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Direct CP violation in $\tau^{\pm} \to K^{\pm}\rho^0(\omega)\nu_\tau \to K^{\pm}\pi^+\pi^-\nu_\tau$

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Abstract We study the direct CP violation in the $\tau^{\pm} \to K^{\pm}\rho^0(\omega)\nu_\tau \to K^{\pm}\pi^+\pi^-\nu_\tau$ decay process in the standard model. An interesting mechanism involving the charge symmetry violating mixing between ρ^0 and ω is applied to enlarge the CP asymmetry. We find that the CP-violating asymmetry can be enhanced greatly via this ρ–ω mixing mechanism when the invariant mass of the $\pi^+\pi^-$ pair is in the vicinity of the ω resonance. With this mechanism, the maximum differential and localized integrated CP asymmetries can reach $-(5.6^{+2.9}_{-1.7}) \times 10^{-12}$ and $6.3^{+2.4}_{-3.3} \times 10^{-11}$, respectively, which is still negligible.

1 Introduction

CP violation was first observed in the neutral kaon system 50 years ago [1]. The violation of CP in the K meson system can be explained by a weak complex phase in the Cabibbo–Kobayashi–Maskawa (CKM) matrix in the standard model (SM) [2,3]. However, the fundamental origin of CP violation is still an open problem and it is not clear if the CKM mechanism is the only source for CP violation. New physics (NP) may exist [4–6] and cause CP violation. To verify the origin of CP violation and look for NP, one needs to collect more information as regards CP violation in as many processes as possible. One such possible process is the τ decay. τ is the only lepton which is heavy enough to decay into hadrons and the pure leptonic and semileptonic character of τ decays provides a clean laboratory to test the structure of the weak currents and the universality of their couplings to the gauge bosons [5]. More importantly, with the establishment of the high-luminosity Super τ-Charm factories, more τ leptons will be produced and its properties will

be measured to a very high precision [4]. After the CLEO-c experiment ceased data collection in March 2008, the BESIII experiment began to collect data, and the luminosity reached 10^{32} cm^{-2} s^{-1} in 2013 [7]. Future high-luminosity Super τ-Charm factories are also being considered in Russia and Italy and may reach the luminosity of 10^{35} cm^{-2} s^{-1} [8–11]. Moreover, Super B-Factories (with the luminosity of 10^{36} cm^{-2} s^{-1}) will produce about 10^{10} τ pairs per year at the $\Upsilon(4S)$ peak [12,13]. The large statistics collected have considerably improved the statistical accuracy of the τ measurements and brought a new level of systematic understanding, allowing us to make sensible tests of the τ properties, provide more information as regards CP asymmetries in τ decay processes, and seek the fundamental origin of CP violation.

Experimental searches for CP-violating asymmetries in τ lepton semileptonic decays have been carried out. The missing evidence for a nonzero CP asymmetry was interpreted in terms of a coupling Λ in the decay $\tau^{\pm} \to \pi^{\pm}\pi^0\nu_\tau$ [14]. Recently, the $\tau^{\pm} \to K_S\pi^{\pm}\nu_\tau$ rate asymmetry was measured to be of order $\mathcal{O}(10^{-3})$ by Belle [15] and BaBar [16]. In order to improve our understanding of CP violation in τ decays, more efforts should be made on the theoretical side. Explicit studies of the decay modes $\tau^{\pm} \to K^{\pm}\pi^+\pi^-\nu_\tau$ [17,18], $\tau^{\pm} \to \pi^{\pm}K^+K^-\nu_\tau$ [17], $\tau^{\pm} \to (3\pi)^{\pm}\nu_\tau$ [19,20], and $\tau^{\pm} \to (4\pi)^{\pm}\nu_\tau$ [19] show that sizeable CP-violating effects could be generated in some models of CP violation involving several Higgs doublets or left–right symmetry.

In the framework of the SM, the direct CP asymmetries come about due to a relative weak (CP-odd) and a relative strong (CP-even) phase. This mechanism is forbidden in τ decays in the leading order of the Fermi coupling constant G_F [21]. The CP violation in τ decays is usually predicted vanishingly small in the SM. For example, the CP violation in the $\tau^{\pm} \to K^{\pm}\pi^0\nu_\tau$ mode is estimated to be of order $\mathcal{O}(10^{-12})$ when one takes higher-order electroweak corrections into account [22]. (Note that for the decay $\tau^{\pm} \to K_S\pi^{\pm}\nu_\tau$, the SM predicts a CP-violating

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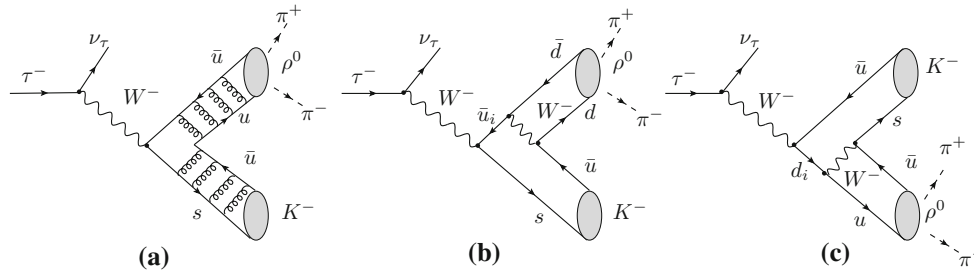c e-mail: yingliubnu@gmail.com

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Fig. 1 The leading-order (**a**) and higher-order diagrams (**b**, **c**) in G_F contributing to the decay $\tau^- \to K^- \rho^0 \nu_\tau \to K^- \pi^+ \pi^- \nu_\tau$. Gluons in **a** are soft ones representing non-perturbative QCD interaction. $u_i = u,\ c,\ t$ in **b** and $d_i = d,\ s,\ b$ in **c**

asymmetry of 3.3×10^{-3} due to the K_0–$\bar{K}_0$ mixing amplitude [23].) In order to obtain a larger CP violation in the SM, one needs to appeal to some phenomenological mechanisms. The charge symmetry violating mixing between ρ^0 and ω (ρ–ω mixing) has been applied in hadron decays for this purpose in the past few years. ρ–ω mixing has the dual advantages that the strong phase difference is large and well known [24,25]. From a series of studies on CP violation, it has already been found that this mechanism can provide a very large strong phase difference (usually 90 degrees) when the mass of the decay product of $\rho^0(\omega)$, $\pi^+\pi^-$, is in the vicinity of the ω resonance in some decay channels of heavy hadrons including B, Λ_b, and D [24–29]. We will apply this mechanism to the τ lepton decay in the present paper.

We will consider the decay process $\tau^\pm \to K^\pm \pi^+ \pi^- \nu_\tau$. The interference between the leading-order diagram in G_F (Fig. 1a) and the second-order weak diagrams (Fig. 1b, c) generates a small CP-violation phase [22]. ρ–ω mixing has been applied for getting a large strong phase when the invariant mass of the $\pi^+\pi^-$ pair is near the ω resonance. Hence one can expect that there could be a bigger CP-violating asymmetry in the $\tau^\pm \to K^\pm \rho^0(\omega)\nu_\tau \to K^\pm \pi^+ \pi^- \nu_\tau$ process. Actually, it will be shown from our explicit calculations that ρ–ω mixing leads to an additional strong phase and enlarges the differential CP-violating asymmetry by a maximum of four orders of magnitude and the localized integrated CP asymmetry by a maximum of three orders of magnitude. Even so, the direct CP asymmetry in the SM is still negligible.

The remainder of this paper is organized as follows. In Sect. 2, we first present the formalism for the CP asymmetry in $\tau^\pm \to K^\pm \rho^0(\omega)\nu_\tau \to K^\pm \pi^+ \pi^- \nu_\tau$ via ρ–ω mixing. Then we give the derivation details of the leading-order and the second-order weak process matrix elements and apply ρ–ω mixing to generate a large CP asymmetry. In Sect. 3, with the expression of meson wave functions and form factors and several parameters we calculate numerical results of the differential and localized integrated CP asymmetries. Our conclusion is included in Sect. 4.

2 CP violation in $\tau^\pm \to K^\pm \rho^0(\omega)\nu_\tau \to K^\pm \pi^+ \pi^- \nu_\tau$

A decay process described by some amplitudes may have CP-even and -odd relative phases. Within the SM, the CP-odd relative phase is always a weak phase difference which is directly determined by the CKM matrix. On the contrary, the CP-even relative phase is usually a strong phase difference due to some complicated phenomenological mechanism. Letting M and $\bar{M}$ be the amplitudes for $\tau^- \to K^- \rho^0(\omega)\nu_\tau \to K^- \pi^+ \pi^- \nu_\tau$ and its CP conjugate one, respectively, we define the two amplitudes as follows:

$$M = g_1 r_1 e^{i\phi_1} + g_2 r_2 e^{i\phi_2}, \tag{1}$$
$$\bar{M} = g_1^* r_1 e^{i\phi_1} + g_2^* r_2 e^{i\phi_2}, \tag{2}$$

where g_1 and g_2 represent CP-odd complex terms which involve coupling constants and CKM matrix elements, and $r_1 e^{i\phi_1}$ and $r_2 e^{i\phi_2}$ terms are even under the CP transformation. Then one has

$$
\begin{aligned}
|M|^2 - |\bar{M}|^2 &= 4 r_1 r_2 \,\mathfrak{Im}(g_1^* g_2) \,\sin(\phi_1 - \phi_2) \\
&= 4 r_1 r_2 |g_1||g_2| \sin[\mathrm{Arg}(g_2/g_1)] \sin(\phi_1 - \phi_2),
\end{aligned} \tag{3}
$$

from which we can see explicitly that both the CP-odd phase difference $\mathrm{Arg}(g_2/g_1)$ and the CP-even phase difference $\phi_1 - \phi_2$ are needed to produce CP violation. It will be shown below that the CP-odd phase difference arises from the second-order weak processes and the CP-even phase difference is determined by the decay widths of intermediate resonances and ρ–ω mixing in the $\tau^- \to K^\pm \rho^0(\omega)\nu_\tau \to K^\pm \pi^+ \pi^- \nu_\tau$ decay mode.

2.1 General formalism for CP asymmetry

The hadronic τ decay amplitude can be factorized into a purely leptonic part including τ lepton and neutrino and a hadronic part, where the hadronic system is created from the vacuum via the charged weak current. Thus, the amplitude of τ^- decaying into the $K^- \pi^+ \pi^- \nu_\tau$ final state through

$K^-\rho^0\nu_\tau$ with the invariant mass of the $\pi^+\pi^-$ pair near the ρ^0 resonance can be written in the following general form:

$$M^\rho = \frac{G_F}{\sqrt{2}} g_{\rho\pi\pi} s_\rho L^\mu H_\mu^\rho, \qquad (4)$$

where $g_{\rho\pi\pi}$ is the effective coupling for $\rho \to \pi\pi$, H_μ^ρ is the hadronic matrix element for $\rho^0 K^-$, L^μ is the lepton transition matrix element which can be written as $\bar{u}_{\nu_\tau}\gamma^\mu(1-\gamma^5)u_\tau$ with u_{ν_τ} and u_τ being the Dirac spinors of ν_τ and τ, respectively, and s_ρ is the propagator of the ρ^0 meson,

$$s_\rho = \frac{1}{s - m_\rho^2 + im_\rho\Gamma_\rho}, \qquad (5)$$

where $\sqrt{s}$ is the invariant mass of the $\pi^+\pi^-$ pair, and m_ρ and Γ_ρ are the mass and width of the ρ^0 meson, respectively. It should be noted that we assume that the ρ^0 meson is on-shell since the invariant mass of the $\pi^+\pi^-$ pair is near the mass of the ρ^0 meson.

Because of the absence of the CP-odd phase, the CP asymmetry is zero in the leading order in G_F in the SM in the τ decay. In order to have a nonzero CP-violating asymmetry, the second-order weak terms corresponding to Fig. 1b, c (with $u_i = u, c, t$ and $d_i = d, s, b$), which provide a CP-odd phase difference, should be taken into account [22]. The leading-order amplitude is denoted by M_0^ρ corresponding to Fig. 1a and the second-order weak terms are denoted by M_1^ρ and M_2^ρ corresponding to Fig. 1b, c, respectively.

As mentioned before, in order to obtain a large CP violation, we intend to apply the $\rho-\omega$ mixing mechanism, which leads to large strong phase differences in heavy hadron decays. In this scenario, to the first order of isospin violation, we have the following total amplitude when the invariant mass of the $\pi^+\pi^-$ pair is near the ω resonance mass:

$$M = M^\rho + M^{\rho-\omega}, \qquad (6)$$

with

$$M^{\rho-\omega} = \frac{G_F}{\sqrt{2}} g_{\rho\pi\pi} s_\rho L^\mu H_\mu^\omega s_\omega \tilde{\Pi}_{\rho\omega}. \qquad (7)$$

where $\tilde{\Pi}_{\rho\omega}$ is the effective $\rho-\omega$ mixing amplitude, s_ω is the propagator of the ω meson, and H_μ^ω includes three ωK annihilation terms $H_\mu^{0\omega}$, $H_\mu^{1\omega}$, and $H_\mu^{2\omega}$ corresponding to Fig. 1a–c, but through the ω intermediate resonance, respectively. We also assume that the ω meson is on-shell. It should be noted that the $\rho \to \omega \to \pi^+\pi^-$ process has been neglected since it is of the second order of isospin violation. The direct coupling $\omega \to \pi^+\pi^-$ has been effectively absorbed into $\tilde{\Pi}_{\rho\omega}$ [30,31]. This leads to the explicit s dependence of $\tilde{\Pi}_{\rho\omega}$. Making the expansion $\tilde{\Pi}_{\rho\omega}(s) = \tilde{\Pi}_{\rho\omega}(m_\omega^2) + (s - m_\omega)\tilde{\Pi}'_{\rho\omega}(m_\omega^2)$, the $\rho-\omega$ mixing parameters were fitted by Gardner and O'Connell [32]:

$$\mathfrak{Re}\tilde{\Pi}_{\rho\omega}(m_\omega^2) = -3500 \pm 300 \text{ MeV}^2,$$
$$\mathfrak{Im}\tilde{\Pi}_{\rho\omega}(m_\omega^2) = -300 \pm 300 \text{ MeV}^2, \qquad (8)$$
$$\tilde{\Pi}'_{\rho\omega}(m_\omega^2) = 0.03 \pm 0.04.$$

We define $M_0 = M_0^\rho + M_0^{\rho-\omega}$, $M_1 = M_1^\rho + M_1^{\rho-\omega}$, and $M_2 = M_2^\rho + M_2^{\rho-\omega}$, where $M_0^{\rho-\omega}$, $M_1^{\rho-\omega}$, and $M_2^{\rho-\omega}$ correspond to $H_\mu^{0\omega}$, $H_\mu^{1\omega}$, and $H_\mu^{2\omega}$, respectively. The CP violation can arise from the interference between M_0 and M_1, M_2. It should be noted that M_1 and M_2 are the second order in G_F. Therefore, to the G_F^3 order, the square of the total amplitude $M = M_0 + M_1 + M_2$ can be written as

$$\begin{aligned}
|M|^2 &= (M_0 + M_1 + M_2)^\dagger (M_0 + M_1 + M_2) \\
&= M_0^\dagger M_0 + (M_0^\dagger M_1 + M_0 M_1^\dagger) \\
&\quad + (M_0^\dagger M_2 + M_0 M_2^\dagger).
\end{aligned} \qquad (9)$$

Then the differential CP asymmetry, which is defined as

$$A_{CP} = \frac{|M|^2 - |\bar{M}|^2}{|M|^2 + |\bar{M}|^2}, \qquad (10)$$

can be written as follows to the order G_F:

$$A_{CP} = \frac{|M|^2 - |\bar{M}|^2}{|M_0|^2 + |\bar{M}_0|^2}, \qquad (11)$$

where the $M_0^\dagger M_1 + M_0 M_1^\dagger$ and $M_0^\dagger M_2 + M_0 M_2^\dagger$ terms are negligible in the denominator since they do not contribute to the second order in G_F. When we take $\rho-\omega$ mixing into account, to the leading order in isospin violation, the three terms in Eq. (9) can be rewritten in the following forms:

$$\begin{aligned}
M_0 M_0^\dagger &= (G_F/\sqrt{2})^2 g_{\rho\pi\pi}^2 s_\rho^2 L^{\mu\nu}\big(H_{\mu\nu}^{0\rho0\rho} + H_{\mu\nu}^{0\rho0\omega}\tilde{\Pi}_{\rho\omega}^* s_\omega^* \\
&\quad + H_{\mu\nu}^{0\omega0\rho}\tilde{\Pi}_{\rho\omega}s_\omega\big), \\
M_0 M_1^\dagger &= (G_F/\sqrt{2})^2 g_{\rho\pi\pi}^2 s_\rho^2 L^{\mu\nu}\big(H_{\mu\nu}^{0\rho1\rho} + H_{\mu\nu}^{0\rho1\omega}\tilde{\Pi}_{\rho\omega}^* s_\omega^* \\
&\quad + H_{\mu\nu}^{0\omega1\rho}\tilde{\Pi}_{\rho\omega}s_\omega\big), \\
M_0 M_2^\dagger &= (G_F/\sqrt{2})^2 g_{\rho\pi\pi}^2 s_\rho^2 L^{\mu\nu}\big(H_{\mu\nu}^{0\rho2\rho} + H_{\mu\nu}^{0\rho2\omega}\tilde{\Pi}_{\rho\omega}^* s_\omega^* \\
&\quad + H_{\mu\nu}^{0\omega2\rho}\tilde{\Pi}_{\rho\omega}s_\omega\big),
\end{aligned} \qquad (12)$$

where $L^{\mu\nu} = L^\mu(L^\nu)^\dagger$ and, for example, $H_{\mu\nu}^{0\rho0\rho} = H_\mu^{0\rho}(H_\nu^{0\rho})^\dagger$.

2.2 Derivation details of matrix elements

The transition from the vacuum to the pseudoscalar meson K^- and the vector one ρ^0 (ω) occurs via weak vector and axial-vector current. Based on Lorentz invariance and parity and time-reversal invariance, one can decompose the
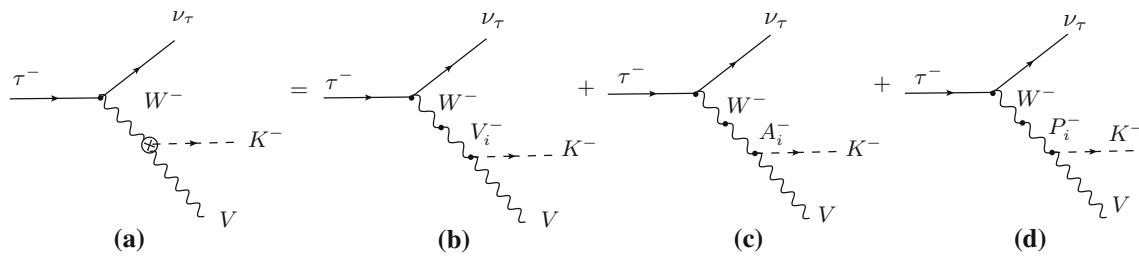
Fig. 2 The Feynman diagrams with the intermediate virtual mesons that connect the weak current and the strong vertex in the decay $\tau^- \to VK^-\nu_\tau$ [V is ρ^0 (or ω)]. **a** This *panel* represents the total effec-

tive strong vertex; **b–d** correspond to the V_i (vector), A_i (axial-vector), and P_i (pseudoscalar) intermediate meson processes, respectively

hadronic matrix element in terms of four form factors in the leading order in G_F [33]:

$$
\begin{aligned}
H_\mu^{0\rho(\omega)} &= -iV_{us}^* \langle \rho^0(\omega)K^-|\bar{s}\gamma_\mu(1-\gamma_5)u|0\rangle \\
&= V_{us}^*[-g\varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}p_1^\alpha p_2^\beta - if\epsilon_\mu^* \\
&\quad - i(a_1 p_{1\mu} + a_2 p_{2\mu})(\epsilon^* \cdot Q)],
\end{aligned}
\tag{13}
$$

where V_{us} is the CKM matrix element, $\bar{s}$ and u are quark field operators, p_1 and p_2 are momenta of ρ^0 (or ω) and K^-, respectively, $Q = p_1 + p_2$ is the momentum transfer to the hadronic system, g is the vector current form factor, f, a_1, and a_2 are axial-vector current form factors, and ϵ_μ denotes the polarization vector of ρ^0 (or ω), which satisfies $p_1 \cdot \epsilon = 0$ and $\sum_{\lambda=0,\pm} \epsilon^{*\mu}(q,\lambda)\epsilon^\nu(q,\lambda) = -g^{\mu\nu} + q^\mu q^\nu/m_V$, where $\lambda = \pm, 0$ represent the transverse and longitudinal polarizations, respectively, and m_V is the mass of the vector meson V ($V = \rho^0$ or ω). The form factors are functions of Q^2 only. They are difficult to relate directly to experimental measurements but can be dealt with in phenomenological models. We will calculate the form factors with the meson dominance model [33]. The pseudoscalar and vector meson annihilation process in the leading order in G_F is generated by the strong interaction. In the meson dominance model it is assumed that intermediate mesons connect the weak current and the strong vertex shown in the Feynman diagrams in Fig. 2. Using the Feynman rules for these diagrams, the following expressions for the form factors are obtained [33]:

$$
\begin{aligned}
f &= -\frac{1}{2}(Q^2 + m_V^2 - m_K^2)\sum_i \frac{h_{A_i}t_{A_i VK}}{D_{A_i}(Q^2)}, \\
g &= \frac{1}{2}\sum_i \frac{h_{V_i}t_{V_i VK}}{D_{V_i}(Q^2)}, \\
a_1 &= \frac{5}{2}\sum_i \frac{h_{P_i}t_{P_i VK}}{D_{P_i}(Q^2)} + \frac{1}{2}\sum_i \frac{h_{A_i}t_{A_i VK}}{D_{A_i}(Q^2)}, \\
a_2 &= \frac{3}{2}\sum_i \frac{h_{P_i}t_{P_i VK}}{D_{P_i}(Q^2)} + \frac{1}{2}\sum_i \frac{h_{A_i}t_{A_i VK}}{D_{A_i}(Q^2)},
\end{aligned}
\tag{14}
$$

where V_i, A_i, and P_i denote vector, axial-vector, and pseudoscalar intermediate meson resonances, respectively, h_{M_i}

($M_i = A_i$, V_i or P_i) denotes the weak coupling of the M_i intermediate meson, $t_{M_i VK}$ is its strong coupling to the VK final state, m_K is the mass of the K meson, and $D_{M_i} \equiv Q^2 - m_{M_i}^2 + im_{M_i}\Gamma_{M_i}$ where m_{M_i} (Γ_{M_i}) is the mass (width) of the corresponding intermediate meson. The details as regards the intermediate mesons and their weak couplings and strong vertex coupling constants will be given in Sect. 3. From Eqs. (12) and (13) it can be found that in the leading order in G_F the CP-odd phase is absent, and the CP-even phase is determined by the decay widths of intermediate resonances when ρ–ω mixing is not considered.

Next, we proceed to evaluate M_1 and M_2 based on the perturbation method. Note that $M_0 M_2^\dagger + M_0^\dagger M_2$ in Eq. (9) is proportional to $|V_{ud_i}|^2|V_{us}|^2$ and will not contribute to CP violation. Hence we only have to consider M_1. In the framework of perturbation method, it can be evaluated in a similar way to B decays [34]. Since the τ mass is much smaller than the W-boson mass M_W, the momenta of all the particles involved in the τ decay are much smaller than M_W. As a result, we can approximate the denominator of the W-boson propagator $(p_1 + p_2)^2 - M_W^2$ by $-M_W^2$ in the numerator of the W-boson propagator. The wave functions including spin factors of pseudoscalar and vector mesons are taken as [35]

$$
\Psi_V(x,p) = -\frac{I}{2\sqrt{2}\sqrt{3}m_V}\phi_V(x)(m_V + \not{p})\not{\epsilon},
\tag{15}
$$

$$
\Psi_P(x,p) = -\frac{I}{2\sqrt{2}\sqrt{3}m_P}\phi_P(x)(m_P + \not{p})\gamma_5,
\tag{16}
$$

where $I = 3$ is an identity in color space, m_P and m_V are the masses of the pseudoscalar and vector mesons, respectively, p represents the momentum of the meson P or V, x is the longitudinal momentum fraction of the constituent quark, and the non-perturbative effects are included in the distribution amplitudes $\phi_V(x)$ and $\phi_P(x)$, which satisfy $\int_0^1 \phi_{V(P)}(x)dx = f_{V(P)}/(2\sqrt{6})$, where $f_{V(P)}$ is the decay constant of $V(P)$. According to the Feynman diagram (b) in Fig. 1, the hadronic matrix elements $H^{1\rho}$ (or $H^{1\omega}$) can be expressed as

$$H_\mu^{1\rho(\omega)} = +(-)\frac{G_F}{\sqrt{2}}(2\pi)^3 I \sum_i V_{u_i s}^* V_{u_i d} V_{ud}^* \sqrt{m_u m_d^2 m_s}$$

$$\times \int_0^1 dx dy \phi_{\rho(\omega)}^*(x)\phi_K^*(y)\frac{1}{2\sqrt{2}m_{\rho(\omega)}}$$

$$\times (m_{\rho(\omega)} - \not{p}_2)\not{\epsilon}^*\gamma_\alpha(1-\gamma_5)\frac{1}{2\sqrt{2}m_K}(m_K - \not{p}_1)$$

$$\times \gamma_5\gamma_\mu(1-\gamma_5)I_{u_i}\gamma^\alpha(1-\gamma_5), \tag{17}$$

where $V_{u_i s}$ and $V_{u_i d}$ are the CKM matrix elements, $+(-)$ corresponds to $\rho^0(\omega)$ and we define $I_{u_i} = i(\not{p}_{u_i} + m_{u_i})/(p_{u_i}^2 - m_{u_i}^2)$ with m_{u_i} and p_{u_i} being the current quark mass and the momentum of the intermediate quark u_i, respectively. We will neglect the difference between the masses of ρ^0 and ω mesons in the following, i.e., we take $m_\rho = m_\omega$.

Using the unitarity of the CKM matrix, we have

$$\sum_i V_{u_i s}^* V_{u_i d} I_{u_i} = V_{ud}^* V_{us}(I_u - I_c) + V_{td}^* V_{ts}(I_t - I_c)$$

$$\approx V_{ud}^* V_{us}(I_u - I_c) - V_{td}^* V_{ts} I_c, \tag{18}$$

where the last line is obtained using the fact that m_t is much larger than masses of other quarks involved in this process. We note that only $V_{td}^* V_{ts}$ provides a weak CP-violation phase, so it is unnecessary to consider the contribution of the first term. As a consequence, the CP asymmetry only depends on $V_{td}^* V_{ts} I_c$. We define

$$A_1 \equiv \int_0^1 dx dy \frac{\phi_K^*(x)\phi_{\rho(\omega)}^*(y)}{x Q^2 + (1-x)m_\rho^2 + (x^2-x)m_K^2 - m_c^2},$$

$$= \int_0^1 dx \frac{f_\rho \phi_K^*(x)}{2\sqrt{6}[x Q^2 + (1-x)m_\rho^2 + (x^2-x)m_K^2 - m_c^2]}, \tag{19}$$

$$B_1 \equiv \int_0^1 dx dy \frac{x\phi_K^*(x)\phi_{\rho(\omega)}^*(y)}{x Q^2 + (1-x)m_\rho^2 + (x^2-x)m_K^2 - m_c^2}$$

$$= \int_0^1 dx \frac{x f_\rho \phi_K^*(x)}{2\sqrt{6}[x Q^2 + (1-x)m_\rho^2 + (x^2-x)m_K^2 - m_c^2]}. \tag{20}$$

Inserting Eqs. (18), (19), and (20) into Eq. (17) and only considering the CP asymmetry term, $H_\mu^{1\rho(\omega)}$ can be simplified as

$$H_\mu^{1\rho(\omega)} = \frac{6\sqrt{2}(2\pi)^3 \sqrt{m_u m_d^2 m_s} G_F V_{ts}^* V_{td} V_{ud}^*}{m_k}$$

$$\times \left\{ -A_1\varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}p_1^\alpha p_2^\beta - i\epsilon^{*\mu}\left[\frac{1}{2}A_1(Q^2 - m_\rho^2 - m_K^2)\right.\right.$$

$$\left.\left. + B_1 m_K^2\right] + iA_1 p_1^\mu(Q \cdot \epsilon^*) + i2B_1 p_2^\mu(Q \cdot \epsilon^*) \right\}. \tag{21}$$

We can see that the weak phase appears but the strong phase is absent in this amplitude if ρ–ω mixing is not included.

Now we take ρ–ω mixing into account and show how ρ–ω mixing enlarges the CP violation. In the meson dominance model, the form factors of the annihilation process are dominated by strong interaction. So, we adopt the same form factors in the $K\rho$ and $K\omega$ annihilation processes. According to Eq. (13), we have $H_\mu^{0\rho} = H_\mu^{0\omega}$. $H_\mu^{1\rho(\omega)}$ is dependent on the hadronic wave functions. Since the wave functions of mesons are determined by strong interaction, which preserves isospin, we assume that the ρ^0 and ω mesons have the same hadronic wave functions. Therefore, from Eq. (17), we have $H_\mu^{1\rho} = -H_\mu^{1\omega}$. Then the first two equations of Eq. (12) can be written as

$$M_0 M_0^\dagger$$
$$= (G_F/\sqrt{2})^2 g_{\rho\pi\pi}^2 s_\rho^2 L^{\mu\nu} H_{\mu\nu}^{0\rho 0\rho}(1 + \tilde{\Pi}_{\rho\omega}^* s_\omega^* + \tilde{\Pi}_{\rho\omega}s_\omega),$$

$$M_0 M_1^\dagger$$
$$= (G_F/\sqrt{2})^2 g_{\rho\pi\pi}^2 s_\rho^2 L^{\mu\nu} H_{\mu\nu}^{0\rho 1\rho}(1 - \tilde{\Pi}_{\rho\omega}^* s_\omega^* + \tilde{\Pi}_{\rho\omega}s_\omega). \tag{22}$$

We can see explicitly that ρ–ω mixing provides additional complex terms to A_{CP}. As will be shown later, these complex terms enlarge the CP-even phase, which leads to a bigger CP asymmetry.

Finally, we will calculate $L^{\mu\nu} H_{\mu\nu}^{0\rho 0\rho}$ and $L^{\mu\nu} H_{\mu\nu}^{0\rho 1\rho}$. For simplicity, we will consider the unpolarized τ decay process. The unpolarized leptonic scattering tensor is

$$L^{\mu\nu} = \frac{1}{2}\sum_{\lambda_3,\lambda_4} tr\left[\bar{u}_{\nu_\tau}(p_3,\lambda_3)\gamma^\mu(1-\gamma_5)u_\tau(p_4,\lambda_4)\right.$$

$$\times \bar{u}_\tau(p_4,\lambda_4)\gamma^\nu(1-\gamma_5)u_{\nu_\tau}(p_3,\lambda_3)\right]$$

$$= 4\left[-g^{\mu\nu}(p_3 \cdot p_4) + p_3^\mu \cdot p_4^\nu + p_3^\nu \cdot p_4^\mu\right.$$

$$\left. + i\varepsilon^{\mu\nu\alpha\beta}p_{3\alpha}p_{4\beta}\right], \tag{23}$$

where p_3 and p_4 represent the momenta of ν_τ and τ, respectively, and λ_3 and λ_4 represent the helicities of ν_τ and τ, respectively. We also sum over the spins of hadrons. Then, from Eqs. (13) and (23), one has

$$L^{\mu\nu} H_{\mu\nu}^{0\rho 0\rho} = 4|V_{us}|^2\left[(-2x_0 - m_\rho^2 x_1 - m_K^2 x_2)(p_3 \cdot p_4)\right.$$

$$+ 2x_1(p_1 \cdot p_3)(p_1 \cdot p_4) + 2x_2(p_2 \cdot p_3)(p_2 \cdot p_4)$$

$$+ (x_+ + x_- + 2gf^* + 2g^*f)(p_1 \cdot p_3)(p_2 \cdot p_4)$$

$$+ (x_+ + x_- - 2gf^* - 2g^*f)(p_1 \cdot p_4)(p_2 \cdot p_3)$$

$$\left. - (x_+ + x_-)(p_1 \cdot p_2)(p_3 \cdot p_4)\right], \tag{24}$$

where

$$x_0 = -\frac{1}{4}g^2(Q^4 + m_\rho^4 + m_K^4 - 2m_\rho^2 Q^2$$

$$- 2m_K^2 Q^2 - 2m_\rho^2 m_K^2) - f^2,$$

$$x_1 = g^2 m_K^2 + a_1^2 \left[-Q^2 + \frac{(p_1 \cdot Q)^2}{m_\rho^2} \right] + \frac{f^2}{m_\rho^2}$$
$$+ \frac{p_1 \cdot p_2}{m_\rho^2} f a_1^* + \frac{p_1 \cdot p_2}{m_\rho^2} f^* a_1,$$

$$x_2 = g^2 m_\rho^2 + a_2^2 \left[-Q^2 + \frac{(p_1 \cdot Q)^2}{m_\rho^2} \right] - (a_2 f^* + a_2^* f),$$

$$x_+ = -\frac{1}{2} g^2 (Q^2 - m_K^2 - m_\rho^2) - a_1 f^* + \frac{p_1 \cdot p_2}{m_\rho^2} f a_2^*$$
$$+ a_1 a_2^* \left[-Q^2 + \frac{(p_1 \cdot Q)^2}{m_\rho^2} \right],$$

$$x_- = -\frac{1}{2} g^2 (Q^2 - m_K^2 - m_\rho^2) - a_1^* f + \frac{p_1 \cdot p_2}{m_\rho^2} f^* a_2$$
$$+ a_1^* a_2 \left[-Q^2 + \frac{(p_1 \cdot Q)^2}{m_\rho^2} \right]. \tag{25}$$

From Eqs. (13), (21), and (23), one has

$$L^{\mu\nu} H_{\mu\nu}^{0\rho 1\rho} = \frac{6\sqrt{2}(2\pi)^3 \sqrt{m_u m_d^2 m_s} G_F V_{ts} V_{td}^* V_{ud} V_{us}^*}{m_K}$$
$$\times \left[2x_1'(p_1 \cdot p_3)(p_1 \cdot p_4) + 2x_2'(p_2 \cdot p_3)(p_2 \cdot p_4) \right.$$
$$+ (x_+' + x_-' + 2f B_1 + 2g\lambda)(p_1 \cdot p_3)(p_2 \cdot p_4)$$
$$+ (x_+' + x_-' - 2f^* B_1 - 2g^*\lambda)(p_1 \cdot p_4)(p_2 \cdot p_3)$$
$$- (2x_0' + x_1' m_\rho^2 + x_2' m_K^2)(p_3 \cdot p_4)$$
$$\left. - (x_+' + x_-')(p_1 \cdot p_2)(p_3 \cdot p_4) \right], \tag{26}$$

where

$$x_0' = -\frac{1}{4} g A_1 (Q^4 + m_\rho^4 + m_K^4 - 2m_\rho^2 Q^2 - 2m_K^2 Q^2$$
$$- 2m_\rho^2 m_K^2) - f\lambda,$$

$$x_1' = -g A_1 m_K^2 + \frac{1}{2} g B_1 (Q^2 - m_\rho^2 - m_K^2) - A_1 f \frac{p_1 \cdot p_2}{m_\rho^2}$$
$$+ a_1 \lambda \frac{p_1 \cdot p_2}{m_\rho^2} - a_1 A_1 \left[-Q^2 + \frac{(p_1 \cdot Q)^2}{m_\rho^2} \right],$$

$$x_2' = -g A_1 m_\rho^2 - 2a_2 B_1 \left[-Q^2 + \frac{(p_1 \cdot Q)^2}{m_\rho^2} \right]$$
$$+ 2B_1 f + a_2 \lambda,$$

$$x_+' = \frac{1}{2} g A_1 (Q^2 - m_K^2 - m_\rho^2) - g A_1 m_K^2$$
$$- 2B_1 f \frac{p_1 \cdot p_2}{m_\rho^2} - a_1 \lambda,$$

$$x_-' = \frac{1}{2} g A_1 (Q^2 - m_K^2 - m_\rho^2) - A_1 f + a_2 \lambda \frac{p_1 \cdot p_2}{m_\rho^2},$$
$$- a_2 A_1 \left[-Q^2 + \frac{(p_1 \cdot Q)^2}{m_\rho^2} \right]$$

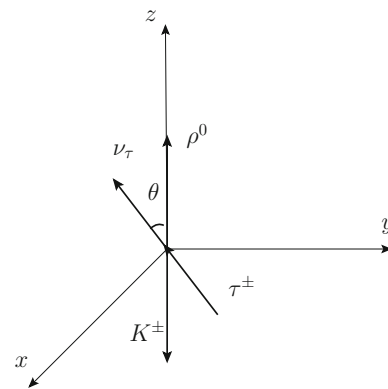$$\lambda = \frac{1}{2} A_1 (Q^2 - m_K^2 - m_\rho^2) + B_1 m_K^2. \tag{27}$$

Fig. 3 The hadronic rest frame. The z axis is chosen to be in the direction of the motion of the ρ^0 (or ω) meson. The three-momentum of K is chosen to be $\boldsymbol{p_2} = -\boldsymbol{p_1}$. The (x, z) plane is aligned with the ρ^0 and ν_τ movement plane, with $\boldsymbol{n}_\perp = (\boldsymbol{p}_1 \times \boldsymbol{p}_3)/|\boldsymbol{p}_1 \times \boldsymbol{p}_3|$ (the normal to the ρ^0 and ν_τ movement plane) pointing along the y axis. The distribution angle θ is the one between the motion direction of ρ^0 (or ω) and the neutrino

2.3 Hadronic rest frame

In the previous subsections, we have given the general expression of CP asymmetry and derivations of the matrix elements. For simplicity, we choose to work in a special reference frame and express the products among vectors p_1, p_2, p_3, p_4, Q, and ϵ in terms of the square of momentum transfer Q^2, the invariant mass of the $\pi^+\pi^-$ pair $\sqrt{s}$, and a distribution angle θ in this subsection. We note that it is convenient to express the momenta of hadrons and leptons and calculate various components of the matrix elements in the hadronic rest frame [36]. This frame is defined in Fig. 3. The z axis is chosen to be in the direction of motion of the ρ^0 (or ω) meson. The three-momentum of K is chosen to be $\boldsymbol{p_2} = -\boldsymbol{p_1}$. The (x, z) plane is aligned with the ρ^0 and ν_τ movement plane, with $\boldsymbol{n}_\perp = (\boldsymbol{p}_1 \times \boldsymbol{p}_3)/|\boldsymbol{p}_1 \times \boldsymbol{p}_3|$ (the normal to the ρ^0 and ν_τ movement plane) pointing along the y axis. The distribution angle θ is the one between the motion direction of ρ^0 (or ω) and the neutrino. Then the momenta of hadrons and leptons in this hadronic rest frame are given as follows:

$$p_1^\mu = (E_1, 0, 0, P),$$
$$p_2^\mu = (E_2, 0, 0, -P),$$
$$p_3^\mu = (K, K\sin\theta, 0, K\cos\theta),$$
$$p_4^\mu = (E_4, K\sin\theta, 0, K\cos\theta),$$
$$Q^\mu = (E_1 + E_2, 0, 0, 0)$$
$$= (E_4 - K, 0, 0, 0), \tag{28}$$

and the polarization vectors of ρ^0 (or ω) in this hadronic rest frame are

$$\epsilon^{\lambda=\pm 1} = (0, 1, \pm i, 0),$$
$$\epsilon^{\lambda=0} = \frac{1}{\sqrt{m_\rho}} (P, 0, 0, E_1), \tag{29}$$

Table 1 The values of h_{M_i}, $t_{M_i V K}$, m_{M_i}, and Γ_{M_i} in the numerical calculations

Intermediate mesons	Pseudoscalar	Axial vector		Vector	
	K^-	$K_1(1270)$	$K_1(1400)$	$K^*(892)$	$K^*(1680)$
h_{M_i} (10^3 MeV^2)	0.159 ± 0.0015 MeV^{-1}	215 ± 25	170 ± 130	188 ± 4	242 ± 25
$t_{M_i V K}$ (10^{-3} MeV^{-1})	-3170 ± 30 MeV	-1.94 ± 0.10	0.48 ± 0.24	8.71 ± 0.95	-3.71 ± 2.60
m_{M_i} (MeV)	494 ± 0.016	1272 ± 7	1403 ± 7	892 ± 0.26	1717 ± 27
Γ_{M_i} (MeV)	0	90 ± 20	174 ± 13	50.8 ± 0.9	322 ± 110

with

$$E_1 = \frac{Q^2 + m_K^2 - m_\rho^2}{2\sqrt{Q^2}}, \qquad E_2 = \frac{Q^2 - m_K^2 - m_\rho^2}{2\sqrt{Q^2}},$$

$$P = \frac{\sqrt{m_K^4 + m_\rho^4 - 2m_K^2 Q^2 - 2m_K^2 m_\rho^2 - 2Q^2 m_\rho^2}}{2\sqrt{Q^2}}, \quad (30)$$

$$K = \frac{m_\tau^2 - Q^2}{2\sqrt{Q^2}}, \qquad E_4 = \frac{m_\tau^2 + Q^2}{2\sqrt{Q^2}}.$$

The above expressions for various hadron and lepton momentum vectors allow us to determine simple expressions for matrix elements which involve products including $p_1 \cdot p_2$, $p_1 \cdot p_3$, $p_1 \cdot p_4$, $p_2 \cdot p_3$, $p_2 \cdot p_4$, $p_3 \cdot p_4$, $p_1 \cdot Q$, $p_2 \cdot Q$, $p_3 \cdot Q$, $p_4 \cdot Q$, and $Q \cdot \epsilon$ in the term of Q^2, $\sqrt{s}$, and θ. We will integrate over the angle θ since we will not consider the angle distribution. Furthermore, by integrating the denominator and numerator of A_{CP}, respectively, in the region Ω in which Q^2 and s vary in some areas, we obtain the localized integrated CP asymmetry, which takes the following form:

$$A_{CP}^\Omega = \frac{\int_\Omega dQ^2 ds (|M|^2 - |\bar{M}|^2)}{\int_\Omega dQ^2 ds (|M_0|^2 + |\bar{M}_0|^2)}. \quad (31)$$

3 Numerical results

From the above discussions, the CP-violating asymmetries depend on the values of Q^2 and s. In this section we give the explicit expressions of meson wave functions and form factors, and values of several parameters in order to calculate the CP-violating asymmetries. We find that significant cancelation occurs as one performs the integration over Q^2. To show more details as regards this cancelation, we calculate both the differential and the integrated CP asymmetries. We also compare CP asymmetries with and without ρ–ω mixing.

3.1 Models for form factors and meson wave functions

The hadronic τ decay is dominated by the meson annihilation diagram, Fig. 1a. As mentioned before, the vector and pseudoscalar meson annihilation form factors in this decay mode are difficult to be related directly to experimental measurements. One therefore needs to adopt phenomenological

models. Following Ref. [33] we use the meson dominance model in our calculation. In this model it is assumed that the vector form factor g is dominated by the $K^*(892)$ and $K^*(1410)$ vector mesons and f and $a_\pm$ are dominated by the exchange of the K^- pseudoscalar meson and the $K_1(1270)$ and $K_1(1400)$ axial-vector mesons [33]. The expressions for the form factors are given in Eq. (14). In Ref. [33], the values of weak couplings and strong vertex couplings were extracted from experiments and fixed by the SU(3) flavor symmetry. We display these values in Table 1.

We use the K meson wave function of the Brodsky–Huang–Lepage prescription which have the following form [37]:

$$\Phi_K(x, \mathbf{k}_\perp)$$
$$= A_K(1-2x)^2 \exp\left[-b_K^2\left(\frac{\mathbf{k}_\perp^2 + m_s'^2}{x} + \frac{\mathbf{k}_\perp^2 + m_u'^2}{1-x} \right) \right], \quad (32)$$

where $\mathbf{k}_\perp$ is the transverse momentum of the constituents of K, m_u', and m_s' are the constituent quark masses of u and s, respectively. Integrating $\Phi_K(x, \mathbf{k}_\perp)$ over $\mathbf{k}_\perp$ one has the following distribution amplitude:

$$\phi_K(x)$$
$$= \frac{A_K}{16\pi^2 b_K^2} x(1-x)(1-2x)^2 \exp\left[-b_K^2\left(\frac{m_s'^2}{x} + \frac{m_u'^2}{1-x} \right) \right]. \quad (33)$$

In the following numerical calculations we use the parameters $A_K = 232$ GeV^{-1}, $b_K^2 = 0.61$ GeV^{-2}, $m_u' = 350$ MeV, $m_s' = 550$ MeV, and $f_\rho = 221$ MeV [37].

3.2 Numerical results for the CP asymmetries

We are now ready to evaluate numerical results of CP asymmetries. We take the meson masses $m_\rho = 770$ MeV and $m_K = 493$ MeV, the lepton mass $m_\tau = 1776$ MeV, the current quark masses $m_u = 2.3$ MeV, $m_d = 4.8$ MeV, $m_s = 95$ MeV, and $m_c = 1275$ MeV [38]. The CKM matrix, of which the elements are determined from experiments, can be expressed in terms of the Wolfenstein parameters A, ρ, λ, and η [38]:

Fig. 4 The differential CP asymmetry as a function of $\sqrt{s}$ and $\sqrt{Q^2}$. The numerical results correspond to central values of the parameters involved in the calculation
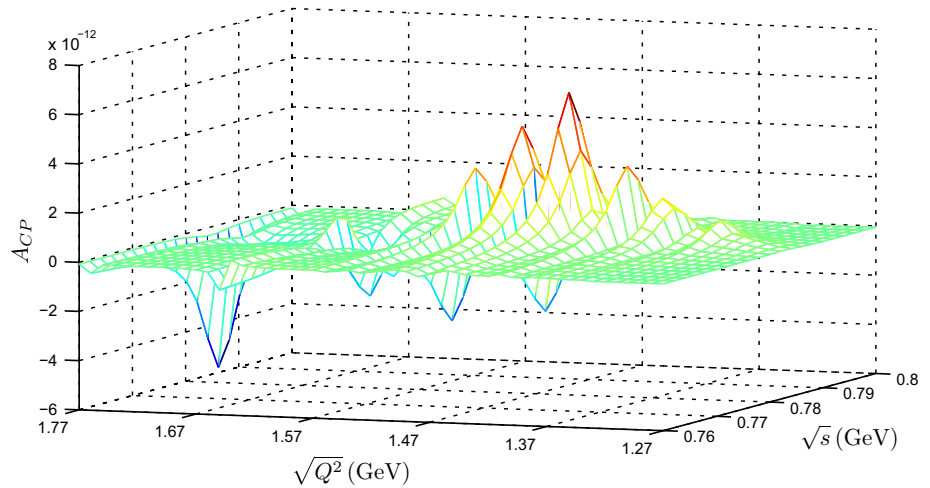
$$\begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \qquad (34)$$

where $\mathcal{O}(\lambda^4)$ corrections are neglected. The latest values for the parameters in the CKM matrix are [38]

$$\lambda = 0.22535 \pm 0.00065, \qquad A = 0.811^{+0.022}_{-0.012},$$
$$\bar{\rho} = 0.131^{+0.0026}_{-0.013}, \qquad \bar{\eta} = 0.345^{+0.013}_{-0.014}, \qquad (35)$$

with

$$\bar{\rho} = \rho\left(1 - \frac{\lambda^2}{2}\right), \qquad \bar{\eta} = \eta\left(1 - \frac{\eta^2}{2}\right). \qquad (36)$$

In our numerical calculations, the most uncertain factors come from the CKM matrix elements and the form factors in the leading-order weak process. In fact, the uncertainties due to the CKM matrix elements are mostly from η since λ is well determined and the CP-violating asymmetries are independent of ρ. Hence in the following we take the central value of λ, 0.225. In the meson dominance model, the uncertainties arising from form factors are dominated by those of the strong and weak coupling constants of the $K_1(1400)$ meson due to the poor quality of measurements. The values of ρ–ω mixing parameters also bring about some uncertainties.

In order to find the details as regards the dependence of the CP violating asymmetries on Q^2 and s, we study the differential CP asymmetries. Since CP asymmetries are calculated around the $\omega(782)$ resonance region, we take the range of $\sqrt{s}$ as 760 MeV $\leq \sqrt{s} \leq$ 800 MeV. From Eqs. (28) and (30), we obtain $(m_\rho + m_K)^2 < Q^2 < m_\tau^2$. Hence we take the range of $\sqrt{Q^2}$ from $(m_\rho + m_K) = 1270$ MeV to $m_\tau = 1770$ MeV. The differential CP asymmetry A_{CP} depending on Q^2 and s is displayed in Fig. 4, where we take central values of the parameters involved in the calculation. We can see that A_{CP} varies from around 10^{-12} to around 10^{-14}. The maximum differential CP-violating asymmetry can reach $-(5.6^{+2.9}_{-1.7}) \times 10^{-12}$,

where the errors come from the uncertainties of the CKM matrix elements, the ρ–ω mixing parameters and the form factors in the leading order in G_F. As we expect, there is a peak for the CP violating parameter A_{CP} when the invariant mass of the $\pi^+\pi^-$ pair is in the vicinity of the ω resonance for a certain $\sqrt{Q^2}$. ρ–ω mixing enlarges A_{CP} by four orders of magnitude in some regions. Furthermore, we also find that the sign of A_{CP} changes frequently in some regions of Q^2. This behavior can easily be understood if one notes that the denominator of A_1 (and B_1), which is defined in Eq. (19) [and (20)], changes its sign when $\sqrt{Q^2}$ crosses the pole. This will lead to cancelations when one performs the integration over Q^2 in some regions. These cancelations are found to be quite obvious around the peak when $\sqrt{s} = 784$ MeV. In order to study the enhancement caused by ρ–ω mixing, we will integrate over Q^2 while keeping $\sqrt{s}$ fixed. We will also compare CP asymmetries with and without ρ–ω mixing in the following.

Firstly, we preform the integration over $\sqrt{Q^2}$ while keeping $\sqrt{s}$ fixed. We divide the integration region into eight equal intervals: (1.30, 1.35 GeV), (1.35, 1.40 GeV), (1.40, 1.45 GeV), (1.45, 1.50 GeV), (1.50, 1.55 GeV), (1.55, 1.60 GeV), (1.60, 1.65 GeV), and (1.65, 1.70 GeV). In each interval, we integrate over $\sqrt{Q^2}$ and calculate the CP asymmetries with and without ρ–ω mixing. The results are denoted by A^s_{CP} and shown in Fig. 5. Our numerical results show that the ρ–ω mixing mechanism enlarges the A^s_{CP} by about one–two orders of magnitude when $\sqrt{s}$ is around 0.784 MeV.

Finally, we integrate A_{CP} over both $\sqrt{Q^2}$ and $\sqrt{s}$ and obtain the localized integrated asymmetries A^Ω_{CP}. Considering the significant region of ρ–ω mixing shown in Fig. 5, we choose the integration interval of $\sqrt{s}$ to be from 0.775 MeV to 0.795 MeV. The numerical results of the localized integrated asymmetries with (without) ρ–ω mixing are shown in Table 2, where the central values of the numerical results correspond to central values of the parameters involved in the calculation and the errors are again from the uncertainties
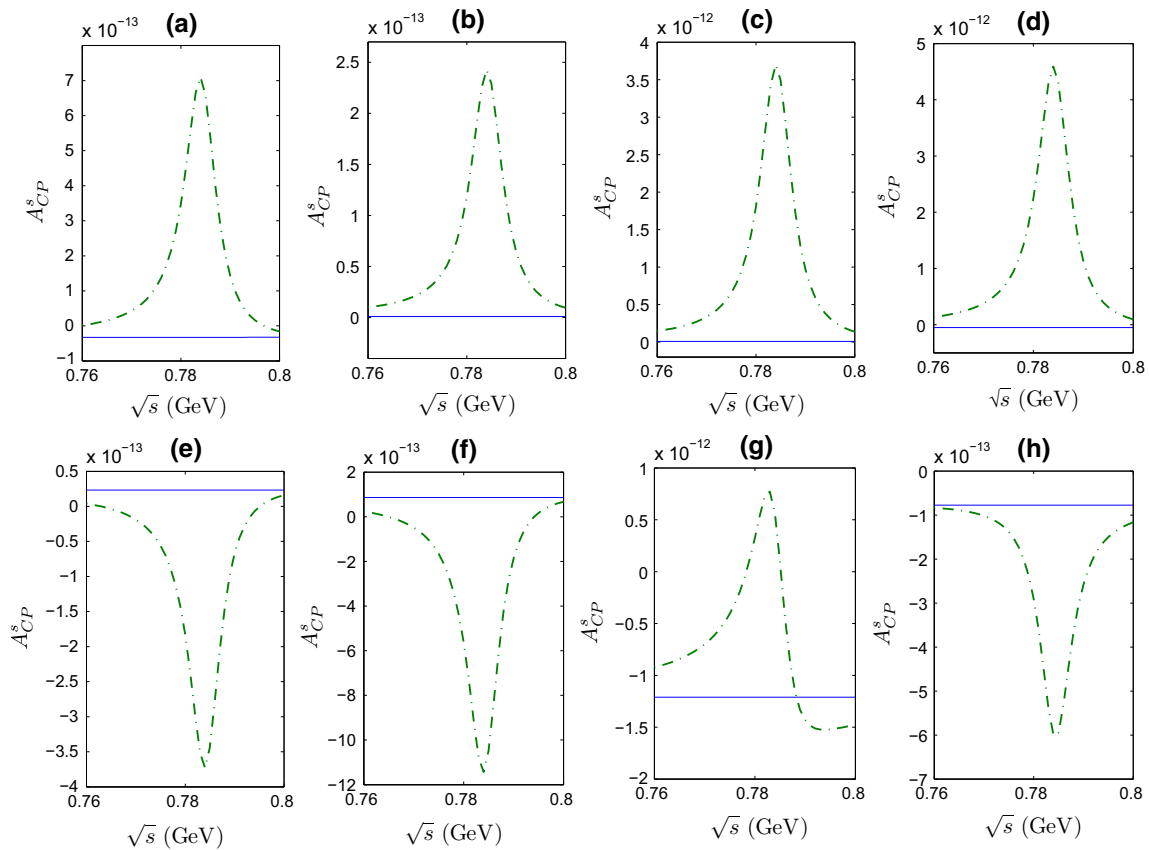
Fig. 5 The localized integrated CP asymmetry A_{CP}^s as a function of $\sqrt{s}$. **a** For integrating over Q^2 in $\sqrt{Q^2} =$ (1.30 GeV, 1.35 GeV): the *dash-dotted line* corresponds to the CP asymmetry including ρ–ω mixing and the *solid line* corresponds to the CP asymmetry without ρ–ω mixing; **b**–**h** correspond to the integration intervals (1.35, 1.40 GeV), (1.40, 1.45 GeV), (1.45, 1.50 GeV), (1.50, 1.55 GeV), (1.55, 1.60 GeV), (1.60, 1.65 GeV), and (1.65, 1.70 GeV), respectively. We take central values of the parameters involved in the calculation

Table 2 The localized integrated asymmetries (in units of 10^{-12}) with (without) ρ–ω mixing. The central values of the numerical results correspond to central values of the parameters involved in the calculation and the errors are from the uncertainties of the CKM matrix elements, the ρ–ω mixing parameters, and the form factors in the leading order in G_F

$\sqrt{Q^2}$ (GeV)	A_{CP}^{Ω}	$\sqrt{Q^2}$ (GeV)	A_{CP}^{Ω}
(1.30, 1.35)	$3.4^{+1.3}_{-2.6}$ $(-0.30^{+0.07}_{-0.19})$	(1.50, 1.55)	$-6.6^{+3.5}_{-4.1}$ $(0.43^{+0.27}_{-0.30})$
(1.35, 1.40)	$9.6^{+3.3}_{-4.9}$ $(0.093^{+0.053}_{-0.041})$	(1.55, 1.60)	$-2.2^{+1.8}_{-0.9}$ $(0.12^{+0.05}_{-0.06})$
(1.40, 1.45)	63^{+24}_{-33} $(0.013^{+0.008}_{-0.004})$	(1.60, 1.65)	$-3.8^{+1.8}_{-2.2}$ (-82^{+47}_{-60})
(1.45, 1.50)	51^{+42}_{-16} $(-0.20^{+0.03}_{-0.09})$	(1.65, 1.70)	$-3.4^{+2.2}_{-1.5}$ $(-0.14^{+0.05}_{-0.01})$

of the CKM matrix elements, the ρ–ω mixing parameters, and the form factors in the leading-order weak process. We can see in most of the intervals ρ–ω mixing enlarges the localized integrated asymmetries. The maximum increase is three orders of magnitude. These predictions lead to a new upper limit of the CP asymmetries based on the SM in this decay channel. Even though the strong phase is enhanced by ρ–ω mixing, the direct CP asymmetry is still negligible. This is just because the small weak relative phase, which comes from the interference between the leading-order and the second-order weak diagrams, is of the third order in G_F.

4 Conclusion

In the framework of the SM, CP violation in the τ lepton decay process arises from a nontrivial phase in the CKM matrix and is predicted to be zero in the leading order in G_F. However, Delepine pointed out that the CP-odd phase can arise from the second-order weak process in the $\tau^{\pm} \to K^{\pm}\pi^0\nu_\tau$ decay mode [22]. Since ρ–ω mixing can provide very large CP asymmetries in some decay channels of heavy hadrons, we have tried to enlarge the CP asymmetry in the $\tau^- \to K^-\rho^0(\omega)\nu_\tau \to K^-\pi^+\pi^-\nu_\tau$ decay via this mechanism.

We have first studied the differential CP asymmetry depending on $\sqrt{Q^2}$ and $\sqrt{s}$. The numerical results show that it varies from around 10^{-12} to around 10^{-14} and the maximum CP-violating asymmetry can reach $-(5.6^{+2.9}_{-1.7}) \times 10^{-12}$. We have found that there is a peak for the CP-violating parameter A_{CP} when the invariant mass of the $\pi^+\pi^-$ pair is in the vicinity of the ω resonance. The advantage of ρ–ω mixing is that it makes the strong phase difference between the hadronic matrix elements of the leading order and the second order in G_F larger at the ω resonance. Consequently, the CP-violating asymmetry reaches the maximum value when the invariant mass of the $\pi^+\pi^-$ pair in the decay product is in the vicinity of the ω resonance. We have also found that A_{CP} changes its sign when $\sqrt{Q^2}$ varies. Then we have calculated the localized integrated CP violating parameter in the τ lepton decay.

After integrating over $\sqrt{Q^2}$ in several intervals, we have shown that the ρ–ω mixing mechanism enlarges the $\sqrt{s}$ dependent CP asymmetry by about one–two orders of magnitude when $\sqrt{s}$ is around 0.784 MeV. The ρ–ω mixing mechanism in present paper can also be considered in the $(3\pi)^{\pm}\nu_\tau$ and $(4\pi)^{\pm}\nu_\tau$ final states in the τ lepton decay.

Finally, we also have performed integration over both $\sqrt{Q^2}$ and $\sqrt{s}$ to obtain A_{CP}^{Ω}. The maximum value of A_{CP}^{Ω} turns out to be $6.3^{+2.4}_{-3.3} \times 10^{-11}$. This value is the largest CP asymmetry in this decay channel within the SM predicted at present. Even though the relatively strong phase is largely enhanced by ρ–ω mixing, the direct CP asymmetry is still negligible. Any measurement of CP violation in this decay channel requires NP, though its effect may be enhanced through the ρ–ω mixing mechanism discussed in this paper.

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