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Open string multi-branched and Kähler potentials

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Abstract: We consider type II string compactifications on Calabi-Yau orientifolds with fluxes and D-branes, and analyse the F-term scalar potential that simultaneously involves closed and open string modes. In type IIA models with D6-branes this potential can be directly computed by integrating out Minkowski three-forms. The result shows a multi-branched structure along the space of lifted open string moduli, in which discrete shifts in special Lagrangian and Wilson line deformations are compensated by changes in the RR flux quanta. The same sort of discrete shift symmetries are present in the superpotential and constrain the Kähler potential. As for the latter, inclusion of open string moduli breaks the factorisation between complex structure and Kähler moduli spaces. Nevertheless, the 4d Kähler metrics display a set of interesting relations that allow to rederive the scalar potential analytically. Similar results hold for type IIB flux compactifications with D7-brane Wilson lines.

Keywords: D-branes, Flux compactifications, Compactification and String Models, Discrete Symmetries

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JHEP09(2016)062

Contents

JHEP09(2016)062

1 Introduction

A substantial effort in the literature of 4d string compactifications is devoted to construct models with a rich chiral gauge sector and where most neutral scalars are stabilised at a sufficiently high scale [1–11]. Achieving both features simultaneously is a non-trivial task, and probably the most developed models in this sense appear in type II orientifold compactifications with either O3/O7 or O6-planes. In this type II scheme the chiral gauge sector arises from space-time filling D-branes, and moduli are fixed by a potential generated by internal background fluxes and non-perturbative effects [2–4]. Remarkably, these two features are usually treated independently, and the presence of the D-branes is ignored when computing the F-term potential that stabilises moduli.[1]

This picture is somewhat reversed for other classes of models at weak coupling, like heterotic/type I or type IIB compactifications with O5-planes. There the techniques to achieve full moduli stabilisation are not so well developed, but on the other hand moduli in the gravity and gauge sector are treated on equal footing. In these setups one can see that, in general, the presence of D-branes/vector bundle sectors modifies the moduli stabilisation

[1] Except when they source non-perturbative effects or implement a vacuum energy uplifting mechanism.

potential [12–16]. Of course, all these different constructions are related to each other by dualities like mirror symmetry, which has been used to match superpotentials involving open and closed string modes in type IIA compactifications with D6-branes. However, in the cases that have so far been explored the source of open string moduli superpotential in the type IIA side is exclusively due to worldsheet instantons, which are very suppressed in the large volume limit in which RR flux potentials are valid. This somehow sustains the perception that D6-brane moduli are to be treated separately from those moduli entering the flux superpotential, and that they should only be considered at a second stage, after the effect of fluxes on the closed string sector has been taken into account. More generally, open string moduli involving D-brane Wilson lines are presumed to only develop superpotentials from either worldsheet or D-brane instantons, being insensitive to and negligible for the dynamics of the flux generated superpotential.

Contrary to this expectation, it was pointed out in [17] that D-brane Wilson line moduli can also develop superpotentials at large volume and weak coupling, comparable in magnitude to the flux generated superpotential for closed string moduli. In those circumstances, one should analyse the process of moduli stabilisation by treating open and closed string modes simultaneously, considering a scalar potential that includes both at the same time.

The purpose of this work is to consider type II compactifications where open strings modes (and in particular Wilson lines) enter the potential generated by fluxes on equal footing to the closed string ones. Such a combined description is not only necessary for consistency, but also provides valuable information regarding the $\mathcal{N} = 1$ compactification data. Indeed, global knowledge of the scalar potential and the superpotential provides stringent constraints on the Kähler potential for 4d chiral fields. In our case it will allow us to elucidate how closed and open string moduli are interrelated and the presence of shift symmetries in the latter, typically a relevant aspect in building models of inflation [18–20].

In order to implement our approach we will seek for compactifications with two basic features, which allow to reverse engineer the Kähler potential dependence on the open string moduli. First, one should be able to compute the F-term scalar potential without prior knowledge of the Kähler or superpotential. Second, one should be able to involve all the open string moduli in the superpotential, which can also be computed independently. We find that type IIA compactifications in Calabi-Yau orientifolds with D6-branes are particularly suitable for this purpose, as they fulfil both requirements when we restrict to tree-level potentials at large compactification volumes. Indeed, for type IIA compactifications with background fluxes the tree-level F-term potential for closed strings can be fully computed by integrating out 4d three-forms [21], and along the lines of [22, 23] one may generalise this computation to include open string moduli into the potential. Moreover one may always generate an F-term for each D6-brane position and Wilson line moduli, by simply adding an internal worldvolume flux to the D6-brane.

In our analysis we find that a particularly important role is played by a series of discrete symmetries manifest both at the level of the scalar potential and superpotential. Whenever a D-brane field appears in the superpotential, closing a non-trivial loop in open string moduli space is not a symmetry by itself, but it must be accompanied by a compensating

JHEP09(2016)062

shift in the RR flux quanta. This unfolding of the open string moduli space and the corresponding discrete shift symmetries determine to large extent how D-brane moduli enter into the superpotential. More precisely, at the level of approximation in which we are working, we find that the superpotential can be written in the form

$$W_{\text{open}} + W_{\text{closed}}(m) \ = \ W_{\text{closed}}(\tilde{m}) \tag{1.1}$$

in which m are the usual RR flux quanta and $\tilde{m}$ are *dressed* fluxes: combinations of flux quanta and open string moduli invariant under the discrete shift symmetries, for which we give a simple geometrical interpretation. A similar statement holds at the level of the flux-generated potential, which displays the multi-branched structure discussed in [21] but now enriched with the open string dependence.

Analysing periodic directions in open string moduli space proves also illuminating to guess the form of the open-closed Kähler potential. This is because one may use such periodic directions to deduce how open string modes redefine closed strings moduli into new holomorphic variables in the 4d effective theory. Such a redefinition dictates in turn how open strings enter into the Kähler potential, from where the educated guess follows. As a direct consequence of this approach we find that open string moduli do not redefine closed string moduli by themselves, but always in combination with other closed string moduli. More precisely, we find that in the presence of open string moduli the complex structure and Kähler moduli spaces no longer factorise. This is a well-known effect for type II toroidal orientifolds, which we are now able to generalise to the Calabi-Yau context. Despite the resulting complication for the Kähler metrics one can still derive interesting relations among them, thanks to the continuous shift symmetries of the tree-level Kähler potential and the fact that it can be expressed in terms of homogeneous functions of real fields. Finally, one can use these relations to show that the F-term scalar potential is indeed reproduced by means of the usual 4d $\mathcal{N} = 1$ supergravity formula.

The rest of the paper is organised as follows. In section 2 we review type IIA compactifications with D6-branes and fluxes, with special emphasis on the description of the open string moduli space. In section 3 we compute the tree-level open-closed scalar potential by direct dimensional reduction, repackaged in the convenient language of Minkowski three-forms. Such a potential displays a set of discrete shift symmetries which in section 4 are also shown to be present at the level of the superpotential. In section 5 we describe how holomorphic variables are redefined in the presence of open string moduli and the implications for the open-closed Kähler potential. In section 6 we use these superpotential and Kähler potential to recover the F-term scalar potential of section 3 via 4d supergravity. Many of these results also apply to type IIB compactifications with O3/O7-planes and D7-brane Wilson lines, as we discuss in section 7. We draw our conclusions and directions for future work in section 8.

Several technical details have been relegated to the appendices. Appendix A discusses aspects of the open-closed Kähler metrics and contains the proof of several identities necessary for the computations of section 6. Appendix B contains a direct derivation of the type IIA flux potential in the democratic formulation of 10d supergravity. Finally, appendix C illustrates the somewhat abstract definitions used along the main text in the simple case of a $\mathbf{T}^2 \times \mathbf{T}^4/\mathbb{Z}_2$ orientifold example.

JHEP09(2016)062

2 D6-branes in type IIA orientifolds

Let us consider a type IIA orientifold compactification on $\mathbb{R}^{1,3} \times \mathcal{M}_6$ with $\mathcal{M}_6$ a compact Calabi-Yau 3-fold. Following the standard construction in the literature [1, 5–7], we take the orientifold action to be given by $\Omega_p(-1)^{F_L}\mathcal{R}$, where Ω_p is the worldsheet parity reversal operator, F_L is the space-time fermion number for the left-movers, and $\mathcal{R}$ is an internal anti-holomorphic involution of the Calabi-Yau. This involution acts on the Kähler 2-form J and the holomorphic 3-form Ω of $\mathcal{M}$ as

$$\mathcal{R}J = -J\,, \qquad \mathcal{R}\Omega = \overline{\Omega} \tag{2.1}$$

The fixed locus $\Pi_{\rm O6}$ of $\mathcal{R}$ is given by one or several 3-cycles of $\mathcal{M}$, in which O6-planes are located. In order to cancel the RR charge of such O6-planes one may introduce 4d space-time filling D6-branes wrapping three-cycles Π_α of $\mathcal{M}_6$,[2] such that the orientifold symmetry is preserved. In the absence of NS background fluxes, RR tadpole cancellation requires that the following equation in $H_3(\mathcal{M}_6, \mathbb{Z})$ is satisfied

$$\sum_\alpha \left([\Pi_\alpha] + [\mathcal{R}\Pi_\alpha]\right) - 4[\Pi_{\rm O6}] \;=\; 0 \tag{2.2}$$

where $\Pi_{\rm O6}$ stands for the O6-plane loci. Finally, for such D6-branes to minimise their energy and preserve the 4d $\mathcal{N} = 1$ supersymmetry of this background they need to wrap special Lagrangian three-cycles. That is they need to satisfy the geometric conditions

$$J|_{\Pi_\alpha} = 0 \qquad \text{and} \qquad \mathrm{Im}\,\Omega|_{\Pi_\alpha} = 0 \qquad \forall\, a \tag{2.3}$$

as well as to have vanishing worldvolume flux $\mathcal{F}$, defined as

$$\mathcal{F} = B|_{\Pi_\alpha} - \sigma F, \tag{2.4}$$

with $\sigma = \frac{l_s^2}{2\pi}$ and the string length given by $l_s = 2\pi\sqrt{\alpha'}$.

In the absence of D-brane moduli, the 4d effective action for the closed string sector of these constructions has been analysed in great detail, see for instance [22, 23, 25]. In particular, the moduli space of closed string deformations and its related Kähler potential can be described as follows. On the one hand there are $h^{1,1}_-(\mathcal{M}_6)$ complexified Kähler moduli defined as

$$J_c \;=\; B + ie^{\phi/2}J \;=\; T^a\omega_a \tag{2.5}$$

where ϕ is the 10d dilaton, J is computed in the Einstein frame and $l_s^{-2}\omega_a$ are harmonic representatives of $H^2_-(\mathcal{M}_6, \mathbb{Z})$ that can be defined as $\omega_a = d\mathrm{Im}J_c/d\mathrm{Im}T^a$. At large volumes compared to the string scale the tree-level Kähler potential for these moduli is given by

$$K_K \;=\; -\log\left(\frac{i}{6}\mathcal{K}_{abc}(T^a - \bar{T}^a)(T^b - \bar{T}^b)(T^c - \bar{T}^c)\right) \tag{2.6}$$

[2] One may additionally consider D8-branes on coisotropic cycles as in [24], but for simplicity we will restrict our discussion to models where only D6-branes are present.

with $\mathcal{K}_{abc} = l_s^{-6} \int_{\mathcal{M}_6} \omega_a \wedge \omega_b \wedge \omega_c \in \mathbb{Z}$ the triple intersection numbers of the compactification manifold. At this level the B-field axions $b^a \equiv \mathrm{Re}\, T^a$ display a continuous shift symmetry only broken by worldsheet instantons and e^{-K_K} is a cubic polynomial on the moduli $t^a \equiv \mathrm{Im}\, T^a$.

On the other hand, the moduli space of complex structure deformations is encoded in terms of the harmonic three-form

$$\Omega_c = C_3 + i\mathrm{Re}(C\Omega) \quad \in \quad \mathcal{H}_+^3(\mathcal{M}_6) \tag{2.7}$$

which is even under the orientifold involution. Here C_3 is the three-form RR potential and $C \equiv e^{-\phi} e^{\frac{1}{2}(K_{\mathrm{CS}} - K_K)}$ stands for a compensator term with $K_{\mathrm{CS}} = -\log\left(\frac{i}{l_s^6} \int \Omega \wedge \bar{\Omega}\right)$. In order to translate this quantity into 4d chiral fields one first describes Ω in terms of a symplectic integer basis $(\alpha_\lambda, \beta^\lambda) \in H^3(\mathcal{M}_6, \mathbb{Z})$ as $\Omega = X^\lambda \alpha_\lambda - \mathcal{F}_\lambda \beta^\lambda$ [26]. Then one uses the orientifold action to split such a basis into even $(\alpha_K, \beta^\Lambda) \in H_+^3(\mathcal{M}_6)$ and odd $(\alpha_\Lambda, \beta^K) \in H_-^3(\mathcal{M}_6)$ three-forms and defines the chiral fields

$$N'^K = l_s^{-3} \int_{\mathcal{M}_6} \Omega_c \wedge \beta^K \qquad\qquad U'_\Lambda = l_s^{-3} \int_{\mathcal{M}_6} \Omega_c \wedge \alpha_\Lambda \tag{2.8}$$

which have a well-defined counterpart in mirror type IIB orientifolds [25]. Finally, one imposes the orientifold constraints to obtain the Kähler potential

$$K_Q = -2\log\left(\frac{1}{4}\left[\mathrm{Re}(C\mathcal{F}_\Lambda)\mathrm{Im}(CX^\Lambda) - \mathrm{Re}(CX^K)\mathrm{Im}(C\mathcal{F}_K)\right]\right) \tag{2.9}$$

which should then be translated into the variables $n'^K \equiv \mathrm{Im}\, N'^K$ and $u'_\Lambda \equiv \mathrm{Im}\, U'_\Lambda$ on which it depends. As pointed out in [23] one can always perform a symplectic transformation so that all moduli are of the kind N'^K. In this case showing such dependence is relatively easy, as we have

$$K_Q = -2\log\left(-\frac{1}{4}\mathrm{Im}(\mathcal{F}_{KL})\, n'^K n'^L\right) \tag{2.10}$$

where $\mathrm{Im}\, \mathcal{F}_{KL}$ is a homogeneous function of zero degree on the n'^K. In general one can show that $e^{-K_Q/2}$ is a homogeneous function of degree two on n'^K and u'_Λ, as we discuss in appendix A.

This effective field theory becomes more involved as soon as we introduce open string degrees of freedom [22, 23, 27, 28]. In particular it was found in [22, 23] that the complex structure moduli are redefined in the presence of D6-brane moduli. Rewriting the Kähler potential (2.9) in terms of the new 4d chiral fields modifies its expression and introduces a dependence in the open string modes. In the following sections we would like to analyse such modifications, how they affect the Kähler potential for closed and open string fields and their implications for the scalar potential governing both. As we will see, a key ingredient of our analysis will be the periodic directions that appear in open string moduli space, and the discrete shift symmetries that they correspond to.

Given a particular compactification, one may describe a point in open string moduli space by considering a set of special Lagrangian three-cycles $\{\Pi_\alpha^0\}$ that satisfy the RR

JHEP09(2016)062

tadpole condition (2.2) and where D6-branes are wrapped. One may now move in this moduli space by deforming the reference three-cycle Π_α^0 to a homotopic special Lagrangian three-cycle Π_α. If the deformation is infinitesimal we can describe it in terms of a normal vector X, and define the open string moduli of a D6-brane wrapping such a three-cycle as

$$\Phi_\alpha^i \,=\, \frac{2}{l_s^4}\int_{\Pi_\alpha^0} (\sigma A - \iota_X J_c)\wedge \rho^i \,=\, \theta_\alpha^i - T^a (\eta_{\alpha\, a}^0)^i{}_j \varphi_\alpha^j \tag{2.11}$$

where A is the D6-brane gauge potential along Π_3^0 containing the Wilson lines degrees of freedom,[3] and $l_s^{-2}\rho^i \in \mathcal{H}^2(\Pi_\alpha^0,\mathbb{Z})$ is a basis of quantised harmonic two-forms in Π_α^0.[4] Here $X = \frac{1}{2} l_s X_j \varphi^j$ is a linear combination of normal vectors to Π_α^0 preserving the special Lagrangian condition, that is such that $[\mathcal{L}_{X_j} J]_{\Pi_\alpha^0} = [\mathcal{L}_{X_j}\Omega]_{\Pi_\alpha^0} = 0$ with $\mathcal{L}_{X_j}$ the Lie derivative along X_j. Following [29] this implies that $\iota_{X_j} J|_{\Pi_\alpha^0}$ is proportional to a harmonic one-form on Π_α^0,[5] and therefore $i, j = 1, \ldots, b_1(\Pi_\alpha^0)$. Finally we define

$$(\eta_{\alpha\, a}^0)^i{}_j \,\equiv\, l_s^{-3}\int_{\Pi_\alpha^0} \iota_{X_j}\omega_a \wedge \rho^i \tag{2.12}$$

to make manifest the implicit dependence of Φ on the Kähler moduli.

For finite homotopic deformations $\Pi_\alpha = \exp_X(\Pi_\alpha^0)$ the dependence on the deformation parameters φ^j should be computed by a normal coordinate expansion [22, 23], which adds to the above linear behaviour a higher order dependence on the φ's. The open string moduli can then be expressed as

$$\Phi_\alpha^i \,=\, \theta_\alpha^i - T^a f_{\alpha\, a}^i \tag{2.13}$$

where the functions $f_{\alpha\, a}^i(\varphi)$ satisfy the differential equation

$$\frac{\partial f_{\alpha\, a}^i}{\partial \varphi^j} \,=\, (\eta_{\alpha\, a})^i{}_j \,\equiv\, l_s^{-3}\int_{\Pi_\alpha} \iota_{X_j}\omega_a \wedge \rho^i \tag{2.14}$$

so that by imposing $f_{\alpha\, a}^i(\varphi^j = 0) = 0$ we recover (2.11) from the leading term in the Taylor expansion $f_{\alpha\, a}^i = (\eta_{\alpha\, a}^0)^i{}_j \varphi_\alpha^j + \ldots$ In general $(\eta_{\alpha\, a})^i{}_j$ and so $f_{\alpha\, a}^i$ may further depend on the Calabi-Yau metric, and therefore on the closed string moduli $\mathrm{Im}\, T^a$, $\mathrm{Im}\, N'^K$, $\mathrm{Im}\, U'_\Lambda$.

In simple compactifications like toroidal orientifolds, the η_a's are independent of φ, and so the definition (2.11) is exact. Moreover, the normal vectors X_j can be chosen such that the η_a's are integer numbers and both θ's and φ's are periodic variables of unit period [30, 31]. This last statement will remain true for the θ's in general Calabi-Yau compactification while for the f_a's things may become more complicated. The best way to

[3] The normalisation for Φ is such that the periodicity $\int_\gamma A \sim \int_\gamma A + \pi$ for D-branes on top of orientifold planes is translated into $\theta \sim \theta + 1$. A similar statement holds for the position moduli.

[4] By taking ρ^i harmonic we are selecting the lowest of a tower of Kaluza-Klein open string modes, but one may extend this definition to the full tower by taking an appropriate basis of quantised two-forms [22]. For most pourposes we will assume a truncation to the lightest states of each open string KK tower.

[5] Notice that the same is not true for $\iota_{X_j} J_c|_{\Pi_\alpha^0}$, whose real part could be a non-harmonic one-form. However, one may always choose the profile for A such that $A - \iota_X J_c|_{\Pi_\alpha^0}$ is harmonic. Alternatively one may take ρ^i to be harmonic two-forms, as we do here.

analyse their periodicity is to define the open string moduli in terms of integration chains, along the lines of [22, 32, 33].

When the three-cycle Π^0_α is homotopically deformed to Π_α, a one-cycle γ^0_i in the Poincaré dual class to ρ^i will sweep a two-chain Γ^i_α in $\mathcal{M}_6$, such that $\partial\Gamma^i_\alpha = \gamma_i - \gamma^0_i$ with γ_i the corresponding one-cycle in Π_α. One can the define the complexified open string coordinates as [32, 33]

$$\Phi^i_\alpha \,=\, \frac{2}{l_s^2}\int_{\Gamma^i_\alpha} \sigma\tilde{F} - J_c \tag{2.15}$$

where $\tilde{F}$ is an extension of the worldvolume field strength $F = dA$ to the two-chain Γ^i_α such that $\int_{\Gamma^i_2}\tilde{F} = \int_{\gamma_i} A - \int_{\gamma^0_i} A$. Using Lefschetz duality one may then rewrite this as

$$\Phi^i_\alpha \,=\, \frac{2}{l_s^4}\int_{\mathcal{C}^\alpha_4}\left(\sigma\tilde{F} - J_c\right)\wedge\tilde{\rho}^i \tag{2.16}$$

as done in [22]. Here $\mathcal{C}^\alpha_4$ is the four-chain swept by the three-cycle Π_α and $\tilde{\rho}^i$ the quantised two-form on $\mathcal{C}^\alpha_4$ that pulls-back to ρ^i on its boundary, while $\tilde{F}$ is now extended to the whole of $\mathcal{C}^\alpha_4$. By comparing these definitions to (2.13) one obtains that

$$f^i_{\alpha\,a} \,=\, \frac{2}{l_s^2}\int_{\Gamma^i_\alpha}\omega_a \,=\, \frac{2}{l_s^4}\int_{\mathcal{C}^\alpha_4}\omega_a\wedge\tilde{\rho}^i \tag{2.17}$$

which clearly satisfies (2.14). From this perspective it is easy to understand when the functions f_a describe periodic coordinates in the D6-brane moduli space. If a homotopic special Lagrangian deformation is such that $\mathcal{C}^\alpha_4$ is a four-cycle in $\mathcal{M}_6$, the D6-brane system has returned to its original position after performing a loop in its moduli space. This should correspond to a discrete symmetry of the theory, just like shifts of Wilson lines by their period. Notice that if $\mathcal{C}^\alpha_4$ is a four-cycle then the f_a's are integer numbers,[6] and so the shifts that are generated by periodic directions in the D6-brane moduli space are

$$\Phi^i_\alpha \,\to\, \Phi^i_\alpha + k^i_\alpha \qquad \text{and} \qquad \Phi^i_\alpha \,\to\, \Phi^i_\alpha - T^a r^i_{\alpha\,a} \tag{2.18}$$

with $k^i_\alpha, r^i_{\alpha\,a} \in \mathbb{Z}$. As we will see later on, such discrete shifts are directly related to the discrete gauge symmetries and the multi-branched structure of F-term scalar potentials in type II compactifications with fluxes and D-branes.

Another set of quantities that will become important in the formulation of the Kähler potential are the functions $\{g^K_{\alpha\,i}, g_{\alpha\,\Lambda\,i}\}$ on the deformation parameters φ's. Such functions provide an alternative parameterisations of the D6-brane moduli space and are defined by the differential equations [32]

$$\frac{\partial g^K_{\alpha\,i}}{\partial\varphi^j} \,=\, (\mathcal{Q}^K_\alpha)_{ij} \qquad \text{and} \qquad \frac{\partial g_{\alpha\,\Lambda\,i}}{\partial\varphi^j} \,=\, (\mathcal{Q}_{\alpha\,\Lambda})_{ij} \tag{2.19}$$

where

$$(\mathcal{Q}^K_\alpha)_{ij} \,=\, l_s^{-3}\int_{\Pi_\alpha}\iota_{X^j}\beta^K\wedge\zeta_i \qquad\qquad (\mathcal{Q}_{\alpha\,\Lambda})_{ij} \,=\, l_s^{-3}\int_{\Pi_\alpha}\iota_{X^j}\alpha_\Lambda\wedge\zeta_i \tag{2.20}$$

[6] In fact they are even integer numbers. However we will also consider the possibility where $\mathcal{C}^\alpha_4 \cup \mathcal{R}\mathcal{C}^\alpha_4$ is a four-cycle without $\mathcal{C}^\alpha_4$ being so, which implies odd f_a's.

Here $\Pi_\alpha = \exp_X(\Pi^0_\alpha)$ is again a homotopic special Lagrangian deformation of Π^0_α, and ζ_i is a basis of quantised harmonic one-forms on it such that $\int_{\Pi^0_\alpha} \zeta_i \wedge \rho^j = l_s^3 \delta_i^j$. Again, these functions can be expressed in terms of chain integrals as [22]

$$\frac{2}{l_s^3}\int_{\Sigma^i_\alpha} \mathrm{Im}\,(C\Omega) \;=\; \frac{2}{l_s^4}\int_{\mathcal{C}_4^\alpha} \mathrm{Im}\,(C\Omega)\wedge\tilde{\zeta}_i \;=\; -g^K_{\alpha\,i}\mathrm{Im}(C\mathcal{F}_K) + g_{\alpha\,\Lambda\,i}\mathrm{Im}(CX^\Lambda) \tag{2.21}$$

where Σ^i_α is the three-chain swept by the two-cycle Poincaré dual to ζ_i, and $\tilde{\zeta}_i$ is the extension of this one-form to the four-chain $\mathcal{C}_4^\alpha$. More explicitly we have that

$$g^K_{\alpha\,i} \;=\; \frac{2}{l_s^4}\int_{\mathcal{C}_4^\alpha} \beta^K\wedge\tilde{\zeta}_i \qquad \text{and} \qquad g_{\alpha\,\Lambda\,i} \;=\; \frac{2}{l_s^4}\int_{\mathcal{C}_4^\alpha} \alpha_\Lambda\wedge\tilde{\zeta}_i . \tag{2.22}$$

3 The scalar potential from Minkowski three-forms

The space of background and D-brane deformations described in the last section will be subject to a scalar potential in certain type IIA compactifications. In particular, both Kähler and complex structure moduli will develop an F-term scalar potential when NSNS and RR background fluxes are present [25, 34]. In the absence of open string deformations, this potential can been reproduced by applying the usual 4d supergravity expression or by direct dimensional reduction. The latter method involves integrating out the degrees of freedom associated to three-form fields in $\mathbb{R}^{1,3}$ which give a non-vanishing contribution to the potential [25, 34]. In fact, as shown in [21] one may describe the full F-term scalar potential purely in terms of contributions coming from Minkowski three-forms if one performs the dimensional reduction in the democratic formulation of type IIA supergravity. In the following we will adopt this latter approach, as it allows to incorporate the D-brane moduli into the computation and derive a scalar potential for open and closed string modes simultaneously [22, 23].

For simplicity let us consider a Calabi-Yau orientifold compactification where only RR background fluxes are present, ignoring for now the presence of localised sources. Working in the democratic formulation we have the RR p-form potentials C_p with $p = 1, 3, 5, 7, 9$ that can be arranged in the polyforms

$$\mathbf{C} \;=\; C_1 + C_3 + C_5 + C_7 + C_9 \qquad \text{or} \qquad \mathbf{A} \;=\; \mathbf{C}\wedge e^{-B} \tag{3.1}$$

respectively dubbed C and A-basis for the RR potentials in [35]. The corresponding gauge invariant field strengths are given by [35, 36]

$$\mathbf{G} \;=\; d\mathbf{C} - H\wedge\mathbf{C} + \bar{\mathbf{G}}\wedge e^B \;=\; \left(d\mathbf{A} + \bar{\mathbf{G}}\right)\wedge e^B \tag{3.2}$$

with $\bar{\mathbf{G}}$ a formal sum of harmonic $(p+1)$-forms of $\mathcal{M}_6$ to be thought as the background value for the internal RR fluxes defined in the A-basis. This basis is particularly adequate to apply Dirac quantisation, since a D$(p-1)$-brane will couple to the potential A_p via its Chern-Simons action, and so the standard reasoning gives the quantisation condition [37]

$$\frac{1}{l_s^p}\int_{\pi_{p+1}} dA_p + \bar{G}_{p+1} \;\in\; \mathbb{Z} \tag{3.3}$$

for any cycle π_{p+1} in the internal space not intersecting a localised source like a background D-brane. In the A-basis a source wrapping a cycle Π_a enters the Bianchi identity as

$$d\left(e^{-B}\wedge\mathbf{G}\right) = d\left(d\mathbf{A}+\bar{\mathbf{G}}\right) = \sum_a \delta(\Pi_a)\wedge e^{-\sigma F_a} \tag{3.4}$$

with $\delta(\Pi_a)$ the delta-function with support on Π_a and indices transverse to it, while F_a is the quantised worldvolume flux threading Π_a. In the absence of localised sources the r.h.s. of (3.4) vanishes and the A_p are globally well-defined, so they do not contribute to the integral in (3.3) which becomes a quantisation condition for the fluxes $\bar{G}_{p+1}$. We can then define flux quanta in terms of the integer cohomology of $\mathcal{M}_6$, namely as

$$m = l_s\bar{G}_0, \qquad m^a = \frac{1}{l_s}\int_{\tilde{\pi}^a}\bar{G}_2, \qquad e_a = \frac{1}{l_s^3}\int_{\pi_a}\bar{G}_4, \qquad e_0 = \frac{1}{l_s^5}\int_{\mathcal{M}_6}\bar{G}_6 \tag{3.5}$$

where $\tilde{\pi}^a \in H_2^-(\mathcal{M}_6,\mathbb{Z})$ and $\pi_a \in H_4^+(\mathcal{M}_6,\mathbb{Z})$. These definitions need to be generalised if we take into account the effect of localised sources in the above Bianchi identity, as we discuss in the next section. Finally, as we have doubled the p-form degrees of freedom we need to impose the Hodge duality relations

$$G_{2n} = (-)^n \star_{10} G_{10-2n}. \tag{3.6}$$

which can be done either by hand or by adding a series of Lagrange multipliers to the 10d supergravity action, as done in appendix B.

To proceed we define a set of Minkowski four-form field strengths arising from the dimensional reduction of the 10d RR field strengths

$$G_4 = F_4^0 + \dots \quad G_6 = F_4^a\wedge\omega_a + \dots \quad G_8 = \tilde{F}_{4\,a}\wedge\tilde{\omega}^a + \dots \quad G_{10} = \tilde{F}_4\wedge\omega_6 + \dots \tag{3.7}$$

Here the four-forms $(F_4^0, F_4^a, \tilde{F}_{4\,a}, \tilde{F}_4)$ have their indices in $\mathbb{R}^{1,3}$ and $(\omega_a, \tilde{\omega}^a, \omega_6)$ are harmonic forms of $\mathcal{M}_6$.[7] In particular $l_s^{-2}\omega_a$ are the harmonic representatives of $H_-^2(\mathcal{M}_6,\mathbb{Z})$ defined in the last section and $l_s^{-4}\tilde{\omega}^a$ is a dual basis of harmonic forms in $H_+^4(\mathcal{M}_6,\mathbb{Z})$ in the sense that

$$\int_{\mathcal{M}_6}\omega_a\wedge\tilde{\omega}^b = l_s^6\delta_a^b, \qquad a,b,\in\{1,\dots,h_-^{1,1}\}. \tag{3.10}$$

Finally $l_s^{-6}\omega_6 = d\mathrm{vol}_{\mathcal{M}_6}/\sqrt{g_{\mathcal{M}_6}}$ represents the unique harmonic six-form with unit integral over $\mathcal{M}_6$. Following [21], such Minkowski four-forms enter the 4d effective action as

$$\begin{aligned}&-\frac{1}{2}\frac{e^{-K}}{32\kappa_4^2}\int_{\mathbb{R}^{1,3}}\frac{1}{4}F_4^0\wedge *F_4^0 + g_{ab}F_4^a\wedge *F_4^b + \frac{e^{-3\phi}g^{ab}}{16\hat{V}_6^2}\tilde{F}_{4\,a}\wedge *\tilde{F}_{4\,b} + \frac{e^{-3\phi}}{4\hat{V}_6^2}\tilde{F}_4\wedge *\tilde{F}_4\\ &+\frac{1}{4\kappa_4^2}\int_{\mathbb{R}^{1,3}}F_4^0\rho_0 + F_4^a\rho_a + \tilde{F}_{4,a}\tilde{\rho}^a + \tilde{F}_4\tilde{\rho}\end{aligned} \tag{3.11}$$

[7] In terms of the dimensional reduction of the RR potential to Minkowski three-forms

$$C_3 = c_3^0 + \dots \quad C_5 = c_3^a\wedge\omega_a + \dots \quad C_7 = \tilde{d}_{3\,a}\wedge\tilde{\omega}^a + \dots \quad C_9 = \tilde{d}_3\wedge\omega_6 + \dots \tag{3.8}$$

We have that

$$F_4^0 = dc_3^0 \qquad F_4^a = dc_3^a - db^a\wedge c_3^0 \qquad \tilde{F}_{4\,a} = d\tilde{d}_{3\,a} - \mathcal{K}_{abc}db^b\wedge c_3^c \qquad \tilde{F}_4 = d\tilde{d}_3 - db^a\wedge\tilde{d}_{3\,a} \tag{3.9}$$

JHEP09(2016)062

where $\hat{V}_6 = l_s^{-6}\mathrm{Vol}(\mathcal{M}_6)$ stands for the covering space compactification volume in the Einstein frame and in string units. In deriving the above expression we have performed the 4d Weyl rescaling $g_{\mu\nu} \to \frac{g_{\mu\nu}}{\hat{V}_6/2}$, we have used that

$$e^K = \frac{e^{-\phi/2}}{8\hat{V}_6^3} \tag{3.12}$$

and that

$$g_{ab} = \frac{e^{-\phi}}{4\hat{V}_6 l_s^6}\int_{\mathcal{M}_6} \omega_a \wedge \star_6 \omega_b \qquad g^{ab} = \frac{4\hat{V}_6 e^{\phi}}{l_s^6}\int_{\mathcal{M}_6} \tilde{\omega}^a \wedge \star_6 \tilde{\omega}^b \tag{3.13}$$

represent the usual metric for harmonic two-forms and its inverse. Finally we have that

$$\begin{aligned}
l_s \rho_0 &= e_0 + b^a e_a + \frac{1}{2}\mathcal{K}_{abc} m^a b^b b^c + \frac{m}{6}\mathcal{K}_{abc} b^a b^b b^c \\
-l_s \rho_a &= e_a + \mathcal{K}_{abc} m^b b^c + \frac{m}{2}\mathcal{K}_{abc} b^b b^c \\
l_s \tilde{\rho}^a &= m^a + m b^a \\
-l_s \tilde{\rho} &= m
\end{aligned} \tag{3.14}$$

We may now integrate out the Minkowski four-forms from (3.11), obtaining the following scalar potential

$$V_{\mathrm{RR}} = \frac{1}{\kappa_4^2} e^K \left[4\rho_0^2 + g^{ab}\rho_a\rho_b + 16 e^{3\phi}\hat{V}_6^2 g_{ab}\tilde{\rho}^a\tilde{\rho}^b + 4e^{3\phi}\hat{V}_6^2\tilde{\rho}^2\right] \tag{3.15}$$

which is nothing but the usual type IIA RR flux potential [34]

$$\begin{aligned}
l_s^2\kappa_4^2 V_{RR}(e_0, e_a, m^a, m) = {} & e^K 4\left(e_0 + b^a e_a + \frac{1}{2}\mathcal{K}_{abc} m^a b^b b^c + \frac{m}{6}\mathcal{K}_{abc} b^a b^b b^c\right)^2 \\
& + e^K g^{ab}\left(e_a + \mathcal{K}_{acd} m^c b^d + \frac{m}{2}\mathcal{K}_{acd} b^c b^d\right)\left(e_b + \mathcal{K}_{bef} m^e b^f + \frac{m}{2}\mathcal{K}_{bef} b^e b^f\right) \\
& + \frac{4}{9} e^K \mathcal{K}^2 g_{ab}\left(m^a + m b^a\right)\left(m^b + m b^b\right) + \frac{1}{9} e^K \mathcal{K}^2 m^2
\end{aligned} \tag{3.16}$$

where we have used that $\mathcal{K} \equiv \mathcal{K}_{abc} t^a t^b t^c = 6e^{3\phi/2}\hat{V}_6$. Alternatively, one may deduce this scalar potential from the dimensional reduction procedure described in appendix B.

Adding a single D6-brane. This method to obtain the scalar potential has the advantage that it allows to incorporate the open string scalars in a very straightforward way. Indeed, one just needs to add to the 4d action (3.11) the extra four-forms couplings that arise form the dimensional reduction of the D-brane Chern-Simons actions, and then proceed as before. As in [22, 23], let us first consider the effect of a single D6-brane wrapping a three-cycle Π_α which is a homotopic special Lagrangian deformation from the reference cycle Π_α^0. Furthermore although it may break supersymmetry, we also consider the following worldvolume flux threading the three-cycle

$$\sigma F = \sigma dA + n_{F\,i}\,\rho^i \qquad n_{F\,i} \in \mathbb{Z} \tag{3.17}$$

where the harmonic two-forms ρ^i have been defined in (2.11). The CS action reads

$$\begin{aligned} S_{\rm CS}^{\rm D6} &= \mu_6 \int_{\mathbb{R}^{1,3}\times\Pi_\alpha^0} e^{\mathcal{L}_X} A_7 + \sigma F \wedge e^{\mathcal{L}_X} A_5 + \frac{1}{2}\sigma^2 F^2 \wedge e^{\mathcal{L}_X} A_3 + \dots \\ &= \mu_6 \int_{\mathbb{R}^{1,3}\times\Pi_\alpha^0} \exp_X(\mathbf{A}) \wedge e^{\sigma F} = \mu_6 \int_{\mathbb{R}^{1,3}\times\mathcal{C}_4^\alpha} d\mathbf{A} \wedge e^{\sigma \tilde{F}} \end{aligned} \tag{3.18}$$

where $\mu_6 = 2\pi/l_s^7$. As in the previous section $X = \frac{1}{2} l_s \varphi^j X_j$ parametrises the homotopic special Lagrangian deformation $\Pi_\alpha = \exp_X(\Pi_a^0)$, $\mathcal{C}_4^\alpha$ is the corresponding four-chain such that $\partial\mathcal{C}_4^\alpha = \Pi_\alpha - \Pi_\alpha^0$ and $\tilde{F}$ is the extension of the field strength made in (2.15). Expanding the RR potentials as in (3.8), we find the following couplings of the D6-brane scalars to the Minkowski four-forms arising from the bulk

$$\frac{1}{4\kappa_4^2} \int_{\mathbb{R}^{1,3}} F_4^0\, \upsilon_0 + F_4^a \upsilon_a + \tilde{F}_{4\,a}\, \tilde{\upsilon}^a \,. \tag{3.19}$$

Here we have defined

$$\begin{aligned} l_s \upsilon_0 &= n_{F\,i}\theta^i - b^a n_{F\,i} f_a^i - b^a n_{a\,i}\theta^i + n_{a\,i} f_c^i b^a b^c \\ l_s \upsilon_a &= n_{a\,i}\theta^i - n_{a\,i} b^c f_c^i + n_{F\,i} f_a^i - b^c n_{c\,i} f_a^i \\ l_s \tilde{\upsilon}^a &= q^a \end{aligned} \tag{3.20}$$

with f_a^i as in (2.17) and

$$n_{a\,i} = \frac{1}{l_s^3} \int_{\Pi_\alpha} \omega_a \wedge \zeta_i \tag{3.21}$$

$$q^a = \frac{2}{l_s^4} \int_{\mathcal{C}_4^\alpha} \tilde{\omega}^a \tag{3.22}$$

with the harmonic one-forms ζ_i defined as in (2.20). By using that the pull-back and the wedge product commute, one can show that these quantities satisfy

$$n_{a\,i} \frac{\partial f_b^i}{\partial \varphi^j} + n_{b\,i} \frac{\partial f_a^i}{\partial \varphi^j} = \mathcal{K}_{abc} \frac{\partial q^c}{\partial \varphi^j} \qquad \Rightarrow \qquad n_{a\,i} f_b^i + n_{b\,i} f_a^i = \mathcal{K}_{abc} q^c \tag{3.23}$$

which can be used to simplify (3.20).

Incorporating the couplings (3.19) into the 4d effective action and then integrating out the Minkowski four-forms will result into performing the following replacements in the expression (3.15) for the scalar potential

$$\rho_0 \mapsto \varrho_0 = \rho_0 + \upsilon_0 \qquad \rho_a \mapsto \varrho_a = \rho_a - \upsilon_a \qquad \tilde{\rho}^a \mapsto \tilde{\varrho}^a = \tilde{\rho}^a + \tilde{\upsilon}^a \tag{3.24}$$

while leaving $\tilde{\rho}$ invariant. Equivalently, one may replace the flux quanta that enter (3.16) by the following quantities

$$\tilde{e}_0 = e_0 + n_{F\,i}\theta^i \qquad \tilde{e}_a = e_a - n_{a\,i}\theta^i - n_{F\,i} f_a^i \qquad \tilde{m}^a = m^a + q^a \tag{3.25}$$

while leaving the Romans mass untouched. The scalar potential that we then obtain is

$$V_{\rm sc} = V_{\rm RR+CS} + V_{\rm DBI} \tag{3.26}$$

where $V_{\rm DBI}$ stands for the total tension of the D6-branes minus that of the O6-planes, while

$$\begin{aligned} V_{\rm RR+CS} &= \frac{1}{\kappa_4^2}e^K\left[4\varrho_0^2 + g^{ab}\varrho_a\varrho_b + \frac{4}{9}e^K\mathcal{K}^2 g_{ab}\tilde{\varrho}^a\tilde{\varrho}^b + \frac{1}{9}e^K\mathcal{K}^2\tilde{\rho}^2\right] \\ &= V_{\rm RR}(\tilde{e}_0, \tilde{e}_a, \tilde{m}^a, m) \end{aligned} \tag{3.27}$$

with $V_{\rm RR}$ given by (3.16) and $\mathcal{K}$ defined below it. For the explicit expression of $V_{\rm RR+CS}$ in terms of closed and open string fields see eq. (B.6).

The scalar potential (3.27) includes both closed and open string deformations, and reproduces the results found in [22, 23]. Particularly important for our purposes are the series of discrete shift symmetries that it contains both for closed and open string axions. The symmetries of the closed string sector are already present in scalar potential (3.15) and correspond to simultaneous discrete shifts in the B-field axions and backgrounds fluxes, as described in detail in [21]. In this sense, the open-closed scalar potential (3.26) adds new discrete shift symmetries related to the D6-brane Wilson line and position deformations. Indeed, on the one hand by definition $V_{\rm DBI}$ does not depend on the Wilson lines. On the other hand, $V_{\rm RR+CS}$ only depends on them through the shifted fluxes (3.25), dubbed *dressed* fluxes henceforth. The latter are left invariant by the discrete shifts

$$\theta^i \to \theta^i + k^i \qquad e_0 \to e_0 - k^i n_{F\,i} \qquad e_a \to e_a + k^i n_{a\,i} \tag{3.28}$$

with k^i such that $k^i n_{F\,i}, k^i n_{a\,i} \in \mathbb{Z}$. Consequently the full scalar potential is left invariant by these shifts. Because $n_{F\,i}, n_{a\,i} \in \mathbb{Z}$, a particular solution is given by taking $k^i \in \mathbb{Z}$, which corresponds to the first kind of shift in (2.18). Note that such discrete symmetries are nothing but large gauge transformations related to the D6-brane Wilson lines, and therefore discrete gauge symmetries. As such they should be present not only in the scalar potential but also at the level of the superpotential, as we will discuss in the next section.

Similarly, if there are periodic directions in the moduli space of D6-brane positions we may formulate the following discrete shifts

$$f_a^i \to f_a^i + r_a^i \qquad e_a \to e_a + n_{F\,i} r_a^i \qquad m^a\mathcal{K}_{abc} \to m^a\mathcal{K}_{abc} - [n_{b\,i} r_c^i + n_{c\,i} r_b^i] \tag{3.29}$$

where now r_a^i is such that $n_{F\,i} r_a^i \in \mathbb{Z}$ and $n_{(b\,i} r_{c)}^i = s^a\mathcal{K}_{abc}$ with $s^a \in \mathbb{Z}$. These conditions are satisfied whenever the shift correspond to the D6-brane sweeping a four-cycle in $\mathcal{M}_6$, that is to the second kind of shift in (2.18). Again, because the D6-brane is returning to the same position, $V_{\rm DBI}$ is left invariant under these shifts.[8]

In both cases, the presence of these discrete symmetries is directly related to the multi-branched structure of the scalar potential, typical of models with axion-monodromy. The multi-branched structure of the closed string sector of this potential was analysed in [21]. In this sense, the presence of open string modes in the potential and the discrete symmetries (3.28) and (3.29) describe how open string modes are related to these branches of the scalar potential, as already pointed out in [38, 39] for the case of the Wilson line.

[8]Namely because $V_{\rm DBI}$ only depends on the induced metric and B-field. This could change if we were considering compactifications with non-trivial NS flux H_3.

Adding all the D6-branes. Let us now consider a full compactification with several D6-branes, wrapping the special Lagrangian three-cycles Π_α and their orientifold images. Recall that a consistent configuration must satisfy the RR tadpole condition (2.2), which is equivalent to the existence of a four-chain $\Sigma_4 \subset \mathcal{M}_6$ that connects all the D6-branes and O6-planes. Physically, one can interpret this four-chain as follows. If we wrap a D6-brane on it, we will construct a domain wall in 4d connecting two different vacua: one of them with all the D6-branes on top of the O6-planes and the other one with the D6-branes wrapping the three-cycles Π_α and their orientifold images.

Now, considering such a global configuration allows to take into account terms of the Chern-Simons action which we implicitly neglected in the single D6-brane case, when computing the couplings (3.19). These terms are those without derivatives, namely

$$\mu_6 \sum_\alpha \int_{\mathbb{R}^{1,3}\times\Pi_\alpha^0} A_7 + \sigma F\wedge A_5 + \ldots = \frac{\mu_6}{2}\int_{\mathbb{R}^{1,3}\times\Sigma_4^0} d\mathbf{A}\wedge e^{\sigma\tilde{F}} = \frac{\mu_6}{2}\int_{\mathbb{R}^{1,3}\times\Sigma_4^0} (\mathbf{G}\wedge e^{-B} - \bar{\mathbf{G}})\wedge e^{\sigma\tilde{F}} \quad (3.30)$$

where in the first equality we have used that A_7 and F vanish on top of the O6-planes and then applied Stokes' theorem on the reference four-chain Σ_4^0, defined such that $\partial\Sigma_4^0 = \sum_\alpha(\Pi_\alpha^0 - \mathcal{R}\Pi_\alpha^0) - 4\Pi_{\rm O6}$. For this one needs to extend the worldvolume flux F from the boundaries to the four-chain Σ_4^0 connecting them, an extension which we dubbed $\tilde{F}$. In terms of the D6-brane domain wall described above, $\tilde{F}$ would be the actual worldvolume flux of the D6-brane along Σ_4^0.

Setting the Wilson lines on all the D6-branes to zero, the r.h.s. of this equation gives 4d couplings of the form (3.19), which will eventually translate into shifts of the flux quanta. In particular we have the shifts

$$e_0 \to e_0 + \frac{1}{8\pi^2}\int_{\Sigma_4^0}\tilde{F}\wedge\tilde{F} \qquad e_a \to e_a - \frac{1}{2\pi l_s^2}\int_{\Sigma_4^0}\omega_a\wedge\tilde{F} \qquad m^a \to m^a + \frac{1}{l_s^4}\int_{\Sigma_4^0}\tilde{\omega}^a. \quad (3.31)$$

Let us now add the contribution of each D6-brane due to turning on the Wilson lines and deforming its embedding away from the reference cycles Π_α^0. One then obtains

$$\begin{aligned}
\tilde{e}_0 &= e_0 + \sum_\alpha n_{F\,i}^\alpha \theta_\alpha^i + \frac{1}{2}\int_{\Sigma_4^0}\frac{\tilde{F}}{2\pi}\wedge\frac{\tilde{F}}{2\pi}\\
\tilde{e}_a &= e_a - \sum_\alpha [n_{a\,i}^\alpha\theta_\alpha^i + n_{F\,i}^\alpha f_{\alpha\,a}^i] - l_s^{-2}\int_{\Sigma_4^0}\omega_a\wedge\frac{\tilde{F}}{2\pi}\\
\tilde{m}^a &= m^a + \sum_\alpha q_\alpha^a + l_s^{-4}\int_{\Sigma_4^0}\tilde{\omega}^a
\end{aligned} \quad (3.32)$$

where the index α runs over each independent brane but not their orientifold images. Finally, since such contributions can be described in terms of four-chains $\mathcal{C}_4^\alpha$ such that $\partial\mathcal{C}_4^\alpha = \Pi_\alpha - \Pi_\alpha^0$, one may define a new global chain $\Sigma_4 = \Sigma_4^0 + \sum_\alpha \mathcal{C}_4^\alpha \cup \mathcal{R}\mathcal{C}_4^\alpha$ and define the dressed fluxes in (3.27) in terms of it

$$\tilde{e}_0 = e_0 + \frac{1}{2}\int_{\Sigma_4}\frac{\tilde{F}}{2\pi}\wedge\frac{\tilde{F}}{2\pi}, \quad \tilde{e}_a = e_a - l_s^{-2}\int_{\Sigma_4}\omega_a\wedge\frac{\tilde{F}}{2\pi}, \quad \tilde{m}^a = m^a + l_s^{-4}\int_{\Sigma_4}\tilde{\omega}^a \quad (3.33)$$

where we have absorbed the Wilson line dependence in the definition of $\tilde{F}$. Notice that this reproduces and extends the result in [22], in the sense that it gives an expression for the scalar potential globally valid in the open string moduli space. In this last expression the discrete shift symmetries leaving invariant the potential are particularly transparent. Such discrete symmetries will be useful to determine the superpotential that corresponds to this F-term scalar potential, as we discuss next.

4 Open-closed superpotential and axion monodromy

In the absence of D-branes, the superpotential generated by type IIA RR fluxes is [40, 41]

$$l_s W_K = e_0 + e_a T^a + \frac{1}{2}\mathcal{K}_{abc} m^a T^b T^c + m\frac{1}{6}\mathcal{K}_{abc} T^a T^b T^c \tag{4.1}$$

Indeed, one can check that plugging this superpotential and the Kähler potential (2.6) in the standard 4d supergravity formula (3.16) is recovered as an F-term scalar potential [22].

Adding D6-branes degrees of freedom should modify the superpotential as

$$W = W_K + W_{\rm D6} \tag{4.2}$$

where $W_{\rm D6}$ contains open and closed string moduli. By the discussion of the last section we should impose that the full superpotential is invariant under the discrete gauge symmetries related to integer shifts of the Wilson lines and completing loops in the moduli space of special Lagrangians. If we just consider the effect of a single D6-brane on Π_α, such symmetries amount to (3.28) and (3.29), fixing the new superpotential piece to

$$l_s W_{\rm D6}(\Phi) = \Phi^i (n_{F\,i} - n_{a\,i} T^a) + l_s W^0_{\rm D6} \tag{4.3}$$

where we have used (2.13) and defined $W^0_{\rm D6}$ as the superpotential at $\Pi_\alpha = \Pi^0_\alpha$. This superpotential contains a bilinear term of the form $n_{i\,a}\Phi^i T^a$ whose microscopic origin was described in [17] and its applications to large field inflation in [38, 39]. As pointed out in [17] one can derive such a bilinear superpotential from the general expression [42, 43]

$$l_s \left[W_{\rm D6}(\Phi) - W^0_{\rm D6}\right] = \frac{1}{l_s^4}\int_{\mathcal{C}_4^\alpha} (\sigma\tilde{F} - J_c)^2 \tag{4.4}$$

with $\mathcal{C}_4^\alpha$ a four-chain such that $\partial\mathcal{C}_4^\alpha = \Pi_\alpha - \Pi^0_\alpha$. Taking a homotopic deformation and applying the definition (2.16) one sees that the superpotential (4.3) is also recovered.

To consider the full set of D6-branes in the compactification let us follow [17] and set

$$l_s W_{\rm D6} = \frac{1}{2l_s^4}\int_{\Sigma_4} (\sigma\tilde{F} - J_c)^2 \tag{4.5}$$

where Σ_4 is the four-chain connecting all the D6-branes to the O6-planes, as described below (2.2). Now, by splitting this four-chain as $\Sigma_4 = \Sigma_4^0 + \sum_\alpha \mathcal{C}_4^\alpha \cup \mathcal{R}\mathcal{C}_4^\alpha$ we obtain the following generalisation of (4.3)

$$\begin{aligned} l_s W_{\rm D6} &= \frac{1}{l_s^4}\sum_\alpha \int_{\mathcal{C}_4^\alpha} (\sigma\tilde{F} - J_c)^2 + \frac{1}{2l_s^4}\int_{\Sigma_4^0} (\sigma\tilde{F} - J_c)^2 \\ &= \sum_\alpha \Phi^i_\alpha (n^\alpha_{F\,i} - n^\alpha_{a\,i} T^a) + l_s W^0_{\rm D6} \end{aligned} \tag{4.6}$$

where Σ_4^0 the four-chain connecting the reference three-cycles wrapped by the D6-branes, and as usual α only runs over half of the D6-branes, excluding orientifold images. Notice that this superpotential also satisfies the appropriate invariance under discrete shifts. Indeed, either taking the definition of dressed fluxes in (3.32) or (3.33) one can show that

$$W \;=\; W_K(e_0, e_a, m^a, m) + W_{\rm D6} \;=\; W_K(\tilde{e}_0, \tilde{e}_a, \tilde{m}^a, m) \tag{4.7}$$

which is the statement made in (3.27) but now at the level of the superpotential. Therefore we find that the dependence of the superpotential on the open string deformations enters uniquely through the *dressed* fluxes $\{\tilde{e}_0, \tilde{e}_a, \tilde{m}^a\}$. The latter are to be thought as the gauge invariant quantities including flux quanta and axions that typically appear in models with F-term axion monodromy [44]. Therefore, following the philosophy in [45–48], one expects that in the 4d scalar potential the dependence on the open string axions also only appears through $\{\tilde{e}_0, \tilde{e}_a, \tilde{m}^a\}$. This result is clearly true for the scalar potential (3.26) computed at tree-level, but the claim is that it should also hold after all kind of UV corrections have been taken into account, which is particularly important in order to build models of large field inflation.

This dependence on $\{\tilde{e}_0, \tilde{e}_a, \tilde{m}^a\}$ is also directly related to the multi-branched, domain-wall connected structure of the scalar potential. From the viewpoint of Wilson line axion dependence this structure was partly discussed in [38, 39], which considered the case with $n_{F\,i} = 0$. In that case the discrete symmetry (3.28) means that a jump between branches in the Wilson line direction is made by nucleating a 4d domain wall made out of D4-branes wrapping the two-cycle in the homology class $n_{a\,i}$P.D.$[\tilde{\omega}^a]$, as crossing such a domain wall will shift the appropriate internal four-form flux. When $n_{F\,i} \neq 0$ such a D4-brane is magnetised and carries an induced D2-brane charge, which implies a further shift in the internal six-form flux in the amount indicated by (3.28).

A similar statement holds whenever there are closed loops in the moduli space of special Lagrangian deformations, as shows the description of dressed fluxes in terms of chain integrals (3.33). Let us for simplicity take $n_{F\,i} = 0$ and switch off the Wilson line moduli. Then we have that $\tilde{e}_0$ and $\tilde{e}_a$ do not depend on Σ_4 while

$$\tilde{m}^a \;=\; m^a + \frac{1}{l_s^4}\int_{\Sigma_4} \tilde{\omega}^a \;=\; \frac{1}{l_s^4}\int_{\Lambda_4} \tilde{\omega}^a + \frac{1}{l_s^4}\int_{\Sigma_4} \tilde{\omega}^a \tag{4.8}$$

with Λ_4 the appropriate choice of four-cycle in $\mathcal{M}_6$.[9] If a D6-brane closes a loop in moduli space by sweeping a non-trivial four-cycle $\Lambda_{4\,\alpha}$, then there will be a non-trivial change in the integral over Σ_4, which can be compensated by replacing $[\Lambda_4] \to [\Lambda_4] - [\Lambda_{4\,\alpha}]$ in (4.8) or equivalently by shifting m^a by integers. In the 4d effective theory, the latter corresponds to crossing a domain-wall made up of a D6-brane wrapping $\Lambda_{4\,\alpha}$. Finally, when we switch on $n_{F\,i}$ the same will apply, but now such a domain wall D6-brane is magnetised internally and it will have induced D4-brane charge, that shift the e_a quanta as well.

[9] In technical terms, we are computing $\tilde{m}^a$ by integrating $\tilde{\omega}^a$ over a cycle in the relative homology group $H_4(\mathcal{M}_6, \Pi_{\rm D6}, \mathbb{Z})$, with $\Pi_{\rm D6} = \cup_\alpha \Pi_\alpha \cup \Pi_{\rm O6}$. Also, in eq. (4.11) a piece of the open-closed superpotential is computed by integrating J_c^2 over a relative homology cycle. This formulation for the open-closed superpotential is analogous to the one for type IIB compactifications with D5-branes, see e.g. [49–52].

As a final remark[10] let us point out that the open-closed superpotential can be described in the compact form

$$W = \frac{1}{l_s^6}\int_{\mathcal{M}_6} \mathbf{G}\wedge e^{iJ} = \frac{1}{l_s^6}\int_{\mathcal{M}_6} (d\mathbf{A}+\bar{\mathbf{G}})\wedge e^{J_c} \tag{4.9}$$

with $\mathbf{G}$ defined in (3.2). In the absence of D6-branes this is obvious, since then $d\mathbf{A}$ is exact and does not contribute to the integral, so we recover (4.1) directly from the definitions (3.5). When we include D6-branes the flux polyform $d\mathbf{A}+\mathbf{G}$ is still quantised, but it is not closed as it satisfies the Bianchi identity (3.4).[11] In particular we have that

$$d(dA_1+\bar{G}_2) = \sum_\alpha \delta(\Pi_\alpha)+\delta(\mathcal{R}\Pi_\alpha)-4\delta(\Pi_{\rm O6}) \tag{4.10}$$

which has a globally well-defined solution for the two-form flux $dA_1+\bar{G}_2$ due to the tadpole condition (2.2). The corresponding contribution to the superpotential reads

$$l_sW = \frac{1}{2l_s^5}\int_{\mathcal{M}_6}(dA_1+\bar{G}_2)\wedge J_c\wedge J_c+\dots = \frac{1}{2l_s^4}\int_{\Sigma_4+\Lambda_4} J_c\wedge J_c+\dots = \frac{1}{2}\tilde{m}^a\mathcal{K}_{abc}T^bT^c+\dots \tag{4.11}$$

In the first equality we have used the fact that the two-form $dA_1+\bar{G}_2$ is quantised as in (3.3), and that J_c^2 is closed to convert the integral over $\mathcal{M}_6$ into an integral over the four-chain Σ_4, following [54] and also [17, 28]. As before, this chain is such that $\partial\Sigma_4 = \sum_\alpha \Pi_\alpha+\mathcal{R}\Pi_\alpha-4\Pi_{\rm O6}$, and so it is determined only up to a closed four-cycle Λ_4, which can be understood as the contribution to the superpotential coming from W_K. In the second equality we have simply used the expression (4.8) for the dressed two-form flux $\tilde{m}^a$.

In fact, this discussion provides a new interpretation of $\tilde{m}^a$. Indeed, let us use Hodge decomposition to split the two-form $dA_1+\bar{G}_2$ such that G_2 is a purely harmonic two-from, while dA_1 is a sum of an exact and co-exact two-forms. Then we necessarily have that

$$\bar{G}_2 = \tilde{m}^a\omega_a. \tag{4.12}$$

That is, the gauge invariant quantity $\tilde{m}^a$ is nothing but a harmonic component of the two-form flux, computed including the D6-brane backreaction. Notice that due to such backreaction $\tilde{m}^a$ does not need to be quantised, since dA_1 also contributes to integrals over two-cycles $\pi_2\in\mathcal{M}_6$ via its co-exact component.

Finally, a similar analysis can be carried for the other components of the polyform $d\mathbf{A}+\bar{\mathbf{G}}$, obtaining the rest of the open-closed superpotential and an analogous interpretation for the remaining dressed fluxes $\tilde{e}_0$, $\tilde{e}_a$.

5 Holomorphic variables and the Kähler potential

As discussed in [22, 23], by dimensionally reducing the 10d type IIA supergravity and D6-brane actions one finds that the open string moduli and RR bulk axions mix kinematically.

[10] See also the discussion in section 3.3 of [53].

[11] Or in other words $d\mathbf{A}+\mathbf{G}$ does no longer belong to the standard cohomology $H^*(\mathcal{M}_6,\mathbb{Z})$ but it does belong to an element of the relative cohomology group $H^*(\mathcal{M}_6,\Pi_{\rm D6},\mathbb{Z}_6)$. See [28] for other applications of such relative (co)homology groups to type IIA compactifications with D6-branes.

In terms of the 4d $\mathcal{N} = 1$ effective field theory this is interpreted as a redefinition of the chiral superfields containing such RR bulk axions, with the new holomorphic variables depending on the open string fields. This behaviour is analogous to the one observed in type IIB orientifold compactifications [55–65], and dictates how open string fields enter into the Kähler potential. In principle, one may determine what the new holomorphic variables are by demanding that the new Kähler potential reproduces the above kinetic mixing and that quantities like the gauge kinetic function depend holomorphically on the new chiral coordinates. The last requirement is however delicate to implement in generic $\mathcal{N} = 1$ compactifications, as it is known that loop corrections will play an important role [59, 66–68] and these are difficult to compute in general [69].

In the following we would like to apply an alternative approach, based on the discrete symmetries discussed in the previous sections. In particular we will implement in our setup the reasoning of [63], in which the definition of holomorphic variables was related to their transformation under discrete shifts of the open string fields.

To proceed we need to understand how to dualise correctly in 4d the two forms dual to the axions $\mathrm{Re}\, N'^K$. To do so we can add to the 4d effective action the following Lagrange multiplier

$$\frac{1}{4\kappa_4^2}\int_{\mathbb{R}^{1,3}} d\rho_K \wedge d\,\mathrm{Re}\, N'^K \tag{5.1}$$

where ρ_K are the two-forms dual to the 4d axions $\mathrm{Re}\, N'^K$, arising from the RR five-form potential as $C_5 = \rho_K \wedge \beta^K$. Such two-forms couple to the D6-brane moduli via its Chern-Simons action and in particular through the term

$$\mu_6 \int_{\mathbb{R}^{1,3}\times\mathcal{C}_4^\alpha} dC_5 \wedge \sigma \tilde{F} \tag{5.2}$$

from where we obtain the four-dimensional coupling

$$-\frac{1}{4\kappa_4^2}\int_{\mathbb{R}^{1,3}} d\rho_K \wedge d\theta_\alpha^i \left(l_s^{-4}\int_{\mathcal{C}_4^\alpha} \beta^K \wedge \zeta_i \right) = -\frac{1}{8\kappa_4^2}\int_{\mathbb{R}^{1,3}} d\rho_K \wedge g^K_{\alpha\, i} d\theta_\alpha^i \,. \tag{5.3}$$

As a result, we have that the bulk axions transform as follows under discrete shifts of the open string moduli

$$\Phi_\alpha^i \to \Phi_\alpha^i + k_\alpha^i \qquad\qquad \mathrm{Re}\, N'^K \to \mathrm{Re}\, N'^K \tag{5.4}$$

$$\Phi_\alpha^i \to \Phi_\alpha^i - T^a \Delta f^i_{\alpha\, a} \qquad\qquad \mathrm{Re}\, N'^K \to \mathrm{Re}\, N'^K + \frac{1}{2}\theta_\alpha^i \Delta g^K_{\alpha\, i} \tag{5.5}$$

where we have defined

$$\Delta f^i_{\alpha\, a} = \frac{1}{l_s^4}\int_{\Lambda_4^\alpha} \omega_a \wedge \tilde{\rho}^i \qquad \text{and} \qquad \Delta g^K_{\alpha\, i} = \frac{1}{l_s^4}\int_{\Lambda_4^\alpha} \beta^K \wedge \tilde{\zeta}_i \tag{5.6}$$

which for a four-cycle Λ_4^α are integer numbers. It is easy to see that these transformations are not holomorphic. However, as in [63] one may redefine the complex structure moduli N'^K to new variables that transform holomorphically under the above shifts. Indeed, let us define

$$N^K = N'^K - \frac{1}{2} T^a \sum_\alpha \mathbf{H}^K_{\alpha\, a} \tag{5.7}$$

where $\mathbf{H}^K_{\alpha\, a}$ are real functions of the three-cycle position moduli defined by

$$\partial_{\varphi^j_\beta} \mathbf{H}^K_{\alpha\, a} \;=\; (\eta_{\beta\, a})^i{}_j g^K_{\alpha\, i} \delta_{\alpha\beta}\,. \tag{5.8}$$

The functionals $\mathbf{H}^K_{\alpha\, a}$ are similar to those defined by Hitchin in [32] in order to describe the metric on the moduli space of special Lagrangian submanifolds,[12] and were used in [22] to propose a Kähler potential including D6-brane moduli. From the definitions of section 2 one can see that along a periodic direction of the D6-brane position

$$f^i_{\alpha\, a} \;=\; s\, \Delta f^i_{\alpha\, a} \qquad\qquad s\, \in\, \mathbb{R} \tag{5.9}$$

the function $g^K_{\alpha\, i}$ must be of the form

$$g^K_{\alpha\, i} \;=\; s\, \Delta g^K_{\alpha\, i} + M^K_{\alpha\, i}(s) \tag{5.10}$$

where we have defined $\Delta f^i_{\alpha\, a}$ and $\Delta g^K_{\alpha\, i}$ as in (5.6). Here $M^K_{\alpha\, i}$ is a periodic function of period one in s, with mean $m^K_{\alpha\, i}$ and such that $M^K_{\alpha\, i}(0) = 0$. We then have that

$$\mathbf{H}^K_{\alpha\, a} \;=\; \frac{1}{2} s^2\, \Delta f^i_{\alpha\, a} \Delta g^K_{\alpha\, i} + s\, \Delta f^i_{\alpha\, a} m^K_{\alpha\, i} + P^K_{\alpha\, a}(s) \tag{5.11}$$

where $P^K_{\alpha\, a}(s+1) = P^K_{\alpha\, a}(s)$. Hence along this periodic direction $\mathbf{H}^K_{\alpha\, a}$ shifts as

$$\mathbf{H}^K_{\alpha\, a}(s+1) - \mathbf{H}^K_{\alpha\, a}(s) \;=\; f^i_{\alpha\, a} \Delta g^K_{\alpha\, i} + \Delta f^i_{\alpha\, a} \left(\frac{1}{2} \Delta g^K_{\alpha\, i} + m^K_{\alpha\, i} \right) \tag{5.12}$$

and therefore the redefined variables shift as

$$\begin{aligned} \Phi^i_\alpha \;&\to \Phi^i_\alpha + k^i_\alpha & \qquad \mathrm{Re}\, N^K \;&\to \mathrm{Re}\, N^K \\ \Phi^i_\alpha \;&\to \Phi^i_\alpha - T^a \Delta f^i_{\alpha\, a} & \qquad \mathrm{Re}\, N^K \;&\to \mathrm{Re}\, N^K + \frac{1}{2} \Phi^i_\alpha \Delta g^K_{\alpha\, i} - \frac{1}{4} T^a \Delta f^i_{\alpha\, a} \left(\Delta g^K_{\alpha\, i} + 2 m^K_{\alpha\, i} \right) \end{aligned} \tag{5.13}$$

giving the desired holomorphic behaviour.

The same reasoning can be applied to the complex structure moduli U'_Λ, obtaining the redefined variables

$$U_\Lambda \;=\; U'_\Lambda + \frac{1}{2} T^a \sum_\alpha \mathbf{H}_{\alpha\, \Lambda\, a} \qquad \text{with} \qquad \partial_{\varphi^j_\beta} \mathbf{H}_{\alpha\, \Lambda\, a} \;=\; (\eta_{\beta\, a})^i{}_j g_{\alpha\, \Lambda\, i} \delta_{\alpha\beta} \tag{5.14}$$

that show the appropriate holomorphic behaviour.

Performing this change of variables into the Kähler potential will implement its dependence on the open string fields. More precisely we have that the piece (2.6) remains invariant, while (2.9) should be rewritten in terms of the new holomorphic variables. That is, we should again consider

$$K_Q \;=\; -2\log(\mathcal{G}^Q(n'^K, u'_\Lambda)) \tag{5.15}$$

[12] To connect with [32] notice that (5.8) is equivalent to $\partial_{\phi^i_\beta} \left(t^a \mathbf{H}^K_{\alpha\, a} \right) = -g^K_{\alpha\, i} \delta_{\alpha\beta}$, with $\phi^i_\alpha = \mathrm{Im}\, \Phi^i_\alpha$.

but express $n'^K = \mathrm{Im}\, N'^K$ and $u'_\Lambda = \mathrm{Im}\, U'_\Lambda$ in terms of the new holomorphic variables, namely

$$n'^K = n^K + \frac{1}{2} t^a \sum_\alpha \mathbf{H}^K_{\alpha\, a} \qquad \text{and} \qquad u'_\Lambda = u_\Lambda - \frac{1}{2} t^a \sum_\alpha \mathbf{H}_{\alpha\, \Lambda\, a} \tag{5.16}$$

with $n^K = \mathrm{Im}\, N^K$, $u_\Lambda = \mathrm{Im}\, U_\Lambda$ and $t^a = \mathrm{Im}\, T^a$. Although our definition of holomorphic variables is different from the proposal in [22], it embeds the Hitchin functionals into the Kähler potential in a similar fashion, reproducing the same Kähler metrics for the open string fields. Finally, one can check that in the toroidal case this redefinition reduces to

$$n'^K = n^K + \frac{1}{4} \sum_\alpha (\mathcal{Q}^K_\alpha)_{ij} \left[(\mathrm{Im}\, T^a\, \eta_{\alpha\, a})^{-1} \right]^j{}_k\, \mathrm{Im}\, \Phi^i_\alpha \mathrm{Im}\, \Phi^k_\alpha \tag{5.17}$$

and similarly for u_Λ, in agreement with standard result in type IIB toroidal orientifolds [55, 57, 59, 63, 64].

Notice that our reasoning partially relies on the existence of periodic directions in the moduli space of D6-branes, and such may not exist for the position moduli in generic compactifications. In these cases the definition of the functions $\mathbf{H}$ could be different, as they are not constrained by periodic position shifts. Nevertheless, as discussed in appendix A, in order to reproduce the appropriate metrics for the chiral fields one needs a redefinition of the form (5.16) with the $\mathbf{H}$'s defined as above. Hence, at the level of approximation that we are working, the redefinition of the complex structure moduli seems sufficiently constrained.

One important consequence of these results is that, due to the above redefinitions, the piece of the Kähler potential (5.15) also depends on the Kähler moduli, and therefore the complex structure and Kähler moduli spaces no longer factorise. In principle this greatly complicates the computation of quantities in the 4d effective field theory, like for instance the F-term scalar potential. One can nevertheless see that, despite this complication, the redefinition of the holomorphic variables implies several non-trivial identities for the Kähler metrics which will be crucial for the computations of the next section.

For instance, recall that in (5.15), $\mathcal{G}^Q$ is a homogeneous function of degree two on the variables $n'^K = \mathrm{Im}\, N'^K$ and $u'_\Lambda = \mathrm{Im}\, U'_\Lambda$. As shown in appendix A, these two variables are in turn homogeneous functions of degree one in $\{\psi^\beta\} = \{t^a, n^K, u_\Lambda, \phi^i\}$, where for simplicity we have absorbed the D6-brane index α into the index i in $\phi^i = \mathrm{Im}\, \Phi^i_\alpha$. As a result, $\mathcal{G}^Q$ is a homogeneous function of degree two on the variables $\{\psi^\beta\}$, which in turn implies that the full Kähler potential $K = K_K + K_Q$ satisfies

$$K^{\alpha\bar\beta} K_{\bar\beta} = -2i \mathrm{Im}\, \Psi^\alpha \tag{5.18}$$

as well as

$$K^{\alpha\bar\beta} K_\alpha K_{\bar\beta} = 7 \tag{5.19}$$

where in both cases the sum is taken over all fields ψ^α.

In addition, from the same definition of the Kähler potential and some simplifying assumptions several important relations for the elements of the inverse Kähler metric follow. First we have

$$K^{i\bar a} = -f^i_b K^{b\bar a} \tag{5.20}$$

Second it happens that

$$K^{a\bar{b}} = (\partial_a \partial_{\bar{b}} K_K)^{-1} \tag{5.21}$$

or in other words the inverse Kähler metric for the Kähler moduli is exactly the same as in the absence of open string degrees of freedom. Finally for the specific definition of **H** taken above we have that

$$K^{i\bar{j}} - K^{a\bar{b}} f_a^i f_b^j = G_{\rm D6}^{ij} \tag{5.22}$$

where $G_{\rm D6}^{ij}$ is the inverse of the natural metric for one-forms in the three-cycle Π_α

$$G_{ij}^{\rm D6} = \frac{e^{-\phi/4}}{8\hat{V}_6} l_s^{-3} \int_{\Pi_\alpha} \zeta_i \wedge * \zeta_j \tag{5.23}$$

In fact, we have that (5.22) fixes **H** up to a linear function on the ϕ^i's, a freedom that can be used to redefine the reference cycles Π_α^0. We refer the reader to appendix A for further details on all these identities.

6 The scalar potential from 4d supergravity

In this section we combine all the results from the previous sections together. In particular we will show that if we take the superpotential of section 4 and the Kähler potential of section 5, we derive the scalar potential of section 3 from the usual F-term 4d supergravity expression

$$V_{\rm F} = \frac{e^K}{\kappa_4^2} \left(K^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W} - 3|W|^2 \right) \tag{6.1}$$

where α runs over all the fields $\{\Psi^\alpha\} = \{T^a, N^K, U_\Lambda, \Phi^i\}$ the Kähler potential depends on. Now using the relations (5.18) and (5.19) we can rewrite things as

$$K^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \overline{W} - 3|W|^2 = K^{\alpha\bar{\beta}} \partial_\alpha W \partial_{\bar{\beta}} \overline{W} + 4 {\rm Im} \left({\rm Im}\, \Psi^\alpha \partial_\alpha W \overline{W} \right) + 4|W|^2 \,, \tag{6.2}$$

and then analyse this expression term by term.

First we have that

$$\begin{aligned} 4l_s^2 |W|^2 = & \left[2e'_a t^a + 2\mathcal{K}_{ab} m'^a b^b - \frac{1}{3} m\mathcal{K} + m\mathcal{K}_{ab} b^a b^b - 2{\rm Re}\, \Phi^i n_{a\,i} t^a + 2{\rm Im}\, \Phi^i (n_{Fi} - n_{a\,i} b^a) \right]^2 \\ & + \left[2e'_0 + 2e'_a b^a + \mathcal{K}_{abc} m'^a b^b b^c - \mathcal{K}_a m'^a + \frac{1}{3} m\mathcal{K}_{abc} b^a b^b b^c \right. \\ & \left. - m\mathcal{K}_a b^a + 2{\rm Im}\, \Phi^i n_{a\,i} t^a + 2{\rm Re}\, \Phi^i (n_{F\,i} - n_{a\,i} b^a) \right]^2 \end{aligned} \tag{6.3}$$

where we have defined

$$\mathcal{K} = \mathcal{K}_{abc} t^a t^b t^c \qquad \mathcal{K}_a = \mathcal{K}_{abc} t^b t^c \qquad \mathcal{K}_{ab} = \mathcal{K}_{abc} t^c \tag{6.4}$$

and have merged the contribution of $W_{\rm D6}^0$ in (4.6) and the closed string piece of the superpotential W_K (4.1) by defining the primed fluxes

$$e'_0 = e_0 + \frac{1}{8\pi^2} \int_{\Sigma_4^0} \tilde{F} \wedge \tilde{F} \qquad e'_a = e_a - l_s^{-2} \int_{\Sigma_4^0} \omega_a \wedge \frac{\tilde{F}}{2\pi} \qquad m'^a = m^a + l_s^{-4} \int_{\Sigma_4^0} \tilde{\omega}^a \tag{6.5}$$

Second we have that

$$
\begin{aligned}
4l_s^2 \mathrm{Im}\left[\psi^\alpha \partial_\alpha W \overline{W}\right] = &- 4\mathrm{Im}\, W \left[t^a \left(e'_a + \mathcal{K}_{abc} m'^b b^c + \frac{1}{2} m \mathcal{K}_{abc} b^b b^c - \frac{1}{2} m \mathcal{K}_a - \mathrm{Re}\, \Phi^i n_{a\,i}\right)\right. \\
&\left. + \mathrm{Im}\, \Phi^i (n_{Fi} - n_{a\,i} b^a)\right] \\
&+ 4\mathrm{Re}\, W \left[t^a (\mathcal{K}_{ab} m'^b + m \mathcal{K}_{ab} b^b - 2\mathrm{Im}\, \Phi^i n_{a\,i})\right]
\end{aligned}
\tag{6.6}
$$

Hence, summing the two results we find

$$
\begin{aligned}
&4l_s^2 |W|^2 + 4l_s^2 \mathrm{Im}\left[\psi^\alpha \partial_\alpha W \overline{W}\right] \\
&\quad = \left[2e'_0 + 2e'_a b^a + \mathcal{K}_{abc} m'^a b^b b^c + \frac{1}{3} m \mathcal{K}_{abc} b^a b^b b^c + 2\mathrm{Re}\, \Phi^i (n_{F\,i} - n_{a\,i} b^a)\right]^2 \\
&\quad - \left[\mathcal{K}_a m'^a + m \mathcal{K}_a b^a - 2\mathrm{Im}\, \Phi^i n_{a\,i} t^a\right]^2 + \frac{4}{3} m\, \mathcal{K}\, \mathrm{Im}\, W
\end{aligned}
\tag{6.7}
$$

Finally, from the relation (5.20) we can express the remaining term in (6.2) as

$$
K^{\alpha\bar\beta} \partial_\alpha W \partial_{\bar\beta} \overline{W} = K^{a\bar b} \left[\partial_a W - f_a^i \partial_i W\right] \left[\partial_{\bar b} \overline{W} - f_b^i \partial_{\bar\imath} \overline{W}\right] + (K^{i\bar\jmath} - K^{a\bar b} f_a^i f_b^j) \partial_i W \partial_{\bar\jmath} \overline{W} \tag{6.8}
$$

where here a, b run over the Kähler moduli and i, j over the open string moduli that appear in the superpotential. This implies that

$$
\begin{aligned}
&l_s^2 K^{\alpha\bar\beta} \partial_\alpha W \partial_{\bar\beta} \overline{W} \\
&\quad = K^{a\bar b} \left[e'_a + \mathcal{K}_{acd} m'^c b^d + \frac{1}{2} m \mathcal{K}_{acd} b^c b^d - \frac{1}{2} m \mathcal{K}_a - \mathrm{Re}\, \Phi^i n_{a\,i} - f_a^i (n_{F\,i} - n_{c\,i} b^c)\right] \\
&\qquad \times \left[e'_b + \mathcal{K}_{bcd} m'^c b^d + \frac{1}{2} m \mathcal{K}_{bcd} b^c b^d - \frac{1}{2} m \mathcal{K}_b - \mathrm{Re}\, \Phi^i n_{b\,i} - f_b^i (n_{F\,i} - n_{c\,i} b^c)\right] \\
&\quad + K^{a\bar b} \left[\mathcal{K}_{ac} m'^c + m \mathcal{K}_{ac} b^c - \mathrm{Im}\, \Phi^i n_{a\,i} + f_a^i n_{c\,i} t^c\right] \\
&\qquad \times \left[\mathcal{K}_{bc} m'^c + m \mathcal{K}_{bc} b^c - \mathrm{Im}\, \Phi^i n_{b\,i} + f_b^i n_{c\,i} t^c\right] \\
&\quad + \left[K^{i\bar\jmath} - K^{a\bar b} f_a^i f_b^j\right] \partial_i W \partial_{\bar\jmath} \overline{W}\,.
\end{aligned}
\tag{6.9}
$$

Using (3.23) we can simplify the first term in the second line of (6.7) to

$$
-\left[\mathcal{K}_a (m'^a + m b^a + q^a)\right]^2 \;=\; -\left[\mathcal{K}_a (\tilde{m}^a + m b^a)\right]^2 \tag{6.10}
$$

as well as the terms appearing in the third and fourth lines of (6.9), that now read

$$
K^{a\bar b} \mathcal{K}_{ad} \mathcal{K}_{bc} \left[\tilde{m}^d + m b^d\right] \left[\tilde{m}^c + m b^c\right] \tag{6.11}
$$

where we have expressed everything in terms of the dressed fluxes $\tilde{m}^a = m'^a + q^a$. Adding these last two terms we obtain

$$
\left(K^{a\bar b} \mathcal{K}_{ad} \mathcal{K}_{bc} + \mathcal{K}_d \mathcal{K}_c\right) \left[\tilde{m}^d + m b^d\right] \left[\tilde{m}^c + m b^c\right] = 16 e^{3\phi} \hat{V}_6^2 g_{dc} \left[\tilde{m}^d + m b^d\right] \left[\tilde{m}^c + m b^c\right] \tag{6.12}
$$

where we have used (5.21) and more precisely (A.36).

The remaining terms can be arranged as follows. One may first rewrite the first two lines of the r.h.s. of (6.9) as

$$\begin{aligned} &K^{a\bar{b}}\left[\tilde{e}_a + \mathcal{K}_{acd}\tilde{m}^c b^d + \frac{1}{2}m\mathcal{K}_{acd}b^c b^d\right] \times \left[\tilde{e}_b + \mathcal{K}_{bcd}\tilde{m}^c b^d + \frac{1}{2}m\mathcal{K}_{bcd}b^c b^d\right] \\ &\qquad - K^{a\bar{b}}m\mathcal{K}_a\left[\tilde{e}_b + \mathcal{K}_{bcd}\tilde{m}^c b^d + \frac{1}{2}m\mathcal{K}_{bcd}b^c b^d\right] + \frac{1}{4}K^{a\bar{b}}\mathcal{K}_a\mathcal{K}_b m^2 \end{aligned} \tag{6.13}$$

where we have again used (3.23), the definition of $\tilde{m}^a$ and that $\tilde{e}^a = e'^a - n_{a\,i}\theta^i - n_{F\,i}f_a^i$. Then, by using that $K^{a\bar{b}}\mathcal{K}_a = \frac{4}{3}\mathcal{K}t^b$ we can add the second line of this equation to the last term in (6.7) and obtain

$$\begin{aligned} &\frac{4}{3}\mathcal{K}m\left[\left(\tilde{e}_a t^a + \mathcal{K}_{ab}\tilde{m}^a b^b - \frac{1}{6}m\mathcal{K} + \frac{1}{2}m\mathcal{K}_{ab}b^a b^b\right) - t^b\left(\tilde{e}_b + \mathcal{K}_{bcd}\tilde{m}^c b^d + \frac{1}{2}\mathcal{K}_{bcd}b^c b^d\right)\right] \\ &\qquad + \frac{1}{4}K^{a\bar{b}}\mathcal{K}_a\mathcal{K}_b m^2 = \left(\frac{1}{3} - \frac{2}{9}\right)\mathcal{K}^2 m^2 = 4e^{3\phi}\hat{V}_6^2 m^2 \end{aligned} \tag{6.14}$$

Summing all these contributions we find the following F-term scalar potential

$$\begin{aligned} V_F = \frac{e^K}{l_s^2\kappa_4^2}\Bigg\{ &\left[2\tilde{e}_0 + 2\tilde{e}_a b^a + \mathcal{K}_{abc}\tilde{m}^a b^b b^c + \frac{1}{3}m\mathcal{K}_{abc}b^a b^b b^c\right]^2 \\ &+ g^{ab}\left(\tilde{e}_a + \mathcal{K}_{acd}\tilde{m}^c b^d + \frac{m}{2}\mathcal{K}_{acd}b^c b^d\right)\left(\tilde{e}_b + \mathcal{K}_{bef}\tilde{m}^e b^f + \frac{m}{2}\mathcal{K}_{bef}b^e b^f\right) \\ &+ \frac{4}{9}\mathcal{K}^2 g_{ab}\left(\tilde{m}^a + mb^a\right)\left(\tilde{m}^b + mb^b\right) + \frac{1}{9}\mathcal{K}^2 m^2\Bigg\} + V_{\rm DBI} \end{aligned} \tag{6.15}$$

where we have defined

$$V_{\rm DBI} = \frac{e^K}{\kappa_4^2}\left[K^{i\bar{j}} - K^{a\bar{b}}f_a^i f_b^j\right]\partial_i W\partial_{\bar{j}}\overline{W}\,. \tag{6.16}$$

Hence, we do indeed recover an F-term scalar potential which is the sum of two terms. The first one is the potential $V_{\rm RR+CS}$ computed in section 3 which has the form of the usual type IIA scalar potential generated by RR fluxes but with those replaced with the dressed fluxes $\{\tilde{e}_0, \tilde{e}_a, \tilde{m}^a, m\}$ that contain the open string moduli dependence. The second piece (6.16) should then correspond to the DBI contribution to the F-term scalar potential, which is non-trivial when $\sigma F - J_c$ does not vanish over Π_α. Whenever such source of supersymmetry breaking is small in string units, the corresponding excess of energy is given by [38, 39]

$$\begin{aligned} V_{\rm DBI} &= \frac{e^K}{l_s^2\kappa_4^2}8\hat{V}_6 e^{\phi/4}\frac{1}{l_s^3}\int_{\Pi_\alpha}(\sigma F - J_c)\wedge *\left(\sigma F - \overline{J_c}\right) & (6.17)\\ &= \frac{e^K}{l_s^2\kappa_4^2}G_{\rm D6}^{ij}\left(n_{F\,i} - n_{a\,i}T^a\right)\left(n_{F\,j} - n_{a\,j}\bar{T}^a\right) & (6.18) \end{aligned}$$

where $G_{\rm D6}^{ij}$ is the inverse of (5.23). We then see that the choice of Kähler metric taken in the previous section also reproduces the expected contribution from the DBI action to the F-term potential.

7 Type IIB models with D7-brane Wilson lines

In this section we translate our results obtained in type IIA compactifications with O6-planes to type IIB compactifications with O7/O3-planes. We consider compactifications of type IIB string theory on $\mathbb{R}^{1,3} \times \mathcal{M}_6$ where $\mathcal{M}_6$ is taken to be a compact Calabi-Yau 3-fold. The orientifold action is given by the $\Omega_p(-1)^{F_L}\mathcal{R}$ where in this case $\mathcal{R}$ is a holomorphic involution of the Calabi-Yau manifold. In particular the action of $\mathcal{R}$ on the Kähler form J and the holomorphic 3-form Ω of the Calabi-Yau 3-fold $\mathcal{M}_6$ is

$$\mathcal{R}J = J\,, \quad \mathcal{R}\Omega = -\Omega\,. \tag{7.1}$$

At the fixed loci of this involution which can be either points or 4-cycles we find O3-planes and O7-planes respectively. Having introduced an orientifold involution it is necessary to cancel the total RR charge induced by the orientifold planes in the compact space. To cancel the RR tadpole induced by the O7-planes we may introduce spacetime filling D7-branes wrapping 4-cycles $\mathcal{S}_a$ in $\mathcal{M}_6$ satisfying the homological relation

$$\sum_a [\mathcal{S}_a] + [\mathcal{R}\mathcal{S}_a] = 8\,[\pi_{O7}]\,. \tag{7.2}$$

In a similar fashion the tadpole induced by the O3-planes may be cancelled by introducing an adequate number of D3-branes.

Preservation of 4d $\mathcal{N}=1$ supersymmetry forces us to take the cycles $\mathcal{S}_a$ to be holomorphic divisors of $\mathcal{M}_6$ with the worldvolume flux $\mathcal{F} = B|_{\mathcal{S}_a} - \sigma F$ satisfying the conditions

$$\mathcal{F}^{(0,2)} = 0\,, \quad J \wedge \mathcal{F} = 0\,, \tag{7.3}$$

where $\alpha^{(p,q)}$ stands for the Hodge type of the differential form α.

The 4d effective action for the closed string sector contains the axio-dilaton $\tau = C_0 + ie^{-\phi}$, Kähler moduli and complex structure moduli, with a factorised moduli space, see [70]. The Kähler moduli are obtained from the dimensional reduction of the Kähler form J, of the B-field and of the RR forms C_2 and C_4 as

$$\begin{aligned}
J &= v^\alpha\,\omega_\alpha\,, && \omega_\alpha \in H^2_+(\mathcal{M}_6,\mathbb{Z})\,,\\
B &= b^a\,\omega_a\,, && \\
C_2 &= c^a\,\omega_a\,, && \omega_a \in H^2_-(\mathcal{M}_6,\mathbb{Z})\,,\\
C_4 &= C_\alpha\,\tilde\omega^\alpha\,, && \tilde\omega^\alpha \in H^4_+(\mathcal{M}_6,\mathbb{Z})\,.
\end{aligned} \tag{7.4}$$

With these definitions at hand we may define the following 4d chiral multiplets

$$G^a = c^a - \tau b^a\,, \quad T_\alpha = \frac{1}{2}\mathcal{K}_{\alpha\beta\gamma}v^\beta v^\gamma - \frac{i}{2(\tau-\overline{\tau})}\mathcal{K}_{\alpha bc}G^b(G-\overline{G})^c - iC_\alpha\,, \tag{7.5}$$

where we defined the triple intersection numbers

$$\mathcal{K}_{\alpha\beta\gamma} = \frac{1}{l_s^6}\int_{\mathcal{M}_6}\omega_\alpha\wedge\omega_\beta\wedge\omega_\gamma\,, \quad \mathcal{K}_{\alpha bc} = \frac{1}{l_s^6}\int_{\mathcal{M}_6}\omega_\alpha\wedge\omega_b\wedge\omega_c\,. \tag{7.6}$$

The Kähler potential at large volume for these moduli is then written as an implicit function of the chiral multiplets as

$$K_K = -2\log\left[\frac{1}{6}\mathcal{K}_{\alpha\beta\gamma}v^\alpha v^\beta v^\gamma\right]. \tag{7.7}$$

For an explicit expression it is necessary to invert the relation between the volume of 2-cycles v^α and the chiral coordinates T_α. While this is hard to do in general, one can see that e^{-K_K} is a homogeneous function of degree three on the moduli.

The complex structure moduli are obtained by performing the dimensional reduction of the holomorphic 3-form Ω on harmonic 3-forms. Due to the orientifold projection we only need to consider 3-forms odd under the orientifold involution. After taking (α_I, β^I) as a symplectic basis for $H^3_-(\mathcal{M}_6, \mathbb{Z})$ we obtain the following expansion for Ω

$$\Omega = X^I\,\alpha_I - F_I\,\beta^I\,. \tag{7.8}$$

The quantities (X^I, F_I) are called periods of the holomorphic 3-form Ω and depend on the complex structure of $\mathcal{M}_6$. In particular it is possible to prove that the F_I are functions of the X^I implying that the latter may be chosen as good projective coordinates on the complex structure moduli space. Fixing $X^0 = 1$ we will identify the remaining X^I with the complex structure moduli with $I = 1, \ldots, h^{(2,1)}_-(\mathcal{M}_6)$. Additionally it is possible to prove that the $F_I = \partial_I F$ where the function F, usually called the prepotential, is homogeneous of degree 2 in the projective coordinates X^I. The Kähler potential for the complex structure moduli may be written using the periods

$$K_Q = -\log\left[i(X^I\overline{F}_I - \overline{X}^I F_I)\right]. \tag{7.9}$$

Finally we need to consider the axio-dilaton whose Kähler potential is simply

$$K_\tau = -\log\big[-i(\tau - \overline{\tau})\big]\,. \tag{7.10}$$

The introduction of an open string sector will add moduli in the 4d effective field theory. The case of D7-branes was analysed in [60] where it was found that for D7-branes the moduli fall in two separate classes, brane position moduli and Wilson line moduli. In [60] it was found that brane position moduli, counted by the cohomology group $H^{(2,0)}(\mathcal{S}_a)$, give a redefinition of the axio-dilaton and therefore modify the Kähler potential (7.10). Similarly it was found that Wilson line moduli, counted by the cohomology group $H^{(0,1)}(\mathcal{S}_a)$, enter in a redefinition of the Kähler moduli thus modifying the Kähler potential (7.7). In the following we shall revisit the definition of the Kähler potential for the Wilson line moduli.

Adding Wilson lines. The definition of Wilson line moduli for a D7-brane on a 4-cycle $\mathcal{S}_a$ is relatively straightforward, as they come from the dimensional reduction of the D7-brane gauge field on elements of $H^1(\mathcal{S}_a)$. To correctly write 4d chiral fields it is necessary to perform the dimensional reduction using elements of $H^{(0,1)}(\mathcal{S}_a)$ giving the following definition for the Wilson line moduli

$$\xi^a_i = \frac{2}{l_s^5}\int_{\mathcal{S}_a}\sigma A\wedge\chi_i\,, \tag{7.11}$$

where χ_i are elements of $H^{(2,1)}(\mathcal{S}_a)$. According to this definition the internal profile for a Wilson line scalar will be proportional to a suitable element of $H^{(1,0)}(\mathcal{S}_a)$ leading to the following dimensional reduction ansatz for the gauge field on the D7-brane a

$$A = \frac{\pi}{l_s}\mathrm{Im}[\xi^a_i\,\overline{\gamma}^i]\,, \tag{7.12}$$

where the differential forms $\overline{\gamma}^i$ give a basis of $H^{(0,1)}(\mathcal{S}_a)$. At this point it is important to choose properly the normalisation of the differential forms γ^i to ensure that the axionic components of the Wilson line moduli have a fundamental period independent of the complex structure on $\mathcal{S}_a$. To this end it is convenient to introduce a suitable basis $(\tilde{\alpha}_i, \tilde{\beta}^j)$ of $H^1(\mathcal{S}_a, \mathbb{Z})$ which allows us to express the differential forms γ^i as[13]

$$\gamma^i = (\mathrm{Im}\, f^a)^{ij}\left[\tilde{\alpha}_j + f^a_{jk}(z)\,\tilde{\beta}^k\right]\,, \tag{7.13}$$

with $f^a_{ij}(z)$ holomorphic in the complex structure moduli and $(\mathrm{Im}\, f^a)^{ij}$ is the inverse of its imaginary part. Setting $\xi^a_i = \eta_i + f^a_{ij}(z)\theta^j$ we obtain that the dimensional reduction ansatz for the gauge field (7.12) becomes

$$A = \frac{\pi}{l_s}\mathrm{Im}[\xi^a_i\,\overline{\gamma}^i] = \frac{\pi}{l_s}\left[-\eta_i\,\tilde{\beta}^i + \theta^i\tilde{\alpha}_i\right]\,, \tag{7.14}$$

implying that the fundamental periods for the axionic fields (η_i, θ^i) is $\eta_i \sim \eta_i + 1$ and $\theta^i \sim \theta^i + 1$, independent of the complex structure moduli. As already anticipated the presence of Wilson line moduli will give a redefinition of the Kähler moduli of the Calabi-Yau threefold. The resulting redefinition is the following

$$\hat{T}_\alpha = T_\alpha - \frac{i}{4}\sum_a (\mathcal{C}^a_\alpha)^i_k(\mathrm{Im}\, f^a)^{kj}\,\xi^a_i\,\mathrm{Im}\,\xi^a_j\,, \tag{7.15}$$

where we introduced the matrix $(\mathcal{C}^a_\alpha)^i_j = l_s^{-4}\int_{\mathcal{S}_a}\omega_\alpha\wedge\tilde{\alpha}_j\wedge\tilde{\beta}^i$. To motivate this redefinition we can follow a similar logic as the one followed in section 5. Following [60] we add the following Lagrange multiplier term in the effective action

$$-\frac{1}{4\kappa_4^2}\int_{\mathbb{R}^{1,3}} d\rho_2^\alpha\wedge dC_\alpha\,, \tag{7.16}$$

where ρ^α are duals of the RR axions C_α in 4d coming from the dimensional reduction of C_4 along 2-forms ω_α. These two forms have additional couplings to the Wilson line moduli of the D7-branes coming from the term

$$\frac{\mu_7}{2}\int_{\mathbb{R}^{1,3}\times\mathcal{S}_a} C_4\wedge\mathcal{F}\wedge\mathcal{F}\,. \tag{7.17}$$

in the Chern-Simons action of the D7-brane. Performing the dimensional reduction of this term we obtain

$$-\frac{1}{16\kappa_4^2}\int_{\mathbb{R}^{1,3}} d\rho^\alpha\wedge\mathrm{Im}\left[\bar{\xi}^a_i\,d\xi^a_j\right](\mathrm{Im} f^a)^{ik}(\mathcal{C}^a_\alpha)^j_k\,, \tag{7.18}$$

[13]We note that when considering F-theory compactifications a similar ansatz was made in [71–73] for the 3-forms of the Calabi-Yau fourfold. This comes as no surprise as the Wilson line moduli of D7-branes come as 3-forms when considering the uplift to F-theory.

which closely resembles the result obtained in [63]. Following the discussion there, it is possible to see that the variable (7.15) shifts holomorphically under discrete shifts of the Wilson line moduli.

Superpotential and discrete symmetries. When turning on RR fluxes the superpotential obtained in the 4d effective field theory has the simple form

$$l_s W_{\rm F} = X^I e_I + F_I\, m^I\,, \tag{7.19}$$

where the closed string flux F_3 has been expanded as

$$l_s^{-2} F_3 = m^I \alpha_I + e_I\, \beta^I\,. \tag{7.20}$$

Introduction of D7-branes will add some terms in the superpotential. Specifically the D7-brane superpotential is [43, 60]

$$l_s W_{\rm D7} = \frac{1}{l_s^5}\int_{\Sigma_5} \Omega \wedge \mathcal{F} \tag{7.21}$$

where, similarly to the case considered in type IIA, Σ_5 is the five-chain connecting all D7-branes to the O7-planes, and $\mathcal{F}$ an extension of the worldvolume flux in it. Splitting the five-chain as $\Sigma_5 = \Sigma_5^0 + \sum_a \mathcal{C}_5^a + \mathcal{R}\mathcal{C}_5^a$ we obtain the following D7-brane superpotential

$$l_s W_{\rm D7} = \sum_a \frac{2}{l_s^5}\int_{\mathcal{C}_5^a} \Omega \wedge \mathcal{F} + \frac{1}{l_s^5}\int_{\Sigma_5^0} \Omega \wedge \mathcal{F} = \sum_a \frac{2}{l_s^5}\int_{\mathcal{C}_5^a} \Omega \wedge \mathcal{F} + l_s W_{\rm D7}^0\,. \tag{7.22}$$

where the index a runs over the D7-branes but not their orientifold images. In the following we shall focus our attention on the dependence on Wilson line moduli thus neglecting the presence of brane position moduli in the superpotential. In this case the superpotential becomes

$$l_s W_{\rm D7} = \sum_a \frac{2}{l_s^5}\int_{\mathcal{C}_5^a} \Omega\wedge\sigma\tilde{F} + l_s W_{\rm D7}^0 = -\sum_a \frac{2}{l_s^4}\left[\int_{\mathcal{S}_a} \Omega \wedge \sigma A - \int_{\mathcal{S}_a^0} \Omega \wedge \sigma A\right] + l_s W_{\rm D7}^0\,. \tag{7.23}$$

Choosing the configuration of the D7-branes such that on the reference 4-cycles $\mathcal{S}_a^0$ Wilson lines are turned off it is possible to drop the last two terms in (7.23). At this point we may use (7.14) to write the superpotential in terms of the Wilson line scalars as

$$l_s W_{\rm D7} = \sum_a \eta_i \left[(c_a)^i_I X^I - (h_a)^{iI} F_I\right] + \theta^i \left[(d_a)^I_i\, F_I - (p_a)_{iI}\, X^I\right]\,, \tag{7.24}$$

where we have defined the following integer numbers[14]

$$\begin{aligned} (c_a)^i_I &= l_s^{-4} \textstyle\int_{\mathcal{S}_a} \alpha_I \wedge \tilde{\beta}^i\,, \quad (d_a)^I_i = l_s^{-4} \displaystyle\int_{\mathcal{S}_a} \beta^I \wedge \tilde{\alpha}_i\,, \\ (h_a)^{iI} &= l_s^{-4} \textstyle\int_{\mathcal{S}_a} \beta^I \wedge \tilde{\beta}^i\,, \quad (p_a)_{iI} = l_s^{-4} \displaystyle\int_{\mathcal{S}_a} \alpha_I \wedge \tilde{\alpha}_i\,. \end{aligned} \tag{7.25}$$

[14]Notice that for these numbers to be non-vanishing the four-cycle $\mathcal{S}_a$ must contain three-cycles which are non-trivial in the homology of $\mathcal{M}_6$, in agreement with the results of [17].

JHEP09(2016)062

By inspection of the D7-brane superpotential we observe that the combined flux and brane superpotential may be nicely written in terms of dressed fluxes

$$l_s W_{\rm IIB} = l_s W_{\rm F} + l_s W_{\rm D7} = X^I \tilde{e}_I + F_I\, \tilde{m}^I \,, \tag{7.26}$$

where the redefined flux quanta are

$$\begin{aligned} \tilde{e}_I &= e_I + \sum_a (c_a)^i_I\, \eta_i - (p_a)_{iI} \theta^i \,, \\ \tilde{m}^I &= m^I + \sum_a (d_a)^I_i\, \theta^i - (h_a)^{iI} \eta_i \,. \end{aligned} \tag{7.27}$$

This demonstrates that the superpotential is invariant under the discrete shifts

$$\begin{aligned} \eta_i &\to \eta_i + k_i \,, \qquad \begin{cases} e_I \to e_I - (c_a)^i_I\, k_i \\ m^I \to m^I + (h_a)^{iI}\, k_i \end{cases} , \\ \theta^i &\to \theta^i + k^i \,, \qquad \begin{cases} e_I \to e_I + (p_a)_{iI}\, k^i \\ m^I \to m^I - (d_a)^I_i\, k^i \end{cases} , \end{aligned} \tag{7.28}$$

in a quite analogous fashion to their type IIA counterparts (3.28) and (3.29).

While this description has the advantage of making manifest the discrete symmetries of the superpotential, it is not obvious that the superpotential is a holomorphic function of the Wilson line moduli. To write the brane superpotential as a holomorphic function of the Wilson line moduli it is necessary to impose the following condition[15]

$$\left[X^I (c_a)^i_I - F_I\, (h_a)^{iI}\right]\, f^a_{ij}(z) = \left[F_I\, (d_a)^I_j - X^I\, (p_a)_{jI}\right] \tag{7.29}$$

which fixes the function $f^a_{ij}(z)$ for those Wilson lines that appear in the superpotential.

Scalar potential. Knowing the form of the superpotential and the holomorphic variables we may perform a similar computation to the one in section 6 to derive the F-term scalar potential. For simplicity we will work in the large complex structure limit where the prepotential takes the form

$$F = \frac{\kappa_{IJK}}{3!} \frac{X^I X^J X^K}{X^0} \,, \tag{7.30}$$

where from now on $I = 1, \dots, h^{(2,1)}_-(\mathcal{M}_6)$ and moreover we will write the complex structure moduli in terms of their real and imaginary parts as $z^I = X^I/X^0 = u^I + i w^I$. In addition, we will absorb the D7-brane index a into the Wilson line index i. We start by noting that in this limit the relations (5.18), (5.19) continue to hold if we assume that f^a_{ij} is a linear function of the complex structure moduli, which we shall assume henceforth.[16] The

[15] Note that the condition (7.29) is necessary to have $\int_{\mathcal{S}_a} \Omega \wedge \gamma^i = 0$ thus ensuring that the differential forms γ^i are of Hodge type (1,0) even when $\Omega|_{\mathcal{S}_4}$ is non-trivial.

[16] As shown in appendix A these properties rely solely on e^{-K} being a homogeneous function in either the real or imaginary part of the moduli, which is true whenever $\mathrm{Im}\, f^a_{ij}$ is a homogeneous function of degree one on the w's. This can be easily achieved by assuming that f^a_{ij} is linear, and in the particular case of large complex structure by setting $p_a = h_a = 0$ and $(c_a)^i_0 = (d_A)^0_i = 0$, which we assume below. It would be interesting to explore more general cases. This seems compatible with the results of [73] where it was found that the function f^a_{ij} is linear in the large complex structure limit.

JHEP09(2016)062

relation (5.20) is then replaced by

$$K^{i\bar{J}} = \frac{\partial \mathrm{Im}\,\xi_i}{\partial \mathrm{Im}\,z^K}\, K^{K\bar{J}}\,, \tag{7.31}$$

Using these properties it is possible to see that

$$K^{A\bar{B}} D_A W D_{\bar{B}} \overline{W} - 3|W|^2 = K^{A\bar{B}} \partial_A W \partial_{\bar{B}} \overline{W} + 4\mathrm{Im}\,(\mathrm{Im}\,\Psi^A \partial_A W \overline{W}) + 4|W|^2\,. \tag{7.32}$$

where the index A runs over all fields Ψ^A. Since the remaining of the computation is very similar to the one considered in section 6 we will omit most details. We find that

$$\begin{aligned} 4l_s^2|W|^2 &+ 4l_s^2\,\mathrm{Im}\left[\psi^A \partial_A W \overline{W}\right] \\ &= 4\left[e_0 + e_I u^I + \frac{1}{2} m^I \kappa_{IJK} u^J u^K - \frac{1}{6} m^0 \kappa_{IJK} u^I u^J u^K - \mathrm{Re}\,\xi_i\, c^i_I u^I\right]^2 \\ &\quad - \frac{4}{3} m^0 \kappa\, \mathrm{Im}\, W - 4\left[c^i_I w^I\, \mathrm{Im}\,\xi_i - \frac{1}{2} m^I \kappa_I + \frac{1}{2} m^0 \kappa_I u^I\right]^2 \end{aligned} \tag{7.33}$$

where analogously to the type IIA case we have defined $\kappa_{IJ} = \kappa_{IJK} w^K$, $\kappa_I = \kappa_{IJK} w^J w^K$, $\kappa = \kappa_{IJK} w^I w^J w^K$. Moreover we obtain that

$$K^{A\bar{B}} \partial_A W \bar{\partial}_{\bar{B}} \overline{W} = \mathfrak{R}_I\, K^{I\bar{J}}\, \mathfrak{R}_{\bar{J}} + \mathfrak{I}_I\, K^{I\bar{J}}\, \mathfrak{I}_{\bar{J}} + (K^{i\bar{j}} - K^{I\bar{J}} \partial_I\, \mathrm{Im}\,\xi_i\, \bar{\partial}_{\bar{J}}\, \mathrm{Im}\,\xi_j)\, \partial_i W \bar{\partial}_{\bar{j}} \overline{W} \tag{7.34}$$

where

$$\begin{aligned} \mathfrak{R}_I &= e_I + m^J \kappa_{IJK} u^K - \frac{1}{2} m^0 \kappa_{IJK} u^J u^K + \frac{1}{2} m^0 \kappa_I - \mathrm{Re}\,\xi_i\, c^i_I - \partial_I \mathrm{Im}\,\xi_i\, c^i_J u^J \qquad (7.35) \\ \mathfrak{I}_I &= m^J \kappa_{IJ} - \frac{1}{2} m^0 \kappa_{IJ} u^J - \partial_I \mathrm{Im}\,\xi_i\, c^i_J\, w^J\,. \qquad (7.36) \end{aligned}$$

Summing up all contributions we obtain the final form for the scalar potential

$$\begin{aligned} V_F = \frac{e^K}{l_s^2 \kappa_4^2} \Bigg\{ &\left[2e_0 + 2\tilde{e}_I u^I + \kappa_{IJK} \tilde{m}^I u^J u^K - \frac{1}{3} m^0 \kappa_{IJK} u^I u^J u^K\right]^2 \qquad (7.37) \\ &+ g^{IJ} \left(\tilde{e}_I + \kappa_{IKL} \tilde{m}^K u^L - \frac{1}{2} m^0 \kappa_{IKL} u^K u^L\right) \times \\ &\times \left(\tilde{e}_J + \kappa_{JMN} \tilde{m}^M u^N - \frac{1}{2} m^0 \kappa_{JMN} u^M u^N\right) \\ &+ \frac{4}{9} \kappa^2 g_{IJ} \left(\tilde{m}^I - m^0 u^I\right)\left(\tilde{m}^J - m^0 u^J\right) + \frac{1}{9} \kappa^2 (m^0)^2 \Bigg\} + V_{\mathrm{DBI}} \end{aligned}$$

where the modified flux quanta are given by (7.27) and we defined

$$V_{\mathrm{DBI}} = \frac{e^K}{\kappa_4^2} (K^{i\bar{j}} - K^{a\bar{b}} \partial_a\, \mathrm{Im}\,\xi_i\, \bar{\partial}_{\bar{b}}\, \mathrm{Im}\,\xi_j)\, \partial_i W \bar{\partial}_{\bar{j}} \overline{W}\,. \tag{7.38}$$

Similarly to the type IIA case, one could have arrived to the piece within brackets in (7.37) by a very simple procedure. First computing the scalar potential for the closed string modes as if there were no D7-branes, and second substituting the RR flux quanta by the quantities (7.27) including the Wilson lines. Notice in fact that, because such a combination of fluxes and Wilson lines is fixed by a discrete gauge symmetry of the compactification, the same prescription applies for a general type IIB flux compactification with O3/O7-planes. Indeed, following [74] one may compute the scalar potential for compactifications with RR and NS three-form fluxes and at arbitrary regions of complex structure, and then simply substitute the RR flux quanta by (7.27) to obtain the scalar potential including complex structure moduli and Wilson lines. It would be interesting to elucidate how this observation constrains the Kähler potential for closed and open string modes, an endeavour which we plan to undertake in the future.

8 Conclusions

In this paper we have made a general analysis of the scalar potential that simultaneously involves open and closed string modes in type II Calabi-Yau compactifications with fluxes and D-branes. We have mostly focused in type IIA flux compactifications with D6-branes, and analysed the scalar potential generated at tree-level and in the large volume limit, that is when the effects of worldsheet instantons can be neglected. Despite this approximation we have shown that certain D6-brane neutral fields, namely Wilson lines and special Lagrangian deformations do enter the tree-level flux potential in quite a similar fashion as the B-field axions do. More precisely, we have found that the way they enter into the flux potential is dictated by a series of discrete shift symmetries, which consist of simultaneously performing loops in open string moduli space and shifting the values of RR flux quanta. As these symmetries are also manifest at the level of the superpotential and they are ultimately related to how we define gauge invariant fluxes in 10d supergravity, we expect them to be present even after threshold corrections have been taken into account.

The form of the open-closed scalar potential has non-trivial implications for the data of the 4d $\mathcal{N} = 1$ supergravity effective field theory. In particular it gives stringent constraints on how the appearance of open string moduli modifies the well-known Kähler potential for closed string modes. In this respect we have found several general features that the open-closed type IIA Kähler potential must satisfy at this level of approximation. First e^K must be a real homogeneous function of degree seven on the imaginary part of the 4d chiral fields, implying a continuous shift symmetry for the D6-brane Wilson lines. This is a rather strong result but nevertheless in total agreement with the uplift of these compactifications to M-theory in G_2 manifolds [75]. Second we have found that in the presence of open string modes, the moduli spaces of complex structure and Kähler deformations no longer factorise, a result that seems to be mostly overlooked in the Calabi-Yau literature. Nonetheless, this is again in agreement with the well-known cases of type II compactifications in toroidal orientifolds. Notice that if we backreact D-branes sources the resulting warping effects are also expected to break such a factorisation [76], so it would be nice to see if these two effects are actually related.

These general results for the Kähler potential are directly related to how closed string 4d holomorphic variables are redefined in the presence of D-branes degrees of freedom. Although our redefinitions differ from the previous proposals in the Calabi-Yau literature,

JHEP09(2016)062

in terms of Kähler potential modifications in D6-brane models they reproduce the proposal made in [22] to embed Hitchin's functionals into the Kähler potential for complex structure moduli. This form of the open-closed Kähler potential will have important implications for models of large field inflation involving D-brane moduli, as will be discussed in [77].

There are a number of directions in which our analysis can be generalised and that would be interesting to explore in the future. For instance, in our type IIA analysis we have only considered the scalar potential and superpotential generated by RR fluxes. While this is sufficient for the scope of this work, it would be important to also include NS fluxes in order to incorporate open string moduli in type IIA models of moduli stabilisation [78–80]. In particular it would be interesting to analyse the interplay of two different effects of such NS fluxes: how they modify the closed string scalar potential and how they generate a discretum of D-brane positions [81]. In a similar spirit, it would be interesting to generalise our type IIA analysis to non-Kähler flux compactifications. Since in these more general backgrounds the D6-brane deformations are also determined by the number non-trivial one-cycles of the wrapped three-cycle [82, 83], one may again consider applying Hitchin's functionals to describe the open string Kähler metrics. Furthermore, it would be interesting to analyse the different corrections that will modify the scalar potential analysed here. They would include threshold corrections to the Kähler potential [59, 66–69], worldsheet instanton corrections to the type IIA superpotential and D-brane instanton corrections. In particular, it would be nice to incorporate the latter directly into the three-form derivation of the scalar potential of section 3, following the recent proposal in [84].

Finally, while our analysis has been restricted to non-chiral D-brane fields it would be interesting to analyse the consequences of our results for more realistic 4d models in which chiral matter arises from D-branes intersections. In this respect notice that throughout our discussion a key role has been played by the discrete shifts in open string moduli space, and in particular those which leave invariant the open-closed superpotential. Remarkably, the same kind of shifts are the ones generating discrete flavour symmetries in simple orientifold models [85]. Therefore it would be interesting to see if there are semi-realistic string models in which the structure of the moduli stabilisation potential is directly related to the flavour structure of the D-brane sector.

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A Details on the Kähler metrics

The Kähler potential that describes type IIA orientifold compactifications is given by

$$K = K_K + K_Q = -\log\left(\mathcal{G}_K \mathcal{G}_Q^2\right) \tag{A.1}$$

where

$$K_K = -\log\left(\frac{i}{6}\mathcal{K}_{abc}(T^a - \bar{T}^a)(T^b - \bar{T}^b)(T^c - \bar{T}^c)\right) = -\log\left(\mathcal{G}_K\right) \tag{A.2}$$

$$K_Q = -2\log\left(\frac{1}{4}\left[\mathrm{Re}(C\mathcal{F}_\Lambda)\mathrm{Im}(CX^\Lambda) - \mathrm{Re}(CX^K)\mathrm{Im}(C\mathcal{F}_K)\right]\right) = -2\log\left(\mathcal{G}_Q\right) \tag{A.3}$$

with the definitions made in section 2. Due to the orientifold geometry the holomorphic three-form of the Calabi-Yau $\mathcal{M}_6$ takes the form

$$C\Omega = \mathrm{Re}(CX^K)\alpha_K + i\mathrm{Im}(CX^\Lambda)\alpha_\Lambda - \mathrm{Re}(C\mathcal{F}_\Lambda)\beta^\Lambda - i\mathrm{Im}(C\mathcal{F}_K)\beta^K \tag{A.4}$$

where each of the coefficients are functions of the real parameters n'^K and u'_Λ that define the complex structure of $\mathcal{M}_6$. Following [26] one can apply the equality

$$\int_{\mathcal{M}_6} \Omega \wedge \partial_{n'^K}\Omega = \int_{\mathcal{M}_6} \Omega \wedge \partial_{u'_\Lambda}\Omega = 0 \tag{A.5}$$

in the above expression for $C\Omega$ to show that

$$n'^K \partial_{n'^K}\mathcal{G}_Q + u'_\Lambda \partial_{u'_\Lambda}\mathcal{G}_Q = 2\mathcal{G}_Q \tag{A.6}$$

which means that $\mathcal{G}_Q$ is a homogeneous function of degree two on the variables n'^K, u'_Λ

$$\mathcal{G}_Q(\lambda n'^K, \lambda u'_\Lambda) = \lambda^2 \mathcal{G}_Q(n'^K, u'_\Lambda) \tag{A.7}$$

In addition, it is easy to see that $\mathcal{G}_K$ is homogeneous of degree three on the variables $t^a = \mathrm{Im}T^a$. Therefore the real function $\mathcal{G}_K\mathcal{G}_Q^2$ that appears in (A.1) is homogeneous of degree seven on the variables $\{t^a, n'^K, u'_\Lambda\}$.

As discussed in section 5, in order to introduce the open string moduli Φ^i into the Kähler potential one needs to express n'^K and u'_Λ in terms of new variables that depend on the open string position moduli. Recall that such moduli are described in terms of the functions $f^i_{\alpha a}$ as

$$\phi^i_\alpha \equiv \mathrm{Im}\,\Phi^i_\alpha = -t^a f^i_{\alpha a} = t^a \frac{d\phi^i_\alpha}{dt^a} \tag{A.8}$$

where in the last equality we have used the definition (2.17) and that $\omega_a = d\,\mathrm{Im}\,J_c/dt^a$. If we see ϕ^i_a as functions of t^a and φ^j_α, the above relation means that ϕ^i_a is homogeneous function of degree one on the Kähler moduli t^a, or in other words that we have the following scaling behaviour

$$t^a \to \lambda t^a \qquad \text{and} \qquad \phi^k_\alpha \to \lambda\phi^k_\alpha \tag{A.9}$$

for the scaling of open string moduli in terms of Kähler moduli. As a direct consequence $f^i_{\alpha a} \equiv f^i_{\alpha a}(t^b, \varphi^j_\alpha)$ are homogenous functions of zero degree on t^a or, if we see them as

$f^i_{\alpha\, a} \equiv f^i_{\alpha\, a}(t^b, \phi^j_\alpha)$, they should be invariant under the simultaneous rescaling (A.9). These statements are equivalent to

$$t^a \frac{d}{dt^a} f^i_{\alpha\, b} \,=\, t^a \left(\partial_{t^a} + \frac{\partial \phi^j_\alpha}{\partial t^a} \partial_{\phi^j_\alpha} \right) f^i_{\alpha\, b} \,=\, \left(t^a \partial_{t^a} + \phi^j_\alpha \partial_{\phi^j_\alpha} \right) f^i_{\alpha\, b} \,=\, 0 \tag{A.10}$$

Finally, as pointed out in the main text $f^i_{\alpha\, a}$ may also depend on the complex structure moduli n'^K and u'_Λ. Now because the harmonic two-forms ω_a are invariant under an overall rescaling of the holomorphic three-form Ω, they can only depend on quotients of their periods, and so the same should be true for $f^i_{\alpha\, a}$. Therefore these functions should also be invariant under the rescaling

$$n'^K \,\to\, \lambda' n'^K \qquad \text{and} \qquad u'_\Lambda \,\to\, \lambda' u'_\Lambda \tag{A.11}$$

with λ' independent from λ in (A.9). Finally, a similar reasoning can be applied to the functions $g^K_{\alpha\, i}$ and $g_{\alpha\,\Lambda\, i}$, defined in (2.22). Indeed, from such a chain integral expression one can argue that these functions should also be invariant under (A.9) and (A.11) separately, and in particular homogeneous functions of zero degree on the variables $\{t^a, n'^K, u'_\Lambda, \phi^j_\alpha\}$.

These observations are relevant for the redefinition of holomorphic variables proposed in section 5, which imply that we must perform the following replacement in $\mathcal{G}_Q$

$$n'^K \,\to\, n^K + \frac{1}{2} t^a \sum_\alpha \mathbf{H}^K_{\alpha\, a} \qquad u'_\Lambda \,\to\, u_\Lambda - \frac{1}{2} t^a \sum_\alpha \mathbf{H}_{\alpha\,\Lambda\, a} \tag{A.12}$$

with $n^K = \mathrm{Im}\, N^K$, $u_\Lambda = \mathrm{Im}\, U_\Lambda$ the imaginary parts of the new holomorphic variables. Here $\mathbf{H}^K_{\alpha\, a}$ and $\mathbf{H}_{\alpha\,\Lambda\, a}$ must satisfy (5.8) and (5.14), or equivalently

$$\partial_{\phi^j_\beta} (t^a \mathbf{H}^K_{\alpha\, a}) \,=\, -g^K_{\alpha\, i} \delta_{\alpha\beta} \qquad\qquad \partial_{\phi^j_\beta} (t^a \mathbf{H}_{\alpha\,\Lambda\, a}) \,=\, -g_{\alpha\,\Lambda\, i} \delta_{\alpha\beta} \tag{A.13}$$

so we can see them as functions of the variables $\{t^a, n'^K, u'_\Lambda, \phi^j_\alpha\}$. Since $g^K_{\alpha\, i}$ and $g_{\alpha\,\Lambda\, i}$ are homogeneous of zero degree on these variables, the same can be assumed for $\mathbf{H}^K_{\alpha\, a}$, $\mathbf{H}_{\alpha\,\Lambda\, a}$. Finally, by recursively performing the replacement (A.12) we can see $\mathbf{H}^K_{\alpha\, a}$, $\mathbf{H}_{\alpha\,\Lambda\, a}$ as homogeneous functions of zero degree on $\{t^a, n^K, u_\Lambda, \phi^k_\alpha\}$.

We then see that the r.h.s. of (A.12) are homogeneous functions of degree one on the real fields $\{t^a, n^K, u_\Lambda, \phi^k_\alpha\}$ on which the Kähler potential depends. As a consequence, when we perform such replacements we obtain that $\mathcal{G}_Q$ remains a homogeneous function of degree two in the new fields, namely

$$\mathcal{G}_Q(\lambda t^a, \lambda n^K, \lambda u_\Lambda\, \lambda \phi^k_\alpha) \,=\, \lambda^2 \mathcal{G}_Q(t^a, n^K, u_\Lambda\, \phi^k_\alpha) \tag{A.14}$$

Finally, we have that

$$\mathcal{G}(\psi^\alpha) \,=\, \mathcal{G}_K \mathcal{G}_Q^2 \tag{A.15}$$

is homogeneous of degree seven on the whole set of fields $\psi^\alpha \equiv \{t^a, n^K, u_\Lambda, \phi^k\}$, where for simplicity we have absorbed the D6-brane index α into the index i.

From this simple observation several useful relation can be derived [86]. For instance

$$K^{\alpha\bar{\beta}}K_{\bar{\beta}} = -(\Psi^\alpha - \bar{\Psi}^\alpha) \equiv -2i\psi^\alpha \tag{A.16}$$

with Ψ^a any of the complex fields of the compactification. To see this we first we rewrite this relation as

$$-2iK_{\bar{\alpha}\beta}\psi^\beta = K_{\bar{\alpha}} \tag{A.17}$$

which is easier to check. Then we compute

$$K_{\bar{\alpha}} = \frac{1}{2i}\frac{\partial_\alpha \mathcal{G}}{\mathcal{G}} \tag{A.18}$$

$$K_{\bar{\alpha}\beta} = -\frac{1}{4}\left(\frac{\partial_\alpha\partial_\beta\mathcal{G}}{\mathcal{G}} - \frac{\partial_\alpha\mathcal{G}\partial_\beta\mathcal{G}}{\mathcal{G}^2}\right) \tag{A.19}$$

and then use the relations

$$\psi^\beta\partial_\beta\mathcal{G} = 7\mathcal{G} \qquad \psi^\beta\partial_\alpha\partial_\beta\mathcal{G} = 6\partial_\alpha\mathcal{G} \tag{A.20}$$

that arise from the homogeneity of $\mathcal{G}$ to prove (A.17). Moreover, using the first identity in (A.20) again, one can show that the no-scale relation

$$K^{\alpha\bar{\beta}}K_\alpha K_{\bar{\beta}} = 7 \tag{A.21}$$

follows automatically. Finally, one can use these relations to get a simple expression for the inverse Kähler metric

$$K^{\bar{\alpha}\beta} = \frac{2}{3}\psi^\alpha\psi^\beta - 4\mathcal{G}\mathcal{G}^{\alpha\beta} \tag{A.22}$$

where $\mathcal{G}^{\alpha\beta}$ is the inverse of $\partial_\alpha\partial_\beta\mathcal{G}$. For a recent general discussion on no-scale Kähler potentials based on homogeneous functions an their generalisation see [87].

Relations for the inverse metric. We would now like to discuss several identities relating the inverse Kähler metric components which are important for the computations of section 6, and see how they may arise from the above Kähler potential. For simplicity we will work in the symplectic basis defined above eq. (2.10) so that $\mathcal{G}_Q$ is a homogeneous function of degree two purely on the variables n'^K, which in turn depend on the holomorphic fields N^K, T^a and Φ^i_α and their conjugates through (A.12). As this dependence can in general be quite involved, in here we will make the simplifying assumption that the functions $f^i_{\alpha\,a}$ and $g^K_{\alpha\,i}$ only depend on φ^j_α, which will allow us to carry the computations analytically.[17]

[17] This assumption is valid in simple cases like toroidal compactifications, and it should be a good approximation in the large volume and complex structure regions of the Calabi-Yau moduli space. Indeed, in general, $f^i_{\alpha\,a}$ and $g^K_{\alpha\,i}$ are homogeneous functions of zero degree on $\{t^a, n'^K, u'_\Lambda, \phi^j_\alpha\}$, invariant under (A.9) and (A.11). As such they depend on φ^j_α and also on quotients of Kähler moduli t^b/t^a and complex structure moduli n'^K/n'^J, u'_Λ/u'_Σ. The dependence on these quotients is very mild for large values of these bulk fields, that is in the regions of large volume and complex structure, where the bulk harmonic forms ω^a, β^K do not vary significantly with respect to variations of the closed string moduli.

Because $\varphi^j_\alpha \equiv \varphi^j_\alpha(\phi^i_\alpha, t^a)$, we may then see $f^i_{\alpha\, a}$ and $g^K_{\alpha\, i}$ as functions of ϕ^i_α and t^a, but nevertheless such that $df^i_{\alpha\, a}/dt^b = dg^K_{\alpha\, i}/dt^b = 0$. Given the definition (5.8) the same applies to $\mathbf{H}^K_{\alpha\, a}$, and so we have that

$$\frac{d\mathbf{H}^K_{\alpha\, a}}{dt^b} = \left(\partial_{t^b} + \frac{\partial \phi^j_\alpha}{\partial t^b}\partial_{\phi^j_\alpha}\right)\mathbf{H}^K_{\alpha\, a} = 0 \quad \Rightarrow \quad \partial_{t^b}(t^a \mathbf{H}^K_{\alpha\, a}) = \mathbf{H}^K_{\alpha\, b} - f^i_{\alpha\, b} g^K_{\alpha\, i} \tag{A.23}$$

which will be used later.

From the assumption that $\mathbf{H}^K_{\alpha\, i}$ does not depend on the complex structure moduli one can see that the Kähler metric can be written in the form

$$\mathbf{K} = \begin{pmatrix} \mathbb{I} & 0 & 0 \\ \mathbf{T}^\dagger & \mathbb{I} & 0 \\ \mathbf{\Phi}^\dagger & 0 & \mathbb{I} \end{pmatrix} \begin{pmatrix} \mathbf{N} & 0 & 0 \\ 0 & \multicolumn{2}{c}{\multirow{2}{*}{$\mathbf{\Omega}$}} \\ 0 & & \end{pmatrix} \begin{pmatrix} \mathbb{I} & \mathbf{T} & \mathbf{\Phi} \\ 0 & \mathbb{I} & 0 \\ 0 & 0 & \mathbb{I} \end{pmatrix} \tag{A.24}$$

where we have defined the matrices

$$\mathbf{N}_{K\bar{L}} = \partial_{N'^K}\partial_{\bar{N}'^L}\mathbf{K_Q} \qquad \mathbf{T}^{\bar{L}}{}_{\bar{a}} = \partial_{\bar{T}^a}\bar{N}'^L \qquad \mathbf{\Phi}^{\bar{L}}{}_{\bar{i}} = \partial_{\bar{\Phi}^i}\bar{N}'^L \tag{A.25}$$

and

$$\mathbf{\Omega} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} \tag{A.26}$$

with

$$\mathbf{A}_{a\bar{b}} = \partial_{T^a}\partial_{\bar{T}^b}\mathbf{K_K} + (\partial_{n'^K}\mathbf{K_Q})\partial_{T^a}\partial_{\bar{T}^b}n'^K \tag{A.27}$$

$$\mathbf{B}_{a\bar{j}} = (\partial_{n'^K}\mathbf{K_Q})\partial_{T^a}\partial_{\bar{\Phi}^j}n'^K \tag{A.28}$$

$$\mathbf{D}_{i\bar{j}} = (\partial_{n'^K}\mathbf{K_Q})\partial_{\Phi^i}\partial_{\bar{\Phi}^j}n'^K \tag{A.29}$$

and $\mathbf{C} = \mathbf{B}^\dagger$. For simplicity we have absorbed the D6-brane index α into the index i counting open string moduli.

From this expression we find that the inverse metric is given by

$$\mathbf{K}^{-1} = \begin{pmatrix} \mathbb{I} & -\mathbf{T} & -\mathbf{\Phi} \\ 0 & \mathbb{I} & 0 \\ 0 & 0 & \mathbb{I} \end{pmatrix} \begin{pmatrix} \mathbf{N}^{-1} & 0 & 0 \\ 0 & \multicolumn{2}{c}{\multirow{2}{*}{$\mathbf{\Omega}^{-1}$}} \\ 0 & & \end{pmatrix} \begin{pmatrix} \mathbb{I} & 0 & 0 \\ -\mathbf{T}^\dagger & \mathbb{I} & 0 \\ -\mathbf{\Phi}^\dagger & 0 & \mathbb{I} \end{pmatrix} \tag{A.30}$$

where

$$\mathbf{\Omega}^{-1} = \begin{pmatrix} \mathbb{I} & 0 \\ -\mathbf{D}^{-1}\mathbf{C} & \mathbb{I} \end{pmatrix} \begin{pmatrix} (\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1} & 0 \\ 0 & \mathbf{D}^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{I} & -\mathbf{B}\mathbf{D}^{-1} \\ 0 & \mathbb{I} \end{pmatrix}. \tag{A.31}$$

From here we deduce the relation

$$\mathbf{K}^{\bar{a}i} + \mathbf{K}^{\bar{a}b}\mathbf{B}_{b\bar{j}}\mathbf{D}^{\bar{j}i} = 0 \tag{A.32}$$

and so the relation (5.20) can be rephrased as

$$\mathbf{B}_{a\bar{j}} = f^i_a \mathbf{D}_{i\bar{j}} \tag{A.33}$$

or equivalently

$$\mathrm{Im}(C\mathcal{F}_K)\left[\partial_{t^a}+\frac{\partial\phi^i}{\partial t^b}\partial_{\phi^i}\right](\partial_{\phi^j}n'^K) = \mathrm{Im}(C\mathcal{F}_K)\frac{d}{dt^a}(\partial_{\phi^j}n'^K) = 0 \tag{A.34}$$

where as before $t^a = \mathrm{Im}\, T^a$ and $\phi^i = \mathrm{Im}\,\Phi^i$. Using that $\partial_{\phi^j}n'^K = -\frac{1}{2}g_i^K$ and the assumption $dg_i^K/dt^a = 0$ we recover the desired identity.

From these expressions one also obtains that

$$\begin{aligned}\mathbf{A}_{a\bar{b}} - \mathbf{B}_{a\bar{j}}\mathbf{D}^{\bar{j}i}\mathbf{C}_{i\bar{b}} &= \partial_{T^a}\partial_{\bar{T}^b}\mathbf{K_K} + (\partial_{n'^K}\mathbf{K_Q})\left(\partial_{T^a}\partial_{\bar{T}^b} - f_a^i\partial_{\Phi^i}\partial_{\bar{T}^b}\right)n'^K \\ &= \partial_{T^a}\partial_{\bar{T}^b}\mathbf{K_K} + \frac{1}{4}(\partial_{n'^K}\mathbf{K_Q})\frac{d}{dt^a}(\partial_{t^b}n'^K) = \partial_{T^a}\partial_{\bar{T}^b}\mathbf{K_K}\end{aligned} \tag{A.35}$$

where in the last equality we have used (A.23). Hence the inverse Kähler metric for the Kähler moduli is exactly the same as in the absence of open string degrees of freedom. More precisely we have that

$$K^{a\bar{b}} = 2t^at^b - \frac{2}{3}\mathcal{K}\mathcal{K}^{ab} \tag{A.36}$$

where we have defined $\mathcal{K}$ as in (6.4) and $\mathcal{K}^{ab}$ is the inverse of $\mathcal{K}_{ab}$ in there. That is, we have the same inverse metric as we would have if there were no open string moduli. Finally, applying (A.33) we have that

$$K^{i\bar{j}} - K^{a\bar{b}}f_a^if_b^j = \mathbf{D}^{i\bar{j}} \tag{A.37}$$

where $\mathbf{D}_{i\bar{j}}$ is given by

$$\mathbf{D}_{i\bar{j}} = \frac{1}{4}\partial_{n'^K}\mathbf{K_Q}\partial_{\phi^i}\,\partial_{\phi^j}n'^K = -\partial_{n'^K}\mathbf{K_Q}\,\frac{1}{8}\partial_{\phi^i}g^K{}_j \tag{A.38}$$

Now in order to match the DBI potential from section 6 we need that

$$\mathbf{D}^{i\bar{j}} = G^{ij}_{\mathrm{D6}} = 8\hat{V}_6\, e^{\phi/4}\, l_s^{-3}\int_{\Pi_\alpha}\rho^i\wedge *\rho^j \tag{A.39}$$

or equivalently

$$\begin{aligned}\mathbf{D}_{i\bar{j}} = G_{ij}^{\mathrm{D6}} &= \frac{e^{-\phi/4}}{8\hat{V}_6}l_s^{-3}\int_{\Pi_\alpha}\zeta_i\wedge *\zeta_j \\ &= -\frac{1}{8\hat{V}_6}\left(l_s^{-3}\int_{\Pi_\alpha}\iota_{X_k}J\wedge\rho^i\right)^{-1}\left(l_s^{-3}\int_{\Pi_\alpha}\iota_{X^k}\mathrm{Im}\,(C\Omega)\wedge\zeta_j\right) \\ &= \frac{1}{8\hat{V}_6}\mathrm{Im}(C\mathcal{F}_K)(\mathcal{Q}^K)_{jk}\left[(t^a\eta_a)^{-1}\right]_i^k\end{aligned} \tag{A.40}$$

where we have used that $e^{-\phi/4}[\iota_{X^j}J]_{\Pi_\alpha} = -*_3[\iota_{X^j}\mathrm{Im}\,(C\Omega)]_{\Pi_\alpha}$ [29]. Comparing with (A.38) this implies that

$$\partial_{\phi^i}g^K{}_j = -(\mathcal{Q}^K)_{jk}\left[(t^a\eta_a)^{-1}\right]_i^k \quad\Longleftrightarrow\quad \partial_{\varphi^i}g^K{}_j = (\mathcal{Q}^K)_{ji} \tag{A.41}$$

in agreement with (2.19). In fact, notice that we will also reproduce the same set of results if in (A.13) we shift $g^K{}_j$ by a constant, which means that the $t^a\mathbf{H}_a^K$ are only determined up to a linear function on ϕ^j.

B The closed string scalar potential

The literature already offers several approaches to obtain the scalar potential for the Kähler moduli in presence of background RR fluxes through dimensional reduction of (massive) type IIA supergravity, both excluding [21, 25, 34] and including [22, 23] couplings to D-branes. In the absence of D6-branes, the scalar potential arises from the kinetic terms of the RR field strengths upon dimensional reduction of the standard formulation of (massive) type II supergravity. In the presence of D6-branes, the road to follow for the dimensional reduction passes through the democratic formulation of massive type IIA supergravity, where aside from the RR-potentials C_1 and C_3 and Romans mass parameter m also the dual RR-potentials C_5, C_7 and C_9 are taken into consideration and the Hodge duality relations are usually imposed by hand. In this formulation the scalar potential for the Kähler moduli emerges upon dualisation of the four-dimensional four-forms associated to the dimensional reduction of the RR-field strengths in favour of the RR-flux quanta.

This appendix offers a first principle approach to deduce a four-dimensional mother action allowing for the dualisation of the four-forms also in the presence of D6-branes, while taking into account the considerations about Dirac quantisation around equation (3.3). As advertised in [35, 36, 88] background geometries including closed string fluxes and D-branes can be better approached from the A-basis, instead of the C-basis. For this reason we choose to start from the following mother action:

$$\begin{aligned}\mathcal{S}_{RR}^{\text{mother}} =& \frac{1}{2\kappa_4^2 l_s^6}\int \Bigg[-\sum_{p=0}^{5}\frac{1}{4}G_{2p}\wedge \star G_{2p} + \frac{1}{2}A_9\wedge dG_0 - \frac{1}{2}A_7\wedge d\left(G_2 - G_0 B\right) \\ &+\frac{1}{2}A_5\wedge d\left(G_4 - G_2\wedge B + \frac{G_0}{2}B^2\right) \\ &-\frac{1}{2}A_3\wedge d\left(G_6 - G_4\wedge B + \frac{1}{2}G_2\wedge B^2 - \frac{G_0}{3!}B^3\right) \\ &+\frac{1}{2}A_1\wedge d\left(G_8 - G_6\wedge B + \frac{1}{2}G_4\wedge B^2 - \frac{1}{3!}G_2\wedge B^3 + \frac{G_0}{4!}B^4\right)\Bigg],\end{aligned} \tag{B.1}$$

where the potentials $\mathbf{A}$ are playing the rôle of Lagrange multipliers imposing the Bianchi identities for the field strengths in the A-basis. The solutions of the Bianchi identities correspond to the field strengths G_{2p} given in terms of the RR-potentials in the A-basis and constant fluxes as indicated in equation (3.4). Using the basis of harmonic forms on $\mathcal{M}_6$ introduced in section 3 for a background with RR-fluxes only, the dimensional reduction of the field strengths contains the flux quanta (3.5):

$$\begin{aligned}&G_0 = l_s^{-1}m,\\ &G_2 = l_s^{-1}\left(m^a + b^a m\right)\omega_a + \dots,\\ &G_4 = l_s^{-1}\left(\left(e_a + \mathcal{K}_{abc}m^b b^c + \frac{m}{2}\mathcal{K}_{abc}b^b b^c\right)\tilde{\omega}^a + D_4^0 + \dots,\right.\\ &G_6 = l_s^{-1}\left(e_0 + e_a b^a + \frac{1}{2}\mathcal{K}_{abc}m^a b^b b^c + \frac{m}{3!}\mathcal{K}_{abc}b^a b^b b^c\right)\omega_6 + \left(D_4^a + b^a D_4^0\right)\wedge\omega_a + \dots,\end{aligned} \tag{B.2}$$

$$G_8 = \left(\tilde{D}_{4a} + \mathcal{K}_{abc} b^b D_4^c + \frac{1}{2}\mathcal{K}_{abc} b^b b^c D_4^0\right) \wedge \tilde{\omega}^a + \ldots,$$
$$G_{10} = \left(\tilde{D}_4 + \tilde{D}_{4a} b^a + \frac{1}{2}\mathcal{K}_{abc} b^a b^b D_4^c + \frac{1}{3!}\mathcal{K}_{abc} b^a b^b b^c D_4^0\right) \wedge \omega_6 + \ldots.$$

Inserting the expansion back into the mother action (B.1), converting to the Einstein-frame by virtue of a rescaling of the ten-dimensional metric $G_{MN} \to e^{\frac{\phi}{2}} G_{MN}$ and rescaling the four-dimensional metric $g_{\mu\nu} \to \frac{g_{\mu\nu}}{\hat{V}_6/2}$, we obtain the following four-dimensional effective action (exploiting the notations of section 3):

$$\begin{aligned}
4\kappa_4^2 \mathcal{L}_{4d}^{\text{mother}} = &- \frac{1}{4}\frac{4}{\hat{V}_6} e^{\frac{5\phi}{2}} \tilde{\rho}^2 - \frac{1}{4} e^{\frac{5\phi}{2}} \frac{16}{\hat{V}_6} g_{ab} \tilde{\rho}^a \tilde{\rho}^b - \frac{1}{4} e^{-\frac{\phi}{2}} \frac{g^{ab}}{\hat{V}_6^3} \rho_a \rho_b - \frac{1}{4} e^{-\frac{\phi}{2}} \frac{4}{\hat{V}_6^3} \rho_0^2 \\
&- \frac{1}{4}\frac{\hat{V}_6}{4} e^{-\frac{5\phi}{2}} \left(\tilde{D}_4 + \tilde{D}_{4a} b^a + \frac{1}{2}\mathcal{K}_{amn} b^a b^m D_4^n + \frac{1}{3!}\mathcal{K}_{amn} b^a b^m b^n D_4^0\right) \\
&\qquad \wedge * \left(\tilde{D}_4 + \tilde{D}_{4b} b^b + \frac{1}{2}\mathcal{K}_{brs} b^b b^r D_4^s + \frac{1}{3!}\mathcal{K}_{brs} b^b b^r b^s D_4^0\right) \\
&- \frac{1}{4} e^{-\frac{5\phi}{2}} \frac{\hat{V}_6 g^{ab}}{16} \left(\tilde{D}_{4a} + \mathcal{K}_{amn} b^m D_4^n + \frac{1}{2}\mathcal{K}_{amn} b^m b^n D_4^0\right) \\
&\qquad \wedge * \left(\tilde{D}_{4b} + \mathcal{K}_{brs} b^r D_4^s + \frac{1}{2}\mathcal{K}_{brs} b^r b^s D_4^0\right) \\
&- \frac{1}{4} e^{\frac{\phi}{2}} g_{ab} \hat{V}_6^3 \left(D_4^a + b^a D_4^0\right) \wedge * \left(D_4^b + b^b D_4^0\right) - \frac{1}{4} e^{\frac{\phi}{2}} \frac{\hat{V}_6^3}{4} D_4^0 \wedge * D_4^0 \\
&- \frac{1}{2} l_s^{-1} m \tilde{D}_4 + \frac{1}{2} l_s^{-1} m^a \tilde{D}_{4a} - \frac{1}{2} l_s^{-1} e_a D_4^a + \frac{1}{2} l_s^{-1} e_0 D_4^0.
\end{aligned} \tag{B.3}$$

By integrating out the four-forms in the order $\tilde{D}_4 \to \tilde{D}_{4a} \to D_4^a \to D_4^0$ in favour of the flux quanta, the usual four-dimensional effective scalar potential for the Kähler moduli arises as given in equations (3.15) and (3.16). Alternatively, one can rotate the four-forms $(D_4^0, D_4^a, \tilde{D}_{4a}, \tilde{D}_4)$ into the four-forms $(F_4^0, F_4^a, \tilde{F}_{4a}, \tilde{F}_4)$, in which case the four-dimensional mother action reads:

$$\begin{aligned}
4\kappa_4^2 \mathcal{L}_{4d}^{\text{mother}} = &- \frac{1}{4} e^{\frac{5\phi}{2}} \frac{4}{\hat{V}_6} \tilde{\rho}^2 - \frac{1}{4} e^{\frac{5\phi}{2}} \frac{16}{\hat{V}_6} g_{ab} \tilde{\rho}^a \tilde{\rho}^b - \frac{1}{4} e^{-\frac{\phi}{2}} \frac{g^{ab}}{\hat{V}_6^3} \rho_a \rho_b - \frac{1}{4} e^{-\frac{\phi}{2}} \frac{4}{\hat{V}_6^3} \rho_0^2 \\
&- \frac{1}{4} e^{-\frac{5\phi}{2}} \frac{\hat{V}_6}{4} \tilde{F}_4 \wedge * \tilde{F}_4 - \frac{1}{4} e^{-\frac{5\phi}{2}} \frac{\hat{V}_6 g^{ab}}{16} \tilde{F}_{4a} \wedge * \tilde{F}_{4b} \\
&- \frac{1}{4} e^{\frac{\phi}{2}} g_{ab} \hat{V}_6^3 F_4^a \wedge * F_4^b - \frac{1}{4} e^{\frac{\phi}{2}} \frac{\hat{V}_6^3}{4} F_4^0 \wedge * F_4^0 \\
&+ \frac{1}{2} \tilde{\rho} \tilde{F}_4 + \frac{1}{2} \tilde{\rho}^a \tilde{F}_{4a} + \frac{1}{2} \rho_a F_4^a + \frac{1}{2} \rho_0 F_4^0
\end{aligned} \tag{B.4}$$

This change of four-form base boils down to a rewriting of the mother action in the C-basis, where we recombined the flux quanta into the Lagrange-multipliers $(\rho_0, \rho_a, \tilde{\rho}^a, \tilde{\rho})$ as in equation (3.14) for simplicity. Eliminating the four-forms $(F_4^0, F_4^a, \tilde{F}_{4a}, \tilde{F}_4)$ through their equations of motion:

$$\begin{aligned}
\tilde{\rho} &= e^{-\frac{5\phi}{2}} \frac{\hat{V}_6}{4} * \tilde{F}_4, \\
\tilde{\rho}^a &= e^{-\frac{5\phi}{2}} \frac{\hat{V}_6 g^{ab}}{16} * \tilde{F}_{4b}, \\
\rho_a &= e^{\frac{\phi}{2}} g_{ab} \hat{V}_6^3 * F_4^b, \\
\rho_0 &= e^{\frac{\phi}{2}} \frac{\hat{V}_6^3}{4} * F_4^0,
\end{aligned} \tag{B.5}$$

yields the scalar potential from the RR fluxes as in equations (3.15) and (3.16). Note that a mother action is not unique and a (classical) theory expressed in a particular set of degrees of freedom can arise from two different mother actions; the only requisite however is that such mother actions reproduce identical equations of motion (and Bianchi identities). It is trivial to see that the mother action (3.11) reproduces the same equations of motion in (B.5) for the four-forms $(F_4^0, F_4^a, \tilde{F}_{4a}, \tilde{F}_4)$ upon the identification $e^K = (8e^{\phi/2}\hat{V}_6^3)^{-1}$.

The virtue of the mother action (3.11) lies in the straightforward generalisation for backgrounds with RR-fluxes and D6-branes, whose Chern-Simons coupling to the bulk degrees of freedom can be captured by a shift of the flux quanta (3.25). Including the contribution of a single D6-brane in the mother action and following the procedures outlined in section 3 we obtain the RR scalar potential in which Kähler moduli and open string moduli mix:

$$\begin{aligned}
\kappa_4^2 V_{RR} = &\frac{e^{-\frac{\phi}{2}}}{2l_s^2\hat{V}_6^3}\Big(e_0 + b^a e_a + \frac{1}{2}\mathcal{K}_{abc}m^a b^b b^c + \frac{m}{6}\mathcal{K}_{abc}b^a b^b b^c \\
&\qquad + n_{F\,i}\theta^i - n_{a\,i}\theta^i b^a - n_{F\,i} f_a^i b^a + n_{ai} f_c^i b^a b^c\Big)^2 \\
&+ \frac{e^{-\frac{\phi}{2}}}{8l_s^2\hat{V}_6^3} g^{ab}\left(e_a + \mathcal{K}_{acd}m^c b^d + \frac{m}{2}\mathcal{K}_{acd}b^c b^d - n_{a\,i}\theta^i - n_{F\,i} f_a^i + \mathcal{K}_{abc}q^b b^c\right) \\
&\qquad \times \left(e_b + \mathcal{K}_{bef}m^e b^f + \frac{m}{2}\mathcal{K}_{bef}b^e b^f - n_{b\,k}\theta^k - n_{F\,k} f_e^k + \mathcal{K}_{bef}q^e b^f\right) \\
&+ \frac{2e^{\frac{5\phi}{2}}}{l_s^2\hat{V}_6} g_{ab}\left(m^a + m b^a + q^a\right)\left(m^b + m b^b + q^b\right) + \frac{e^{\frac{5\phi}{2}}}{2l_s^2\hat{V}_6} m^2
\end{aligned} \tag{B.6}$$

C A toroidal orbifold example

To clarify the geometric origin of the open-closed superpotential from section 4 and the emergence of the open string moduli in the Kähler potential, we consider an explicit realisation on the orientifold $(T^2 \times K3)/\Omega_p(-)^{F_L}\mathcal{R}$. $K3$ is considered in the orbifold limit $T^4/\mathbb{Z}_2$ [89] inheriting bulk two-cycles from the covering four-torus, while the $\mathbb{Z}_2$ action implies the existence of 16 exceptional two-cycles e_{ij} stuck at the $\mathbb{Z}_2$-fixed points $i, j \in \{1, 2, 3, 4\}$. For simplicity, we choose the T^4 to be factorisable and choose the root-lattice of SU(2)×SU(2) for each separate two-torus T^2. Factorisable bulk three-cycles Π_α^{bulk} on $T_{(1)}^2 \times T^4/\mathbb{Z}_2$ are expressed as linear combinations of basis three-cycles $(\pi_{\alpha_k}, \pi_{\beta^k})$, which represent Poincaré dual three-cycles to the symplectic basis (α_k, β^k) of (bulk) (2,1)-forms in $H^3(T^2 \times K3)$:

$$\begin{array}{ll} \alpha_0 = dx^1 \wedge dx^2 \wedge dx^3, & \beta^0 = -dy^1 \wedge dy^2 \wedge dy^3, \\ \alpha_1 = dx^1 \wedge dy^2 \wedge dy^3, & \beta^1 = -dy^1 \wedge dx^2 \wedge dx^3, \\ \alpha_2 = dy^1 \wedge dx^2 \wedge dy^3, & \beta^2 = -dx^1 \wedge dy^2 \wedge dx^3, \\ \alpha_3 = dy^1 \wedge dy^2 \wedge dx^3, & \beta^3 = -dx^1 \wedge dx^2 \wedge dy^3. \end{array} \tag{C.1}$$

These generators of bulk three-cycles arise by considering $\mathbb{Z}_2$-invariant product cycles of the one-cycle π_1 or π_2 on $T^2_{(1)}$ and factorisable bulk two-cycles on $T^4/\mathbb{Z}_2$:

$$\begin{array}{ll} \pi_{\beta^0} = \pi_1 \otimes \pi_3 \otimes \pi_5, & \pi_{\alpha_0} = -\pi_2 \otimes \pi_4 \otimes \pi_6, \\ \pi_{\beta^1} = \pi_1 \otimes \pi_4 \otimes \pi_6, & \pi_{\alpha_1} = -\pi_2 \otimes \pi_3 \otimes \pi_5, \\ \pi_{\beta^2} = \pi_2 \otimes \pi_3 \otimes \pi_6, & \pi_{\alpha_2} = -\pi_1 \otimes \pi_4 \otimes \pi_5, \\ \pi_{\beta^3} = \pi_2 \otimes \pi_4 \otimes \pi_5, & \pi_{\alpha_3} = -\pi_1 \otimes \pi_3 \otimes \pi_6. \end{array} \tag{C.2}$$

The generators $(\pi_{\alpha_k}, \pi_{\beta^k})$ in $H_3(T^2 \times K3, \mathbb{Z})$ form a symplectic basis of bulk three-cycles with $\mathcal{R}$-even three-cycles π_{β^k} and $\mathcal{R}$-odd three-cycles π_{α_k}, and were chosen in such a way that all complex structure moduli are of the N^k-kind as discussed in section 2. Exceptional three-cycles Π_α^{ex} can be expressed in terms of the generators $(\varepsilon_{ij}, \tilde{\varepsilon}_{ij})$, constructed as $\mathbb{Z}_2$-invariant direct products of the one-cycle π_1 or π_2 on $T^2_{(1)}$ and an exceptional divisor e_{ij} on $T^4/\mathbb{Z}_2$:

$$\varepsilon_{ij} = \pi_1 \otimes e_{ij}, \qquad \tilde{\varepsilon}_{ij} = \pi_2 \otimes e_{ij}, \qquad i,j \in \{1,2,3,4\}. \tag{C.3}$$

Under the orientifold projection, the exceptional divisors pick up a minus sign,[18] more explicitly $\mathcal{R}(e_{ij}) = -e_{ij}$, such that exceptional three-cycles can be decomposed into $\mathcal{R}$-even exceptional three-cycles $\tilde{\varepsilon}_{ij}$ and $\mathcal{R}$-odd exceptional three-cycles ε_{ij}. The lattice of three-cycles generated by $\{\pi_{\alpha_k}, \pi_{\beta^k}, \varepsilon_{ij}, \tilde{\varepsilon}_{ij}\}$ has to be supplemented with fractional three-cycles:

$$\Pi_\alpha^{\text{frac}} = \frac{1}{2}\Pi_\alpha^{\text{bulk}} + \frac{1}{2}\Pi_\alpha^{\text{ex}}, \tag{C.4}$$

to obtain the full lattice of three-cycles $H_3(T^2 \times K3, \mathbb{Z})$.

Next, we consider three D6-branes stacks a, b and c supported by three different fractional three-cycles Π_a, Π_b and Π_c respectively, with torus wrapping numbers given by

$$\begin{aligned} \Pi_a &: (1,0)_{(\pi_1,\pi_2)} \times (1,0)_{(\pi_3,\pi_4)} \times (1,0)_{(\pi_5,\pi_6)}, \\ \Pi_b &: (1,0)_{(\pi_1,\pi_2)} \times (0,1)_{(\pi_3,\pi_4)} \times (0,-1)_{(\pi_5,\pi_6)}, \\ \Pi_c &: (1,0)_{(\pi_1,\pi_2)} \times (1,1)_{(\pi_3,\pi_6)} \times (1,1)_{(\pi_5,\pi_4)}. \end{aligned} \tag{C.5}$$

Note the bulk part of the fractional three-cycle Π_c is non-factorisable and should be written as the linear combination $\Pi_c^{\text{bulk}} = \pi_{\beta^0} - \pi_{\beta^1} + \pi_1 \otimes \pi_3 \otimes \pi_4 - \pi_1 \otimes \pi_5 \otimes \pi_6$. Such non-factorisable three-cycles can be related [24] to coisotropic D8-branes through T-dualities,

[18] This minus sign can be traced back to the involution operation $J \xrightarrow{\mathcal{R}} -J$, translated to the de Rahm dual $(1,1)$-forms situated at the blown-up singularities.

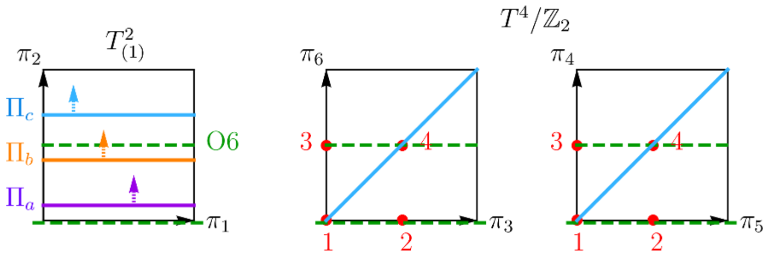

Figure 1. Geometric representation of the orientifold $(T^2 \times K3)/\Omega_p(-)^{F_L}\mathcal{R}$ in terms of factorised two-tori. The red points $1, 2, 3$ and 4 correspond to the $\mathbb{Z}_2$ fixed points, while the green dashed lines indicate the fixed planes under the anti-holomorphic involution $\mathcal{R}$. The torus wrapping numbers for the three-cycles Π_a, Π_b and Π_c are given in equation (C.5) and the factorisation of $T^4/\mathbb{Z}_2$ has been chosen to easily depict Π_c^{bulk}.

and share with the latter the property that the lagrangian condition $J|_{\Pi_c} \neq 0$ is violated for anisotropic untwisted Kähler moduli, i.e. $T^2 \neq T^3$. This potential violation is the source for the bilinear coupling in the open-closed D6-brane superpotential in this toy model. Since the non-factorisable three-cycles $\pi_1 \otimes \pi_3 \otimes \pi_4$ and $\pi_1 \otimes \pi_5 \otimes \pi_6$ are $\mathcal{R}$-odd under the orientifold projection, they do not yield non-vanishing RR tadpoles. Each fractional three-cycle is frozen at four separate fixed points (ij) on $T^4/\mathbb{Z}_2$, such that the D6-brane position moduli along $T^4/\mathbb{Z}_2$ are projected out. These fractional three-cycles thus only retain the position-moduli $\Phi^1_{\alpha=a,b,c}$ along the first two-torus, as indicated in figure 1 by virtue of arrows. Furthermore, the exceptional part for each fractional three-cycle forms a linear combination of four $\mathcal{R}$-odd exceptional basis cycles ε_{ij}, such that the sum of a fractional three-cycle with its orientifold image only wraps bulk three-cycle generators. This consideration implies that the twisted RR tadpoles automatically vanish and drastically simplifies the bulk RR tadpole cancelation conditions (2.2),

$$N_a + N_c = 16, \qquad N_b + N_c = 16, \tag{C.6}$$

which are satisfied for the gauge group choice $\mathrm{SO}(32-2N)_a \times \mathrm{USp}(32-2N)_b \times \mathrm{U}(N)_c$. The $\mathrm{SO}(2N_a)$ enhancement of the a-stack gauge group follows when the a-stack lies on top of the $\Omega\mathcal{R}$-plane along $T^2_{(1)}$, while the $\mathrm{USp}(2N_b)$-enhancement of the b-stack gauge group occurs for a b-stack on top of the $\Omega\mathcal{R}\mathbb{Z}_2$-plane along $T^2_{(1)}$. In case the cycles are pulled away from the O6-planes along $T^2_{(1)}$, the gauge group supported by the three fractional three-cycles corresponds to $\mathrm{U}(16-N)_a \times \mathrm{U}(16-N)_b \times \mathrm{U}(N)_c$. Given that all three fractional cycles are parallel to the $\Omega\mathcal{R}\mathbb{Z}_2$-plane along the first two-torus $T^2_{(1)}$, the K-theory constraints are trivially satisfied.

For this consistent D6-brane configuration, we can now infer the structure of the Kähler potential and superpotential. We therefore introduce local coordinates $z^i = x^i + i\, y^i$ on each two-torus $T^2_{(i)}$ with periodicity $x^i \sim x^i + 1$ and $y^i \sim y^i + 1$ along the basis one-cycles. In first instance, we compute the quantities $(\mathcal{Q}^K_\alpha)_{ij}$ defined in (2.20) by identifying the harmonic one-form $l_s^{-1}\zeta_1 = dx^1 \in \mathcal{H}^1(\Pi^0_\alpha, \mathbb{Z})$ compatible with the D-brane normal deformation $X = \frac{1}{2} l_s \partial_{y^1}$ parallel to π_2 for all fractional three-cycles. An explicit computation for the three

D6-brane stacks then shows:

$$\begin{array}{ll} (\mathcal{Q}_a^0)_{11} = 0, & (\mathcal{Q}_a^1)_{11} = -1, \\ (\mathcal{Q}_b^0)_{11} = 1, & (\mathcal{Q}_b^1)_{11} = 0, \\ (\mathcal{Q}_c^0)_{11} = 1, & (\mathcal{Q}_c^1)_{11} = -1. \end{array} \tag{C.7}$$

The components $(\mathcal{Q}_\alpha^{K=2,3})_{11}$ vanish as the interior products of $\beta^{K=2,3}$ with respect to X^1 vanish, implying that only the complex structure moduli N^0 and N^1 will be redefined by the open string moduli Φ_α^1. The rigidity of the fractional three-cycles along $T^4/\mathbb{Z}_2$ also implies that components $(\mathcal{Q}_\alpha^K)_{ij}$ with $i,j \in \{2,3\}$ vanish. Secondly, we have to compute the quantities $(\eta^0_{\alpha\, a})^i{}_j$ as defined in (2.12). To this end, we introduce the harmonic two-form $l_s^{-2}\rho_\alpha^1 \in \mathcal{H}^2(\Pi_\alpha^0, \mathbb{Z})$ for each cycle $\alpha \in \{a,b,c\}$, such that it forms the Poincaré dual to ζ^1 on Π_α^0. A straightforward computation then reveals:

$$(\eta^0_{a\,1})^1{}_1 = -1, \qquad (\eta^0_{b\,1})^1{}_1 = -1, \qquad (\eta^0_{c\,1})^1{}_1 = -1, \tag{C.8}$$

while all other components vanish. Hence, the full Kähler potential for the bulk moduli is given by:

$$\begin{aligned} K_Q + K_K = &-\log\left(N^0 - \overline{N}^0 - \frac{1}{4}\frac{(\Phi_b^1 - \overline{\Phi}_b^1)^2 + (\Phi_c^1 - \overline{\Phi}_c^1)^2}{T^1 - \overline{T}^1}\right) \\ &-\log\left(N^1 - \overline{N}^1 + \frac{1}{4}\frac{(\Phi_a^1 - \overline{\Phi}_a^1)^2 + (\Phi_c^1 - \overline{\Phi}_c^1)^2}{T^1 - \overline{T}^1}\right) \\ &-\sum_{k=2}^{3}\log\left(N^k - \overline{N}^k\right) - \sum_{i=1}^{3}\log\left(i(T^i - \overline{T}^i)\right), \end{aligned} \tag{C.9}$$

where we used the redefinition (5.17) for the complex structure moduli N'^0 and N'^1. And lastly, in order to determine the open-closed superpotential W_{D6} in (4.6) for this D6-brane configuration we have to calculate the geometric quantities n_{ai}^α defined through:

$$n_{a1}^\alpha = l_s^{-3}\int_{\Pi_\alpha} \omega_a \wedge \zeta_1. \tag{C.10}$$

Keeping in mind the special Lagrangian condition for the fractional three-cycles enables us to pull-back the two-form ω_α with respect to Π_α and to obtain the geometric quantities n_{ai}^α:

$$n_{11}^c = 0, \quad n_{21}^c = 1 = -n_{31}^c, \qquad n_{ai}^\alpha = 0 \quad \alpha \in \{a,b\}\,(\forall\, a,i). \tag{C.11}$$

The full open-closed superpotential W_{D6}, including a D-brane worldvolume flux F supported by the de Rahm dual two-cycles to the two-forms ρ_α^1, thus reads:

$$l_s W_{\mathrm{D6}} = -\Phi_c^1(T^2 - T^3). \tag{C.12}$$

Given the Kähler potential and the open-closed superpotential one can determine the scalar potential explicitly along the lines of section 6. The RR part of the scalar potential V_{RR+CS}

is given by (3.27) with the redefined flux quanta given by:

$$\begin{aligned} \tilde{e}_0 &= e_0 && \\ \tilde{e}_1 &= e_1, & \tilde{m}^1 &= m^1, \\ \tilde{e}_2 &= e_2 - \theta_c^1, & \tilde{m}^2 &= m^2 + \varphi_c^1 \\ \tilde{e}_3 &= e_3 + \theta_c^1, & \tilde{m}^3 &= m^3 - \varphi_c^1. \end{aligned} \tag{C.13}$$

The contributions to the redefined flux quanta $\tilde{m}^a$ can be easily computed using equation (3.23) and the quantities n_{ai}^α and $(\eta^0_{\alpha\, a})^i{}_j$ calculated above. The DBI-part follows from expression (6.16) and the open-closed superpotential (C.12):

$$V_{\text{DBI}} = \frac{e^K}{l_s^2 \kappa_4^2} G^{11}_{\text{D6}_c} \left(T^2 - T^3\right)\left(\overline{T}^2 - \overline{T}^3\right). \tag{C.14}$$

The inverse metrics $G^{11}_{\text{D6}_\alpha}$ on the open string moduli spaces have to be determined for each D6-brane Π_α separately and depend implicitly on the Kähler modulus T^1 and the complex structure moduli N^0 and N^1.

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JHEP09(2016)062

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