Generation-Recombination Noise: The Fundamental Sensitivity Limit for Kinetic Inductance Detectors

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Abstract We present measurements of quasiparticle generation-recombination noise in aluminium Microwave Kinetic Inductance Detectors, the fundamental noise source for these detectors. Both the quasiparticle lifetime and the number of quasiparticles can be determined from the noise spectra. The number of quasiparticles saturates to $10~\mu m^{-3}$ at temperatures below 160 mK, which is shown to limit the quasiparticle lifetime to 4 ms. These numbers lead to a generation-recombination noise limited noise equivalent power (NEP) of 1.5×10^{-19} W/Hz$^{1/2}$. Since $NEP \propto N_{qp}$, lowering the number of remnant quasiparticles will be crucial to improve the sensitivity of these detectors. We show that the readout power now limits the number of quasiparticles and thereby the sensitivity.

1 Introduction

Microwave Kinetic Inductance Detectors [1] (KIDs) are the most promising candidate for future space- and ground based large array imaging and imaging spectroscopy at sub-millimeter wavelengths. Several groups have already successfully demonstrated operation on a telescope [2, 3]. As in any pair breaking detector, the fundamental limit to the sensitivity is governed by random generation and recombination of quasiparticles, which we have recently demonstrated with aluminium microwave resonators [4]. Here we discuss generation-recombination noise in Al KIDs,

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based on the same dataset, showing that this most fundamental noise limit is now reached. We discuss the implications for detector optimisation.

2 Quasiparticle Number Fluctuations

In a superconductor in thermal equilibrium, the density of quasiparticles per unit volume is given by

$$n_{qp} = 2N_0\sqrt{2\pi k_B T \Delta}\exp(-\Delta/k_B T), \tag{1}$$

valid at $k_B T < \Delta$, with N_0 the single spin density of states at the Fermi level ($1.72 \times 10^{10}\,\mu\mathrm{m}^{-3}\,\mathrm{eV}^{-1}$ for Al), k_B Boltzmann's constant, T the temperature and Δ the energy gap of the superconductor. Assuming a thermal distribution of quasiparticles and phonons, the average quasiparticle recombination time is given by [5]

$$\tau_r = \frac{\tau_0}{\sqrt{\pi}}\left(\frac{k_B T_c}{2\Delta}\right)^{5/2}\sqrt{\frac{T_c}{T}}\exp(\Delta/k_B T) = \frac{\tau_0}{n_{qp}}\frac{N_0(k_B T_c)^3}{2\Delta^2}, \tag{2}$$

where T_c is the critical temperature of the superconductor and τ_0 a material dependent, characteristic electron-phonon interaction time.

In thermal equilibrium, the generation and recombination rates are equal and the variance of the random number fluctuations $\sigma^2 = \langle \delta N_{qp}^2 \rangle = N_{qp} = n_{qp}V$, with V the volume of the system. The power spectral density of these fluctuations shows a Lorentzian spectrum, given by [6, 7]

$$S_N(\omega) = \frac{4N_{qp}\tau_r}{1 + (\omega\tau_r)^2}, \tag{3}$$

with ω the angular frequency. Equations (1) and (2) show that the product $N_{qp}\tau_r$, which gives the level of the spectrum, is constant over temperature. One can show [5] that $N_{qp}\tau_r \propto 1/b\Delta^2$, which shows that it is beneficial for observing generation-recombination noise to take a material with low b, indicating the strength of the electron-phonon coupling, and low Δ. As an example, Al has 2–3 orders of magnitude higher $N_{qp}\tau_r$ than Ta and Nb.

3 Microwave Resonator Experiment

We measure the quasiparticle number fluctuations using a high-quality microwave resonator. The high frequency response of the superconductor is controlled by the quasiparticle density through the complex conductivity $\sigma_1 - i\sigma_2$ [8]. Quasiparticle number fluctuations will show up as fluctuations in the complex conductivity. To measure the complex conductivity, a 40 nm thick Al film was patterned into microwave resonators. The film was sputter-deposited onto a C-plane sapphire substrate. The critical temperature is 1.11 K, from which the energy gap is $\Delta = 1.76 k_B T_c = 168\,\mu\mathrm{eV}$. The low temperature resistivity $\rho = 0.8\,\mu\Omega\,\mathrm{cm}$ and the residual resistance
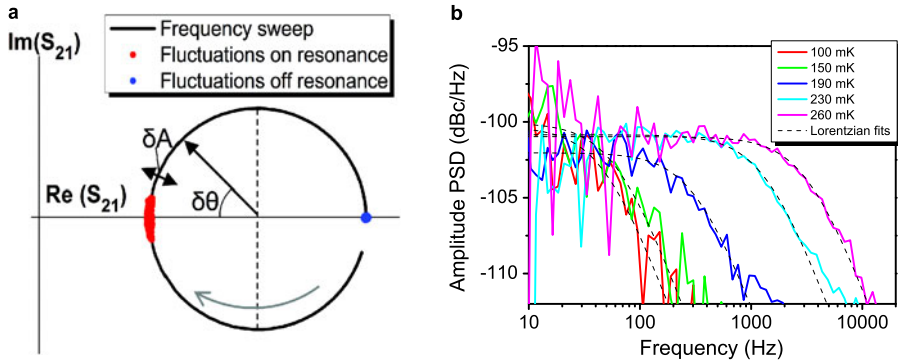
Fig. 1 (Color online) (**a**) The real and imaginary parts of the complex transmission S_{21} as a function of frequency. The *gray arrow* indicates increasing frequency. A resonator amplitude A and phase θ are defined with respect to the resonance circle centre. (**b**) Power spectral density of the resonator amplitude fluctuations as a function of frequency for six different temperatures, corrected for system noise [4]. The *dashed lines* are Lorentzian fits to the spectra

ratio RRR = 5.2. The film was patterned by wet etching into distributed, half wavelength, coplanar waveguide resonators, with a defined central line width of 3.0 μm and gaps of 2.0 μm wide. The resonator under consideration shows its lowest order resonance at 6.61924 GHz and has a central strip volume of $1.0 \cdot 10^3$ μm^3. The resonance curve at 100 mK shows a coupling limited quality factor of 3.87×10^4. The samples are cooled in a pulse tube pre-cooled adiabatic demagnetization refrigerator. Special care has been taken to make the setup light tight [4, 9], presented in detail in an accompanying paper [10].

The complex transmission of the microwave circuit is measured with a quadrature mixer and traces out a circle in the complex plane [1, 11]. We define a resonator amplitude and phase with respect to the resonance circle, as depicted in Fig. 1a. The amplitude responsivity to quasiparticles, dA/dN_{qp}, was determined experimentally [11]. For similar resonators it is known that the sensitivity in phase is limited by two-level fluctuators [12] and that the sensitivity in amplitude, which is limited by the cryogenic low noise amplifier, is up to a factor 10 better [11].

The power spectral density due to quasiparticle number fluctuations in the resonator amplitude (also called dissipation quadrature) is given by

$$S_A(\omega) = S_N(\omega) \frac{(dA/dN_{qp})^2}{1 + \omega^2 \tau_{res}^2}, \tag{4}$$

where τ_{res} is the resonator ringtime given by $\tau_{res} = \frac{Q}{\pi f_0}$. In this experiment $\tau_r \gg \tau_{res} \approx 2$ μs, meaning that the roll-off in the noise spectrum will be determined solely by τ_r if S_A is dominated by quasiparticle number fluctuations.

We have measured the fluctuations in the resonator amplitude in equilibrium at the resonant frequency using a microwave power of -77 dBm (20 pW). Measurements at this power serve as an example, the power dependence is shown later on. The power spectral density is shown in Fig. 1b for various temperatures. We have corrected the power spectral density for system noise contributions by subtracting a spectrum taken
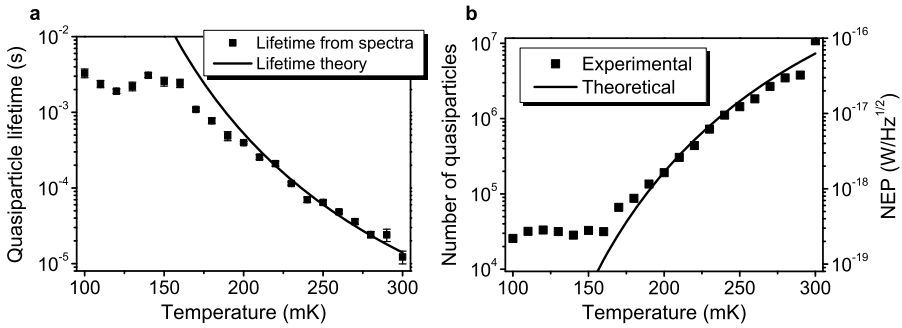
Fig. 2 (**a**) The quasiparticle lifetime as a function of temperature, obtained from the roll-off frequency in the resonator amplitude spectrum. The *solid line* represents the lifetime calculated from theory. (**b**) The measured N_{qp} and corresponding NEP due to generation-recombination noise as a function of temperature. The *solid line* shows the theoretical expectation

off-resonance (see Fig. 1a). Before calculating the spectrum, the time-domain trace was filtered for large energy impacts [4]. We observe that the power spectral density has a constant level for all temperatures, with a roll-off frequency that increases with temperature. From these two properties we conclude that we directly observe quasiparticle generation-recombination noise.

The measured recombination time, τ_r, is extracted from the resonator amplitude noise spectra and shown in Fig. 2a as a function of temperature. At temperatures from 180–300 mK, we find the expected exponential temperature dependence. Equation (2) is used to fit for the characteristic electron-phonon interaction time τ_0 and we find a value of 458 ± 10 ns. At temperatures <160 mK we measure a temperature independent quasiparticle recombination time of 2.2 ms.

The measured level of the power spectral density and the quasiparticle lifetime are combined to obtain N_{qp}, using (3) and (4) together with the measured dA/dN_{qp}. The result is plotted in Fig. 2b. For temperatures above 160 mK the expected exponential temperature dependence is found. Below 160 mK, the number of quasiparticles saturates to around 30,000 (30 μm^{-3}). The saturation in the quasiparticle lifetime (Fig. 2a) is consistent with the saturation in N_{qp}, from which we conclude that the saturation in τ_r, observed earlier [13, 14], is here due to a saturation in N_{qp}, consistent with calculations for superconducting qubits [15].

The noise spectra are converted into noise equivalent power (NEP) through [11]

$$NEP(\omega) = \sqrt{S_A} \left(\frac{\eta \tau_r}{\Delta} \frac{\delta A}{\delta N_{qp}} \right)^{-1} \sqrt{1 + \omega^2 \tau_r^2} \sqrt{1 + \omega^2 \tau_{res}^2}, \qquad (5)$$

with $\eta = 0.57$ an efficiency factor. The thus calculated electrical NEP at 100 mK is plotted in Fig. 3a. We find that the generation-recombination noise contribution to the NEP is 2×10^{-19} W/Hz$^{1/2}$, which leads to $NEP = 3 \times 10^{-19}$ W/Hz$^{1/2}$ including system noise. We plot the low-frequency NEP as a function of temperature in Fig. 2b (right axis), in the same plot as N_{qp}, which shows the measurements are consistent
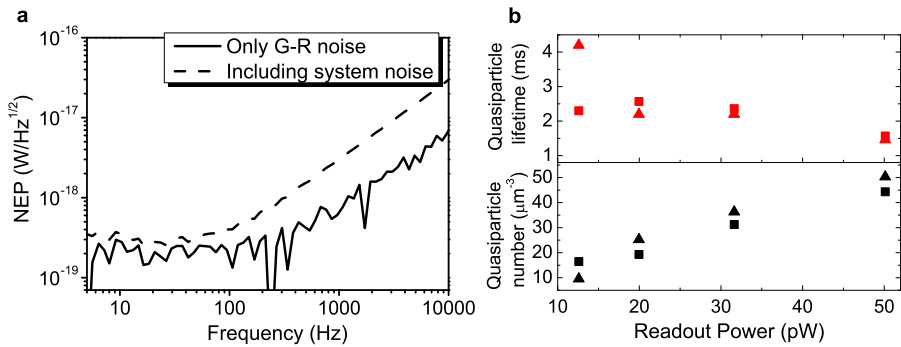
Fig. 3 (Color online) (**a**) The measured Noise Equivalent Power as a function of frequency at 100 mK. The *dashed curve* shows the NEP without corrections, the *solid line* shows the NEP for the noise spectrum corrected for system noise, giving the pure generation-recombination noise contribution to the NEP. (**b**) Number of quasiparticles and quasiparticle lifetime at the lowest temperatures as a function of microwave readout power, extracted from both the amplitude power spectral density (*squares*) and the amplitude-phase cross-PSD (*triangles*)

with the theoretically expected generation-recombination limited NEP:

$$NEP_{g-r}(\omega) = \frac{2\Delta}{\eta} \sqrt{\frac{N_{qp}}{\tau_r}}. \tag{6}$$

We have now reached the fundamental noise limit for KIDs, but the NEP can still be reduced. The observation that the quasiparticle lifetime is limited by the quasiparticle density reduces (6) to $NEP_{g-r} \propto N_{qp}$, thus to reduce *NEP*, we have to reduce N_{qp}. As discussed earlier, we have thoroughly excluded stray light, proven in this experiment by the fact that we see no extra photon-noise on top of the generation-recombination noise. We also exclude large energy impacts visible in the time-domain data [4]. Our most recent data shows that N_{qp} and τ_r are now limited by the microwave readout power, as is shown in Fig. 3b. We find a close to linear dependence of N_{qp} on readout power. At the lowest power $n_{qp} = 10$ μm^{-3} and $\tau_r = 4$ ms, leading to $NEP \approx 1.5 \times 10^{-19}$ W/Hz$^{1/2}$. Although this particular set of (preliminary) results might be explained by a simple heating model [16], we emphasise that on a microscopic level a more rigorous treatment will be needed, since the microwave field will significantly alter the quasiparticle distribution function [17, 18], which we hope to investigate deeper. Several other groups have shown remnant quasiparticle densities lower than 1 μm^{-3} in experiments with qubits [19, 20] and single-electron islands [21] using aluminium, which would make $NEP \approx 1 \times 10^{-20}$ W/Hz$^{1/2}$ possible.

We have recently shown [22] that if a detector in the dark is limited by generation-recombination noise, it will necessarily be limited by photon-noise under illumination, bringing two fundamental limits within reach at once.

4 Conclusions

We have measured generation-recombination noise in Al KIDs. Both the quasiparticle lifetime and the number of quasiparticles can be determined from the noise

spectra. The number of quasiparticles saturates to 10 μm^{-3} at temperatures below 160 mK at the lowest microwave power. We show that the quasiparticle lifetime is limited by that number of quasiparticles to 4 ms. These numbers lead to a generation-recombination noise limited NEP of 1.5×10^{-19} W/Hz$^{1/2}$, which can be lowered by decreasing the quasiparticle density and the volume.

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