JHEP03(2011)133

Different SO(10) paths to fermion masses and mixings

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ABSTRACT: Recently SO(10) models with type-II see-saw dominance have been proposed as a promising framework for obtaining Grand Unification theories with approximate Tri-bimaximal (TB) mixing in the neutrino sector. We make a general study of SO(10) models with type-II see-saw dominance and show that an excellent fit can be obtained for fermion masses and mixings, also including the neutrino sector. To make this statement more significant we compare the performance of type-II see-saw dominance models in fitting the fermion masses and mixings with more conventional models which have no built-in TB mixing in the neutrino sector. For a fair comparison the same input data and fitting procedure is adopted for all different theories. We find that the type-II dominance models lead to an excellent fit, comparable with the best among the available models, but the tight structure of this framework implies a significantly larger amount of fine tuning with respect to other approaches.

KEYWORDS: GUT, Quark Masses and SM Parameters, Neutrino Physics

Contents

1 Introduction

In this article we make a quantitative comparison of the performance of different types of SO(10) Grand Unification Theories (GUT's) in reproducing the observed values of fermion masses and mixing, also including the neutrino sector. By now we have a rather precise knowledge of the leptonic mixing angles [1–9] which, within the experimental accuracy, are consistent with the Tri-Bimaximal (TB) pattern [10–16] and, as such, are very different from the quark mixing angles. A still open and challenging problem is that of formulating a natural model of Grand Unification based on SO(10), leading not only to a good description of quark masses and mixing but also, in addition, of charged lepton masses and to approximate TB neutrino mixing. In SO(10) the main added difficulty with respect to SU(5) is clearly that all fermions in one generation belong to a single 16-dimensional representation, so that one cannot separately play with the properties of the SU(5)-singlet right-handed neutrinos in order to explain the striking difference between quark and neutrino mixing. There are a number of rather complete SO(10) models, with different architectures, that, without having a built-in TB mixing yet are able to reproduce the data on neutrino mixing angles [31–33, 35, 36, 38, 67, 70]. These models fall in different classes: renormalizable or not, with lopsided or with symmetric mass matrices, with various assumed flavour symmetry, with different types of see-saw and so on. In most of these models some dedicated parameters are available to fit the observed neutrino masses and mixing angles. In these models TB mixing appears as accidental, and if the data would become somewhat different, the new values of the mixing angles could as well be fitted by simply changing the values of the parameters. These models are certainly interesting and, in this article, a number of

them will be confronted with the data and their respective performances will be compared and taken as a reference. A more difficult goal would be the construction of SO(10) models where TB mixing is built in and is automatic in a well defined first approximation, due, for example, to an underlying (broken) flavour symmetry. The leading approximation in these models is particularly constrained and the neutrino mixing angles are fixed in this limit. There are a number of GUT models of this type based on SU(5) (see, for example, [17, 19–22]), but, as mentioned, the SO(10) case is more difficult and the existing attempts, in our opinion, are still not satisfactory in all respects.

A promising strategy in order to separate charged fermions and neutrinos in SO(10) is to assume the dominance of type-II see-saw [23–25] (with respect to type-I see-saw [26–30]) for the light neutrino mass matrix. Grand Unified SO(10) models based on type-II see-saw dominance have been studied in refs. [39–44, 49–51, 57, 58, 60–64]. If type-II seesaw is responsible for neutrino masses, then the neutrino mass matrix (proportional to) f (see eqs. (2.1), (2.3), (2.4)) is separated from the dominant contributions to the charged fermion masses and can therefore show a completely different pattern. This is to be compared with the case of type-I see-saw where the neutrino mass matrix depends on the neutrino Dirac and Majorana matrices and, in SO(10), the relation with the charged fermion mass matrices is tighter. Here we do not consider the problem of formulating a flavour symmetry or another dynamical principle that can lead to approximate TB mixing, but rather study the performance of the type-II see-saw SO(10) models in fitting the data on fermion masses in comparison with other model architectures. Actually, we will show in section 2 that, without loss of generality, we can always go to a basis where the matrix f is of the TB type. In fact, since the TB mixing matrix is independent of the mass eigenvalues, the most general neutrino mass matrix, which is a symmetric complex matrix, can always be transformed into a TB mixing mass matrix by a change of the charged lepton basis. The observed deviations from TB mixing will then be generated by the diagonalisation of charged leptons and, in order to agree with the data, must be small. In turn the charged lepton mixings are related in SO(10) to the quark mixings. Thus, in this class of models TB mixing is exact in the approximation of neglecting charged fermion mixings. When a symmetry guarantees TB mixing in first approximation the corrections from the diagonalisation of charged leptons are automatically small, while in general could be large. The main purpose of our analysis is to see to which extent this particular structure models is supported by the data among different types of SO(10) .

In renormalizable SO(10) models (a non necessary assumption which is only taken here in some cases for simplicity) the fermion masses are generated by Yukawa couplings with Higgs fields transforming as **10**, **$\overline{126}$** (both symmetric) and **120** (antisymmetric). Alternatively, in non renormalizable SO(10) models the large representations **$\overline{126}$** and **120** can be effectively obtained from the tensor products of smaller Higgs representations. The **10** Yukawa couplings contributing to up, down and charged lepton masses in most models have a large 33 term, corresponding to the large third generation masses, while all other entries are smaller and lead by themselves to zero CKM mixing (because the **10** contributes equally to up and down mixing). Quark mixings arise from small corrections due to **$\overline{126}$**, the same Higgs representation that determines f which, in models with type-II see-saw, is

dominant in the neutrino sector, and to **120**. Thus, in this approach, in the absence of **120**, there is a strict relation between quark masses and mixings and the neutrino mass matrix. The presence of **120** dilutes this connection which however still remains important. In particular the small deviations from TB mixing induced by the diagonalisation of the charged lepton mass matrix, barring cancelations, are of the same order as the quark mixing angles and most of the parameters appear in both.

An interesting question is to see to which extent the data are compatible with the constraints implied by this interconnected structure. A goal of this work is precisely to study how the inclusion of TB mixing along these lines is reflected in the ability of the model in reproducing the data in comparison with different structures. Thus, independent of the problem of determining a flavour symmetry that can fix the parameters to their required values, we study SO(10) models based on the dominance of type-II see-saw with respect to their performance in fitting the fermion masses and mixings in comparison with alternative realistic SO(10) models. Some other analyses have appeared in the literature where SO(10) models with dominance of type-II see-saw have been confronted with the data (see, for example, [57–59, 67]). The present analysis is different and, in some respect, more general and, in addition, also includes the comparison with the most established models based on SO(10) that can be considered both realistic (i.e. that have been worked out to a level that a comparison with the data is possible) and complete (i.e. that also include the neutrino sector). Each model will be compared with the same set of data on masses and mixings given at the GUT scale. For this purpose we specify a set of data for reference. We are not much concerned with the uncertainties which for sure exist in evolving the physical quantities measured at the electroweak scale up to the corresponding ones at the GUT scale. We will adopt a reasonable evolution up to the GUT scale, as can be found in the literature [59, 71–73], and consider the resulting set of data as the truth and fit all relevant models to that set (to be precise, we actually consider two sets of data, one for models with small $\tan\beta$ and one for those with large $\tan\beta$). We argue that if a particular model is sizably better than another in fitting these idealized data it will also score better on the real data. For our comparison the first quality factor is the χ^2 or the χ^2/d.o.f. obtained from the fit for each model. We also introduce a parameter $d_{\rm FT}$ for a quantitative measure of the amount of fine-tuning of parameters which is needed in each model. We find that SO(10) models based on dominance of type-II see-saw can fit the data remarkably well, both in absolute terms and in comparison with other types of models, but at the price of a substantial amount of fine-tuning needed to compensate for the tension introduced by the double role of f in determining both the neutrino mass matrix and the charged fermion masses and mixings. In particular the smallness of the first generation masses requires a precise cancellation of larger terms.

This article is organised as follows. In section 2 we define our reference model, renormalizable with type-II dominance. In section 3 we discuss the relations between parameters of the model and measured quantities. In section 4 we specify the set of data that we adopt at the GUT scale. In section 5 we describe our fitting procedure and the quality indicators that we consider to compare different models. In section 6 we briefly describe a number of alternative SO(10) models and we fit them to the same data as we did for our refer-

ence model. Finally in section 7 we present our summary and conclusion. The best fit parameters and observables are listed in appendix A for all the competing models.

2 A class of SO(10) models

We consider the class of renormalizable Supersymmetric (SUSY) SO(10) models with dominance of type-II see-saw for neutrino masses [39–44, 49–51, 57, 58, 60–64]. In the following we indicate the generic model of this class by T-IID (from Type-II Dominance). In renormalizable SO(10) models the Higgs fields that contribute to fermion masses are in **10** (denoted by H), $\overline{\textbf{126}}$ ($\overline{\Delta}$) and **120** (Σ). The Yukawa superpotential of this model is then given by:

$$W_Y \;=\; h\,\psi\psi H + f\,\psi\psi\bar{\Delta} + h'\,\psi\psi\Sigma, \tag{2.1}$$

where the symbol ψ stands for the **16** dimensional representation of SO(10) that includes all the fermion fields in one generation. The coupling matrices h and f are symmetric, while h' is anti-symmetric. The representations H and Δ have two standard model (SM) doublets in each of them whereas Σ has four such doublets. At the GUT scale M_{GUT}, once the GUT and the $B - L$ symmetry are broken, one linear combination of the up-type and one of down-type doublets remain almost massless whereas the remaining combinations acquire GUT scale masses. The electroweak symmetry is broken after the light Minimal Supersymmetric Standard Model (MSSM) doublets (to be called $H_{u,d}$) acquire vacuum expectation values (vevs) and they then generate the fermion masses. The resulting mass formulae for different fermion masses are given by (see, for example, [61–63]):

$$\begin{aligned}
Y_u &= h + r_2 f + r_3 h', \\
Y_d &= r_1(h + f + h'), \\
Y_e &= r_1(h - 3f + c_e h'), \\
Y_{\nu^D} &= h - 3r_2 f + c_\nu h',
\end{aligned} \tag{2.2}$$

where Y_a are mass matrices divided by the electro-weak vev's $v_{u,d}$ and r_a ($a = 1, 2, 3$ and c_b ($b = e, \nu$) are the mixing parameters which relate the $H_{u,d}$ to the doublets in the various GUT multiplets.

In generic SO(10) models of this type, the neutrino mass formula has a type-II [23–25] and a type-I [26–30] contributions:

$$\mathcal{M}_\nu \;=\; f v_L - M_D \frac{1}{f v_R} M_D^T, \tag{2.3}$$

where v_L is the vev of the $B - L = 2$ triplet in the $\overline{\textbf{126}}$ Higgs field. Note that in general, the two contributions to neutrino mass depend on two different parameters, v_L and v_R, and it is possible to have a symmetry breaking pattern in SO(10) such that the first contribution (the type-II term) dominates over the type-I term. The possible realisation of this dominance and its consistency with coupling unification has been studied in the

literature [44, 49–54] and found tricky but not impossible [56]. The neutrino mass formula then becomes

$$\mathcal{M}_\nu \ \sim \ f v_L. \tag{2.4}$$

Note that f is the same coupling matrix that appears in the charged fermion masses in eq. (2.2), up to factors from the Higgs mixings and the Clebsch-Gordan coefficients. Also note that the neutrino Dirac mass, proportional to $Y_{\nu D}$ in eqs. (2.2), only enters in the neglected type-I see-saw terms and does not play a role in the following analysis. The equations (2.2) and (2.4) are the key equations in this approach.

For generic eigenvalues m_i, the most general matrix that is diagonalised by the TB unitary transformation is given by:

$$f = U_{\text{TB}}^* \text{diag}(m_1, m_2, m_3) U_{\text{TB}}^\dagger \quad . \tag{2.5}$$

where, with a specific phase convention, we can take:

$$U_{\text{TB}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{2.6}$$

In this convention U_{TB} is a real orthogonal matrix and all phases can be included in the eigenvalues m_i. Then the matrix f is symmetric with complex entries and, from eq. (2.5), one obtains:

$$f = \begin{pmatrix} f_2 & f_1 & f_1 \\ f_1 & f_2 + f_0 & f_1 - f_0 \\ f_1 & f_1 - f_0 & f_2 + f_0 \end{pmatrix}, \tag{2.7}$$

with: $m_1 = f_2 - f_1$, $m_2 = f_2 + 2f_1$ and $m_3 = f_2 - f_1 + 2f_0$.

An important observation is that, for a generic neutrino mass matrix f', we can always go to a basis where f' is diagonalised by the TB unitary transformation in eq. (2.6) and is of the form in eq. (2.7). In fact, if we start from a complex symmetric matrix f' not of that form, it is sufficient to diagonalise it by a unitary transformation U: $f'_{\text{diag}} = U^T f' U$ and then take the matrix

$$f = U_{\text{TB}}^* f'_{\text{diag}} U_{\text{TB}}^\dagger = U_{\text{TB}}^* U^T f' U U_{\text{TB}}^\dagger \tag{2.8}$$

As a result the matrices f and f' are related by a change of the charged lepton basis induced by the unitary matrix $O = U U_{\text{TB}}^\dagger$ (in SO(10) the matrix O rotates the whole fermion representations $\mathbf{16_i}$). Since TB mixing is a good approximation to the data we argue that this basis is a good starting point. In other words, for the physical neutrino mass matrix, TB mixing is a good approximation so that the unitary transformation O is close to the identity. In this basis the deviations from TB mixing will be generated by the diagonalisation of charged leptons which, in order to agree with the data, must be small. At the same time also the quark mixings must be small in order to correspond to the data.

In the selected basis, the parameterisations adopted for the matrices h (symmetric) and h' (antisymmetric) are given by:

$$h = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{12} & h_{22} & h_{23} \\ h_{13} & h_{23} & Y \end{pmatrix}, \qquad (2.9)$$

and

$$h' = i \begin{pmatrix} 0 & \sigma_{12} & \sigma_{13} \\ -\sigma_{12} & 0 & \sigma_{23} \\ -\sigma_{13} & -\sigma_{23} & 0 \end{pmatrix}, \qquad (2.10)$$

The h_{33} element of h has been emphasised with the special notation Y because usually it is the dominant term: $Y \gg h_{ij}$. All the matrix elements f_{ij}, Y, h_{ij} and σ_{ij}, as well as r_a and c_b that appear in eq. (2.2), will be taken as real. This leads to a crucial economy of parameters (justified by the fact that, as we shall see, the resulting fit is very good) and can be seen to correspond to an underlying "parity" symmetry [60] that implies that all mass matrices obtained from h, h' and f are hermitian. Note that, due to the imaginary unit factor i in front of the h' matrix, this purely imaginary matrix will in general induce CP violation in the quark and lepton sectors.

The charged fermion mass matrices are given by:

$$M_u = v_u \begin{pmatrix} r_2 f_2 + h_{11} & r_2 f_1 + h_{12} + i r_3 \sigma_{12} & r_2 f_1 + h_{13} + i r_3 \sigma_{13} \\ r_2 f_1 + h_{12} - i r_3 \sigma_{12} & r_2(f_0 + f_2) + h_{22} & r_2(-f_0 + f_1) + h_{23} + i r_3 \sigma_{23} \\ r_2 f_1 + h_{13} - i r_3 \sigma_{13} & r_2(-f_0 + f_1) + h_{23} - i r_3 \sigma_{23} & Y + r_2(f_0 + f_2) \end{pmatrix},$$
$$(2.11)$$

$$M_d = \frac{r_1 v_u}{\tan \beta} \begin{pmatrix} f_2 + h_{11} & f_1 + h_{12} + i \sigma_{12} & f_1 + h_{13} + i \sigma_{13} \\ f_1 + h_{12} - i \sigma_{12} & f_0 + f_2 + h_{22} & -f_0 + f_1 + h_{23} + i \sigma_{23} \\ f_1 + h_{13} - i \sigma_{13} & -f_0 + f_1 + h_{23} - i \sigma_{23} & Y + f_0 + f_2 \end{pmatrix}, \qquad (2.12)$$

$$M_e = \frac{r_1 v_u}{\tan \beta} \begin{pmatrix} -3 f_2 + h_{11} & -3 f_1 + h_{12} + i c_e \sigma_{12} & -3 f_1 + h_{13} + i c_e \sigma_{13} \\ -3 f_1 + h_{12} - i c_e \sigma_{12} & -3(f_0 + f_2) + h_{22} & -3(-f_0 + f_1) + h_{23} + i c_e \sigma_{23} \\ -3 f_1 + h_{13} - i c_e \sigma_{13} & -3(-f_0 + f_1) + h_{23} - i c_e \sigma_{23} & Y - 3(f_0 + f_2) \end{pmatrix},$$
$$(2.13)$$

where $\tan \beta = v_u / v_d$, with $v_{u,d}$ being the vacuum expectation values of $H_{u,d}$, and M_u, M_d and M_e refer to up, down and charged leptons, respectively. Note that all mass matrices are hermitian, hence each of them can be diagonalised by a unitary matrix. We have 6 real parameters in h, 3 in h', 3 in f plus $r_1/tan\beta$, r_2, r_3, c_e and the ratio v_L/v_u, or a total of 17 parameters. The (in principle) measurable quantities are 12 fermion masses, 6 mixing angles, 2 CP violating phases (we do not include Majorana phases) or a total of 20. Only 18 of these observables have been measured (the PNMS phase and one of the neutrino masses are unconstrained). In the following more predictive versions of the model with less free parameters will also be considered (for example, the JK2 model in section 6).

3 Observables and parameters

Starting from the neutrino mass matrix in eqs. (2.4), (2.7) we obtain:

$$m_{\nu 1} = (f_2 - f_1)v_L, \quad m_{\nu 2} = (f_2 + 2f_1)v_L, \quad m_{\nu 3} = (f_2 - f_1 + 2f_0)v_L. \qquad (3.1)$$

The oscillation frequencies are given by:

$$\Delta m_{\text{sol}}^2 = m_{\nu 2}^2 - m_{\nu 1}^2 = 3f_1(f_1 + 2f_2)v_L^2$$
$$\Delta m_{\text{atm}}^2 = m_{\nu 3}^2 - m_{\nu 1}^2 = 4f_0(f_0 + f_2 - f_1)v_L^2 \qquad (3.2)$$

and their ratio r is:

$$r = \frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2} = \frac{3f_1(f_1 + 2f_2)}{4f_0(f_0 + f_2 - f_1)} \qquad (3.3)$$

The experimental smallness of $r \sim 1/30$ suggests that f_1/f_0 is small, $|f_1/f_0| \sim 0.1 - 0.2$ (barring an accidental cancellation with $f_1 \sim -2f_2$).

The deviations of the leptonic mixing angles from the TB values are induced by the diagonalisation of the charged lepton mass matrix given in eq. (2.13). As M_e is hermitian it is diagonalised by a unitary transformation:

$$M_e = U_e M_e^{\text{diag}} U_e^\dagger \qquad (3.4)$$

where M_e^{diag} stands for a diagonal matrix with real non-negative elements m_e, m_μ, m_τ and U_e is the relevant unitary matrix. Any unitary matrix can in general be written as (see, for example, [18]):

$$U = e^{i\phi_0} \text{diag}(e^{i(\phi_1 + \phi_2)}, e^{i\phi_2}, 1)\tilde{U}\text{diag}(e^{i(\phi_3 + \phi_4)}, e^{i\phi_4}, 1), \qquad (3.5)$$

where ϕ_i (i=0,...,4) run from 0 to 2π and $\tilde{U}$ is the standard parameterization for the CKM mixing matrix, namely

$$\tilde{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad (3.6)$$

where all the mixing angles belong to the first quadrant and δ to $[0, 2\pi]$. An approximate form of m_e that follows from eq. (3.4) for $m_\tau \gg m_\mu \gg m_e$, in a linear approximation in the small mixing angles unless non linear terms are rescued by large mass factors, is given by:

$$M_e = \begin{pmatrix} m_e + m_\mu s_{12}^{e2} + m_\tau s_{13}^{e2} & (m_\mu s_{12}^e + m_\tau s_{13}^e s_{23}^e e^{i\delta^e})e^{i\phi_1^e} & m_\tau s_{13}^e e^{i(\delta^e + \phi_1^e + \phi_2^e)} \\ (m_\mu s_{12}^e + m_\tau s_{13}^e s_{23}^e e^{-i\delta^e})e^{-i\phi_1^e} & m_\mu + m_\tau s_{23}^{e2} & m_\tau s_{23}^e e^{i\phi_2^e} \\ m_\tau s_{13}^e e^{-i(\delta^e + \phi_1^e + \phi_2^e)} & m_\tau s_{23}^e e^{-i\phi_2^e} & m_\tau \end{pmatrix}. \qquad (3.7)$$

Going back to eq. (2.13) and similar ones for the heavier generations, in the above approximation, we can write down simple analytic expressions for the mass eigenvalues (except for the first generation mass which is fine tuned in this class of models) and the mixing

angles and phases. Note, however, that exact expressions are used in our numerical fits. Starting from the 2-3 family sector we have:

$$m_\tau \approx k[Y - 3(f_0 + f_2)] \tag{3.8}$$
$$m_\mu \approx k[h_{22} - 3(f_0 + f_2)] - m_\tau s_{23}^{e2}$$
$$s_{23}^e e^{i\phi_2^e} \approx \frac{k}{m_\tau}[h_{23} + 3(f_0 - f_1) + ic_e\sigma_{23}].$$

with

$$k = \frac{r_1 v_u}{\tan\beta} \tag{3.9}$$

We also obtain:

$$s_{13}^e e^{i(\delta^e + \phi_1^e + \phi_2^e)} \approx \frac{k}{m_\tau}[h_{13} - 3f_1 + ic_e\sigma_{13}] \tag{3.10}$$

The following expression for s_{12}^e is less accurate but, in general, still sufficiently good, at least for indicative purposes. It is obtained from the 11 entries of the matrices in eqs. (2.13), (3.7) by neglecting m_e:

$$s_{12}^e \approx \sqrt{\frac{k}{m_\mu}(h_{11} - 3f_2) - \frac{m_\tau}{m_\mu}s_{13}^{e2}} \tag{3.11}$$

A remaining phase can be derived from the equation (obtained from the 12 entries of the matrices in eqs. (2.13), (3.7):

$$(m_\mu s_{12}^e + m_\tau s_{13}^e s_{23}^e e^{i\delta^e})e^{i\phi_1^e} \approx k(h_{12} - 3f_1 + ic_e\sigma_{12}) \tag{3.12}$$

From fitting the data one finds that indeed Y is the largest parameter, followed by h_{13}, h_{22}, h_{23}, σ_{13}, σ_{23} and f_0 while f_1, f_2, h_{11}, h_{12} and σ_{12} are still smaller.

In a linear approximation in the leptonic mixing angles s_{ij}^e the corrections to $U_{\rm TB}$ from the diagonalization of charged leptons are given by (for each matrix element we omit an overall phase):

$$U_{12} \approx \frac{1}{\sqrt{3}}(1 - s_{12}^e e^{i\phi_1^e} - s_{13}^e e^{i(\delta^e + \phi_1^e + \phi_2^e)}) \tag{3.13}$$
$$U_{13} \approx \frac{1}{\sqrt{2}}(s_{12}^e - s_{13}^e e^{i(\delta^e + \phi_2^e)})$$
$$U_{23} \approx \frac{-1}{\sqrt{2}}(1 + s_{23}^e e^{i\phi_2^e}).$$

The corresponding corrected mixing angles are:

$$\sin\theta_{12} = |U_{12}| \approx \frac{1}{\sqrt{3}}(1 - s_{12}^e \cos\phi_1^e - s_{13}^e \cos(\delta^e + \phi_1^e + \phi_2^e)) \tag{3.14}$$
$$\theta_{13} = |U_{13}| \approx \frac{1}{\sqrt{2}}\sqrt{s_{12}^{e2} + s_{13}^{e2} - 2s_{12}^e s_{13}^e \cos(\delta^e + \phi_2^e)}$$
$$\sin\theta_{23} = |U_{23}| \approx \frac{1}{\sqrt{2}}(1 + s_{23}^e \cos\phi_2^e).$$

It turns out, however, that the above linear approximations are often not sufficiently accurate because the leptonic mixing angles s_{ij}^e are not small enough. This is particularly true for $\sin \theta_{12}$ and $\sin \theta_{13}$. So the above linearised formulae are only given for indicative purposes and, in our fit, we used the exact expressions for the mixing, obtained from $U_e^\dagger U_{\rm TB}$ from eqs. (3.5), (3.6). But the approximate analytic formulae are useful to understand the need of fine tuning to reproduce the observed masses for the light generations of leptons and the neutrino masses and mixing. We see in fact from eqs. (3.2), (3.3) that neutrino masses impose a strong constraint on the values of f_i. But the same f_i enter in the charged lepton masses and the leptonic mixing angles eqs. (3.9), (3.10), (3.11) and they must conspire with the h_{ij} and σ_{ij} parameters in order to reproduce the observed values. And the same is also true for the quark masses that we now discuss.

In the quark sector the approximate formulae for the 2-3 families, analoguous to eqs. (3.9) for charged leptons, are derived from eqs. (2.11), (2.12):

$$m_b \approx k[Y + f_0 + f_2] \tag{3.15}$$
$$m_s \approx k[h_{22} + f_0 + f_2] - m_b s_{23}^{d2}$$
$$s_{23}^d e^{i\phi_2^d} \approx \frac{k}{m_\tau}[h_{23} - f_0 + f_1 + i\sigma_{23}].$$

$$m_t \approx v_u[Y + r_2(f_0 + f_2)] \tag{3.16}$$
$$m_c \approx v_u[r_2 f_0 + r_2 f_2 + h_{22}] - m_t s_{23}^{u2}$$
$$s_{23}^u e^{i\phi_2^u} \approx \frac{v_u}{m_\tau}[h_{23} + r_2(f_1 - f_0) + i r_3 \sigma_{23}].$$

Note that, as $Y \gg |f_i|$, $m_b = m_\tau$ approximately holds at the GUT scale (bottom-tau unification). Relations analogous to eqs. (3.10), (3.11), (3.12) can be readily written down for the up and down quark sectors. For the CKM mixing matrix we go back to eq. (3.5) and derive U_u and U_d that diagonalise m_u and m_d, given in eqs. (2.11), (2.12), respectively. We then construct $V_{\rm CKM} = U_u^\dagger U_d$.

4   A set of idealized data at the GUT scale

For the charged fermion masses and CKM mixings at the GUT scale we used the values given in table 1(a)-table 1(b) as input for our fitting procedure. These values are based on the analysis of ref. [72], in the MSSM framework, and were obtained from a two-loop Renormalization Group Evolution (RGE) from the SUSY scale at about $1\ TeV$ to the GUT scale $\sim 2 \times 10^{16}\ GeV$. Ref. [72] can be considered as an upgrade of a previous work [71]. This analysis has been repeated for two different values of the supersymmetric (SUSY) parameter $\tan \beta$: $\tan \beta = 10$ and $\tan \beta = 55$. In fact $\tan \beta$ is the most important SUSY parameter that directly affects the RGE. So these tables have been used to fit two typical classes of models with small or large $\tan \beta$. Actually for both values of $\tan \beta$ we have used in the fit an updated version of the data which was derived in refs. [59, 73] shown in table 1. In particular, refs. [59, 73] use more recent values for m_u, m_d and m_s at low energy with respect to the ones used in ref. [72]. The errors taken on the data

Observables	Input data		Observables	Values
$m_u[MeV]$	0.55 ± 0.25		$m_u[MeV]$	0.45 ± 0.2
$m_c[MeV]$	210 ± 21		$m_c[MeV]$	217 ± 35
m_t [GeV]	$82.4^{+30.3}_{-14.8}$		$m_t[GeV]$	97 ± 38
$m_d[MeV]$	1.24 ± 0.41		$m_d[MeV]$	1.3 ± 0.6
$m_s[MeV]$	21.7 ± 5.2		$m_s[MeV]$	23 ± 6
$m_b[GeV]$	$1.06^{+0.14}_{-0.09}$		$m_b[GeV]$	1.4 ± 0.6
$m_e[MeV]$	0.3585 ± 0.0003		$m_e[MeV]$	0.3565 ± 0.001
$m_\mu[MeV]$	75.672 ± 0.058		$m_\mu[MeV]$	75.3 ± 0.12
$m_\tau[GeV]$	1.2922 ± 0.0013		$m_\tau[GeV]$	1.629 ± 0.037
V_{us}	0.2243 ± 0.0016		V_{us}	0.2243 ± 0.0016
V_{cb}	0.0351 ± 0.0013		V_{cb}	0.0351 ± 0.0013
V_{ub}	0.0032 ± 0.0005		V_{ub}	0.0032 ± 0.0005
$J \times 10^{-5}$	2.2 ± 0.6		$J \times 10^{-5}$	2.2 ± 0.6
(a) $tg\beta = 10$			(b) $\tan\beta = 55$	

Table 1. GUT scale data for charged fermions for $tg\beta = 10$ (ref. [59]) and $\tan\beta = 55$ (ref. [73])

points at the GUT scale are those shown in ref. [59] or in ref. [73] and do not include an estimate of the additional ambiguities associated with the chosen procedure. For example, the SUSY spectrum parameters, which mainly enter in the evolution through the threshold corrections, are subject to considerable ambiguities [74–76]. Moreover at the GUT scale other threshold corrections appear, but they are again affected by large uncertainties and model dependent. As we are only interested in defining a set of idealized data in order to make a meaningful performance test for a number of different models, we decided to ignore the uncertainties on both the SUSY- and GUT-scale threshold corrections. As already discussed we are not much concerned with these uncertainties, rather we prefer to fit all models on the same set of data, arguing that if a particular model is better than another in fitting these idealized data it would also score better on the real data if they were known.

Concerning the neutrino sector, we have ignored the effects of the evolution from the low energy scale to the GUT scale. In fact for all the models analysed here the mass spectrum is non degenerate, so that the evolution can be considered negligible to a good approximation [79]. The low energy data are taken from [8, 9] and the corresponding values are given in table 2.

5 Fitting procedure and quality factors

As already discussed, the class of models T-IID described in section 2 contains a total of 17 independent parameters. We are going to compare these models with the set of 18 "measurements" described in the previous section. As shown in eq. (2.12), (2.13) the parameter $\tan\beta$ always enters in the combination $r_1/\tan\beta$ in all the observables. So in these models it is possible to obtain a large ratio m_t/m_b without making $\tan\beta$ large

Observable	Input data
$\Delta m_{21}^2 \times 10^{-5} [eV^2]$	7.65 ± 0.23
$\Delta m_{31}^2 \times 10^{-3} [eV^2]$	2.40 ± 0.12
$sin^2\theta_{13}$	0.010 ± 0.016
$sin^2\theta_{12}$	0.304 ± 0.022
$sin^2\theta_{23}$	0.50 ± 0.07

Table 2. Neutrino masses and mixing ([8, 9])

(in fact, large $\tan\beta$ can be problematic in the MSSM with a GUT Yukawa $t - b - \tau$ unification [77, 78]). Here we are going to fit the model presented in section 2 on the data in table 1(a), which refer to the $\tan\beta = 10$ case. We use the numerical minimization tool Minuit2 developed at CERN.

We introduce a parameter d_{FT} for a quantitative measure of the amount of fine-tuning needed in the models. This adimensional quantity is obtained as the sum of the absolute values of the ratios between each parameter and its "error", defined for this purpose as the shift from the best fit value that changes χ^2 by one unit, with all other parameters fixed at their best fit values (this is not the error given by the fitting procedure because in that case all the parameters are varied at the same time and the correlations are taken into account):

$$d_{\mathrm{FT}} = \sum \left| \frac{\mathrm{par}_i}{\mathrm{err}_i} \right| \tag{5.1}$$

It is clear that d_{FT} gives a rough idea of the amount of fine-tuning involved in the fit because if some $|\mathrm{err}_i/\mathrm{par}_i|$ are very small it means that it takes a minimal variation of the corresponding parameters to make a large difference on χ^2. The value of d_{FT} for our best fit output is shown in table 3(a). To get a better idea of the significance of this number, one can compare it with a similar number d_{Data} based on the data, i.e. the sum of the absolute values of the ratios between each observable and its error as derived from the input data:

$$d_{\mathrm{Data}} = \sum \left| \frac{obs_i}{\mathrm{err}_i} \right| \tag{5.2}$$

In particular for the set of data in table 1(a) $d_{\mathrm{Data}} \sim 3800$ and for table 1(b) $d_{\mathrm{Data}} \sim 1300$.

The best fit results for the models T-IID are shown in table 3. In table 3(a) we indicate the values of the observables obtained at the GUT scale, the χ^2 calculated in each sector (quarks, charged leptons, neutrinos), the χ^2/d.o.f. (in the case of T-IID we have one degree of freedom, so the reduced χ^2/d.o.f. is the same as the χ^2) and the fine-tuning parameter d_{FT}. In table 3(b) we show the best fit parameters.

We caution that, due to the non linearity of the problem and the large number of parameters, many local minima are present in all the fits we have performed. Although we have carefully tested the selected minimum for possible improvements, still we cannot be sure that it indeed is the global minimum, within reasonable ranges for the parameters. But, since we obtain an excellent agreement with the data, we are not much concerned

Observable	Best fit value
$m_u[MeV]$	0.553
$m_c[MeV]$	210
$m_t[GeV]$	82.6
$m_d[MeV]$	1.15
$m_s[MeV]$	22.4
$m_b[GeV]$	1.08
$m_e[MeV]$	0.3585
$m_\mu[MeV]$	75.67
$m_\tau[GeV]$	1.292
V_{us}	0.224
V_{cb}	0.0351
V_{ub}	0.00320
$J \times 10^{-5}$	2.19
$\Delta m_{21}^2 \times 10^{-5}[eV^2]$	7.65
$\Delta m_{32}^2 \times 10^{-3}[eV^2]$	2.40
$sin^2\theta_{13}$	0.0126
$sin^2\theta_{12}$	0.305
$sin^2\theta_{23}$	0.499
χ^2 quark	0.0959
χ^2 charged fermions	0.0959
χ^2 neutrino	0.0316
χ^2 totale	0.127
χ^2/dof totale	0.127
d_{FT}	469777

(a)

Parameter	Best fit value
$h_{11}v_u[GeV]$	0.808
$h_{12}v_u[GeV]$	1.17
$h_{13}v_u[GeV]$	6.06
$h_{22}v_u[GeV]$	5.37
$h_{23}v_u[GeV]$	5.64
$Yv_u[GeV]$	85.0
$f_0v_u[GeV]$	-2.20
$f_1v_u[GeV]$	-0.276
$f_2v_u[GeV]$	-0.228
$\sigma_{12}v_u[GeV]$	-0.270
$\sigma_{13}v_u[GeV]$	2.27
$\sigma_{23}v_u[GeV]$	6.37
$r_1/\tan\beta$	0.0129
r_2	1.66
r_3	0.612
c_e	3.85
$v_L/v_u \times 10^{-9}$	0.0112

(b)

Table 3. Fit result for the model T-IID described in section 2

with this problem. In fact the resulting $\chi^2 \sim 0.13$ is very good. We note however that a substantial level of fine tuning is needed. We in fact obtain $d_{\mathrm{FT}} \sim 4.7 \; 10^5$ from the fit, to be compared with $d_{\mathrm{Data}} \sim 3.8 \; 10^3$. As explained in section 3, this is due to the fact that the neutrino oscillation frequency data impose strong constraints on the f_i values. Those also enter in the expressions of the mass values and the mixings of charged fermions and in the deviations from TB neutrino mixing. As a result, the strong suppression of the first generation masses and the observed values of mixing angles can only be obtained by a fine tuning among the f_i parameters with those of the **10** and **120** matrices.

An important conclusion that we can already give at this stage is that models of the T-IID type formulated in section 2 in the general framework of type-II see-saw dominance can lead at a very good fit of the data but at the price of a pronounced level of fine tuning. In the following section we will compare this class of models with some other "realistic" SO(10) models in the literature.

6 Comparison with other SO(10) models

In order to appreciate the performance of the above class of models in fitting the fermion masses and mixings we present a comparison with some other realistic SO(10) models present in literature. For a meaningful comparison we applied the same fitting procedure to each model, using the same set of data as described in the previous section. A list of realistic SO(10) theories can be found in [31]. Here we consider a number of models, with different structure and type (renormalizable, non renormalizable, lopsided or symmetric, with type-I and/or type-II see-saw and so on).

The model introduced and discussed in a series of papers by Dermisek and Raby (DR) [32, 33] is an example of non-renormalizable SO(10) theory, with Higgs multiplets in the **10**, **45** and **$\overline{16}$**, based on the flavour symmetry $S_3 \times U(1) \times Z_2 \times Z_2$. This model can be considered as a descendant of the model by Barbieri et al, [34], with the two lightest generations in a doublet of U(2) and the third generation in a singlet. In fact, S_3, the permutation group of 3 objects, is a discrete subgroup of SO(3) (and SU(2) is the covering group of SO(3)) with inequivalent irreducible representations **2**, **1** and **1'**. In the S_3 symmetry limit only the third generation masses are allowed. Then the second generation masses are generated by a symmetry breaking stage with intensity proportional to a parameter ϵ, and finally the first generation masses only arise when an additional stage with intensity ϵ' is switched on. The magnitudes of ϵ and ϵ' are determined by a Froggatt-Nielsen mechanism, which is induced by a set of heavy fields that are then integrated away. In the neutrino sector a type-I see-saw mechanism with a hierarchical Majorana mass matrix is adopted. New SO(10)-singlet neutrino and scalar fields are introduced, so that, enough freedom is allowed that is essential to reproduce the observed neutrino properties. In this model the ratio of top to bottom masses is of order $\tan\beta$, which must be large: $m_t(m_t)/m_b(m_t) \sim \tan\beta \sim 50$.

A good realisation of the lopsided idea is given by the model by Albright, Babu and Barr (ABB) [35, 36], also non-renormalizable and with type-I see-saw. In minimal SU(5) the down quark and the charged lepton mass matrices are connected by a transposition, as the roles of **$\bar{5}$** and **10** are interchanged in the respective mass matrices. So, if the mass matrices are asymmetric (lopsided), left-handed mixings of charged leptons can be large without implying large left-handed mixings for quarks (in fact, this only implies large right-handed quark mixings which are not observable). In turn large charged lepton left-handed mixings contribute to the observed large neutrino mixings. In the ABB model the breaking of SO(10) in fact preserves SU(5) in a first stage, which makes this connection with lopsid-edness relevant. This model is based on a flavour symmetry $U(1) \times Z_2 \times Z_2$, which, however, plays a different role than in the DR model. In fact, while in the DR model the flavour symmetry and its breaking are mainly used to reproduce the hierarchy of fermion masses and mixings, in the ABB construction, the main goal is to select the Lagrangian terms which are desired and reject those that would not reproduce the data. In particular, in the Higgs sector the symmetry is crucial in order to implement and preserve the mechanism of Dimopoulos and Wilczek for the solution of the doublet-triplet splitting problem [37]. As is often the case in non renormalizable models, the Higgs sector is inspired by a minimality

requirement that demands the smallest possible representations compatible with realistic properties. Accordingly the Higgs sector contains **10**, **16** + $\overline{\mathbf{16}}$, **45** representations and a few SO(10) singlets. The ABB model was originally formulated when the neutrino frequencies and mixing angles were not as precisely measured as now. Later the lepton sector has been revised in ref. [36] with some ad hoc arbitrary ingredients and we have adopted this last description here as it can be fitted to the present data.

We have also considered a variation of this model proposed by Ji, Li, Mohapatra (JLM) [38] which was motivated by a less ad-hoc treatment of the neutrino sector. Precisely, the down quark and the charged lepton mass matrices are the same as in ABB, while the up and Dirac neutrino mass matrices are modified by introducing some new vertices. The structure of neutrino mixing is not attributed to the Majorana matrix, which in this model is diagonal, but rather to the modified Dirac matrix. The model is again based on type-I see-saw and the new added operators introduce a sufficient number of new free parameters to accommodate the neutrino mixing angles.

Turning now to renormalizable models we have analysed the model referred to as BSV in table 4. In this version one is introducing the minimal Higgs content in the Yukawa sector (**10**, $\overline{\mathbf{126}}$). This minimal model has been discussed in ref. [42, 43] and compared with the data in ref. [59] (where also the cases of type-I and mixed type-I and type-II were considered). A different perspective on this model is presented in ref. [48], also including some comparison with the data (with mixed type-I and type-II see-saw). With this restricted Higgs content one cannot impose the L-R parity that leads to real hermitian h and f matrices, otherwise there is no CP violation and the too restricted number of parameters does not allow a good fit. Thus in ref. [59] complex h and f matrices were taken. As a consequence there are more parameters than observables. We have repeated the fit of that model within our procedure and the results are listed in table 4. We see from our fit in table 4 that, in spite of this multitude of parameters, with type-II dominance, in the absence of the **120**, no good fit of the data can be obtained.

Another particular class of renormalizable models with type-II see-saw dominance has been discussed by Joshipura and Kodrani (JK) [67]. This model comes in two versions, one with type-I dominance (JK1) and one with type-II dominance (JK2). The Higgs coupled to fermions are in **10**, $\overline{\mathbf{126}}$ and **120**, like in the T-IID model. The characteristic feature of the model is the presence of a broken $\mu - \tau$ symmetry (an explicit breaking is present in the **10** mass terms) in addition to the parity symmetry which leads to hermitian mass matrices. In the case of type-II see-saw dominance this model is a particular case of T-IID with some restrictions on the parameters imposed by the ansatz of broken $\mu - \tau$ symmetry.

The renormalizable model proposed by Grimus and Kuhbock [70] is of particular interest for us because it closely corresponds to the T-IID model (the fermions masses arise from Higgs in **10**, $\overline{\mathbf{126}}$ and **120** and the parity symmetry of ref. [60] is assumed) except for the fact that it is based on type-I see-saw dominance. There is one more parameter, v_R (see eq. (2.3)), than in the model T-IID. The only other difference is that this model was fitted in the basis where the matrix h from the **10** is diagonal and real, while for the T-IID we worked in the TB basis for the matrix f of the $\overline{\mathbf{126}}$.

Model	d.o.f.	χ^2	$\chi^2/$d.o.f.	d_{FT}	d_{Data}
DR [32, 33]	4	0.41	0.10	$7.0\ 10^3$	$1.3\ 10^3$
ABB [35–37]	6	2.8	0.47	$8.1\ 10^3$	$3.8\ 10^3$
JLM [38]	4	2.9	0.74	$9.4\ 10^3$	$3.8\ 10^3$
BSV [59]	< 0	6.9	-	$2.0\ 10^5$	$3.8\ 10^3$
JK2 [67]	3	3.4	1.1	$4.7\ 10^5$	$3.8\ 10^3$
GK [70]	0	0.15	-	$1.5\ 10^5$	$3.8\ 10^3$
T-IID	1	0.13	0.13	$4.7\ 10^5$	$3.8\ 10^3$

Table 4. Comparison of different SO(10) models fitted to the data. The double lines mark three sectors: the DR model, non renormalizable with type-I see-saw and the only one with large $\tan\beta$, the non renormalizable models ABB and JLM, lopsided with type-I see-saw and the renormalizable models BSV, JK2, T-IID, with type-II see-saw, and GK, renormalizable with type-I see-saw

The model by Dermisek and Raby is the only one that demands a large value of $\tan\beta$, so we fit it on the data in table 1(b), while all other models are fitted on the values in table 1(a). A collection of our results on comparing the different models is shown in table 4. In appendix A we show the mass matrices of the different models in terms of the corresponding parameters and the results of the fitting procedure.

The results in table 4 are in some cases somewhat different than those from the fits described by the authors of the various models. One obvious reason is that the set of the input data for the fit is different, as explained in section 4. In some cases (e.g. for JK2 and GK) the difference also comes from the fact that, for hermitian matrices, we fitted the eigenvalues of m_i, that is of the mass matrices and not those of $m_i^\dagger m_i$. Indeed, from fitting the squares of masses one can end up with solutions of negative mass which we discard.

From the results in table 4 we see that the most established realistic SO(10) models, DR, ABB, JLM, which are non renormalizable with type-I see-saw, achieve a $\chi^2/$d.o.f. smaller than 1 with a moderate level of fine tuning, defined by the parameter d_{FT}. In the above list of models DR is special as it has large $\tan\beta$, so that it was fitted to a different set of data, given in the right panel of table 1, which were obtained by evolving up to the GUT scale with large $\tan\beta$. The model T-IID which we have introduced and described in section 2, realizes an excellent fit, but with a level of fine tuning considerably larger than in the DR, ABB and JLM models. As already mentioned, this large fine tuning arises from the difficulty of fitting the light 1st generation charged fermion masses, together with the neutrino oscillation frequencies and mixing angles. In fact the neutrino oscillation frequencies and mixing angles lead to f matrix elements of comparable size that need cancelations to occur with the parameters in h and h' in order to reproduce the light quark and lepton masses. Note that in the DR, ABB, JLM the fine tuning is less pronounced because in all these models new parameters appear in the neutrino sector, so that the neutrino masses and mixings are more independent from the charged fermion sectors. In the JK2 model the constraints from the broken $\mu - \tau$ symmetry reduce the number of parameters within the general framework of the T-IID model. As a consequence

the quality of the fit worsens and the level of fine tuning is the same. Instead the poor result of the BSV model, both in terms of χ^2 and of d_{FT} shows that the presence of the **120** is crucial [49–51, 57, 58]. In ref. [44–47, 53, 54, 59] it was also shown that the contribution from the **120** is essential for consistency with the existing proton decay bounds. Finally the GK model shows that if one takes the same framework of T-IID, except that the assumption of type-I see-saw dominance is made, an excellent fit is also obtained, still with a substantial level of fine tuning.

We conclude this section by recalling that in this work we have compared the models only on the basis of their ability to fit the fermion masses and mixings. Clearly for a model to be complete and satisfactory many more aspects are important, like the mechanism of the GUT symmetry breaking, the running of the couplings towards the GUT unification, the compatibility of the model with the proton decay bounds and so on (see ref. [53, 54] and refs therein). Also, for the models based on type-II see-saw dominance we assumed here that the dominance is absolute, while in reality one should estimate the corrections to this approximation or include the normal type-I see-saw terms [59]. Additional quality factors could be considered. In models with an underlying flavour symmetry the smallness of some of the parameters is guaranteed by the broken flavour symmetry. Thus an important quality factor is the percentage of small quantities that are predicted to be small. For some of the models there is a broken flavour symmetry that reproduces, at least in part, the observed hierarchies. But this is not the case for T-IID where the problem of an underlying flavour symmetry was not addressd here.

7 Summary and conclusion

We have made a quantitative comparison of the performance of different types of SO(10) Grand Unification Theories (GUT's) in reproducing the observed values of fermion masses and mixing, also including the neutrino sector. All models, chosen among the most complete and sufficiently realistic, have been confronted with the same set of data (except for DR that requires a large value of $\tan\beta$), using the same fitting procedure. We have shown that a SO(10) model with type-II see-saw dominance can achieve a very good fit of fermion masses and mixings also including the neutrino sector (provided that the representations **10**, **$\overline{126}$** and **120** are all included). The quality of the fit in terms of χ^2 and χ^2/d.o.f. is comparable with the best realistic SO(10) model that we have tested. However, the tight structure of the T-IID model implies a significantly larger amount of fine tuning with respect to more conventional models like the DR or the ABB and JLM models. But those models have no built-in TB mixing and in fact could accommodate a wide range of mixing angle values. A model with type-II see-saw dominance can offer a convenient framework for obtaining a GUT model with approximate TB mixing in the neutrino sector. For this goal a suitable flavour symmetry should be introduced in order to enforce TB mixing as a first approximation. Such a flavour symmetry construction was attempted in ref. [64] but the resulting model is very sketchy and can only be considered as a first step. In fact, when the model of ref. [64] is submitted to our fitting procedure it leads to $\chi^2 \sim 342$, χ^2/d.o.f ~ 38 and $d_{\text{FT}} \sim 1.2\ 10^4$ which is quite far from the performance of all models that we

considered. The formulation of a natural and elegant model along these lines is not an easy task and more work is demanded.

Acknowledgments

A   Model parameters and their best fit values

In this section we show the Yukawa matrices for every model that we have analysed together with the parameter values obtained from our fitting procedure. $M_{u/d/e/\nu}$ are the Dirac matrices for up-quarks/down-quarks/charged leptons/neutrinos. $M_{R/L}$ are the Majorana matrices for the right/left neutrinos.

- Model DR

$$M_u = \begin{pmatrix} 0 & \epsilon' \rho & -\epsilon\,\varepsilon \\ -\epsilon' \rho & \tilde{\epsilon}\,\rho & -\epsilon \\ \epsilon\,\varepsilon & \epsilon & 1 \end{pmatrix} \lambda \sin\beta$$

$$M_d = \begin{pmatrix} 0 & \epsilon' & -\epsilon\,\varepsilon\,\sigma \\ -\epsilon' & \tilde{\epsilon} & -\epsilon\,\sigma \\ \epsilon\,\varepsilon & \epsilon & 1 \end{pmatrix} \lambda \cos\beta$$

$$M_e = \begin{pmatrix} 0 & -\epsilon' & 3\,\epsilon\,\varepsilon \\ -\epsilon' & 3\,\tilde{\epsilon} & 3\,\epsilon \\ -3\,\epsilon\,\varepsilon\,\sigma & -3\,\epsilon\,\sigma & 1 \end{pmatrix} \lambda \cos\beta$$

$$M_\nu = \begin{pmatrix} 0 & -\epsilon'\,\omega & \frac{3}{2}\,\epsilon\,\varepsilon\,\omega \\ -\epsilon'\,\omega & 3\,\tilde{\epsilon}\,\omega & \frac{3}{2}\,\epsilon\,\omega \\ -3\,\epsilon\,\varepsilon\,\sigma & -3\,\epsilon\,\sigma & 1 \end{pmatrix} \lambda \sin\beta$$

$$M_R = \begin{pmatrix} M_{R_1} & 0 & 0 \\ 0 & M_{R_2} & 0 \\ 0 & 0 & M_{R_3} \end{pmatrix}$$

Parameter	Best fit value
λ	0.464
$\lambda\epsilon$	-0.0215
$|\sigma|$	0.256
δ_σ	0.120
$|\rho|$	0.0565
δ_ρ	-1.59
$\lambda|\tilde{\epsilon}|$	0.00656
$\delta_{\tilde{\epsilon}}$	-0.57
$\lambda\epsilon'$	-0.00157
$\lambda|\varepsilon|$	0.00286
δ_ε	-2.45
$M_{R_1} \times 10^{13}[GeV]$	0.000226
$M_{R_2} \times 10^{13}[GeV]$	0.0144
$M_{R_3} \times 10^{13}[GeV]$	-17.5

As specified by the authors of the model we have taken $\tan\beta = 50.34$. Some of the parameters are complex and, in these cases, we have set $z = |z|e^{i\delta_z}$ and all the phases are expressed in radiants. The quantity ω is given by $\omega = \frac{2\sigma}{2\sigma-1}$.

- Model ABB

$$M_u = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} M_U$$

$$M_d = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma + \epsilon/3 \\ \delta' e^{i\phi} & -\epsilon/3 & 1 \end{pmatrix} M_D$$

$$M_e = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon \\ \delta' e^{i\phi} & \sigma + \epsilon & 1 \end{pmatrix} M_D$$

$$M_\nu = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} M_U$$

$$M_R = \begin{pmatrix} c^2\eta^2 & -b\epsilon\eta & a\eta \\ -b\epsilon\eta & \epsilon^2 & -\epsilon \\ a\eta & -\epsilon & 1 \end{pmatrix} \Lambda_R$$

Parameter	Best fit value
ϵ	0.140
$M_U[GeV]$	95.5
η	0.00000576
σ	1.72
$M_D[GeV]$	0.612
δ	0.00795
δ'	-0.00836
ϕ	-1.12
a	-1.48
b	-2.55
c	2.95
$\Lambda_R \times 10^{14}[GeV]$	2.04

- Model JLM

$$M_u = \begin{pmatrix} \eta & 0 & k+\rho/3 \\ 0 & 0 & \omega \\ k-\rho/3 & \omega & 1 \end{pmatrix} M_U$$

$$M_d = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma + \epsilon/3 \\ \delta' e^{i\phi} & -\epsilon/3 & 1 \end{pmatrix} M_D$$

$$M_e = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon \\ \delta' e^{i\phi} & \sigma + \epsilon & 1 \end{pmatrix} M_D$$

$$M_\nu = \begin{pmatrix} \eta & 0 & k-\rho \\ 0 & 0 & \omega \\ k+\rho & \omega & 1 \end{pmatrix} M_U$$

$$M_R = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{pmatrix} \Lambda_R$$

Parameter	Best fit value
ϵ	0.144
$M_U[GeV]$	87.6
η	0.00000739
σ	1.81
$M_D[GeV]$	0.589
δ	0.00991
δ'	0.0141
ϕ	0.468
ω	-0.0451
ρ	0.00899
k	0.0188
a	-0.00159
b	-0.00193
$\Lambda_R \times 10^{13}[GeV]$	2.07

- Model BSV

$$M_u = v_u \begin{pmatrix} r_2 f_2 + h_{11} & r_2 f_1 + h_{12} & r_2 f_1 + h_{13} \\ r_2 f_1 + h_{12} & r_2(f_0 + f_2) + h_{22} & r_2(-f_0 + f_1) + h_{23} \\ r_2 f_1 + h_{13} & r_2(-f_0 + f_1) + h_{23} & Y + r_2(f_0 + f_2) \end{pmatrix}$$

$$M_d = \frac{r_1 v_u}{\tan \beta} \begin{pmatrix} f_2 + h_{11} & f_1 + h_{12} & f_1 + h_{13} \\ f_1 + h_{12} & f_0 + f_2 + h_{22} & -f_0 + f_1 + h_{23} \\ f_1 + h_{13} & -f_0 + f_1 + h_{23} & Y + f_0 + f_2 \end{pmatrix}$$

$$M_e = \frac{r_1 v_u}{\tan \beta} \begin{pmatrix} -3f_2 + h_{11} & -3f_1 + h_{12} & -3f_1 + h_{13} \\ -3f_1 + h_{12} & -3(f_0 + f_2) + h_{22} & -3(-f_0 + f_1) + h_{23} \\ -3f_1 + h_{13} & -3(-f_0 + f_1) + h_{23} & Y - 3(f_0 + f_2) \end{pmatrix}$$

$$M_L = \begin{pmatrix} f_2 & f_1 & f_1 \\ f_1 & f_2 + f_0 & f_1 - f_0 \\ f_1 & f_1 - f_0 & f_2 + f_0 \end{pmatrix} v_L$$

Parameter	Best fit value
$|h_{11}|v_u[GeV]$	1.42
$\delta_{h_{11}}$	-0.496
$|h_{12}|v_u[GeV]$	0.435
$\delta_{h_{12}}$	3.09
$|h_{13}|v_u[GeV]$	10.7
$\delta_{h_{13}}$	-0.614
$|h_{22}|v_u[GeV]$	0.793
$\delta_{h_{22}}$	1.22
$|h_{23}|v_u[GeV]$	3.20
$\delta_{h_{23}}$	2.80
$|Y|v_u[GeV]$	79.9
δ_Y	-0.732
$f_0 v_u[GeV]$	-2.13
$f_1 v_u[GeV]$	0.369
$f_2 v_u[GeV]$	0.110
$r_1/\tan \beta$	0.0148
$|r_2|$	0.515
δ_{r_2}	1.52
$v_L/v_u \times 10^{-9}$	0.0108

Here again the complex parameters are understood as $z = |z|e^{i\delta_z}$ and the phases are in radiants.

- Model JK2

$$M_u = \begin{pmatrix} rh_{11} + sf_{11} & rh_{12} + sf_{12} + itg_{12} & rh_{12} + sf_{12} - itg_{12} \\ rh_{12} + sf_{12} - itg_{12} & rh_{22} + sf_{22} & rh_{23} + sf_{23} + itg_{23} \\ rh_{12} + sf_{12} + itg_{12} & rh_{23} + sf_{23} - itg_{23} & rh_{33} + sf_{22} \end{pmatrix} v$$

$$M_d = \begin{pmatrix} h_{11} + f_{11} & h_{12} + f_{12} + ig_{12} & h_{12} + f_{12} - ig_{12} \\ h_{12} + f_{12} - ig_{12} & h_{22} + f_{22} & h_{23} + f_{23} + ig_{23} \\ h_{12} + f_{12} + ig_{12} & h_{23} + f_{23} - ig_{23} & h_{33} + f_{22} \end{pmatrix} v$$

$$M_e = \begin{pmatrix} h_{11} - 3f_{11} & h_{12} - 3f_{12} + ipg_{12} & h_{12} - 3f_{12} - ipg_{12} \\ h_{12} - 3f_{12} - ipg_{12} & h_{22} - 3f_{22} & h_{23} - 3f_{23} + ipg_{23} \\ h_{12} - 3f_{12} + ipg_{12} & h_{23} - 3f_{23} - ipg_{23} & h_{33} - 3f_{22} \end{pmatrix} v$$

$$M_L = \begin{pmatrix} f_{11} & f_{12} & f_{12} \\ f_{12} & f_{22} & f_{23} \\ f_{12} & f_{23} & f_{22} \end{pmatrix} r_L$$

Parameter	Best fit value
$h_{11}v[GeV]$	0.00204
$h_{22}v[GeV]$	0.576
$h_{23}v[GeV]$	0.120
$h_{33}v[GeV]$	0.619
$f_{11}v[GeV]$	-0.000960
$f_{12}v[GeV]$	-0.00398
$f_{22}v[GeV]$	-0.0282
$f_{23}v[GeV]$	0.0381
$g_{12}v[GeV]$	0.00514
$g_{23}v[GeV]$	0.522
r	74.0
s	141
t	71.6
p	1.17
$r_L/v \times 10^{-9}$	-0.741

- Model GK

$$M_u = \begin{pmatrix} r_H h_{11} + r_F f_{11} & r_F f_{12} + ir_u g_{12} & r_F f_{13} + ir_u g_{13} \\ r_F f_{12} - ir_u g_{12} & r_H h_{22} + r_F f_{22} & r_F f_{23} + ir_u g_{23} \\ r_F f_{13} - ir_u g_{13} & r_F f_{23} - ir_u g_{23} & r_H h_{33} + r_F f_{33} \end{pmatrix} v$$

$$M_d = \begin{pmatrix} h_{11} + f_{11} & f_{12} + ig_{12} & f_{13} + ig_{13} \\ f_{12} - ig_{12} & h_{22} + f_{22} & f_{23} + ig_{23} \\ f_{13} - ig_{13} & f_{23} - ig_{23} & h_{33} + f_{33} \end{pmatrix} v$$

$$M_e = \begin{pmatrix} h_{11} - 3f_{11} & -3f_{12} + ir_l g_{12} & -3f_{13} + ir_l g_{13} \\ -3f_{12} - ir_l g_{12} & h_{22} - 3f_{22} & -3f_{23} + ir_l g_{23} \\ -3f_{13} - ir_l g_{13} & -3f_{23} - ir_l g_{23} & h_{33} - 3f_{22} \end{pmatrix} v$$

$$M_\nu = \begin{pmatrix} r_H h_{11} - 3r_F f_{11} & -3r_F f_{12} + ir_D g_{12} & -3r_F f_{13} + ir_D g_{13} \\ -3r_F f_{12} - ir_D g_{12} & r_H h_{22} - 3r_F f_{22} & -3r_F f_{23} + ir_D g_{23} \\ -3r_F f_{13} - ir_D g_{13} & -3r_F f_{23} - ir_D g_{23} & r_H h_{33} - 3r_F f_{33} \end{pmatrix} v$$

$$M_R = \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{12} & f_{22} & f_{23} \\ f_{13} & f_{23} & h_{33} \end{pmatrix} \frac{1}{r_R} \tag{A.1}$$

Parameter	Best fit value
$h_{11}v[GeV]$	2.91
$h_{22}v[GeV]$	36.4
$h_{33}v[GeV]$	1130
$f_{11}v[GeV]$	-1.40
$f_{12}v[GeV]$	0.779
$f_{13}v[GeV]$	6.29
$f_{22}v[GeV]$	-15.1
$f_{23}v[GeV]$	40.6
$f_{33}v[GeV]$	-48.2
$g_{12}v[GeV]$	-2.06
$g_{13}v[GeV]$	-1.77
$g_{23}v[GeV]$	0.427
r_H	78.0
r_F	146
r_u	0.190
r_l	-9.19
r_D	-6780
$r_R/v \times 10^{-10}$	2.27

References

[1] G. Altarelli and F. Feruglio, *Models of neutrino masses and mixings*, *New J. Phys.* **6** (2004) 106 [hep-ph/0405048] [SPIRES].

[2] G. Altarelli, *Status of neutrino masses and mixing in 2009*, *Nuovo Cim.* **C32N5-6** (2009) 91 [arXiv:0905.3265] [SPIRES].

[3] R.N. Mohapatra and A.Y. Smirnov, *Neutrino mass and new physics*, *Ann. Rev. Nucl. Part. Sci.* **56** (2006) 569 [hep-ph/0603118] [SPIRES].

[4] W. Grimus, *Neutrino physics: models for neutrino masses and lepton mixing*, *PoS* **P 2GC** (2006) 001 [hep-ph/0612311] [SPIRES].

[5] M.C. Gonzalez-Garcia and M. Maltoni, *Phenomenology with massive neutrinos*, *Phys. Rept.* **460** (2008) 1 [arXiv:0704.1800] [SPIRES].

[6] G.L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A.M. Rotunno, *Hints of $\theta_{13} > 0$ from global neutrino data analysis*, *Phys. Rev. Lett.* **101** (2008) 141801 [arXiv:0806.2649] [SPIRES].

[7] G.L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A.M. Rotunno, *What we (would like to) know about the neutrino mass*, arXiv:0809.2936 [SPIRES].

[8] T. Schwetz, M.A. Tortola and J.W.F. Valle, *Three-flavour neutrino oscillation update*, *New J. Phys.* **10** (2008) 113011 [arXiv:0808.2016] [SPIRES].

[9] M. Maltoni and T. Schwetz, *Three-flavour neutrino oscillation update and comments on possible hints for a non-zero θ_{13}*, *PoS* **IDM2008** (2008) 072 [arXiv:0812.3161] [SPIRES].

[10] P.F. Harrison, D.H. Perkins and W.G. Scott, *Tri-bimaximal mixing and the neutrino oscillation data*, *Phys. Lett.* **B 530** (2002) 167 [hep-ph/0202074] [SPIRES].

[11] P.F. Harrison and W.G. Scott, *Symmetries and generalisations of tri-bimaximal neutrino mixing*, *Phys. Lett.* **B 535** (2002) 163 [hep-ph/0203209] [SPIRES].

[12] P.F. Harrison and W.G. Scott, *$\mu - \tau$ reflection symmetry in lepton mixing and neutrino oscillations*, *Phys. Lett.* **B 547** (2002) 219 [hep-ph/0210197] [SPIRES].

[13] P.F. Harrison and W.G. Scott, *Permutation symmetry, tri-bimaximal neutrino mixing and the S3 group characters*, *Phys. Lett.* **B 557** (2003) 76 [hep-ph/0302025] [SPIRES].

[14] P.F. Harrison and W.G. Scott, *Status of tri-/bi-maximal neutrino mixing*, hep-ph/0402006 [SPIRES].

[15] P.F. Harrison and W.G. Scott, *The simplest neutrino mass matrix*, *Phys. Lett.* **B 594** (2004) 324 [hep-ph/0403278] [SPIRES].

[16] G. Altarelli and F. Feruglio, *Discrete flavor symmetries and models of neutrino mixing*, *Rev. Mod. Phys.* **82** (2010) 2701 [arXiv:1002.0211] [SPIRES].

[17] G. Altarelli, F. Feruglio and C. Hagedorn, *A SUSY* SU(5) *grand unified model of tri-bimaximal mixing from A_4*, *JHEP* **03** (2008) 052 [arXiv:0802.0090] [SPIRES].

[18] G. Altarelli, F. Feruglio and I. Masina, *Can neutrino mixings arise from the charged lepton sector?*, *Nucl. Phys.* **B 689** (2004) 157 [hep-ph/0402155] [SPIRES].

[19] M.-C. Chen and K.T. Mahanthappa, *CKM and tri-bimaximal MNS matrices in a* SU(5) × (d)T *model*, *Phys. Lett.* **B 652** (2007) 34 [arXiv:0705.0714] [SPIRES];

[20] C. Hagedorn, S.F. King and C. Luhn, *A SUSY GUT of flavour with S_4 ×* SU(5) *to NLO*, *JHEP* **06** (2010) 048 [arXiv:1003.4249] [SPIRES].

[21] H. Ishimori, K. Saga, Y. Shimizu and M. Tanimoto, *Tri-bimaximal mixing and Cabibbo angle in S_4 flavor model with SUSY*, *Phys. Rev.* **D 81** (2010) 115009 [`arXiv:1004.5004`] [SPIRES].

[22] T.J. Burrows and S.F. King, $A_4 \times$ SU(5) *SUSY GUT of flavour in 8d*, *Nucl. Phys.* **B 842** (2011) 107 [`arXiv:1007.2310`] [SPIRES].

[23] G. Lazarides, Q. Shafi and C. Wetterich, *Proton lifetime and fermion masses in an* SO(10) *model*, *Nucl. Phys.* **B 181** (1981) 287 [SPIRES].

[24] J. Schechter and J.W.F. Valle, *Neutrino Masses in* SU(2) $\times$ U(1) *theories*, *Phys. Rev.* **D 22** (1980) 2227 [SPIRES].

[25] R.N. Mohapatra and G. Senjanović, *Neutrino masses and mixings in gauge models with spontaneous parity violation*, *Phys. Rev.* **D 23** (1981) 165 [SPIRES].

[26] P. Minkowski, $\mu \to e\gamma$ *at a rate of one out of 1-billion muon decays?*, *Phys. Lett.* **B 67** (1977) 421 [SPIRES].

[27] T. Yanagida, in the proceedings of the *Workshop on Unified Theories, KEK Report 79-18*, O. Sawada and A. Sugamoto eds., Tsukuba Japan (1979) p. 95.

[28] M. Gell-Mann, P. Ramond and R. Slansky, *Supergravity*, North Holland, Amsterdam Netherlands (1979) p. 315.

[29] S.L. Glashow, *The future of elementary particle physics*, in the proceedings of the *1979 Cargese Summer Institute on Quarks and Leptons*, M. Lévy et al. eds., Plenum, New York U.S.A. (1980) p. 687.

[30] R.N. Mohapatra and G. Senjanović, *Neutrino mass and spontaneous parity nonconservation*, *Phys. Rev. Lett.* **44** (1980) 912 [SPIRES].

[31] C.H. Albright and W. Rodejohann, *Model-independent analysis of tri-bimaximal mixing: a softly-broken hidden or an accidental symmetry?*, *Phys. Lett.* **B 665** (2008) 378 [`arXiv:0804.4581`] [SPIRES].

[32] R. Dermisek and S. Raby, *Bi-large neutrino mixing and CP-violation in an* SO(10) *SUSY GUT for fermion masses*, *Phys. Lett.* **B 622** (2005) 327 [`hep-ph/0507045`] [SPIRES].

[33] R. Dermisek, M. Harada and S. Raby, SO(10) *SUSY GUT for fermion masses: lepton flavor and CP-violation*, *Phys. Rev.* **D 74** (2006) 035011 [`hep-ph/0606055`] [SPIRES].

[34] R. Barbieri, L.J. Hall, S. Raby and A. Romanino, *Unified theories with* U(2) *flavor symmetry*, *Nucl. Phys.* **B 493** (1997) 3 [`hep-ph/9610449`] [SPIRES].

[35] C.H. Albright and S.M. Barr, *Construction of a minimal Higgs* SO(10) *SUSY GUT model*, *Phys. Rev.* **D 62** (2000) 093008 [`hep-ph/0003251`] [SPIRES].

[36] C.H. Albright and S.M. Barr, *Realization of the large mixing angle solar neutrino solution in an* SO(10) *supersymmetric grand unified model*, *Phys. Rev.* **D 64** (2001) 073010 [`hep-ph/0104294`] [SPIRES].

[37] S.M. Barr and S. Raby, *Minimal* SO(10) *unification*, *Phys. Rev. Lett.* **79** (1997) 4748 [`hep-ph/9705366`] [SPIRES].

[38] X.-d. Ji, Y.-c. Li and R.N. Mohapatra, *An* SO(10) *GUT model with lopsided mass matrix and neutrino mixing angle* θ_{13}, *Phys. Lett.* **B 633** (2006) 755 [`hep-ph/0510353`] [SPIRES].

[39] K.S. Babu and R.N. Mohapatra, *Predictive neutrino spectrum in minimal* SO(10) *grand unification*, *Phys. Rev. Lett.* **70** (1993) 2845 [`hep-ph/9209215`] [SPIRES].

[40] K.S. Babu and C. Macesanu, *Neutrino masses and mixings in a minimal* SO(10) *model*, *Phys. Rev.* **D 72** (2005) 115003 [hep-ph/0505200] [SPIRES].

[41] B. Brahmachari and R.N. Mohapatra, *Unified explanation of the solar and atmospheric neutrino puzzles in a minimal supersymmetric* SO(10) *model*, *Phys. Rev.* **D 58** (1998) 015001 [hep-ph/9710371] [SPIRES].

[42] B. Bajc, G. Senjanović and F. Vissani, *How neutrino and charged fermion masses are connected within minimal supersymmetric* SO(10), hep-ph/0110310 [SPIRES].

[43] B. Bajc, G. Senjanović and F. Vissani, $b - \tau$ *unification and large atmospheric mixing: A case for non-canonical see-saw*, *Phys. Rev. Lett.* **90** (2003) 051802 [hep-ph/0210207] [SPIRES].

[44] B. Bajc, A. Melfo, G. Senjanović and F. Vissani, *Fermion mass relations in a supersymmetric* SO(10) *theory*, *Phys. Lett.* **B 634** (2006) 272 [hep-ph/0511352] [SPIRES].

[45] T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, *General formulation for proton decay rate in minimal supersymmetric* SO(10) *GUT*, *Eur. Phys. J.* **C 42** (2005) 191 [hep-ph/0401213] [SPIRES].

[46] T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, SO(10) *group theory for the unified model building*, *J. Math. Phys.* **46** (2005) 033505 [hep-ph/0405300] [SPIRES].

[47] T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, *Higgs masses in the minimal SUSY* SO(10) *GUT*, *Phys. Rev.* **D 72** (2005) 051701 [hep-ph/0412348] [SPIRES].

[48] B. Bajc, I. Dorsner and M. Nemevšek, *Minimal* SO(10) *splits supersymmetry*, *JHEP* **11** (2008) 007 [arXiv:0809.1069] [SPIRES].

[49] H.S. Goh, R.N. Mohapatra and S.-P. Ng, *Minimal SUSY* SO(10)*, $b\tau$ unification and large neutrino mixings*, *Phys. Lett.* **B 570** (2003) 215 [hep-ph/0303055] [SPIRES].

[50] H.S. Goh, R.N. Mohapatra and S.-P. Ng, *Minimal SUSY* SO(10) *model and predictions for neutrino mixings and leptonic CP-violation*, *Phys. Rev.* **D 68** (2003) 115008 [hep-ph/0308197] [SPIRES].

[51] H.S. Goh, R.N. Mohapatra and S. Nasri, SO(10) *symmetry breaking and type-II seesaw*, *Phys. Rev.* **D 70** (2004) 075022 [hep-ph/0408139] [SPIRES].

[52] C.S. Aulakh, B. Bajc, A. Melfo, G. Senjanović and F. Vissani, *The minimal supersymmetric grand unified theory*, *Phys. Lett.* **B 588** (2004) 196 [hep-ph/0306242] [SPIRES].

[53] C.S. Aulakh and S.K. Garg, *MSGUT: From bloom to doom*, *Nucl. Phys.* **B 757** (2006) 47 [hep-ph/0512224] [SPIRES].

[54] C.S. Aulakh and S.K. Garg, *The new minimal supersymmetric GUT*, hep-ph/0612021 [SPIRES].

[55] C.S. Aulakh and S.K. Garg, *Nmsgut II: Pinning the Nmsgut@LHC*, arXiv:0807.0917 [SPIRES].

[56] A. Melfo, A. Ramirez and G. Senjanović, *Type II see-saw dominance in* SO(10), *Phys. Rev.* **D 82** (2010) 075014 [arXiv:1005.0834] [SPIRES].

[57] S. Bertolini, M. Frigerio and M. Malinsky, *Fermion masses in SUSY* SO(10) *with type-II seesaw: A non-minimal predictive scenario*, *Phys. Rev.* **D 70** (2004) 095002 [hep-ph/0406117] [SPIRES].

[58] S. Bertolini and M. Malinsky, *On CP-violation in minimal renormalizable SUSY* SO(10) *and beyond*, *Phys. Rev.* **D 72** (2005) 055021 [hep-ph/0504241] [SPIRES];

[59] S. Bertolini, T. Schwetz and M. Malinsky, *Fermion masses and mixings in* SO(10) *models and the neutrino challenge to SUSY GUTs*, *Phys. Rev.* **D 73** (2006) 115012 [`hep-ph/0605006`] [SPIRES].

[60] B. Dutta, Y. Mimura and R.N. Mohapatra, *Neutrino masses and mixings in a predictive* SO(10) *model with CKM CP-violation*, *Phys. Lett.* **B 603** (2004) 35 [`hep-ph/0406262`] [SPIRES].

[61] B. Dutta, Y. Mimura and R.N. Mohapatra, *Suppressing proton decay in the minimal* SO(10) *model*, *Phys. Rev. Lett.* **94** (2005) 091804 [`hep-ph/0412105`] [SPIRES].

[62] B. Dutta, Y. Mimura and R.N. Mohapatra, *Neutrino mixing predictions of a minimal* SO(10) *model with suppressed proton decay*, *Phys. Rev.* **D 72** (2005) 075009 [`hep-ph/0507319`] [SPIRES].

[63] B. Dutta, Y. Mimura and R.N. Mohapatra, *Origin of quark-lepton flavor in* SO(10) *with Type II seesaw*, *Phys. Rev.* **D 80** (2009) 095021 [`arXiv:0910.1043`] [SPIRES].

[64] B. Dutta, Y. Mimura and R.N. Mohapatra, *An* SO(10) *grand unified theory of flavor*, *JHEP* **05** (2010) 034 [`arXiv:0911.2242`] [SPIRES];

[65] B. Dutta and Y. Mimura, *Properties of fermion mixings in intersecting D-brane models*, *Phys. Lett.* **B 633** (2006) 761 [`hep-ph/0512171`] [SPIRES].

[66] T. Fukuyama and N. Okada, *Neutrino oscillation data versus minimal supersymmetric* SO(10) *model*, *JHEP* **11** (2002) 011 [`hep-ph/0205066`] [SPIRES].

[67] A.S. Joshipura, B.P. Kodrani and K.M. Patel, *Fermion masses and mixings in a $\mu - \tau$ symmetric* SO(10), *Phys. Rev.* **D 79** (2009) 115017 [`arXiv:0903.2161`] [SPIRES].

[68] L. Lavoura, H. Kuhbock and W. Grimus, *Charged-fermion masses in* SO(10)*: Analysis with scalars in 10+120*, *Nucl. Phys.* **B 754** (2006) 1 [`hep-ph/0603259`] [SPIRES].

[69] W. Grimus and H. Kuhbock, *Fermion masses and mixings in a renormalizable* SO(10) $\times Z_2$ *GUT*, *Phys. Lett.* **B 643** (2006) 182 [`hep-ph/0607197`] [SPIRES].

[70] W. Grimus and H. Kuhbock, *A renormalizable* SO(10) *GUT scenario with spontaneous CP-violation*, *Eur. Phys. J.* **C 51** (2007) 721 [`hep-ph/0612132`] [SPIRES].

[71] H. Fusaoka and Y. Koide, *Updated estimate of running quark masses*, *Phys. Rev.* **D 57** (1998) 3986 [`hep-ph/9712201`] [SPIRES].

[72] C.R. Das and M.K. Parida, *New formulas and predictions for running fermion masses at higher scales in SM, 2HDM and MSSM*, *Eur. Phys. J.* **C 20** (2001) 121 [`hep-ph/0010004`] [SPIRES].

[73] M. Heinze and M. Malinsky, *Flavour structure of supersymmetric* SO(10) *GUTs with extended matter sector*, *Phys. Rev.* **D 83** (2011) 035018 [`arXiv:1008.4813`] [SPIRES].

[74] D.M. Pierce, J.A. Bagger, K.T. Matchev and R.-j. Zhang, *Precision corrections in the minimal supersymmetric standard model*, *Nucl. Phys.* **B 491** (1997) 3 [`hep-ph/9606211`] [SPIRES].

[75] G. Ross and M. Serna, *Unification and fermion mass structure*, *Phys. Lett.* **B 664** (2008) 97 [`arXiv:0704.1248`] [SPIRES].

[76] T. Blazek, S. Raby and S. Pokorski, *Finite supersymmetric threshold corrections to CKM matrix elements in the large* $\tan\beta$ *regime*, *Phys. Rev.* **D 52** (1995) 4151 [`hep-ph/9504364`] [SPIRES].

[77] W. Altmannshofer, D. Guadagnoli, S. Raby and D.M. Straub, *SUSY GUTs with Yukawa unification: A Go/no-go study using FCNC processes*, *Phys. Lett.* **B 668** (2008) 385 [arXiv:0801.4363] [SPIRES].

[78] K. Tobe and J.D. Wells, *Revisiting top-bottom-tau Yukawa unification in supersymmetric grand unified theories*, *Nucl. Phys.* **B 663** (2003) 123 [hep-ph/0301015] [SPIRES].

[79] P.H. Chankowski and S. Pokorski, *Quantum corrections to neutrino masses and mixing angles*, *Int. J. Mod. Phys.* **A 17** (2002) 575 [hep-ph/0110249] [SPIRES].