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The modulated annual cycle: an alternative reference frame for climate anomalies

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Abstract In climate science, an anomaly is the deviation of a quantity from its annual cycle. There are many ways to define annual cycle. Traditionally, this annual cycle is taken to be an exact repeat of itself year after year. This stationary annual cycle may not reflect well the intrinsic nonlinearity of the climate system, especially under external forcing. In this paper, we re-examine the reference frame for anomalies by re-examining the annual cycle. We propose an alternative reference frame for climate anomalies, the modulated annual cycle (MAC) that allows the annual cycle to change from year to year, for defining anomalies. In order for this alternative reference frame to be useful, we need to be able to define the instantaneous annual cycle: we therefore also introduce a new method to extract the MAC from climatic data. In the presence of a MAC, modulated in both amplitude and frequency, we can then define an alternative version of an anomaly, this time with respect to the instantaneous MAC rather than a permanent and unchanging AC. Based on this alternative definition of anomalies, we re-examine some familiar physical processes: in particular SST re-emergence and ENSO phase locking to the annual cycle. We find that the re-emergence mechanism may be alternatively interpreted as an explanation of the change of the annual cycle instead of an explanation of the interannual to interdecadal persistence of SST anomalies. We also find that the ENSO phase locking can largely be attributed to the residual annual cycle (the difference of the MAC and the corresponding traditional annual cycle) contained in the traditional anomaly, and, therefore, can be alternatively interpreted as a part of the annual cycle phase locked to the annual cycle itself. In addition to the examples of reinterpretation of physics of well known climate phenomena, we also present an example of the implications of using a MAC against which to define anomalies. We show that using MAC as a reference framework for anomaly can bypass the difficulty brought by concepts such as "decadal variability of summer (or winter) climate" for understanding the low-frequency variability of the climate system. The concept of an amplitude and frequency modulated annual cycle, a method to extract it, and its implications for the interpretation of physical processes, all may contribute potentially to a more consistent and fruitful way of examining past and future climate variability and change.

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Decadal climate variability of winter (summer) temperature · Nonlinear non-stationary time series · Empirical mode decomposition · Ensemble empirical mode decomposition

1 Introduction

In the past few decades, a set of major research foci in climate studies have been climate variability of interannual (or longer) timescales and climate change (NRC 1996, 1998; IPCC 2001, and the references therein). The starting point of the derived variability and change of climate is the climate anomaly, which is defined as the departure of an individual climate quantity during a particular period from the climatology of that quantity, the characteristic or typical value that is representative of the majority of cases over the period of several decades or more (American Meteorological Society 2000).

This concept of climate anomaly seems so self-evident that most climatologists take it for granted and few bother to examine it. For example, commonly in the climate literature, a monthly anomaly is derived from the deviation of monthly averaged climate quantities, such as temperature, pressure, wind speed, etc., from their long term traditional annual cycles (TAC), which is defined as the component of variability that is a function of the month of the year but is independent of the year (American Meteorological Society 2000). In such a definition of anomaly, an implicit a priori assumption is that a set of stationary processes produce a constant annual cycle (AC). Since the validity of this assumption is often questionable in the face of the response of a nonlinear climate system to external forcing, the anomaly so defined may not be physically adequate, and consequently, the proposed physical mechanisms based on the analysis of the anomaly that are used to explain the underlying physical processes generating such anomaly may be misleading or even erroneous. In addition, due to possible long term climate change, the reference frame that is used to define the monthly anomalies should also include the effect of these slow climate changes. Fedorov and Philander (2000) provides an example in which the SST anomaly can be better physically explained when it is defined as the deviation from its TAC and the component of decadal change combined. The definition of an anomaly should take into account the nonlinearity and non-stationarity of the reference climatology since the climate is changing.

In this study, we will explore two related issues. The first is to reexamine the concept of the annual cycle of a nonlinear, non-stationary climate system, to propose a definition of a modulated annual cycle (MAC), an annual cycle that can be modulated in both amplitude and frequency, and to introduce a method for extracting the MAC from climate data. The proposition of MAC is not completely physically based; rather it is based on the general properties of nonlinear dynamical systems in response to periodic forcing. The reason for us to use the term "modulated annual cycle", rather than use terms such as "nonlinear non-stationary annual cycle" (Shen et al. 2005) and "changing seasonal cycle" (Pezzulli et al. 2005), is to emphasize the changing characteristics of the annual cycle (in amplitude and frequency) from a data analysis perspective, rather than the physical and mathematical reasons for these changes which are often hard to evaluate quantitatively.

The second issue we want to explore is related to the reference frame for climate anomalies. Numerous papers on climate sciences using monthly anomalies with respect to their TACs have been published, and physical mechanisms have been proposed to explain these climate anomalies, especially their interannual or longer timescale variability. However, in many cases, the mechanisms proposed have been tailored to fit the anomalous data without questioning the rationale of the definition of anomalous data. In some of these cases, the seemingly well fitted mechanism for the early part of data may not work well for the later data for reasons that may have more to do with the annual cycle than with the process of interest itself. An illustrative example is the phase locking of ENSO to the annual cycle. The phases of ENSO before 1980 analyzed by Rasmusson and Carpenter (1982) indicated the locking. However, most of the later ENSO events seem not to lock to the annual cycle well (Torrence and Webster 1998). One possibility for this inconsistency is that the physics of ENSO changed. Another possibility is that the physics of ENSO did not change, but the AC is changing. In the latter case, it would be the reference frame which differs in the earlier and later parts of the record.

The paper is arranged as the follows: In Sect. 2, the physical origin of the AC and the traditional methods to define the AC, and their drawbacks, are discussed. This will lead to an argument that the annual cycle should be defined adaptively. Section 3 will introduce an adaptive method to extract the MAC. In Sect. 4, examples of using the MAC as an alternative reference frame for climate anomalies will be presented, which will demonstrate the usefulness of the MAC. A summary and discussion will be presented in the final section.

2 Definition of the modulated annual cycle

In atmospheric and climate sciences, there are several well known cycles, such as the diurnal cycle related to the change of local solar radiation associated with the Earth's daily rotation and yearly revolution around the Sun; the solar cycle related to the eleven year variation of sunspots,

and the Milankovitch cycle in paleoclimatology related to the long term variation of solar radiation reaching the position of the earth. In most cases, the "cycle" refers to an interval during which a recurring sequence of events occurs, although the recurrence may not necessary be exact. The most popular use of "cycle" in atmospheric and climate sciences is annual cycle or seasonal cycle[1] of a climate variable, a reference frame for the definition of anomalies.

The annual cycle is often assumed invariant from year to year, with physical justification that the solar radiation incident at the top of the atmosphere repeats year after year almost perfectly at any location of the Earth, at least on human time scales. However, we are mostly concerned with the variables (such as temperature and pressure) characterizing the *response* of the climate system to such periodic forcing. Therefore, the concept of annual cycle should relate the response of the nonlinear climate system to the periodic incoming solar radiation and it is most likely that nonlinearities prevent the response from being perfectly periodic.

A convenient example is the Lorenz model (1963), which has been widely used to illustrate many important concepts in atmospheric and climate sciences, forced by a periodic forcing, as described in the following equations:

$$\frac{dx_1}{dt} = s(-x_1 + x_2) + a \sin\left(\frac{2\pi t}{T}\right), \tag{1a}$$

$$\frac{dx_2}{dt} = rx_1 - x_2 - x_1 x_3, \tag{1b}$$

$$\frac{dx_3}{dt} = -bx_3 + x_1 x_2, \tag{1c}$$

where s, r, and b are standard parameters of the Lorenz model, a is the amplitude of the forcing term, and T is the period of forcing corresponding to a 1 year period. Figure 1 shows the solution x_1 with specified parameters. Although the solution of x_1 is noisy (caused by nonlinear chaotic behavior of the system), a visual inspection of this solution leads to the impression that the annual cycle is modulated in both amplitude and frequency along the time axis. Indeed, a traditionally fitted annual cycle using a relatively narrow band-pass filter centered at a 1 year period or an adaptively fitted one using the EEMD method (Wu and Huang 2005b, 2008) further confirms the presence of these modulations. It should be noted here that the properties of the solution shown in this example are not special to this particular nonlinear system—rather, this type of modulation is present in nonlinear systems generally. Therefore, if the nonlinearity of the climate system is recognized, the

[1] According to Pezzulli et al. (2005), Trenberth pointed out that "seasonal cycle" may not be an accurate terminology for a cycle that repeats yearly. We certainly agree with his argument, and hence, from now on, we will only use "annual cycle" in this context.
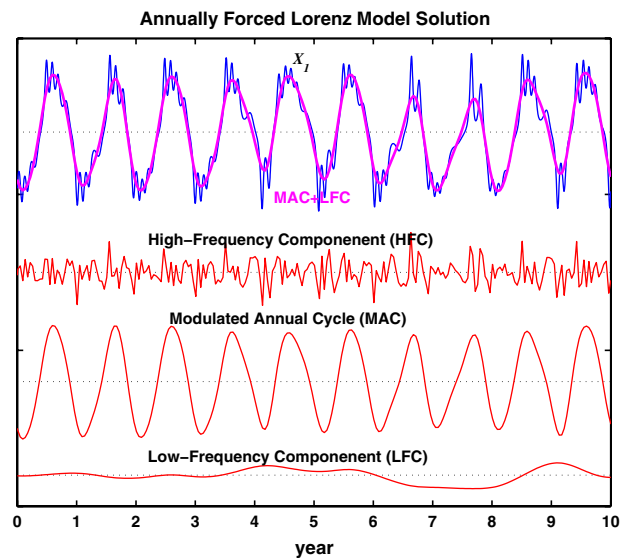

Fig. 1 x_1 from the forced Lorenz model (*blue line*) with parameters s = 10, r = 25.5, b = 8/3, and a = 150. The decomposition of x_1 into a high-frequency component, a modulated annual cycle, and a low frequency component are plotted as the *top red line*, the *middle red line*, and the *lower red line*, respectively. The *bold magenta line* overlapping with x_1 is the sum of the modulated annual cycle and the low-frequency component

MAC should be more general than the TAC; at least, the former allows for the possibility of change and the latter is a special case of the former. Indeed, the MAC of the climate system, which is the response of the climate system to the external forcing, reflects the internal complexity of the climate system as well as the forcing.

The concept of the MAC is not new (although perhaps the nomenclature is). It is not uncommon to experience an unusually cold winter followed by a very hot summer and another cold winter, all of which comprises an instantaneously stronger AC than usual. That the AC may vary from year to year implies that the response of the Earth's climate system to the solar forcing may be neither linear nor stationary.

The recognition of the MAC in climate studies is also not new. Previous work related to the MAC is well summarized in a recent study of the variability of seasonality by Pezzulli et al. (2005). The work summarized includes the temporal change of the AC and its spatial unevenness, as well as possible causal mechanisms. For example, Gu et al. (1997) document the change of annual cycle of the sea surface temperature (SST) at 140 W on the Equator. Van Loon et al. (1993) document that the changing ACs of pressure and wind in the Southern Hemisphere have zonally asymmetric structures. The MAC was even used to explain the phase locking of ENSO to the annual cycle in a simple model (Xie 1995), which is closely related to our later reexamination of ENSO and annual cycle phase-locking based on the analysis of observational data.

While the concept of the MAC is simple, defining the MAC of a particular variable from data is difficult. Since the climate system is nonlinear and may have internal chaotic behavior, it is impossible to have a rigorous definition of MAC with a priori determined characteristics. Therefore, the extraction of MAC will be data-oriented, based on some of the assumed general properties of responses of nonlinear systems to periodic forcing. The preliminary verification of the appropriateness of the MAC can then be carried out by borrowing Drazin's first step of data analysis (Drazin 1992): examining the decomposition visually. With the above caveats, we are now ready to discuss the annual cycle from a data analysis viewpoint.

2.1 Extrinsic and pre-determined annual cycle

The simple TAC is defined as the component of long term variability that is a function of the day (perhaps month) of the year but is independent of the year (American Meteorological Society 2000). The TAC of a climate variable is obtained by averaging values of that variable of the same day (month) of different years over a long period. The TAC would then include not only the cycle of annual timescale, but also its sub-harmonics, for example, semianuual cycle, 4-, 3-, and 2-month cycle, for monthly data. Such an annual cycle may not reflect the actual annual cycle in nature. When the issue is interannual or longer timescale variability, defining anomalies with respect to the TAC often distorts the anomaly. To illustrate the point, take an idealized model of a climate variable to be the following:

$$V(t) = A\left[1 + a\cos\left(b\frac{2\pi t}{T}\right)\right]\cos\left(\frac{2\pi t}{T} + \varphi\right) + C(t) + N(t), \tag{2}$$

where A is a positive constant, a and b are positive values significantly smaller than 1, T is the length of year, φ is the initial phase of the annual cycle, $C(t)$ is the climate variability and change of interannual or longer timescales, and $N(t)$ is related to the weather noise. The term inside the square brackets is then the annual cycle amplitude modulation. For convenience, we will refer to the model described by Eq. (2) as model (2) in the later context. Figure 2 plots one realization of model (2) when $C(t)$ is set to zero.

If the annual cycle is taken as the TAC, anomalies with respect to it are:

$$V'(t) = Aa\cos\left(b\frac{2\pi t}{T}\right)\cos\left(\frac{2\pi t}{T} + \varphi\right) + C(t) + N(t). \tag{3}$$

The first term of the RHS of (3), which can be reasonably interpreted as part of the annual cycle, is then attributed to the anomaly.
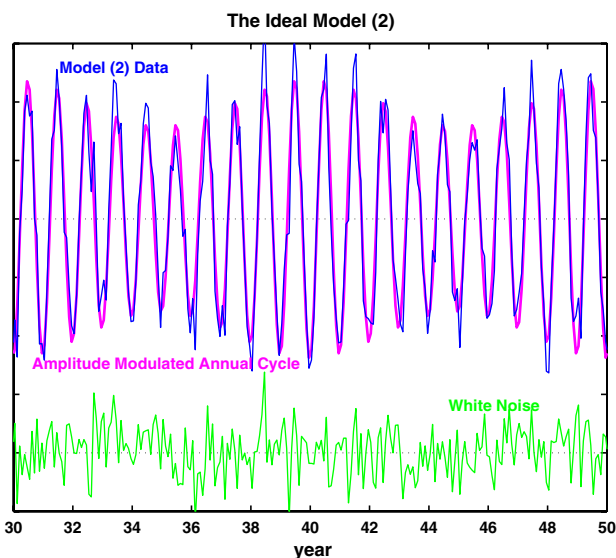

Fig. 2 The synthetic climate model output (*blue line*) and their components: the prescribed amplitude modulated annual cycle (*the magenta line*) and white noise (*lower green line*). In the model, $A = 1$, $a = 0.2$, $b = 0.1$, and $\varphi = 13\pi/12$

Another commonly used AC is the running mean of the inputted data directly (e.g., Gu et al. 1997; Pezzulli et al. 2005), or a running mean of TAC over a relatively long period (e.g., 30 years) (Kirtman et al. 2002). The pre-determined timescale needed to define the running mean has little rational basis, since non-stationary processes have no a priori local timescale. More complicated methods of extracting an AC, such as Fourier-based filtering (Bacastow et al. 1985; Thoning and Tans 1989), are also widely used in climate studies, especially in the description of the global carbon cycle. For the Fourier component based annual cycles, the stationarity is naturally inherited from Fourier transform. In addition, the band of the Fourier components needs to be subjectively determined.

2.2 Intrinsic and adaptive annual cycle

The definitions of the AC and the algorithms for extracting the AC discussed above generally involve prescribed parameters or functions, which are extrinsic and subjective. To overcome the aforementioned drawbacks, we address the problem of how to determine the AC for data from non-stationary and nonlinear processes without relying on extrinsic functions or simplifying assumptions.

Before proceeding further, we note five points related to the AC of a climate variable:

1. For the nonlinear, non-stationary climate system, the AC of a climate variable can never be defined unambiguously, since it is an inseparable part of the system's evolution. For the reason, there are many

ways to define annual cycle, such as those mentioned in the previous subsection.

2. The extrinsic annual cycles discussed earlier are all subjectively defined. There is no particular plausible reason to support the use of any of these extrinsic annual cycles other than convenience.

3. The AC of a data record should be an intrinsic property of the data. Since in the climate system, the interaction between different timescales is not negligible, and the decomposition of data in terms of components of subjectively determined timescales may not catch the physical essence. A more appropriate approach is to consider the AC to be intrinsic to the nonlinear climate system forced by external forcing. Being intrinsic requires that the method used in defining AC be adaptive, so that the AC extracted is derived from and based on the data, maximizing the separation of AC from variability of other timescales.

4. The AC should be a temporally local quantity. Indeed, temporal locality should be the first principle to guide all the time series analysis, even for a purely periodic time series in which a short piece of data contains all the necessary information to describe the time series. This requirement reflects the evolution of time series analysis: from the Fourier transform to the windowed Fourier transform (Gabor 1946) and to wavelet analysis (Daubechies 1992). *The general idea behind this is that the latter evolution cannot change the reality that happened earlier; and the corresponding physical interpretation should also not change with the addition of new data.* With this requirement satisfied, the extracted AC for a particular temporal location (a particular year) would not change when new data of later years are added and the same extraction method is applied to the data with the new addition. Therefore, the TAC, at least logically, is not the best quantity to describe a quasi-periodic phenomenon, as the idealized model (2) indicates.

5. Fifth, the AC should allow amplitude and frequency modulation, as the forced Lorenz model demonstrated, since both the amplitude and the frequency of a periodically forced nonlinear system is not necessary periodic. This property can indeed be easily demonstrated for any given nonlinear differential equation under periodic forcing.

With these considerations, we take the AC of climate data as the amplitude and frequency modulated annual cycle, which is *an adaptively and temporally locally determined intrinsic component of the climate data that contains frequency and amplitude modulations and has quasi-annual period.* To determine such an AC from data, the previous well-known "adaptive" methods using the autocorrelation matrices, such as cyclostationary empirical orthogonal functions (Kim et al. 1996) and singular spectrum analysis (Ghil et al. 2002), have limitations in satisfying the temporal locality requirement, since the calculation of autocorrelations uses the whole data and has an implicit stationarity assumption. This drawback can easily be demonstrated with a time series of different statistical properties in its first and second halves. For such a time series, the analysis of the first half and the analysis of the whole data using an autocorrelation-based method would lead to different results for the first half.

In this study, we will take advantage of recent advances in adaptive and temporally local time series analysis to analyze MAC, particularly, the empirical mode decomposition (Huang et al. 1998, 1999, 2003; Flandrin et al. 2004; Wu and Huang 2004, 2005a; Wu et al. 2007; Huang and Wu 2008) and its most recent improvement, the Ensemble EMD (Wu and Huang 2005b, 2008).

3 Extracting MAC using the ensemble EMD

3.1 The empirical mode decomposition (EMD) and the ensemble EMD (EEMD)

The detailed description of the EMD method can be found in Huang et al. (1998, 1999). In contrast to most previous methods of data analysis, the EMD method is adaptive and temporally local, with the basis of the decomposition derived from the data. In the EMD, the data $x(t)$ is decomposed in terms of the intrinsic mode functions (IMFs), c_j, that is,

$$x(t) = \sum_{j=1}^{n} c_j(t) + r_n(t), \tag{4a}$$

where

$$c_j(t) = a_j(t) \cos\left[\int \omega_j(t) dt\right], \tag{4b}$$

and r_n is the residual of the data $x(t)$, after n intrinsic mode functions (IMFs) are extracted. The IMFs expressed in Eq. (4b) are simple oscillatory functions with relatively slowly varying and non-negative amplitude and relatively fast changing and non-negative frequency at any temporal location.

In practice, the EMD is implemented through a sifting process that uses only local extrema. From any data r_{j-1}, say, the procedure is as follows: (1) identify all the local extrema (the combination of both maxima and minima) and connect all these local maxima (minima) with a cubic spline as the upper (lower) envelope; (2) obtain the first component h by taking the difference between the data and

the local mean of the two envelopes; and (3) treat h as the data and repeat steps 1 and 2 as many times as is required until the envelopes are symmetric with respect to zero mean under certain criteria. The final h is designated as c_j. A complete sifting process stops when the residue, r_n, becomes a monotonic function or a function only containing one internal extremum from which no more IMF can be extracted.

The EMD has some unique properties. It is a temporally local analysis method (Huang and Wu 2008; Wu and Huang 2008). It behaves as a dyadic filter bank[2] in which the Fourier spectra of various IMFs collapse to a single shape along the axis of logarithm of period or frequency (Flandrin et al. 2004; Wu and Huang 2004, 2005a). For a time series with length N, the EMD usually results in close to but no more than $\log_2 N$ components. Therefore, it is also a sparse decomposition method, naturally improving the efficiency of representing signals in data.

While the EMD has these wonderful properties, it also has a serious "mode mixing" problem, which is defined as any IMF consisting of oscillations of dramatically disparate scales. An annoying implication of such mode mixing is related to unstableness and lack of the "physical uniqueness" of the decomposition using the EMD. Since real data almost always contain a certain amount of random noise or intermittences that are not known to us, an important issue, therefore, is whether the decomposition is sensitive to noise. If the decomposition is insensitive to added noise of small but finite amplitude and bears little quantitative and no qualitative change, the decomposition is generally considered stable and satisfies the physical uniqueness; and otherwise, the decomposition is unstable and does not satisfy the physical uniqueness. The result from decomposition that does not satisfy the physical uniqueness may not be reliable and may not be suitable for physical interpretation. Unfortunately, the EMD in general does not satisfy this requirement due to the decomposition is solely based on the distribution of extrema.

To solve this problem, the Ensemble EMD (EEMD) was developed (Wu and Huang 2005b, 2008). In this method, counter-intuitively, multiple noise realizations are added to one time series of observations to mimic a scenario of multiple trials of observation for a single trial of observation so as to carry out ensemble average approach for corresponding IMFs and extract scale-consistent signals. The major steps of the EEMD method is as the following:

1. add a white noise series to the targeted data;
2. decompose the data with added white noise into IMFs;
3. repeat step 1 and step 2 again and again, but with different white noise series each time; and
4. obtain the (ensemble) means of corresponding IMFs of the decompositions as the final result.

The effects of the decomposition using the EEMD are that the added white noise series cancel each other, and the mean IMFs stays within the natural dyadic filter windows as discussed in Flandrin et al. (2004) and Wu and Huang (2004, 2005a), significantly reducing the chance of mode mixing and preserving the dyadic property. The role of noise in the EEMD is similar to the catalyst in a chemical reaction, not taking place in the reaction but making it easier to accomplish. Therefore, the EEMD is a truly *noise-assisted data analysis* method and keeps the "physical uniqueness". Since the EEMD is based on the EMD method, the adaptiveness and the temporal locality are preserved, making it an ideal method consistent with the temporal locality requirement of MAC that was discussed in previous secion.

3.2 Extracting MAC using the EEMD

As pointed out by Wu and Huang (2005b, 2008), the EEMD components are not necessarily IMFs, i.e, they may not be expressed in terms of simple waves as an IMF by Eq. (4b). Therefore, further processing of the EEMD components is necessary to obtain a simple wave form of the AC.

For monthly climate data (such as wind, temperature, pressure), the first component of an EEMD decomposition is almost always a high frequency component, and the second and the third components often have similar numbers of extremas which are phase locked. As the EMD is a dyadic filter bank with its Fourier spectrum bands tailed off and neighboring bands having Fourier spectrum overlapped at spectrum tail regions (Flandrin et al. 2004; Wu and Huang 2004, 2005a), the second component is almost all from the annual cycle and the third component contains some annual cycle as well as some longer timescale variation. To obtain a relatively narrow band annual cycle, the components two and three are combined and then subjected to a single EMD decomposition. The combined component has well defined maximums (minimums) with their neighboring maximums having a distance of about 12 month. In such a case, an additional single EMD decomposition gives an amplitude-frequency modulated AC, as expressed in Eq. (4b), and a lower frequency component.

The MAC of model (2) using the above approach is displayed in Fig. 3. Clearly from visual inspection, the MAC catches the prescribed annual cycle (ANNU) very

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[2] A dyadic filter bank is a collection of band pass filters that have a constant band pass shape (e.g., a Gaussian distribution) but with neighboring filters covering half or double of the frequency range of any single filter in the bank. The frequency ranges of the filters can be overlapped. For example, a simple dyadic filter bank can include filters covering frequency windows such as 50–120, 100–240, 200–480 Hz, and et al.
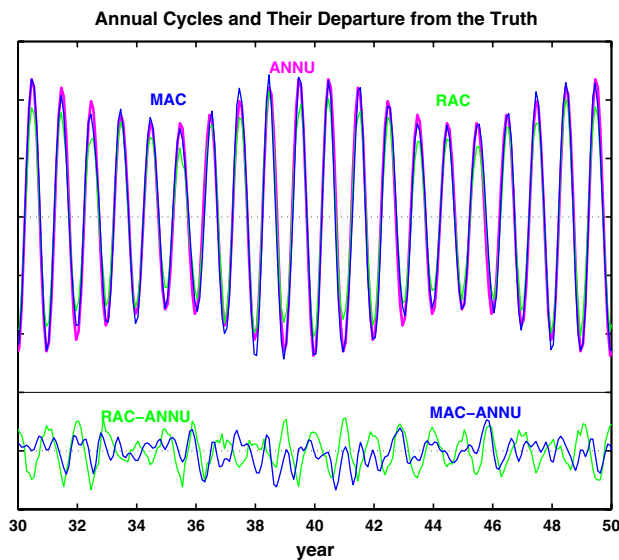
Annual Cycles and Their Departure from the Truth

Fig. 3 The amplitude–frequency modulated annual cycle (*MAC, the top blue line*) extracted using the EEMD and its departure (*the bottom blue line*) from the prescribed amplitude modulated annual cycle (*ANNU, magenta line*). The corresponding green lines are the running mean annual cycle (*RAC, the top green line*) and its departure (*the bottom red line*) from the prescribed ANNU

well. In some years, the MAC can either slightly underestimate or overestimate the strength of the annual cycle. The correlation between the ANNU and the difference of the MAC and the ANNU, which we call error, is only −0.1064 for the entire 100 years (1,200 data points) of data, essentially implying that this error is close to random). The anomaly with respect to the MAC is also close to random and has a 1-month-lag autocorrelation of −0.1086. This anomaly catches the true anomaly, which is the specified noise, very well: its correlation with the added noise is 0.82, and its standard deviation is 0.1727, a value slightly smaller than the standard deviation of the true anomaly. It should be noted here that the smaller deviation of 0.1727 is not really a drawback of the method. If we consider the data from model (2) as something being observed in the real case, the true MAC is unknown. The part of the anomaly that behaves like the true MAC is naturally being considered as part of the true MAC. It can be verified that if we indeed have multiple observation of data sets, mimicked by multiple one hundred year long time series with the same true MAC but different realizations of noise, the true MAC can be recovered by taking the average of the MACs corresponding to each time series. In this sense, the slight underestimation of the anomaly standard deviation is not the problem of the method but is caused by the lack of multiple observations.

To demonstrate the advantage of the MAC over annual cycles defined using other methods, the TAC and the running mean annual cycle, the RAC, which is defined in a

way identical to the method used in Pezzulli et al. (2005), are obtained. The comparisons of the related quantities are displayed in Table 1. The TAC does not contain any amplitude modulation and the anomaly defined with respect to it has much larger standard deviation (0.2283). The small correlation between the ANNU and the difference of the TAC and the ANNU [expressed by Eq. (3)] is a little bit misleading. If correlation between two is calculated for every 5-year episode after dividing the whole length into twenty 5-year episodes, the correlation coefficients regularly jump between two larger values (around 0.5 and −0.5). Therefore, the small coefficient is about the average of the two relatively large values and does not reflect the regularity of the residual annual cycle. This characteristic is also echoed by the relatively large standard deviation of the anomaly with respect to the TAC. The RAC catches the annual cycle amplitude change to a large degree. However, it systematically underestimates the strength of the annual cycle. This is understandable; the running mean has a tendency to smooth the peaks and fast changes. Therefore, the correlation of the ANNU and the difference of the RAC and the ANNU has a large (absolute) value; the standard deviation of the anomaly with respect to RAC is large; and the 1-month-lag autocorrelation of this anomaly is also large.

It should be noted here that the MAC obtained using the method outlined above is totally based on local timescales. Some annually occurring events, such as Indian Ocean dipole (Saji et al. 1999; Xie et al. 2002) peaks exclusively during September–November, are not interpreted as contributing to the annual cycle; rather they are interpreted as high frequency events with their amplitudes being close to zero between neighboring events. Figure 4 presents such an example. In Fig. 4, the SST (Rayner et al. 1996) at (15°W, 60 N) is decomposed in terms of the high frequency part, the MAC, and the low frequency part (including trend). It is clear that the high frequency part consists of mostly the semi-annual oscillations (contaminated by higher frequency components). The MAC has a larger amplitude from 1974 to 1979 than the other years in that decade.

It should also be noted here that the amplitude modulation of the annual cycle of a climate variable is often comparable to the amplitude of the low frequency component (the sum of all the variability on timescales longer than the annual) of the same climate variable. In the example shown in Fig. 4, for the whole second half of the twenty century, the component of interannual or longer timescales has a value range from 9.8 to 11.4°C (a 1.6 C difference), while the amplitude of the MAC changes within a range from from 1.4 to 2.1 C, which corresponds to a change range of 1.4 C for the summer–winter temperature difference. Our examinations of surface

Table 1 Comparisons of different annual cycles and anomalies with respect to them

	ANNU	TAC	MAC	RAC
STD of the Anomaly wrt. an_AC	0.2000	0.2283	0.1727	0.2409
One-month-lag autocorrelation of the anomaly wrt. an_AC	0.0319	0.2150	−0.1086	0.2440
STD of the difference of an_AC and ANNU	0.0000	0.1000	0.1125	0.1621
Correlation of ANNU and the difference of an_AC and ANNU	0.0000	−0.1400	−0.1064	−0.8166

'an_AC' represents an annual cycle among the prescribed annual cycle (ANNU), the traditional annual cycle (TAC), the amplitude-frequency modulated annual cycle (MAC), and the annual cycle calculated from running means (RAC). "STD" represents the standard deviation. The first row of values are standard deviations of anomalies with respect to different annual cycles; the second row of values are 1-month-lag auto-correlations for different anomalies; The third row of values are the standard deviation of errors in annual cycle with respect to different annual cycles; and the fourth row of values are the correlation between true annual cycle and errors in annual cycle with respect to different annual cycle
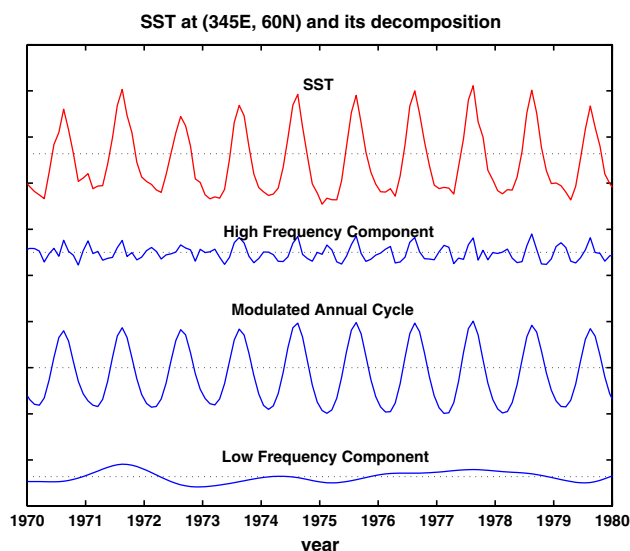

Fig. 4 Decomposition of the sea surface temperature (SST) at (15 W, 60 N). The *red line* is the SST, the *blue lines* from the top to the bottom is the high frequency component, the MAC, and the low frequency component

temperatures from different locations over the globe (not plotted here) shows that the ratio of the range of low-frequency temperature variability and the range of the amplitude of MAC in this Fig. 4 is typical in the regions over the globe other than the tropical Pacific where inter-annual variability has a relatively larger value. These values indicate that the anomaly w.r.t. TAC and the anomaly w.r.t. MAC can have a difference that could be as large as the variability of interannual or/and longer time-scales, and consequently, leading possibly to significantly different physical explanations for the low frequency variability when these anomalies are analyzed.

4 Implication of the MAC for climate studies

With the concept of MAC clarified and an extracting method developed, we are now ready to study climate

variability with respect to an alternative reference frame of MAC. The anomalies with respect to this alternative reference frame are different from their corresponding counterparts with respect to TAC. As a consequence, a physical mechanism that can explain the anomaly with respect to one reference frame well may not work to explain the anomaly with respect to another reference frame. Therefore, we anticipate new mechanisms may emerge. In the following, we will examine some well-studied physical mechanisms, such as reemergence and the phase locking of ENSO to annual cycle, as well as concepts and methodologies used to study interannual to decadal climate variability. We will demonstrate some advantages of this alternative reference frame, especially in leading to more logically consistent understandings of climate variability.

4.1 Reemergence of SST anomalies

The reemergence mechanism is used to explain the recurrence and persistence of SST anomalies on interannual to decadal timescale (e.g., Sarachik and Vimont 2003). In this subsection, we will demonstrate that reemergence can be alternatively interpreted as a mechanism for explaining annual cycle modulation, rather than for explaining anomaly persistence.

Reemergence is a mechanism proposed to explain the recurrence of the sea surface temperature anomaly with respect to TAC from one winter to the next without persisting through the summer, and was first noted in the mid 1970s by Namias and Born (1970) and Namias (1974). The phenomena did not attract much attention until two decades later when Alexander and Deser (1995) connected the recurring SST anomaly with respect to the TAC with mixed layer shoaling in summer and reemergence upon deepening in winter. In the following years, many areas of SST anomaly with respect to TAC in the North Pacific and the North Atlantic were studied (Bhatt et al. 1998; Alexander and Timlin 1999; Watanabe and Kimoto 2000; Alexander et al. 2001; Timlin et al. 2002), and the reemergence mechanism was firmly established.

The reemergence mechanism can be summarized as follows: Suppose that there is positive SST anomaly with respect to TAC in the winter season when the mixed layer is deep. The mixing brings the anomaly down to the bottom of the mixed layer. At the surface level, due to strong damping (by air–sea heat flux, see, e.g., Frankignoul and Hasselmann 1977), this anomaly begins to weaken very fast (within a few months). As the summer sets on, the mixed layer begins to shoal. At the level of the bottom of winter mixed layer, the anomaly is isolated from communicating to the surface layer and is preserved due to weak damping in summer time. When the winter season comes, the mixed layer deepens and reaches the level where the isolated anomaly resides, and mixes that anomaly into the whole mixed layer again, and the surface positive anomaly reemerges. In this mechanism, the annual evolution of the mixed layer is used to explain the persistence of the anomaly from one year to the next.

It is believed that the reemergence has large-scale structures in both North Pacific and North Atlantic. In North Atlantic, the spatial structures represented by the leading empirical orthogonal functions (EOFs) associated with the reemergence, as demonstrated by Timlin et al. (2002), are similar to the dominant structures of the SST variability (again represented by leading EOFs) as revealed by Wallace et al. (1993), Deser and Blackmon (1993), and Watanabe and Kimoto (2000). These results suggest that the North Atlantic low-frequency variability could be, at least partly, explained by the reemergence mechanism.

The mechanism implicitly involves almost annually repeated progression of mixed layer, and hence raises the question of whether the persistence of the SST anomaly with respect to the TAC is part of the annual cycle if the annual cycle can have amplitude and frequency modulation.

We analyze the EOFs and corresponding principle components (PCs) of the SST. The SST is the Hadley Center analyzed monthly SST from January 1951 to December 2000 (Rayner et al. 1996). After extracting the TAC at each grid, the calculations of the EOFs and their corresponding PCs are carried out. The first two leading EOFs (explained 20.0 and 13.8% of the total variance, respectively) and corresponding PCs are plotted in Fig. 5. The first EOF and the first PC resemble those displayed in Fig. 5 of Timlin et al. (2002). The minor differences may be caused by the different lengths of data used in this study and in their study, and by not taking into account the spatially denser grid points in higher latitudes in this study.

Since the SST anomaly with respect to TAC contains a considerable amount of high frequency variability (noise), the recurrence of the SST anomaly in neighboring winters can hardly be visually seen. The alternative method is to examine the lagged autocorrelation of a PC with different starting month. Since the second EOF is significantly different from those argued for the reemergence in the abovementioned references, we expect that the lag correlation of the corresponding PC does not contain peaks apart with a temporal distance for about 12 months for any starting month. Indeed, this is the case (figure not shown). Therefore, our focus is put on the first EOF and its corresponding PC, of which the lag correlation is displayed in Fig. 6. From Fig. 6, we see that the lag correlation has peaks in winter season if the starting month is not July. If only winter anomaly is concerned, it is clear that the lag correlation of the first PC has maximums around 12 month lag and 24 month lag, implying the reemergence of SST anomaly with respect to TAC that is consistent with the results described in the studies referred above. However, the peaks of the lag correlation that are almost phase locked to the winter season regardless of the starting month, especially those corresponding to autumn starting month, can hardly be explained by the reemergence mechanism. From Fig. 6, we also see that the local maxima of lag correlation peak annually, showing that the anomaly with respect to TAC still has an strong annual cycle component if it is viewed locally in temporal domain, which implies the existence of amplitude modulation of the annual cycle.

To answer the question of whether the persistence of the SST anomaly is caused by the residual part of changing annual cycle contained in the anomaly with respect to the TAC, the low frequency variability with respect to the MAC are analyzed. This part of anomaly is defined as the SST after removing the MAC and higher frequency variability. The leading two EOFs and their corresponding PCs are displayed in Fig. 7. Since the higher frequency part is removed and the total variance of the low frequency anomaly with respect to MAC is smaller, the leading EOFs explain relatively larger percentage of variance, 28.1 and 16.8%, respectively, for the first and the second EOFs. The spatial structures of the EOFs in this case are almost identical to those analyzed from the anomaly with respect to TAC, and the PCs are the smoothed versions of corresponding PCs of the anomaly with respect to TAC.

The lagged correlations of the PCs of the anomaly with respect to the MAC for selected starting month are displayed in Fig. 8. Clearly, the reemergence inferred by the peaks of lagged autocorrelation around 12 month lag disappears for the first PC of the anomaly with respect to MAC.

The above result essentially implies that the reemergence may be associated with the residual part of annual cycle (the difference between the MAC and the TAC) that is not eliminated in the anomaly with respect to TAC. Indeed, such an argument can be more clearly illustrated by the idealized model (2) we discussed earlier. Here, instead
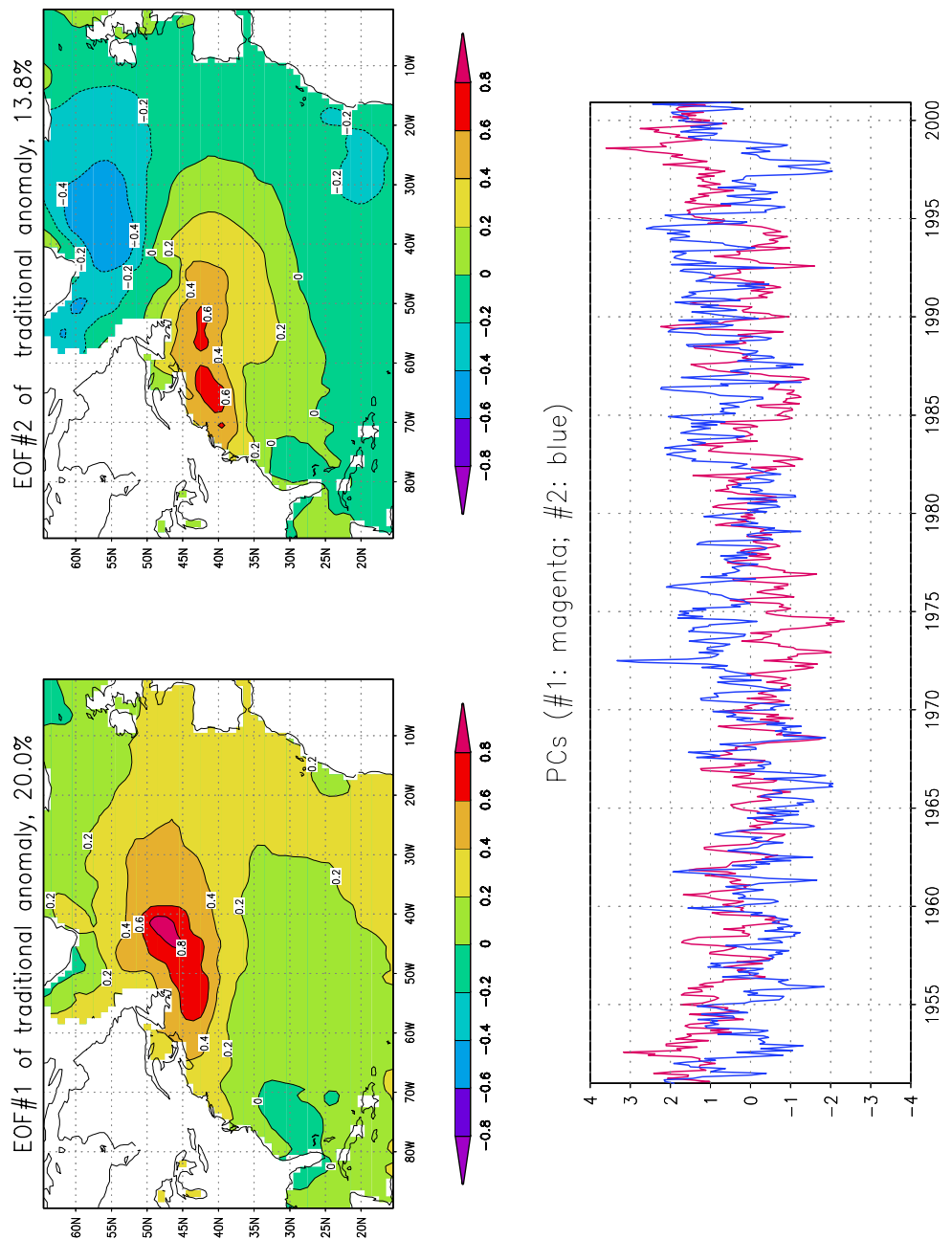
of assuming the noise $N(t)$ to be white, we assume it to be red. In this model, if the annual cycle is taken as the TAC, anomalies with respect to it are expressed by Eq. (3), and correspondingly, the anomalies with respect to its MAC are

$$V''(t) = C(t) + N(t). \tag{5}$$

If $C(t)$ is further set to zero, the anomaly with respect to MAC is the red noise prescribed, and its lagged autocorrelation decays exponentially without noticeably peaking around 12 month lag. However, when TAC is used as a reference, the anomaly contains residual annual cycle. The

lagged autocorrelation of such an anomaly has a peak at 12 months lag, as Fig. 9 displays.

To support the relevance of the simple model, the SST at (15 W, 60 N) is analyzed, a decade of which is shown in Fig. 4. In Fig. 9, the lagged autocorrelation of the anomalous SST with respect to its TAC is displayed. The autocorrelation has a peak at around 12 month. The peak is relatively small, since the amplitude modulation for the annual cycle is relatively small compared to the amplitude of low-frequency variability and noise. However, the small peak around 12 month lag in the lagged correlation is
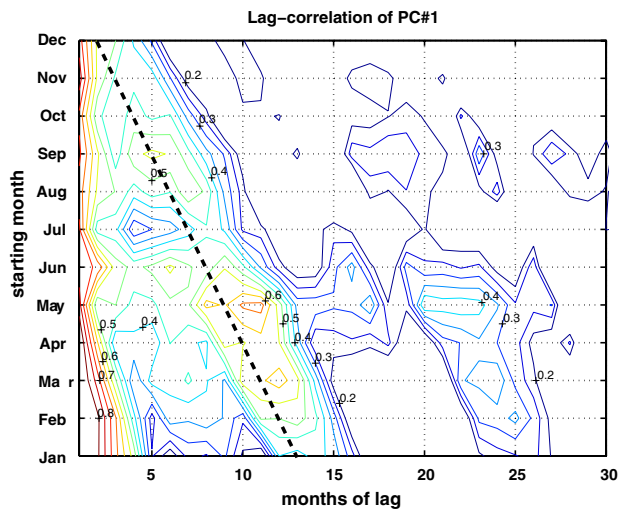
Fig. 6 The contoured lag correlation of the first PC plotted in Fig. 5 as a function of the starting month and the months of lag. The *bold dashed line* corresponds to the location of the January following the starting month. The contour interval is 0.05°C, and only contours of values equal to or larger than 0.2 are plotted

caused mainly by the residual annual cycle contained in the anomaly with respect to TAC. The lagged autocorrelation of the low frequency anomaly after removing MAC and high frequency component, in contrast, does not peak at around 12 month. It should be noted here that, for this SST time series, the high frequency component is dominated by the semiannual cycle, which is largely contained in the TAC. If the high frequency part is taken as part of the anomaly, the lagged autocorrelation peaks semiannually.

It should also be noted that the spatial structure associated with reemergence is independent of whether the reemergence mechanism appears to operate (w.r.t. TAC) or not (w.r.t. MAC). The similar evolutions (PCs) of the corresponding EOFs of anomaly with respect to TAC and of low frequency anomaly indicate that the coherent structures of values of the cross-correlation of SST anomalies at various grid points are dominated by their low frequency variability, not the high frequency part. The EOFs of the low frequency part of SST variability are robust, and not sensitive to filtering method. In our study, we filtered out MAC and high frequency part using the EEMD. Indeed, it can be verified that Fourier Transform based low pass and running mean based low pass of the SST in North Atlantic would lead to almost identical EOFs and their corresponding PCs. Therefore, such EOF structures are structures of low frequency variability of the SST, not the annual recurrence of SST anomaly.

The results presented in Figs. 5, 6, 7, 8 and 9 indicate that the apparent reemergence of SST anomaly in winter season is more likely the persistence and peaking of the anomaly with respect to TAC that contains the residual part of changing annual component. An alternative view of the role of reemergence in climate variability is that the reemergence modifies the annual cycle rather than leading to the long term SST anomaly persistence.

4.2 ENSO phase locking to annual cycle

The second well-known physical problem that we are going to examine is the ENSO phase locking to the annual cycle. The concept of this phase locking can be traced back to Rasmusson and Carpenter (1982), who found that for the majority of the ENSO events happening before 1980, the composite ENSO phases tend to peak in the winter season. Since then, various theories have been developed to explain the interaction between the annual cycle and the ENSO cycle (Jin et al. 1994, 1996; Tziperman et al. 1994, 1995; Chang et al. 1994, 1995, among others). One of these theories focused particularly on the issue of phase locking of ENSO to the annual cycle (Xie 1995; Jin et al. 1996; Tziperman et al. 1997, 1998; Neelin et al. 2000).

Among these studies, Neelin et al. (2000) questioned the concept of the phase locking of ENSO to annual cycle, and pointed out that the concept may be misleading. Xie (1995) schematically illustrated that the superposition of amplitude modulated annual cycle and ENSO cycle can lead to apparent ENSO phase locking to annual cycle when the anomaly is defined with respect to the TAC. However, due to the lack of a dependable method to extract the MAC, Xie's view was not further elaborated.

In the following, we are going to explore whether the superposition of MAC and ENSO cycles indeed leads to apparent ENSO phase locking to the annual cycle when the anomaly is defined with respect to the TAC. Our results, based on the analysis using the MAC reference frame for anomaly, will echo Neelin et al.'s sentiment that the ENSO phase locking to annual cycle is misleading, and support Xie's view that the locking is caused by superposition of MAC onto interannual variability. The essential argument is that, in the previous studies of observational SST, the apparent phase locking is caused largely by the residual annual cycle (the difference between MAC and TAC) that is considered as part of the anomaly with respect to TAC.

The data for the analysis of observational record in this study is the modified cold tongue index (MCTI). The original CTI is defined as the averaged SST anomalies with respect to TAC over 6 N–6 S, 180–90 W (Deser and Wallace 1987, 1990). To include the possible MAC, the MCTI is defined as the SST over the same region. The SST used is the Hadley Center analyzed SST (Rayner et al. 1996) from January 1900 to December 1999. A section of the decomposition from January 1970 to December 1989 of the MCTI into sub-annual component, the MAC, the interannual variability, and decadal variability using the method

Fig. 7 The first two empirical orthogonal functions (*EOFs*) (*upper panels*) and their corresponding principle components (*PCs*) (*lower panel*, with the *magenta line* the first PC and the *blue line* the second PC) of the interannual or longer timescale variability of the North Atlantic SST
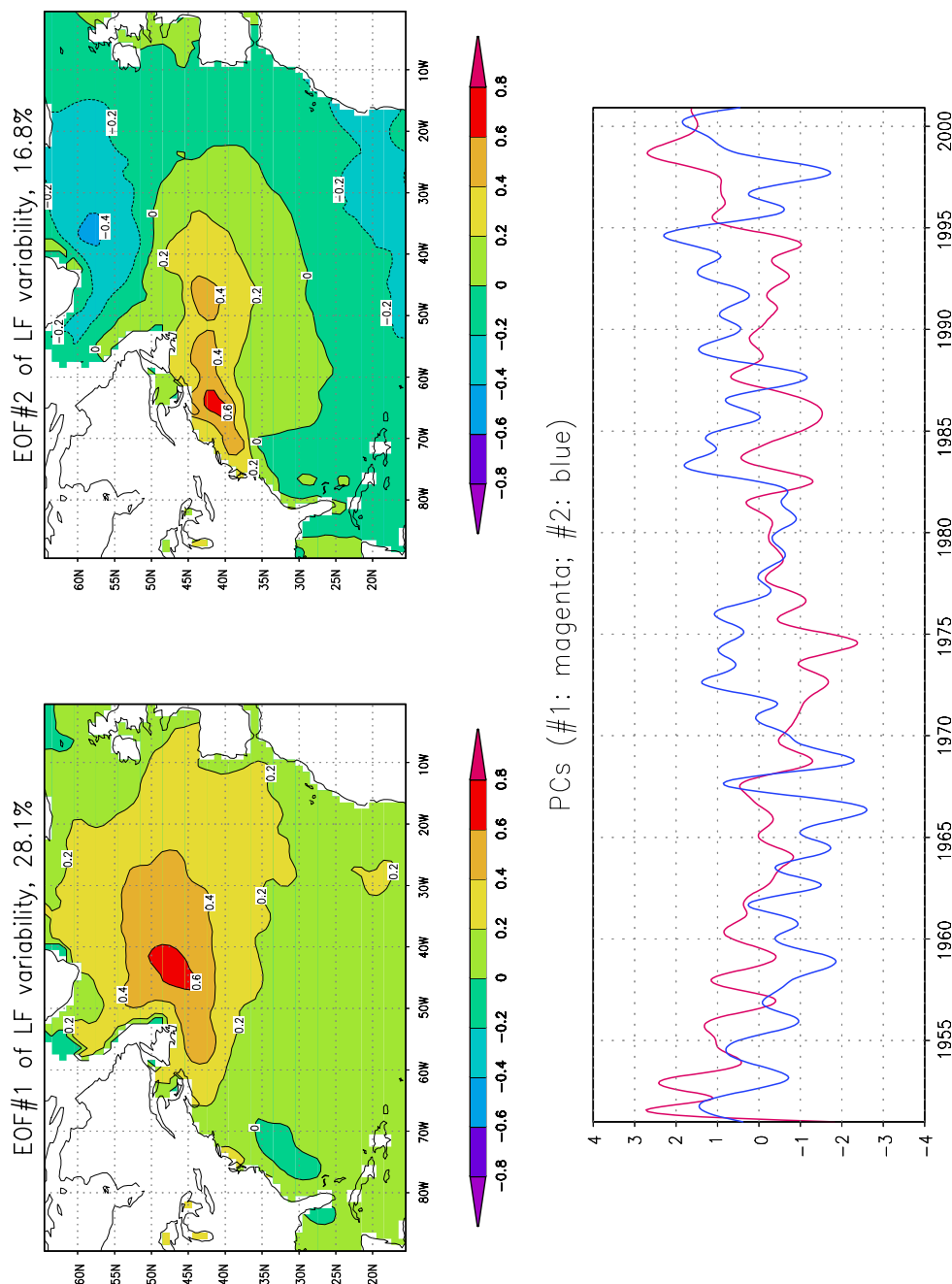
described in Sect. 3 is presented in Fig. 10. From a visual inspection, it can be concluded that the decomposition using the EEMD separates components of different timescales well. It should be noted that the episode displayed in Fig. 10 is randomly selected. The results corresponding to any episode from the 100 year would provide the same visual impression of temporal scale separation.

To facilitate a comparison, an independent decomposition of the same index using Singular Spectrum Analysis (SSA, see Ghil et al. 2002) is also presented. In the SSA, an embedding dimension of 60 is selected. For the MCTI, there are nine outstanding components. The first two

components are related to annual cycle, while the next seven components are all related to the interannual variability. Clearly, the interannual component resulted from SSA are very close to that from the EEMD, especially in that their peaks are almost always aligned along the time axis. This comparison provides evidence that the interannual variability is not sensitive to a decomposition method. However, the annual cycle from the SSA has some amplitude and frequency modulation but is closer to the TAC of the data defined over the whole data length.

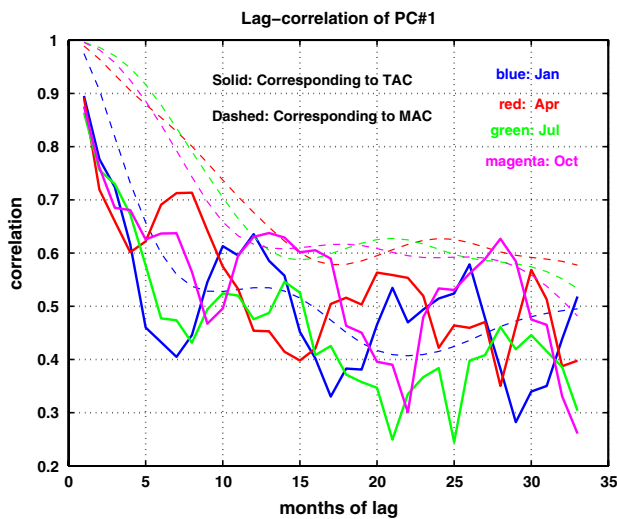The anomaly with respect to TAC contains noise and variability of all timescales. The actual temporal location

Fig. 8 The lagged autocorrelations of the first PCs of anomalies with respect to TAC and to MAC. The *blue, red, green, and magenta lines* are the lag correlations of the first PC with January, April, July, and October as the starting month, respectively. The *solid line* group is corresponding to the TAC as a reference frame for anomaly, and the *dashed line* group is corresponding to the MAC as a reference frame for anomaly
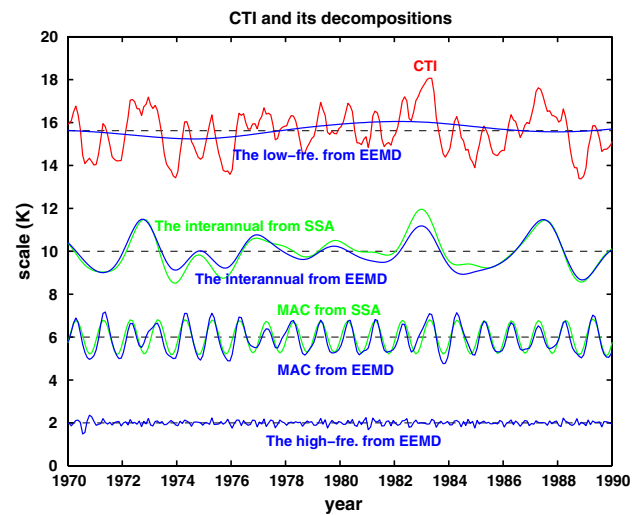

Fig. 9 The lagged autocorrelations of the synthetic climate model (2) (*left panel*) and of the SST at (15 W, 60 N) (*right panel*). In these plots, red noise of 1 month lag autocorrelation of 0.8 replaces the white noise of the same standard deviation in the earlier (e.g., in Fig. 2). The *black lines* are corresponding annually averaged lag correlation for anomalies with respect to TAC (*solid line*) and Mac (*dashed line*) in both panels. The *blue, red, green, and magenta lines* in the *right panel* are the lag correlations of the first PC with January, April, July, and October as the starting month, respectively. The *solid line* group is corresponding to the TAC as a reference from for anomaly, and the *dashed line* group is corresponding to the MAC as a reference from for anomaly

of peaks of interannual variability is shadowed by the high frequency variability and noise, and can not be determined. Therefore, the traditional method to examine the phase

Fig. 10 A section of the modified cold tongue index (*MCTI*), which is the averaged SST over 6 N–6 S, 180–90 W, and its decompositions using the EEMD. The *red line* is the modified CTI; the *blue lines* from the bottom to the top are the noise, the modulated annual cycle (*MAC*), the interannual variability, and the decadal and longer timescale variability of the MCTI, respectively. The *two green lines* are the annual cycle (the sum of the first two SSA components) and the interannual variability (the sum of the components three to nine) of the MCTI obtained using the singular spectrum analysis (*SSA*). The values appeared along the vertical axis are not the true values for the decomposition but serve as a scale for different components
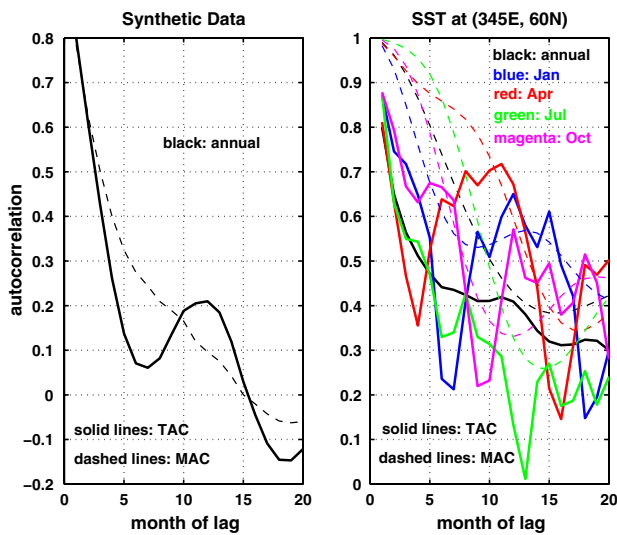
locking of ENSO to annual cycle is through the examination of standard deviation of the anomaly (with respect to its TAC) for different months, since the deviation from TAC is relatively larger in the months in which interannual variability reaches an extrema. In this study, a similar approach is also adopted for the components decomposed by using the EEMD. In Fig. 11, the standard deviations of the anomaly with respect to its TAC and of the interannual component of the EEMD decomposition are plotted as a function of month. The larger values of the standard deviation appear in the winter months for the anomaly with respect to TAC. In contrast, when the MAC is used as an alternative reference frame for the definition of anomaly, the values of standard deviation are much flatter from January to December. Further analysis shows that the sub-annual component and the residual annual cycle contained in the anomaly with respect to TAC have their standard deviations peak in winter season, which explains most of larger standard deviation of the anomaly with respect to TAC in winter months.

With the smooth evolution of the interannual variability of MCTI, the temporal locations of extrema of the inter-annual variability, corresponding to the peaking of ENSO warm/cold events can be uniquely determined. Figure 12 plots the extremas of interannual ENSO component as a function of Julian month. The maxima and minima spread over all months rather than concentrate in winter months,
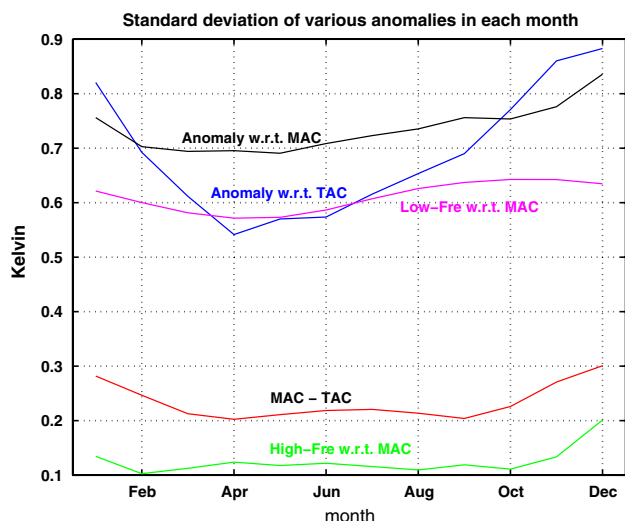
Fig. 11 The standard deviations of the anomalies with respect to TAC (*blue line*) and with respect to MAC (*black line*) for each month. The standard deviations of the interannual component of the MCTI, of the residual annual cycle (MAC–TAC), and of the high-frequency component are plotted as *the magenta line, the red line, and the green line*, respectively. The high-frequency component of MCTI is displayed as the *bottom blue line* in Fig. 10

Fig. 12 The temporal appearance of the extrema of the interannual component of the modified CTI as a function of month. The *red asterisks* correspond to maxima; and *blue circles* to minima. The *horizontal red and blue lines* are the one standard deviation level for maxima and for minima, respectively. The events outside the region bordered by these two lines can be considered significant El Niño or La Niña events

although the springtime corresponds to the extrema of relatively small absolute values. Even for the extrema corresponding to traditionally recognized El Niño/La Niña events (with the extrema values exceeding thresholds of one standard deviation for warm and cold events, respectively), the extrema are still spread over all months except the spring time, consistent with the results from Neelin et al. (2000).

The results presented in Figs. 10, 11, and 12 indicate that the phase locking of ENSO to the annual cycle is due mainly to the residual annual component when the TAC is used as the reference frame for the definition of anomaly. An alternative view for the interaction between ENSO and annual cycle is that ENSO evolves mainly with its own agenda, and the apparent more frequent occurrences of the ENSO in a particular season is due not to the phase locking to annual cycle, but rather to inclusion of the residual annual cycle in the ENSO interannual evolution.

4.3 Concepts of interannual and decadal climate variability

In this subsection, we will discuss the implication of reference annual cycle to the definitions of climate variability on interannual to decadal timescale. In literatures, we often encounter concepts of low frequency variability of the averaged climate over a particular sub-annual period, such as the decadal variability of winter temperature, of January temperature, etc. When such concepts are used, the exact
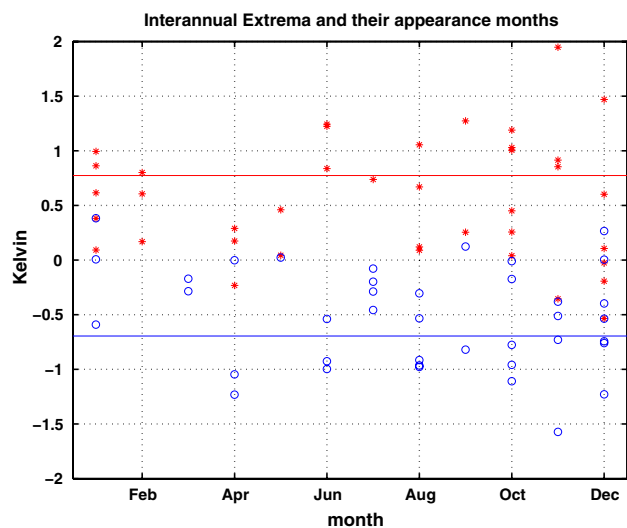
meanings of these concepts are often unclarified; and whether 'the decadal variability of winter temperature' refers to the limited scope directly defined by the words of the phrase, that is, winter temperature variability only, or implicitly to the extended meaning of 'the decadal variability of the climate system manifested by winter temperature' is often unclear. In both interpretations, such concepts seem to have unsatisfying consequences when the reference frame of TAC is used; and the use of MAC as a reference frame can reconcile these two interpretations.

From the standpoint of the limited scope definition, the 'decadal variability of winter temperature' is explicit and has unambiguous meaning, focusing the variability on winter temperature only. Under this interpretation, a physical mechanism that explains the winter temperature variability consider winter temperature in an isolated fashion and leave the temperature of other seasons between two neighboring winters unconcerned, implicitly introducing the temporal disruption of the physical processes of the climate system. However, a climate system is a physical system having temporal continuity. From observations, we see the variability of winter temperature in the reference framework of TAC is more closely related to the temperature of prior season than the temperature of the same season but one year earlier. Some evidence to support this argument has already been presented in Figs. 6, 8, and 9 in which winter SST anomaly with respect to TAC shows a larger correlation (of a value greater than 0.7) with the prior autumn SST anomaly than that (of a value smaller

than 0.7) with the following winter temperature. Therefore, when physical understanding is pursued from winter only data, this limited interpretation may lead to potentially logical inconsistency between the physical mechanism for an arbitrarily constructed temporally discrete climate system (i.e., winter temperature) and the temporally continuous nature.

If 'the decadal variability of winter temperature' is interpreted as 'the decadal variability of the temperature manifested by winter temperature', the winter temperature should be considered as the sampling at a certain set of evenly spaced temporal locations of the slowly varying (decadal change) temperature. Similarly, 'the decadal variability of summer temperature' should be considered another set of evenly spaced temporal locations of the same temperature evolution. Since the temporal sampling distance of the temperature (characterized by the neighboring summers or neighboring winters) are much smaller than the concerned time scales of the variability, the different versions of decadal variability from winter and from summer temperature should be qualitatively and quantitatively identical, or at least, very close to each other. However, this requirement seems to be hardly satisfied in realistic world. An example of SST at (15 W, 60 N) is displayed in Fig. 13 (the SST data as displayed as the red line in Fig. 4). Clearly, the different versions of interannual and longer timescale variability derived from summer SST (the red line in Fig. 13) and from winter SST (the green line in Fig. 13) are significantly different. When these two versions of SST are analyzed using the Fourier spectra to identify the dominant average decadal scales, it is seen that the winter temperature is almost exclusively dominated by a 50 year cycle while the summer temperature consists of a less dominant 50 year cycle as well as a dominant 25 year cycle, requiring significantly different physical mechanisms to explain such cycles. This discrepancy could be even worse when more versions of decadal variability of temperature are defined, such as decadal variability of January temperature, decadal variability of July 1 temperature, which may all require different physical mechanisms to explain. In such a case, the reconciliation of the physical mechanisms for decadal variability of temperature of 'winter' and of it sub-period, for example, January, become a daunting task.

From above discussions, it seems that we have been facing a dilemma: either to use an interpretation of a concept of climate variability inconsistent with physical nature or to use an interpretation that will lead to inconsistent physical mechanisms of climate variability. However, when we carefully reconsider this dilemma, we find that the dilemma is largely caused by the hidden annual cycles in the winter only data. The 'decadal variability of winter temperature' is derived based on the
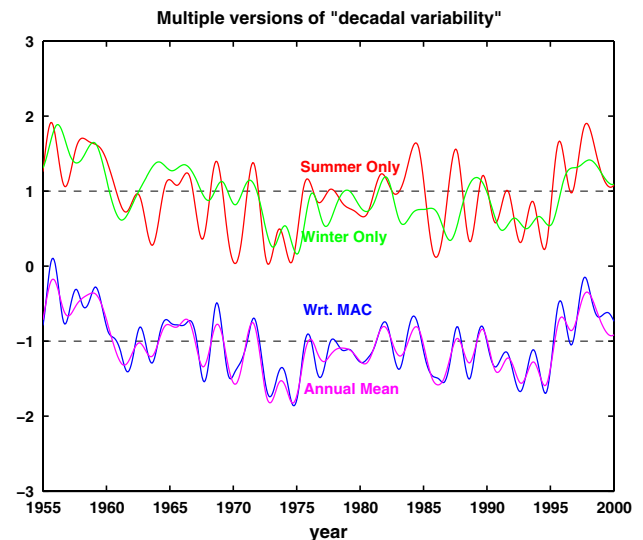

Fig. 13 The various versions of interannual or longer timescale variability of the SST at (15 W, 60 N). The *green line* corresponds to the variability of winter SST; the *red line* corresponds to the variability of summer SST. For these two lines, the data defined yearly have been interpolated monthly using the cubic splines. The *magenta line* is the average of *the green and the red lines* shifted down to align with the *blue line*. The *blue line* is the variability of the monthly SST after excluding its high frequency component and MAC. The vertical axis only provides the relative scale rather the value of variability

averaged temperature of winter. In such a derived time series, since the temporal distance between any two neighboring data points is 1 year, it is widely perceived that the variability of period shorter than one year, as well as the possible annual cycles, have already been extracted, leaving only interannual or long timescale variability. However, this perception is problematic. To illustrate, we again use an ideal model modified from model (2) presented earlier to represent temperature changes along the time. To make it simpler, we assume the noise $N(t)$ to be zero, leaving only the amplitude modulated annual cycle and the interannual or longer timescale variability $C(t)$. The time series is plotted in Fig. 14. From this timeseries, the winter (summer) temperature is defined as the average temperature over December, January, and February (June, July, and August). These averaged temperatures are also plotted in Fig. 14.

Clearly from Fig. 14, the low frequency variability defined by the winter temperature only and the decadal variability defined by the summer temperature only of the synthetic model have large difference due to the implicit inclusion of the annual cycle change. However, when the low frequency variability is derived from the anomaly with respect to MAC, the multiple versions of the low-frequency variability are only the results of the uniquely defined decadal variability sampled at different temporal locations
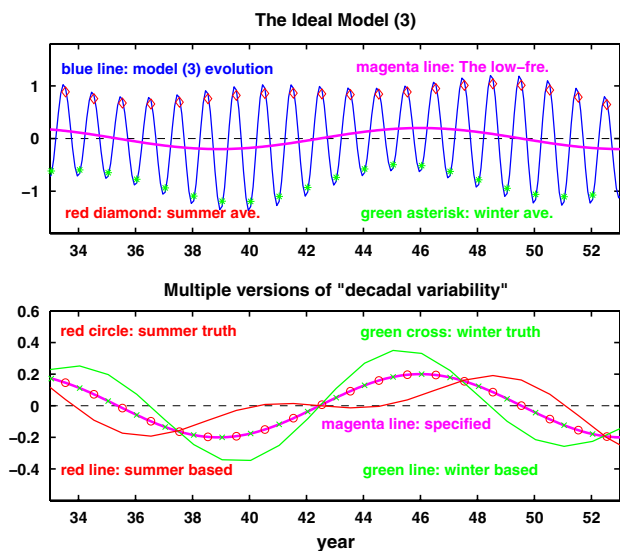
Fig. 14 The different versions of decadal variability of a synthetic model composed of only the low frequency component and the MAC of model (2). In the *upper panel*, the blue line is the synthetic model data; the *magenta line* is the low frequency component; the *red diamonds* represent summer temperature; and the *green asterisks* represent winter temperature. In the *lower panel, the magenta line* is the same as the one in the *upper panel; the red (green) line* is the interpolated version of the summer (winter) temperature in the upper panel; and the *red circles* represent the low frequency component sampled in summer time; and *green circles* represent the low frequency component sampled in summer time

(e.g., summer, winter, January, July 1, etc.). Since the sampling timescale is much smaller than the timescale of the low frequency variability, these multiple versions of low frequency variability become all consistent with the specified low frequency variability in the synthetic model. In this way, one can avoid the different versions of low frequency variability and focus only on a uniquely defined version of the low frequency variability, leading to *potential simplification* of our understanding of the low frequency variability of the system through pursuing a single version of theory for the uniquely defined low frequency variability with respect to MAC instead of pursuing numerous versions of theories for the explanation of the corresponding numerous versions of low-frequency variability.

A real world example of the low frequency variability with respect to MAC is displayed as the blue line in the lower panel of Fig. 13. Overlapped on it is the average of the winter only variability and summer only variability, both interpolated from yearly to monthly using the cubic spline. It is clear that the average of the variability of summer and of winter is quite consistent with the low frequency variability in the MAC reference frame on decadal timescales, although the interannual variability of the latter is sharper.

5 Summary and discussions

Anomaly is the deviation from the standard or expected, and is an irregularity which may be difficult to explain using existing rules or theory. Since any definition of deviation involves a reference frame from which the deviation can be determined, changing a reference frame would change the corresponding anomaly. As a consequence, a physical mechanism that can explain well the anomaly with respect to one reference frame may not work well to explain the anomaly with respect to another reference frame.

In this study, we have proposed an alternative reference frame, the amplitude-frequency MAC, to the traditional reference frame of the annually repeating annual cycle (TAC) for climate anomaly. Since the climate system is a nonlinear system, the response of this system to a purely periodic forcing is often not periodic, rather it contains amplitude–frequency modulation. Therefore, MAC as a reference frame for climate anomalies may be worthwhile to pursue. Indeed, MAC is a more general reference frame for climate anomaly than the TAC since it includes TAC. We have also developed a method based on the ensemble empirical mode decomposition to extract MAC in climate data. The method was tested with both synthetic and real climate data and was demonstrated to be useful.

With MAC as a reference frame, we defined alternative anomaly. Based on the anomaly with respect to MAC, we reexamined some widely accepted physical mechanisms as well as concepts and methodologies used to study interannual to decadal climate variability. One of the physical mechanisms reexamined is the reemergence mechanism that was often used to explain the long term persistence of SST anomaly in the midlatitude oceans. The reemergence mechanism explains the recurrence of anomaly with respect to TAC. When anomaly is defined with respect to MAC, the annual recurrence disappears, and hence, no reemergence happens. However, the result does not indicate reemergence is an artificial mechanism. Rather, the recurring part of the anomaly with respect to TAC is alternatively interpreted as a part of MAC. We also found that the previously recognized spatial structure of SST variability associated with the reemergence is more likely a spatial structure associated with the low frequency variability of SST rather than the reemergence.

A widely held notion being examined is the phase locking of ENSO evolution to annual cycle. The anomaly with respect to TAC (i.e., CTI) contains the residual annual cycle (MAC–TAC). This part of anomaly is phase locked to the annual cycle. If the MAC and the higher frequency variability are removed from the MCTI, the remaining (interannual and longer timescale) part peaks in widely spread months rather than only in winter season, that is, no

phase locking. Therefore, the ENSO phase locking can be alternatively interpreted as the residual annual cycle contained in the traditional anomaly phase locked to annual cycle itself.

The implications of the reference frame of MAC to the methodology of climate study were also presented. We argued that concepts such as 'decadal variability of summer (winter) temperature' lead to numerous versions of the interannual to interdecadal climate variability of a climate variable averaged over an arbitrarily sub-annual period (such as a season, month, day). To understand physically these versions of the interannual to interdecadal climate variability, many different physical mechanisms may be needed. How to reconcile these physical mechanisms remains a problem to be solved. We demonstrated that, with MAC as a reference frame for climate anomaly, the problems associated with the multiple definitions of the interannual to interdecadal climate variability of a climate variable can be largely bypassed, and multiple physical mechanisms may no longer be a necessity.

The concept of amplitude–frequency modulated annual cycle, a method to extract it, and the implications of amplitude–frequency modulated annual cycle in climate study constitute our efforts to construct an alternative framework for climate study. The examples presented earlier have demonstrated, to some extent, the advantages of using MAC as a reference frame for climate anomaly over the using TAC as a reference frame. However, it is arguable that in some of examples shown above, the difficulty in understanding interannual to decadal variability is transformed to the difficulty in understanding the change of annual cycle. Then, what is the rational for choosing a new reference frame?

The complete answer to this question may lie far ahead and out of our reach now. When many more physical phenomena in climate are studied using the MAC framework, we will be able to tell, from experimental perspective, of the usefulness of the MAC framework. However, at this moment, we have to use philosophical arguments for the MAC framework. It is widely recognized that when all else being equal, the simpler the theory, the greater the elegance of the theory. As stated by Richter (2006):

> "trend of the path to understanding … is always toward the reductionist—understanding complexity in terms of an underlying simplicity."

Our previous examples have demonstrated that the physical explanation for climate variability other than the annual cycle may be simplified and become more logically consistent in the reference frame of MAC. For example, to use annual recurring phenomenon to explain annual cycle change rather than to explain persistence on interannual to decadal timescale makes the physical explanation for the climate variability easier to reconcile with human intuition; and to simplify the apparent ENSO phase locking to be the "residual annual cycle" phase locked to annual cycle also makes intuitive sense rather than to use complicated phase locking mechanism in nonlinear dynamics to explain the same phenomenon. Of course, the most evident example of potential simplicity of climate variability in the reference frame work is to define a unique version of the low frequency variability. In this way, numerous versions of low frequency variability and their physical explanations in the reference frame of TAC can be avoided. This make a case that the alternative reference frame of MAC is indeed worthwhile to pursue.

Before we close this paper, we would like to acknowledge that the reference frame of MAC for climate anomaly itself needs to be further studied. The physics of how and why the annual cycle of a climate variable change remains to be further understood. As we mentioned earlier in the discussion that led to our definition of MAC, the derivation of the MAC is not based solely on the physics but rather more on the temporal locality constraint in light of the principle of the physical world that the later evolution of a physical system could not change what already happen. Accordingly, the *physical* interpretations derived from the analysis of a climate time series of a given length, especially those interpretations of the phenomena of a timescale significantly smaller than the length of the data, should not be changed when the same climate time series with added record of later evolution is reanalyzed. The traditional annual cycles discussed in Sect. 2 do not satisfy this constraint, but the MAC as we derived using the EEMD does. In our ongoing research, we are investigating why and how the annual cycle changes. The preliminary results show that the cause of the change of annual cycle of the surface temperature is tied to both the change of forcing (such as the short wave radiation absorbed at the surface) and the nonlinearity of the climate system (such as advection). Due to the length limit, these results will be reported elsewhere.

Finally, we would like to point out that the ultimate verification of the advantage of MAC reference frame should come from the prediction of future climate. In our ongoing research, we examine the ENSO dynamics using the delayed oscillator model forced by annual cycle, which is a simplified analogue of the Zebiak–Cane model (Zebiak and Cane 1987) for ENSO, the preliminary results show that predictability of such a model is indeed enhanced dramatically when the model parameters are specified within a certain (realistic) range. When the observed Niño3.4 index is concerned, we also found that the ENSO spring prediction barrier disappears in the reference frame of MAC. These results will also be reported elsewhere.

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