma d_{q_j} s \\
 & - \frac{t}{T-\eta} \int_{t_m}^T \int_{t_m}^s f(\sigma, x(\sigma)) d_{q_m} \sigma d_{q_m} s \\
 & + \sum_{0 < t_k < t} \left(\int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^s f(\sigma, x(\sigma)) d_{q_{k-1}} \sigma d_{q_{k-1}} s + I_k(x(t_k)) \right) \\
 & + \sum_{0 < t_k < t} \left(\int_{t_{k-1}}^{t_k} f(s, x(s)) d_{q_{k-1}} s + I_k^*(x(t_k)) \right) (t - t_k) \\
 & + \int_{t_k}^t \int_{t_k}^s f(\sigma, x(\sigma)) d_{q_k} \sigma d_{q_k} s, \tag{3.1}
 \end{aligned}$$

with $\sum_{0 < 0}(\cdot) = 0$.

Proof For $t \in J_0$, taking the q_0 -integral for the first equation of (1.1), we get

$$D_{q_0} x(t) = D_{q_0} x(0) + \int_0^t f(s, x(s)) d_{q_0} s, \tag{3.2}$$

which yields

$$D_{q_0} x(t_1) = D_{q_0} x(0) + \int_0^{t_1} f(s, x(s)) d_{q_0} s. \tag{3.3}$$

For $t \in J_0$ we obtain by q_0 -integrating (3.2),

$$\begin{aligned}
 x(t) & = x(0) + D_{q_0} x(0)t + \int_0^t \int_0^s f(\sigma, x(\sigma)) d_{q_0} \sigma d_{q_0} s \\
 & := A + Bt + \int_0^t \int_0^s f(\sigma, x(\sigma)) d_{q_0} \sigma d_{q_0} s \quad (x(0) = A, D_{q_0} x(0) = B).
 \end{aligned}$$

In particular, for $t = t_1$

$$x(t_1) = A + Bt_1 + \int_0^{t_1} \int_0^s f(\sigma, x(\sigma)) d_{q_0} \sigma d_{q_0} s. \tag{3.4}$$

For $t \in J_1 = (t_1, t_2]$, q_1 -integrating (1.1), we have

$$D_{q_1} x(t) = D_{q_1} x(t_1^+) + \int_{t_1}^t f(s, x(s)) d_{q_1} s.$$

Using the third condition of (1.1) with (3.3), it follows that

$$D_{q_1} x(t) = B + \int_0^{t_1} f(s, x(s)) d_{q_0} s + I_1^*(x(t_1)) + \int_{t_1}^t f(s, x(s)) d_{q_1} s. \tag{3.5}$$

Taking the q_1 -integral to (3.5) for $t \in J_1$, we obtain

$$\begin{aligned}
 x(t) &= x(t_1^+) + \left[B + \int_0^{t_1} f(s, x(s)) d_{q_0} s + I_1^*(x(t_1)) \right] (t - t_1) \\
 &\quad + \int_{t_1}^t \int_{t_1}^s f(\sigma, x(\sigma)) d_{q_1} \sigma d_{q_1} s.
 \end{aligned} \tag{3.6}$$

Applying the second equation of (1.1) with (3.4) and (3.6), we get

$$\begin{aligned}
 x(t) &= A + Bt_1 + \int_0^{t_1} \int_0^s f(\sigma, x(\sigma)) d_{q_0} \sigma d_{q_0} s + I_1(x(t_1)) \\
 &\quad + \left[B + \int_0^{t_1} f(s, x(s)) d_{q_0} s + I_1^*(x(t_1)) \right] (t - t_1) \\
 &\quad + \int_{t_1}^t \int_{t_1}^s f(\sigma, x(\sigma)) d_{q_1} \sigma d_{q_1} s \\
 &= A + Bt + \int_0^{t_1} \int_0^s f(\sigma, x(\sigma)) d_{q_0} \sigma d_{q_0} s + I_1(x(t_1)) \\
 &\quad + \left[\int_0^{t_1} f(s, x(s)) d_{q_0} s + I_1^*(x(t_1)) \right] (t - t_1) \\
 &\quad + \int_{t_1}^t \int_{t_1}^s f(\sigma, x(\sigma)) d_{q_1} \sigma d_{q_1} s.
 \end{aligned}$$

Repeating the above process, for $t \in J$, we get

$$\begin{aligned}
 x(t) &= A + Bt \\
 &\quad + \sum_{0 < t_k < t} \left(\int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^s f(\sigma, x(\sigma)) d_{q_{k-1}} \sigma d_{q_{k-1}} s + I_k(x(t_k)) \right) \\
 &\quad + \sum_{0 < t_k < t} \left(\int_{t_{k-1}}^{t_k} f(s, x(s)) d_{q_{k-1}} s + I_k^*(x(t_k)) \right) (t - t_k) \\
 &\quad + \int_{t_k}^t \int_{t_k}^s f(\sigma, x(\sigma)) d_{q_k} \sigma d_{q_k} s.
 \end{aligned} \tag{3.7}$$

The first boundary condition of (1.1) implies $A = 0$. The second boundary condition of (1.1) yields

$$\begin{aligned}
 &\sum_{k=1}^m \left(\int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^s f(\sigma, x(\sigma)) d_{q_{k-1}} \sigma d_{q_{k-1}} s + I_k(x(t_k)) \right) \\
 &\quad + \sum_{k=1}^m \left(\int_{t_{k-1}}^{t_k} f(s, x(s)) d_{q_{k-1}} s + I_k^*(x(t_k)) \right) (T - t_k) \\
 &\quad + \int_{t_m}^T \int_{t_m}^s f(\sigma, x(\sigma)) d_{q_m} \sigma d_{q_m} s + BT \\
 &= \sum_{k=1}^j \left(\int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^s f(\sigma, x(\sigma)) d_{q_{k-1}} \sigma d_{q_{k-1}} s + I_k(x(t_k)) \right)
 \end{aligned}$$

$$\begin{aligned}
 & + \sum_{k=1}^j \left(\int_{t_{k-1}}^{t_k} f(s, x(s)) d_{q_{k-1}} s + I_k^*(x(t_k)) \right) (\eta - t_k) \\
 & + \int_{t_j}^{\eta} \int_{t_j}^s f(\sigma, x(\sigma)) d_{q_j} \sigma d_{q_j} s + B\eta,
 \end{aligned}$$

which implies

$$\begin{aligned}
 B = & - \sum_{k=1}^j \left(\int_{t_{k-1}}^{t_k} f(s, x(s)) d_{q_{k-1}} s + I_k^*(x(t_k)) \right) \\
 & - \frac{1}{T - \eta} \sum_{k=j+1}^m \left(\int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^s f(\sigma, x(\sigma)) d_{q_{k-1}} \sigma d_{q_{k-1}} s + I_k(x(t_k)) \right) \\
 & - \frac{1}{T - \eta} \sum_{k=j+1}^m \left(\int_{t_{k-1}}^{t_k} f(s, x(s)) d_{q_{k-1}} s + I_k^*(x(t_k)) \right) (T - t_k) \\
 & + \frac{1}{T - \eta} \int_{t_j}^{\eta} \int_{t_j}^s f(\sigma, x(\sigma)) d_{q_j} \sigma d_{q_j} s - \frac{1}{T - \eta} \int_{t_m}^T \int_{t_m}^s f(\sigma, x(\sigma)) d_{q_m} \sigma d_{q_m} s.
 \end{aligned}$$

Substituting the constant B into (3.7), we obtain (3.1) as required. □

In view of Lemma 3.1, we define an operator $\mathcal{A} : PC(J, \mathbb{R}) \rightarrow PC(J, \mathbb{R})$ by

$$\begin{aligned}
 (\mathcal{A}x)(t) = & -t \sum_{k=1}^j \left(\int_{t_{k-1}}^{t_k} f(s, x(s)) d_{q_{k-1}} s + I_k^*(x(t_k)) \right) \\
 & - \frac{t}{T - \eta} \sum_{k=j+1}^m \left(\int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^s f(\sigma, x(\sigma)) d_{q_{k-1}} \sigma d_{q_{k-1}} s + I_k(x(t_k)) \right) \\
 & - \frac{t}{T - \eta} \sum_{k=j+1}^m \left(\int_{t_{k-1}}^{t_k} f(s, x(s)) d_{q_{k-1}} s + I_k^*(x(t_k)) \right) (T - t_k) \\
 & + \frac{t}{T - \eta} \int_{t_j}^{\eta} \int_{t_j}^s f(\sigma, x(\sigma)) d_{q_j} \sigma d_{q_j} s \\
 & - \frac{t}{T - \eta} \int_{t_m}^T \int_{t_m}^s f(\sigma, x(\sigma)) d_{q_m} \sigma d_{q_m} s \\
 & + \sum_{0 < t_k < t} \left(\int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^s f(\sigma, x(\sigma)) d_{q_{k-1}} \sigma d_{q_{k-1}} s + I_k(x(t_k)) \right) \\
 & + \sum_{0 < t_k < t} \left(\int_{t_{k-1}}^{t_k} f(s, x(s)) d_{q_{k-1}} s + I_k^*(x(t_k)) \right) (t - t_k) \\
 & + \int_{t_k}^t \int_{t_k}^s f(\sigma, x(\sigma)) d_{q_k} \sigma d_{q_k} s.
 \end{aligned} \tag{3.8}$$

It should be noticed that problem (1.1) has solutions if and only if the operator \mathcal{A} has fixed points.

For convenience, we set

$$\Phi_k = \left[(t_k - t_{k-1})(T - t_k) + \frac{(t_k - t_{k-1})^2}{1 + q_{k-1}} \right] M_1 + M_2 + (T - t_k)M_3, \tag{3.9}$$

$$\Psi_k = \left[(t_k - t_{k-1})(T - t_k) + \frac{(t_k - t_{k-1})^2}{1 + q_{k-1}} \right] L_1 + L_2 + (T - t_k)L_3, \tag{3.10}$$

for $k = 1, \dots, m$.

Theorem 3.2 *Assume that:*

- (H₁) *The function $f : [0, T] \times \mathbb{R} \rightarrow \mathbb{R}$ is a continuous and there exists a constant $L_1 > 0$ such that $|f(t, x) - f(t, y)| \leq L_1|x - y|$, for each $t \in J$ and $x, y \in \mathbb{R}$.*
- (H₂) *The functions $I_k, I_k^* : \mathbb{R} \rightarrow \mathbb{R}$ are continuous and there exist constants $L_2, L_3 > 0$ such that $|I_k(x) - I_k(y)| \leq L_2|x - y|$ and $|I_k^*(x) - I_k^*(y)| \leq L_3|x - y|$ for each $x, y \in \mathbb{R}, k = 1, 2, \dots, m$.*

If

$$\begin{aligned} \Delta := & T \sum_{k=1}^j [(t_k - t_{k-1})L_1 + L_3] + \frac{T}{T - \eta} \sum_{k=j+1}^m \Psi_k + \frac{TL_1}{T - \eta} \left(\frac{(\eta - t_j)^2}{1 + q_j} + \frac{(T - t_m)^2}{1 + q_m} \right) \\ & + \sum_{k=1}^m \Psi_k + \frac{(T - t_m)^2}{1 + q_m} L_1 \leq \delta < 1, \end{aligned} \tag{3.11}$$

then the impulsive q_k -difference boundary value problem (1.1) has a unique solution on J .

Proof First, we transform the problem (1.1) into a fixed-point problem, $x = \mathcal{A}x$, where the operator \mathcal{A} is defined by (3.8). By using Banach’s contraction principle, we shall show that \mathcal{A} has a fixed point which is the unique solution of problem (1.1).

Set $\sup_{t \in J} |f(t, 0)| = M_1 < \infty$, $\sup\{|I_k(0)| : k = 1, 2, \dots, m\} = M_2 < \infty$, $\sup\{|I_k^*(0)| : k = 1, 2, \dots, m\} = M_3 < \infty$ and a constant

$$\begin{aligned} \rho = & T \sum_{k=1}^j [(t_k - t_{k-1})M_1 + M_3] + \frac{T}{T - \eta} \sum_{k=j+1}^m \Phi_k \\ & + \frac{TM_1}{T - \eta} \left(\frac{(\eta - t_j)^2}{1 + q_j} + \frac{(T - t_m)^2}{1 + q_m} \right) \\ & + \sum_{k=1}^m \Phi_k + \frac{(T - t_m)^2}{1 + q_m} M_1. \end{aligned} \tag{3.12}$$

Choosing $r \geq \frac{\rho}{1 - \delta}$, where $\delta \leq \varepsilon < 1$, we show that $\mathcal{A}B_r \subset B_r$, where $B_r = \{x \in PC(J, \mathbb{R}) : \|x\| \leq r\}$. For $x \in B_r$, we have

$$\begin{aligned} & \|\mathcal{A}x\| \\ & \leq \sup_{t \in J} \left\{ t \sum_{k=1}^j \left(\int_{t_{k-1}}^{t_k} |f(s, x(s))| d_{q_{k-1}} s + |I_k^*(x(t_k))| \right) \right\} \end{aligned}$$

$$\begin{aligned}
 & + \frac{t}{T-\eta} \sum_{k=j+1}^m \left(\int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^s |f(\sigma, x(\sigma))| d_{q_{k-1}} \sigma d_{q_{k-1}} s + |I_k(x(t_k))| \right) \\
 & + \frac{t}{T-\eta} \sum_{k=j+1}^m \left(\int_{t_{k-1}}^{t_k} |f(s, x(s))| d_{q_{k-1}} s + |I_k^*(x(t_k))| \right) (T-t_k) \\
 & + \frac{t}{T-\eta} \int_{t_j}^{\eta} \int_{t_j}^s |f(\sigma, x(\sigma))| d_{q_j} \sigma d_{q_j} s + \frac{t}{T-\eta} \int_{t_m}^T \int_{t_m}^s |f(\sigma, x(\sigma))| d_{q_m} \sigma d_{q_m} s \\
 & + \sum_{0 < t_k < t} \left(\int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^s |f(\sigma, x(\sigma))| d_{q_{k-1}} \sigma d_{q_{k-1}} s + |I_k(x(t_k))| \right) \\
 & + \sum_{0 < t_k < t} \left(\int_{t_{k-1}}^{t_k} |f(s, x(s))| d_{q_{k-1}} s + |I_k^*(x(t_k))| \right) (t-t_k) \\
 & + \left. \int_{t_k}^t \int_{t_k}^s |f(\sigma, x(\sigma))| d_{q_k} \sigma d_{q_k} s \right\} \\
 \leq & T \sum_{k=1}^j \left(\int_{t_{k-1}}^{t_k} (|f(s, x(s)) - f(s, 0)| + |f(s, 0)|) d_{q_{k-1}} s + |I_k^*(x(t_k)) - I_k^*(0)| + |I_k^*(0)| \right) \\
 & + \frac{T}{T-\eta} \sum_{k=j+1}^m \left(\int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^s (|f(\sigma, x(\sigma)) - f(\sigma, 0)| + |f(\sigma, 0)|) d_{q_{k-1}} \sigma d_{q_{k-1}} s \right. \\
 & \left. + |I_k(x(t_k)) - I_k(0)| + |I_k(0)| \right) \\
 & + \frac{T}{T-\eta} \sum_{k=j+1}^m \left(\int_{t_{k-1}}^{t_k} (|f(s, x(s)) - f(s, 0)| + |f(s, 0)|) d_{q_{k-1}} s \right. \\
 & \left. + |I_k^*(x(t_k)) - I_k^*(0)| + |I_k^*(0)| \right) (T-t_k) \\
 & + \frac{T}{T-\eta} \int_{t_j}^{\eta} \int_{t_j}^s (|f(\sigma, x(\sigma)) - f(\sigma, 0)| + |f(\sigma, 0)|) d_{q_j} \sigma d_{q_j} s \\
 & + \frac{T}{T-\eta} \int_{t_m}^T \int_{t_m}^s (|f(\sigma, x(\sigma)) - f(\sigma, 0)| + |f(\sigma, 0)|) d_{q_m} \sigma d_{q_m} s \\
 & + \sum_{k=1}^m \left(\int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^s (|f(\sigma, x(\sigma)) - f(\sigma, 0)| + |f(\sigma, 0)|) d_{q_{k-1}} \sigma d_{q_{k-1}} s \right. \\
 & \left. + |I_k(x(t_k)) - I_k(0)| + |I_k(0)| \right) \\
 & + \sum_{k=1}^m \left(\int_{t_{k-1}}^{t_k} (|f(s, x(s)) - f(s, 0)| + |f(s, 0)|) d_{q_{k-1}} s \right. \\
 & \left. + |I_k^*(x(t_k)) - I_k^*(0)| + |I_k^*(0)| \right) (T-t_k) \\
 & + \int_{t_m}^T \int_{t_m}^s (|f(\sigma, x(\sigma)) - f(\sigma, 0)| + |f(\sigma, 0)|) d_{q_m} \sigma d_{q_m} s \\
 \leq & T \sum_{k=1}^j ((t_k - t_{k-1})(L_1 r + M_1) + L_3 r + M_3)
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{T}{T-\eta} \sum_{k=j+1}^m \left(\frac{(t_k - t_{k-1})^2}{1 + q_{k-1}} (L_1 r + M_1) + L_2 r + M_2 \right) \\
 & + \frac{T}{T-\eta} \sum_{k=j+1}^m ((t_k - t_{k-1})(L_1 r + M_1) + L_3 r + M_3)(T - t_k) \\
 & + \frac{T}{T-\eta} \left(\frac{(\eta - t_j)^2}{1 + q_j} + \frac{(T - t_m)^2}{1 + q_m} \right) (L_1 r + M_1) \\
 & + \sum_{k=1}^m \left(\frac{(t_k - t_{k-1})^2}{1 + q_{k-1}} (L_1 r + M_1) + L_2 r + M_2 \right) \\
 & + \sum_{k=1}^m ((t_k - t_{k-1})(L_1 r + M_1) + L_3 r + M_3)(T - t_k) + \frac{(T - t_m)^2}{1 + q_m} (L_1 r + M_1) \\
 & = r\Lambda + \rho \leq (\delta + 1 - \varepsilon)r \leq r.
 \end{aligned}$$

It follows that $AB_r \subset B_r$.

For $x, y \in PC(J, \mathbb{R})$ and for each $t \in J$, we have

$$\begin{aligned}
 & \|Ax - Ay\| \\
 & \leq \sup_{t \in J} \left\{ t \sum_{k=1}^j \left(\int_{t_{k-1}}^{t_k} |f(s, x(s)) - f(s, y(s))| d_{q_{k-1}} s + |I_k^*(x(t_k)) - I_k^*(y(t_k))| \right) \right. \\
 & \quad + \frac{t}{T-\eta} \sum_{k=j+1}^m \left(\int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^s |f(\sigma, x(\sigma)) - f(\sigma, y(\sigma))| d_{q_{k-1}} \sigma d_{q_{k-1}} s \right. \\
 & \quad \left. \left. + |I_k(x(t_k)) - I_k(y(t_k))| \right) \right. \\
 & \quad + \frac{t}{T-\eta} \sum_{k=j+1}^m \left(\int_{t_{k-1}}^{t_k} |f(s, x(s)) - f(s, y(s))| d_{q_{k-1}} s + |I_k^*(x(t_k)) - I_k^*(y(t_k))| \right) (T - t_k) \\
 & \quad + \frac{t}{T-\eta} \int_{t_j}^{\eta} \int_{t_j}^s |f(\sigma, x(\sigma)) - f(\sigma, y(\sigma))| d_{q_j} \sigma d_{q_j} s \\
 & \quad + \frac{t}{T-\eta} \int_{t_m}^T \int_{t_m}^s |f(\sigma, x(\sigma)) - f(\sigma, y(\sigma))| d_{q_m} \sigma d_{q_m} s \\
 & \quad + \sum_{0 < t_k < t} \left(\int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^s |f(\sigma, x(\sigma)) - f(\sigma, y(\sigma))| d_{q_{k-1}} \sigma d_{q_{k-1}} s + |I_k(x(t_k)) - I_k(y(t_k))| \right) \\
 & \quad + \sum_{0 < t_k < t} \left(\int_{t_{k-1}}^{t_k} |f(s, x(s)) - f(s, y(s))| d_{q_{k-1}} s + |I_k^*(x(t_k)) - I_k^*(y(t_k))| \right) (t - t_k) \\
 & \quad \left. + \int_{t_k}^t \int_{t_k}^s |f(\sigma, x(\sigma)) - f(\sigma, y(\sigma))| d_{q_k} \sigma d_{q_k} s \right\} \\
 & \leq T \|x - y\| \sum_{k=1}^j [(t_k - t_{k-1})L_1 + L_3] + \frac{T \|x - y\|}{T - \eta} \sum_{k=j+1}^m \left(\frac{(t_k - t_{k-1})^2}{1 + q_{k-1}} L_1 + L_2 \right) \\
 & \quad + \frac{T \|x - y\|}{T - \eta} \sum_{k=j+1}^m ((t_k - t_{k-1})L_1 + L_3)(T - t_k)
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{T\|x - y\|}{T - \eta} \left(\frac{(\eta - t_j)^2}{1 + q_j} + \frac{(T - t_m)^2}{1 + q_m} \right) L_1 \\
 & + \sum_{k=1}^m \left(\frac{(t_k - t_{k-1})^2}{1 + q_{k-1}} L_1 + L_2 \right) \|x - y\| + \sum_{k=1}^m ((t_k - t_{k-1})L_1 + L_3)(T - t_k) \|x - y\| \\
 & + \frac{(T - t_m)^2}{1 + q_m} L_1 \|x - y\| \\
 & = \Lambda \|x - y\|.
 \end{aligned}$$

As $\Lambda < 1$, \mathcal{A} is a contraction. Hence, by Banach's contraction mapping principle, we find that \mathcal{A} has a fixed point which is the unique solution of problem (1.1). \square

Our next result is based on Krasnoselskii's fixed-point theorem.

Lemma 3.3 (Krasnoselskii's fixed-point theorem) [19] *Let M be a closed, bounded, convex and nonempty subset of a Banach space X . Let A, B be the operators such that (a) $Ax + By \in M$ whenever $x, y \in M$; (b) A is compact and continuous; (c) B is a contraction mapping. Then there exists $z \in M$ such that $z = Az + Bz$.*

Further, we use the notation

$$\begin{aligned}
 \theta_1 & = T \sum_{k=1}^j (t_k - t_{k-1}) + \frac{T}{T - \eta} \sum_{k=j+1}^m \frac{(t_k - t_{k-1})^2}{1 + q_{k-1}} \\
 & + \frac{T}{T - \eta} \sum_{k=j+1}^m (T - t_k)(t_k - t_{k-1}) + \frac{T(\eta - t_j)^2}{(T - \eta)(1 + q_j)} \\
 & + \frac{T(T - t_m)^2}{(T - \eta)(1 + q_m)} + \sum_{k=1}^{m+1} \frac{(t_k - t_{k-1})^2}{1 + q_{k-1}} + \sum_{k=1}^m (T - t_k)(t_k - t_{k-1}), \tag{3.13}
 \end{aligned}$$

and

$$\theta_2 = jTN_2 + \frac{(m - j)TN_1}{T - \eta} + mN_1 + N_2 \sum_{k=1}^m (T - t_k) + \frac{TN_2}{T - \eta} \sum_{j+1}^m (T - t_k). \tag{3.14}$$

Theorem 3.4 *Let $f : J \times \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function. Assume that (H_2) holds and in addition suppose that:*

(H_3) $|f(t, x)| \leq \mu(t), \forall (t, x) \in J \times \mathbb{R}$, and $\mu \in C(J, \mathbb{R}^+)$.

(H_4) *There exist constants $N_1, N_2 > 0$ such that $|I_k(x)| \leq N_1$ and $|I_k^*(x)| \leq N_2$ for all $x \in \mathbb{R}$, for $k = 1, 2, \dots, m$.*

Then the impulsive q_k -difference boundary value problem (1.1) has at least one solution on J provided that

$$jTL_3 + mL_2 + \frac{T(m - j)L_2}{T - \eta} + L_3 \sum_{k=1}^m (T - t_k) < 1. \tag{3.15}$$

Proof Firstly, we define $\sup_{t \in J} |\mu(t)| = \|\mu\|$. Choosing a suitable ball $B_R = \{x \in PC(J, \mathbb{R}) : \|x\| \leq R\}$, where

$$R \geq \|\mu\| \theta_1 + \theta_2, \tag{3.16}$$

and θ_1, θ_2 are defined by (3.13), (3.14), respectively, we define the operators \mathcal{S}_1 and \mathcal{S}_2 on B_R by

$$\begin{aligned} (\mathcal{S}_1 x)(t) &= -t \sum_{k=1}^j \int_{t_{k-1}}^{t_k} f(s, x(s)) d_{q_{k-1}} s - \frac{t}{T-\eta} \sum_{k=j+1}^m \int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^s f(\sigma, x(\sigma)) d_{q_{k-1}} \sigma d_{q_{k-1}} s \\ &\quad - \frac{t}{T-\eta} \sum_{k=j+1}^m (T-t_k) \int_{t_{k-1}}^{t_k} f(s, x(s)) d_{q_{k-1}} s + \frac{t}{T-\eta} \int_{t_j}^{\eta} \int_{t_j}^s f(\sigma, x(\sigma)) d_{q_j} \sigma d_{q_j} s \\ &\quad - \frac{t}{T-\eta} \int_{t_m}^T \int_{t_m}^s f(\sigma, x(\sigma)) d_{q_m} \sigma d_{q_m} s + \sum_{0 < t_k < t} \int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^s f(\sigma, x(\sigma)) d_{q_{k-1}} \sigma d_{q_{k-1}} s \\ &\quad + \sum_{0 < t_k < t} (t-t_k) \int_{t_{k-1}}^{t_k} f(s, x(s)) d_{q_{k-1}} s + \int_{t_k}^t \int_{t_k}^s f(\sigma, x(\sigma)) d_{q_k} \sigma d_{q_k} s, \quad t \in [0, T], \end{aligned}$$

and

$$\begin{aligned} (\mathcal{S}_2 x)(t) &= -t \sum_{k=1}^j I_k^*(x(t_k)) - \frac{t}{T-\eta} \sum_{k=j+1}^m I_k(x(t_k)) - \frac{t}{T-\eta} \sum_{k=j+1}^m (T-t_k) I_k^*(x(t_k)) \\ &\quad + \sum_{0 < t_k < t} I_k(x(t_k)) + \sum_{0 < t_k < t} (t-t_k) I_k^*(x(t_k)), \quad t \in [0, T]. \end{aligned}$$

For any $x, y \in B_R$, we have

$$\begin{aligned} \|\mathcal{S}_1 x + \mathcal{S}_2 y\| &\leq \|\mu\| \left[T \sum_{k=1}^j (t_k - t_{k-1}) + \frac{T}{T-\eta} \sum_{k=j+1}^m \frac{(t_k - t_{k-1})^2}{1 + q_{k-1}} \right. \\ &\quad + \frac{T}{T-\eta} \sum_{k=j+1}^m (T-t_k)(t_k - t_{k-1}) + \frac{T(\eta - t_j)^2}{(T-\eta)(1 + q_j)} \\ &\quad + \left. \frac{T(T-t_m)^2}{(T-\eta)(1 + q_m)} + \sum_{k=1}^{m+1} \frac{(t_k - t_{k-1})^2}{1 + q_{k-1}} + \sum_{k=1}^m (T-t_k)(t_k - t_{k-1}) \right] \\ &\quad + jTN_2 + \frac{(m-j)TN_1}{T-\eta} + mN_1 + N_2 \sum_{k=1}^m (T-t_k) + \frac{TN_2}{T-\eta} \sum_{j+1}^m (T-t_k) \\ &= \|\mu\| \theta_1 + \theta_2 \\ &\leq R. \end{aligned}$$

Hence, $\mathcal{S}_1 x + \mathcal{S}_2 y \in B_R$.

To show that \mathcal{S}_2 is a contraction, for $x, y \in PC(J, \mathbb{R})$, we have

$$\begin{aligned} \|\mathcal{S}_2x - \mathcal{S}_2y\| &\leq T \sum_{k=1}^j |I_k^*(x(t_k)) - I_k^*(y(t_k))| + \frac{T}{T-\eta} \sum_{k=j+1}^m |I_k(x(t_k)) - I_k(y(t_k))| \\ &\quad + \sum_{k=1}^m |I(x(t_k)) - I(y(t_k))| + \sum_{k=1}^m (t-t_k) |I_k^*(x(t_k)) - I_k^*(y(t_k))| \\ &\leq \left[jTL_3 + mL_2 + \frac{T(m-j)L_2}{T-\eta} + L_3 \sum_{k=1}^m (T-t_k) \right] \|x-y\|. \end{aligned}$$

From (3.15), it follows that \mathcal{S}_2 is a contraction.

Next, the continuity of f implies that the operator \mathcal{S}_1 is continuous. Further, \mathcal{S}_1 is uniformly bounded on B_R by

$$\|\mathcal{S}_1x\| \leq \|\mu\|\theta_1.$$

Now we shall prove the compactness of \mathcal{S}_1 . Setting $\sup_{(t,x) \in J \times B_R} |f(t,x)| = f^* < \infty$, then for each $\tau_1, \tau_2 \in (t_l, t_{l+1})$ for some $l \in \{0, 1, \dots, m\}$ with $\tau_2 > \tau_1$, we have

$$\begin{aligned} &|(\mathcal{S}_1x)(\tau_2) - (\mathcal{S}_1x)(\tau_1)| \\ &\leq |\tau_2 - \tau_1| \sum_{k=1}^j \int_{t_{k-1}}^{t_k} |f(s, x(s))| d_{q_{k-1}}s \\ &\quad + \frac{|\tau_2 - \tau_1|}{T-\eta} \sum_{k=j+1}^m \int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^s |f(\sigma, x(\sigma))| d_{q_{k-1}}\sigma d_{q_{k-1}}s \\ &\quad + \frac{|\tau_2 - \tau_1|}{T-\eta} \sum_{k=j+1}^m (T-t_k) \int_{t_{k-1}}^{t_k} |f(s, x(s))| d_{q_{k-1}}s \\ &\quad + \frac{|\tau_2 - \tau_1|}{T-\eta} \int_{t_j}^{\eta} \int_{t_j}^s |f(\sigma, x(\sigma))| d_{q_j}\sigma d_{q_j}s \\ &\quad + \frac{|\tau_2 - \tau_1|}{T-\eta} \int_{t_m}^T \int_{t_m}^s |f(\sigma, x(\sigma))| d_{q_m}\sigma d_{q_m}s + |\tau_2 - \tau_1| \sum_{k=1}^l \int_{t_{k-1}}^{t_k} |f(s, x(s))| d_{q_{k-1}}s \\ &\quad + \left| \int_{t_l}^{\tau_2} \int_{t_l}^s |f(\sigma, x(\sigma))| d_{q_l}\sigma d_{q_l}s - \int_{t_l}^{\tau_1} \int_{t_l}^s |f(\sigma, x(\sigma))| d_{q_l}\sigma d_{q_l}s \right| \\ &\leq |\tau_2 - \tau_1| f^* \left[\sum_{k=1}^j (t_k - t_{k-1}) + \frac{1}{T-\eta} \sum_{k=j+1}^m \frac{(t_k - t_{k-1})^2}{1 + q_{k-1}} + \frac{(\eta - t_j)^2}{(T-\eta)(1 + q_j)} \right. \\ &\quad + \frac{(T - t_m)^2}{(T-\eta)(1 + q_m)} + \frac{1}{T-\eta} \sum_{k=j+1}^m (T-t_k)(t_k - t_{k-1}) \\ &\quad \left. + \sum_{k=1}^l (t_k - t_{k-1}) + \frac{(\tau_1 + \tau_2 + 2t_l)}{1 + q_l} \right]. \end{aligned}$$

As $\tau_1 \rightarrow \tau_2$, the right hand side above (which is independent of x) tends to zero. Therefore, the operator \mathcal{S}_1 is equicontinuous. Since \mathcal{S}_1 maps bounded subsets into relatively compact

subsets, it follows that \mathcal{S}_1 is relative compact on B_R . Hence, by the Arzelá-Ascoli theorem, \mathcal{S}_1 is compact on B_R . Thus all the assumptions of Lemma 3.3 are satisfied. Hence, by the conclusion of Lemma 3.3, the impulsive q_k -difference boundary value problem (1.1) has at least one solution on J . \square

4 Examples

Example 4.1 Consider the following nonlinear second-order impulsive q_k -difference equation with three-point boundary condition:

$$\begin{cases} D_{\frac{4}{5+k}}^2 x(t) = \frac{e^{-\cos^2 t} |x(t)|}{(6+t)^2(1+|x(t)|)}, & t \in J = [0, 1], t \neq t_k = \frac{k}{10}, \\ \Delta x(t_k) = \frac{|x(t_k)|}{8(7+|x(t_k)|)}, & k = 1, 2, \dots, 9, \\ D_{\frac{4}{5+k}} x(t_k^+) - D_{\frac{4}{4+k}} x(t_k) = \frac{1}{6} \tan^{-1}\left(\frac{1}{8}x(t_k)\right), & k = 1, 2, \dots, 9, \\ x(0) = 0, & x(1) = x\left(\frac{1}{4}\right). \end{cases} \quad (4.1)$$

Here $q_k = 4/(5 + k)$ for $k = 0, 1, 2, \dots, 9$, $m = 9$, $T = 1$, $\eta = 1/4$, $j = 2$, $f(t, x) = (e^{-\cos^2 t} |x|) / ((6 + t)^2(1 + |x|))$, $I_k(x) = |x| / (8(7 + |x|))$ and $I_k^*(x) = (1/6) \tan^{-1}(x/8)$. Since

$$\begin{aligned} |f(t, x) - f(t, y)| &\leq (1/36)|x - y|, \\ |I_k(x) - I_k(y)| &\leq (1/56)|x - y| \quad \text{and} \quad |I_k^*(x) - I_k^*(y)| \leq (1/48)|x - y|, \end{aligned}$$

then (H_1) and (H_2) are satisfied with $L_1 = (1/36)$, $L_2 = (1/56)$, $L_3 = (1/48)$. We can show that

$$\Lambda \approx 0.5730986482 < 1.$$

Hence, by Theorem 3.2, the three-point impulsive q_k -difference boundary value problem (4.1) has a unique solution on $[0, 1]$.

Example 4.2 Consider the following nonlinear second-order impulsive q_k -difference equation with three-point boundary condition:

$$\begin{cases} D_{\frac{3}{6+k}}^2 x(t) = \frac{\sin^2(\pi t)}{(t+4)^2} \frac{|x(t)|}{(1+|x(t)|)}, & t \in J = [0, 1], t \neq t_k = \frac{k}{10}, \\ \Delta x(t_k) = \frac{|x(t_k)|}{9(7+|x(t_k)|)}, & k = 1, 2, \dots, 9, \\ D_{\frac{3}{6+k}} x(t_k^+) - D_{\frac{3}{5+k}} x(t_k) = \frac{|x(t_k)|}{4(5+|x(t_k)|)}, & k = 1, 2, \dots, 9, \\ x(0) = 0, & x(1) = x\left(\frac{9}{20}\right). \end{cases} \quad (4.2)$$

Set $q_k = 3/(6 + k)$ for $k = 0, 1, 2, \dots, 9$, $m = 9$, $T = 1$, $\eta = 9/20$, $j = 4$, $f(t, x) = (\sin^2(\pi t) |x|) / ((t + 4)^2(1 + |x|))$, $I_k(x) = |x| / (9(7 + |x|))$ and $I_k^*(x) = |x| / (4(5 + |x|))$. Since

$$|I_k(x) - I_k(y)| \leq (1/63)|x - y| \quad \text{and} \quad |I_k^*(x) - I_k^*(y)| \leq (1/20)|x - y|,$$

then (H_2) is satisfied with $L_2 = (1/63)$, $L_3 = (1/20)$. It is easy to verify that $|f(t, x)| \leq \mu(t) \equiv 1$, $I_k(x) \leq N_1 = 1/9$ and $I_k^*(x) \leq N_2 = 1/4$ for all $t \in [0, 1]$, $x \in \mathbb{R}$, $k = 1, \dots, m$. Thus (H_3) and (H_4) are satisfied. We can show that

$$jTL_3 + mL_2 + \frac{T(m-j)L_2}{T-\eta} + L_3 \sum_{k=1}^m (T-t_k) = \frac{19741}{27720} < 1.$$

Hence, by Theorem 3.3, the three-point impulsive q_k -difference boundary value problem (4.2) has at least one solution on $[0, 1]$.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

Both authors contributed equally in this article. They read and approved the final manuscript.

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