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Superspace with manifest T-duality from type II superstring

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Abstract: A superspace formulation of type II superstring background with manifest T-duality symmetry is presented. This manifestly T-dual formulation is constructed in a space spanned by two sets of nondegenerate super-Poincaré algebras. Supertorsion constraints are obtained from consistency of the κ-symmetric Virasoro constraints. All superconnections and vielbein fields are solved in terms of a prepotential which is one of the vielbein components. $AdS_5\times S^5$ background is explained in this formulation.

Keywords: Supersymmetry and Duality, Superspaces, Supergravity Models, String Duality

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Contents

1 Introduction

T-duality symmetry, which is the symmetry of the low energy effective theory of string, was manifestly realized by spacetime coordinates doubled firstly in [1–3]. T-duality has brought a variety of studies on the generalized complex geometry triggered by [4, 5]. Both aspects of T-duality are characterized by generalized diffeomorphism with manifest T-duality. Various topics in the early stage are for example in [6–9], and recent development is in review articles for example [10, 11]. Supersymmetric T-duality theories with Ramond-Ramond fields have been also developed [12–16].

In this paper we propose a superspace formulation of type II superstring backgrounds with the manifest T-duality symmetry. The space is spanned by two sets of nondegenerate super-Poincaré algebras. The nondegeneracy of the group is required by consistency of the affine Lie algebra. A nondegenerate pair of supertranslation generators $(D_\mu,\ \Omega^\mu)$ was introduced in [17–19], while a nondegenerate pair of Lorentz generators $(S_{mn},\ \Sigma^{mn})$ was introduced in [20–24]. Type II supergravity theories in superspace approach were presented in [25–27] where different superspaces are used; (D, P) superspace is used in [25, 26], (S, D, P, Ω) superspace is used in [27]. We use $(S, D, P, \Omega, \Sigma)$ superspace. The difference leads to different sets of auxiliary fields for the different symmetries, despite of the same physical degrees of freedom.

There are gauge symmetries generated by the generalized Lie bracket [1–3]. The gauge symmetry of the doubled nondegenerate super-Poincaré space includes the general coordinate invariance, the gauge symmetry of NS-NS B field, two local supersymmetries, and two local Lorentz's. Curved backgrounds are introduced in a covariant way by a vielbein superfield $E_{\underline{AM}}$. Supersymmetric vielbein components include the R-R field strength $E_{\underline{\Omega\Omega}}$ in addition to the gravitational field $E_{(\underline{PP})}$, the NS-NS B field $E_{[\underline{PP}]}$ and the NS-NS field strength $E_{\underline{P\Sigma}}$, which are mixed under the T-duality transformation. The T-duality symmetry acts linearly on the vielbein field in a string or a brane Hamiltonian [28–30], while it acts fractional linearly on the gravitational field G and the B field, $G + B$, in the low energy effective theory. Therefore we begin by the superstring mechanics to construct a manifestly T-dual formulation of the type II supergravity.

In a superspace approach supersymmetry is manifest, but a consistent set of torsion constraints is nontrivial. Torsion constraints are derived from the κ-symmetry of the superstring. It was shown that the κ-symmetric Virasoro condition in curved superspace is equal to the supergravity equation of motion [31–33]. We construct κ-symmetric Virasoro constraints in the nondegenerate super-Poincaré space. Torsion constraints are solved dimension by dimension. The gauges are chosen by solving torsion constraints so that all of vielbein are fixed in terms of a prepotential, $E_{\underline{DD}} = B_{\underline{\mu\nu}}$, which is a spinor-spinor component of the B field with dimension -1.

We examine an AdS space case as an curved background example. The covariant derivatives of superstring in the $\mathrm{AdS}_5{\times}\mathrm{S}^5$ background were computed in [20, 21] showing the κ-symmetric Virasoro constraints up to the Lorentz constraint. In this paper we recompute the algebra with manifest Lorentz current S, and reconstruct Ω and Σ currents in such a way that supertorsions are totally graded antisymmetric. Then torsion constraints in the manifestly T-dual formulation are confirmed by comparing with obtained supertorsions and supercurvature tensors.

The procedure of the manifestly T-dual formulation [24] in superspace version is the following:

1. Extend a Lie algebra to an affine Lie algebra.

   Begin with a coset G/H where G must have a nondegenerate group metric. Double the generators, which become the basis of the T-duality symmetry rotation. Generalized diffeomorphism is generated by the zero-mode of the affine Lie algebra.

2. Make covariant derivatives with vielbein E.

   Superconnection fields in the covariant derivatives are recognized as H components of the vielbein, $E_{\underline{A}}{}^{\mathrm{H}}$. Impose coset constraints and an orthogonal condition on vielbein field to reduce vielbein field contents.

3. Constrain torsions from the κ-symmetry.

   Construct a set of κ-symmetric Virasoro constraints, then examine its consistency. Supercurvature tensors are recognized as H components of torsion, $T_{\underline{AB}}{}^{\mathrm{H}}$.

4. Break manifest T-duality symmetry to hidden one.

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Impose dimensional reduction conditions using with symmetry generators which commute with covariant derivatives. Gauge fixings allow to obtain the usual set of coordinates.

The organization of the paper is the following: in section 2 notation of derivatives and indices is summarized. Then the affine super-Poincaré algebra generated by $(S, D, P, \Omega, \Sigma)$ is presented. In section 3 the κ-symmetry constraints as well as Virasoro constraints for type II superstrings are presented. In section 4 the manifestly T-dual formalism of type II superstring background is presented. The vielbein field, embedded in an element of OSp(d^2,d^2 | $2^{d/2}$, $2^{d/2}$), includes superconnections which are usually treated separately from vielbein. Torsion constraints and supercurvature tensors are obtained, where all superconnections are solved in terms of a prepotential. The prepotential superfield is a component of the vielbein with dimension -1 which is the lowest dimension of fields. In section 5 the superstring current algebra in the $AdS_5 \times S^5$ background is presented. Supertorsions and supercurvature tensors are determined. Dimensional reduction constraints are imposed as well as gauge fixing conditions of local Lorentz constraints and the section condition in order to reduce to the usual space.

2 Type II nondegenerate superspace

We consider a superspace which is defined by covariant derivatives of two sets of nondegenerate super-Poincaré algebras. At first a notation of several kinds of covariant derivatives is listed. Then notation of algebras and indices is listed. Next the nondegenerate super-Poincaré algebras are presented.

2.1 Notation

Covariant derivatives. Several kinds of covariant derivatives are denoted as

$$\begin{array}{cccccc} & \text{Lie algebra} \rightarrow & \text{particle} & \rightarrow & \text{string} & \\ \text{Flat} & G_I & \mathring{\nabla}_I & & \mathring{\triangleright}_I & \\ \downarrow & & & & & \\ \text{Curved} & & \nabla_A = E_A{}^I \mathring{\nabla}_I & & \triangleright_A = E_A{}^I \mathring{\triangleright}_I & \end{array} \tag{2.1}$$

Hamiltonians for a particle and a string in curved background are given by bilinears of ∇_A's and $\triangleright_A$'s respectively, where $E_A{}^I$ is vielbein field.

- Particle algebra in flat space

  For a Lie algebra G_I

  $$[G_I, G_J\} = if_{IJ}{}^K G_K \quad ,$$

  symmetry generator $\tilde{\nabla}$ and covariant derivative $\mathring{\nabla}$ are constructed by introducing a group element $g(Z^I)$ with particle coordinates Z^I. The coefficient of Right invariant one form is denoted by $L_M{}^I$ used in the left action generator and vice versa by $R_M{}^I$

used in the right action generator;

$$
\begin{aligned}
\text{Symmetry generator}: \tilde{\nabla}_I &= L_I{}^M \frac{1}{i}\partial_M\ ,\ (dg)g^{-1} = idZ^M L_M{}^I G_I \\
\text{Covariant derivative}: \mathring{\nabla}_I &= R_I{}^M \frac{1}{i}\partial_M\ ,\ g^{-1}(dg) = idZ^M R_M{}^I G_I = J^I G_I \\
&\text{Algebra} \\
[\tilde{\nabla}_I, \tilde{\nabla}_J\} = if_{IJ}{}^K \tilde{\nabla}_K\ \ ,\ \ &[\mathring{\nabla}_I, \mathring{\nabla}_J\} = -if_{IJ}{}^K \mathring{\nabla}_K\ ,\ [\mathring{\nabla}_I, \tilde{\nabla}_J\} = 0
\end{aligned}
$$

where $[*,*\}$ is a graded bracket which is anticommuting for two fermions and commuting for others.

- String algebra in flat space

  Affine Lie algebra is obtained by stringy extension as $Z(\tau,\sigma)$

$$
\begin{aligned}
\text{Symmetry generator}\ &:\ \tilde{\triangleright}_I = L_I{}^M \left(\frac{1}{i}\partial_M + \partial_\sigma Z^N B_{NM}\right) - \partial_\sigma Z^M L_M{}^J \mathring{\eta}_{JI} \\
\text{Flat current}\ &:\ \mathring{\triangleright}_I = R_I{}^M \left(\frac{1}{i}\partial_M + \partial_\sigma Z^N B_{NM}\right) + \partial_\sigma Z^M R_M{}^J \mathring{\eta}_{JI} \\
&\text{Algebra} \\
[\tilde{\triangleright}_I(1), \tilde{\triangleright}_J(2)\} &= if_{IJ}{}^K \tilde{\triangleright}_K \delta(2-1) + i\mathring{\eta}_{IJ}\partial_\sigma\delta(2-1) \\
[\mathring{\triangleright}_I(1), \mathring{\triangleright}_J(2)\} &= -if_{IJ}{}^K \mathring{\triangleright}_K \delta(2-1) - i\mathring{\eta}_{IJ}\partial_\sigma\delta(2-1) \\
[\mathring{\triangleright}_I(1), \tilde{\triangleright}_J(2)\} &= 0
\end{aligned}
$$

  with $1 = \sigma_1$, $2 = \sigma_2$, $\delta(2-1) = \delta(\sigma_1 - \sigma_2)$ and $\partial_\sigma\delta(2-1) = \partial_{\sigma_2}\delta(\sigma_2 - \sigma_1)$. There appear Schwinger terms including $\partial_\sigma\delta$. A group metric $\mathring{\eta}_{IJ}$ is a graded symmetric nondegenerate constant matrix.

- Particle algebra in curved space

  By introducing background fields through vielbein $E_A{}^J(Z)$, the covariant derivatives in a curved background and the algebra become

$$
\begin{aligned}
\text{Covariant derivative}: \nabla_A &= E_A{}^J(Z)\mathring{\nabla}_J \\
\text{Algebra}: [\nabla_A, \nabla_B\} &= -iT_{AB}{}^C \nabla_C \quad .
\end{aligned}
$$

  The torsion $T_{AB}{}^C$ is a function of vielbein fields whose flat limit is the structure constant $f_{IJ}{}^K$.

- String algebra in curved space

  The covariant derivatives in a curved background and the affine algebra become

$$
\begin{aligned}
\text{Curved current}: \triangleright_A &= E_A{}^J(Z(\tau,\sigma))\mathring{\triangleright}_J \\
\text{Algebra}\quad [\triangleright_A, \triangleright_B\} &= -iT_{AB}{}^C \triangleright_C \delta(2-1) - i\eta_{AB}\partial_\sigma\delta(2-1)\ .
\end{aligned}
$$

  The torsion $T_{AB}{}^C$ and the group metric η_{AB} are functions of vielbein fields in general. We will impose η_{AB} to be constant $\mathring{\eta}_{IJ}$.

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Nondegenerate super-Poincaré. We consider the super-Poincaré group with introducing a nondegenerate group metric. Closure of the Jacobi identity of an affine algebra requires a totally graded antisymmetric structure constant. The existence of nondegenerate group metric allows to totally antisymmetric structure constant, $f_{IJK} \equiv f_{IJ}{}^{L}\eta_{LK} = \frac{1}{3!}f_{[IJK]}$.

For a superstring in a flat space the NS-NS three form is given by a closed three form, $H = J^D \wedge J^D \wedge J^P$. Contrast to the fact that B_{NM} field, defined by $\int H = dZ^M dZ^N B_{NM}$, cannot be constant in conventional superspace, the two form potential can be constructed as $B = J^D \wedge J^\Omega$ with $H = dB$ in the nondegenerate superspace.

We begin with a space generated by translation generator p, then we add supersymmetry generator D and Lorentz generator S. They are extended to affine algebras, and further extended in nondegenerate manner as

$$
\begin{array}{llll}
 & \text{translation} & \to\ \text{supertranslation} \to & \text{super}-\text{Poincaré} \\
\text{particle} & p_m & D_\mu, p_m & S_{mn}, D_\mu, p_m \\
\downarrow & & & \\
\begin{array}{l}\text{open}\\ \text{string}\end{array} & P_m = p + \partial_\sigma x & D_\mu, P_m, \Omega^\mu & S_{mn}, D_\mu, P_m\, \Omega^\mu, \Sigma^{mn} \\
\downarrow & & & \\
\begin{array}{l}\text{type II}\\ \text{string}\end{array} & \begin{cases} P_m = p + \partial_\sigma x \\ P_{m'} = p - \partial_\sigma x \end{cases} & \begin{cases} D_\mu, P_m, \Omega^\mu \\ D_{\mu'}, P_{m'}, \Omega^{\mu'} \end{cases} & \begin{cases} S_{mn}, D_\mu, P_m, \Omega^\mu, \Sigma^{mn} \\ S_{m'n'}, D_{\mu'}, P_{m'}, \Omega^{\mu'}, \Sigma^{m'n'} \end{cases}
\end{array}
\tag{2.2}
$$

For type II strings we double whole set of generators in the T-dual formalism. A set of dimensional reduction constraints, coset constraints and the section condition are imposed to remove unphysical degrees of freedom in the end.

- Open superstring

  Nondegenerate super-Poincaré covariant derivatives for a particle $\mathring{\nabla}_M$, the one for a string $\mathring{\triangleright}_M$ and coordinates Z^M are followings:

$$
\begin{array}{lrl}
\text{Indices} & {}_M = & ({}_{mn},\ {}_\mu,\ {}_m,\ {}^\mu,\ {}^{mn}) \\
\text{Flat supercovariant derivative :} & \mathring{\nabla}_M = & (s,\ d,\ p,\ \omega,\ \sigma) \\
\text{Flat super current :} & \mathring{\triangleright}_M = & (S,\ D,\ P,\ \Omega,\ \Sigma) \\
\text{Supercoordinate :} & Z^M = & (u,\ \theta,\ x,\ \varphi,\ v)
\end{array}
\tag{2.3}
$$

- Type II superstring

  Now we double all currents and coordinates in order to construct manifestly T-dual formulation of the type II theory. Left and right currents are denoted by unprimed

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indices and primed indices respectively:

$$
\begin{array}{llr}
\text{Indices} & {}_{M} = ({}_{mn},{}_{\mu},{}_{m},{}^{\mu},{}^{mn}),\ {}_{M'} = ({}_{m'n'},{}_{\mu'},{}_{m'},{}^{\mu'},{}^{m'n'}) & \\
\text{Flat supercovariant derivative :} & \mathring{\triangleright}_{\underline{M}} = (\mathring{\triangleright}_{M},\ \mathring{\triangleright}_{M'}) & \\
\qquad\qquad\text{flat left :} & \mathring{\triangleright}_{M} = (S_{mn},\ D_{\mu},\ P_{m},\ \Omega^{\mu},\ \Sigma^{mn}) & (2.4) \\
\qquad\qquad\text{flat right :} & \mathring{\triangleright}_{M'} = (S_{m'n'},\ D_{\mu'},\ P_{m'},\ \Omega^{\mu'},\ \Sigma^{m'n'}) & \\
\text{Supercoordinates :} & Z^{\underline{M}} = (Z^{M},\ Z^{M'}) &
\end{array}
$$

Type IIA or type IIB is determined by the choice of two kinds of fermions. In ten dimension the chiral representation is used as

$$
\{\Gamma_m,\Gamma_n\} = 2\eta_{mn}\ ,\quad \Gamma_m = \begin{pmatrix} & (\gamma_m)^{\mu\nu} \\ (\gamma_m)_{\mu\nu} & \end{pmatrix}\ ,\quad \Gamma_{mn} = \begin{pmatrix} (\gamma_{mn})^{\mu}{}_{\nu} & \\ & (\gamma_{mn})_{\mu}{}^{\nu} \end{pmatrix}
$$

$$
\Psi = \begin{pmatrix} \psi^{\mu} \\ \chi_{\mu} \end{pmatrix}\ ,\quad [S_{mn},\Psi] = \frac{i}{2}\begin{pmatrix} (\gamma_{mn})^{\mu}{}_{\nu}\psi^{\nu} \\ (\gamma_{mn})_{\mu}{}^{\nu}\chi_{\nu} \end{pmatrix}\ .
$$

Type IIA/IIB fermions are assigned as

$$
\begin{array}{l}
\text{type IIA } (Z^{\mu},\ Z^{\mu'}) = (\theta^{\mu},\ \theta_{\mu}) \\
\text{type IIB } (Z^{\mu},\ Z^{\mu'}) = (\theta_1{}^{\mu},\ \theta_2{}^{\mu})\ ,
\end{array}
$$

with respect to a common Lorentz symmetry generator which is defined only after the dimensional reduction.

2.2 Affine nondegenerate super-Poincaré algebra

The affine nondegenerate super-Poincaré algebra generated by (2.4) is given by

$$
[\mathring{\triangleright}_{\underline{M}}(1), \mathring{\triangleright}_{\underline{N}}(2)\} = -if_{\underline{MN}}{}^{\underline{K}}\mathring{\triangleright}_{\underline{M}}\delta(2-1) - i\eta_{\underline{MN}}\partial_{\sigma}\delta(2-1) \tag{2.5}
$$

where structure constants and nondegenerate metrics in left and right modes are given as

$$
\begin{aligned}
[\mathring{\triangleright}_{M}(1), \mathring{\triangleright}_{N}(2)\} &= -if_{MN}{}^{K}\mathring{\triangleright}_{K}\delta(2-1) - i\eta_{MN}\partial_{\sigma}\delta(2-1) \\
[\mathring{\triangleright}_{M'}(1), \mathring{\triangleright}_{N'}(2)\} &= if_{MN}{}^{K}\mathring{\triangleright}_{K'}\delta(2-1) + i\eta_{MN}\partial_{\sigma}\delta(2-1) \\
[\mathring{\triangleright}_{M}(1), \mathring{\triangleright}_{N'}(2)\} &= 0\ .
\end{aligned}
$$

Canonical dimensions of $(S, D, P, \Omega, \Sigma)$ are $0, \frac{1}{2}, 1, \frac{3}{2}, 2$ respectively. The σ-derivative ∂_{σ} in the Schwinger term carry canonical dimension 2 (string tension is abbreviated). The affine nondegenerate super-Poincaré algebra with $m = 0, 1, \cdots, 9$ and $\mu = 1, \cdots, 16$ is given by

$$\begin{array}{ll}
\text{dim } 0: & [S_{mn}(1), S_{lk}(2)] = -i\eta_{[k|[m}S_{n]|l]}\delta(2-1) \\
\text{dim } \frac{1}{2}: & [S_{mn}(1), D_\mu(2)] = -\frac{i}{2}(D\gamma_{mn})_\mu\delta(2-1) \\
\text{dim } 1: & [S_{mn}(1), P_l(2)] = -iP_{[m}\eta_{n]l}\delta(2-1) \\
& \{D_\mu(1), D_\nu(2)\} = 2P_m\gamma^m{}_{\mu\nu}\delta(2-1) \\
\text{dim } \frac{3}{2}: & [S_{mn}(1), \Omega^\mu(2)] = \frac{i}{2}(\gamma_{mn}\Omega)^\mu\delta(2-1) \\
& [D_\mu(1), P_n(2)] = 2(\gamma_n\Omega)_\mu\delta(2-1) \\
\text{dim } 2: & [S_{mn}(1), \Sigma^{lk}(2)] = -i\delta^{[k}_{[m}\Sigma_{n]}{}^{l]}\delta(2-1) + i\delta^l_{[m}\delta^k_{n]}\partial_\sigma\delta(2-1) \\
& \{D_\mu(1), \Omega^\nu(2)\} = -\frac{i}{4}\Sigma^{mn}(\gamma_{mn})^\nu{}_\mu\delta(2-1) + i\delta^\nu_\mu\partial_\sigma\delta(2-1) \\
& [P_m(1), P_n(2)] = i\Sigma_{mn}\delta(2-1) + i\eta_{mn}\partial_\sigma\delta(2-1)
\end{array} \quad (2.6)$$

Commutators with dimension greater than 5/2 are zero. The gamma matrices satisfy

$$(\gamma^m)_{\mu\nu} = (\gamma^m)_{\nu\mu}\,,\ (\gamma^{(m|})^{\mu\rho}(\gamma^{|n)})_{\rho\nu} = 2\eta^{mn}\delta^\mu_\nu\,,\ (\gamma_m)_{(\mu\nu}(\gamma^m)_{\rho)\lambda} = 0 \quad .$$

The right currents $(S', D', P', \Omega', \Sigma')$ satisfy the same algebra with opposite sign.

The nondegenerate metric $\eta_{\underline{MN}}$ is denoted as:

$$\begin{aligned} \eta_{\underline{MN}} &= \begin{pmatrix} \eta_{MN} & 0 \\ 0 & \eta_{M'N'} \end{pmatrix} \\ &= \begin{pmatrix} \eta_{MN} & 0 \\ 0 & -\eta_{MN} \end{pmatrix} \end{aligned}, \quad \eta_{MN} = \begin{array}{c} S \\ D \\ P \\ \Omega \\ \Sigma \end{array} \begin{pmatrix} & & & & \delta^l_{[m}\delta^k_{n]} \\ & & & \delta^\nu_\mu & \\ & & \eta_{mn} & & \\ & -\delta^\mu_\nu & & & \\ \delta^{[m}_l\delta^{n]}_k & & & & \end{pmatrix} \quad (2.7)$$

3 κ-symmetric Virasoro constraints

The background of a bosonic string is determined by the Virasoro constraints. The Green-Schwarz superstring has κ-symmetry which is necessary to eliminate a half of fermionic degrees of freedom. So the background of the Green-Schwarz superstring is determined by κ-symmetry covariant Virasoro constraints. In this section a consistent set of the κ-symmetry constraints and the Virasoro constraints are obtained.

3.1 Virasoro constraints

The Virasoro constraints are the Hamiltonian constraint $\mathcal{H}_\tau$ and the σ diffeomorphism constraint $\mathcal{H}_\sigma$, which are written in bilinear of brane currents to generate generalized gauge symmetries [28–30], as

$$\begin{aligned} \mathcal{H}_\tau &= \frac{1}{2}\mathring{\triangleright}_{\underline{M}}\hat{\delta}^{\underline{MN}}\mathring{\triangleright}_{\underline{N}} \\ \mathcal{H}_\sigma &= \frac{1}{2}\mathring{\triangleright}_{\underline{M}}\eta^{\underline{MN}}\mathring{\triangleright}_{\underline{N}} \end{aligned} \quad (3.1)$$

$\hat{\delta}_{\underline{MN}}$ and $\eta^{\underline{MN}}$ are

$$
\begin{aligned}
\hat{\delta}^{\underline{MN}} &= \begin{pmatrix} \eta^{MN} & 0 \\ 0 & \eta^{M'N'} \end{pmatrix} = \begin{pmatrix} \eta^{MN} & 0 \\ 0 & \eta^{MN} \end{pmatrix} \\
\eta^{\underline{MN}} &= \begin{pmatrix} \eta^{MN} & 0 \\ 0 & \eta^{M'N'} \end{pmatrix} = \begin{pmatrix} \eta^{MN} & 0 \\ 0 & -\eta^{MN} \end{pmatrix} \\
\eta^{MN} &= \begin{array}{c} S \\ D \\ P \\ \Omega \\ \Sigma \end{array} \begin{pmatrix} & & & & \delta_l^{[m}\delta_k^{n]} \\ & & & -\delta_\nu^\mu & \\ & & \eta^{mn} & & \\ & \delta_\mu^\nu & & & \\ \delta^l_{[m}\delta^k_{n]} & & & & \end{pmatrix}
\end{aligned}
$$

satisfying

$$
\eta^{\underline{MN}}\eta_{\underline{NK}}\eta^{\underline{KL}} = \hat{\delta}^{\underline{MN}}\eta_{\underline{NK}}\hat{\delta}^{\underline{KL}} = \eta^{\underline{ML}} \quad . \tag{3.2}
$$

Virasoro algebra is given by

$$
\begin{aligned}
[\mathcal{H}_\sigma(1),\ \mathcal{H}_\sigma(2)] &= i\left(\mathcal{H}_\sigma(1) + \mathcal{H}_\sigma(2)\right)\partial_\sigma\delta(2-1) \\
[\mathcal{H}_\tau(1),\ \mathcal{H}_\tau(2)] &= i\left(\mathcal{H}_\sigma(1) + \mathcal{H}_\sigma(2)\right)\partial_\sigma\delta(2-1) \\
[\mathcal{H}_\sigma(1),\ \mathcal{H}_\tau(2)] &= i\left(\mathcal{H}_\tau(1) + \mathcal{H}_\tau(2)\right)\partial_\sigma\delta(2-1)
\end{aligned} \tag{3.3}
$$

where (3.2) is used.

The diffeomorphism constraint $\mathcal{H}_\sigma$ generates σ derivative and $\mathring{\triangleright}_{\underline{M}}$ generates covariant derivatives as

$$
\begin{aligned}
\partial_\sigma\Phi(Z^{\underline{N}}) = [\textstyle\int i\mathcal{H}_\sigma, \Phi(Z^{\underline{N}})] \ , \quad [i\mathring{\triangleright}_{\underline{M}}(1), \Phi(Z^{\underline{N}}(2))] = D_{\underline{M}}\Phi(Z^{\underline{N}})\delta(2-1) \\
\Rightarrow \quad \partial_\sigma\Phi(Z^{\underline{N}}) = \mathring{\triangleright}_{\underline{M}}\eta^{\underline{MK}}\left(D_{\underline{K}}\Phi(Z^{\underline{N}})\right) \quad .
\end{aligned} \tag{3.4}
$$

Virasoro constraints $\mathcal{H}_\tau$ and $\mathcal{H}_\sigma$ are separated into left and right Virasoro constraints $\mathcal{A}$ and $\mathcal{A}'$ respectively

$$
\begin{aligned}
\mathcal{A} &= \frac{1}{2}\left(\mathcal{H}_\tau + \mathcal{H}_\sigma\right) = \frac{1}{2}\mathring{\triangleright}_M\eta^{MN}\mathring{\triangleright}_N \\
\mathcal{A}' &= \frac{1}{2}\left(\mathcal{H}_\tau - \mathcal{H}_\sigma\right) = -\frac{1}{2}\mathring{\triangleright}_{M'}\eta^{MN}\mathring{\triangleright}_{N'} \quad .
\end{aligned} \tag{3.5}
$$

3.2 Fermionic constraints and κ-symmetry

The covariant expression of the Green-Schwarz superstring has fermionic constraints, $D_{\underline{\mu}} = 0$. The κ-symmetry is generated by first class constraints $\mathcal{B}^\mu = D_\nu P_m(\gamma^m)^{\nu\mu}$ and $\mathcal{B}^{\mu'} = D_{\nu'}P_{m'}(\gamma^m)^{\nu\mu}$ which is a half of $D_{\underline{\mu}}$. Another half is second class constraint. Instead of imposing second class constraints, first class constraints were constructed by bilinear of the second class constraints [19] as $\mathcal{C}_{\mu\nu}, \mathcal{D}_m = 0$.

Two approaches of fermionic constraints are;

- Second class approach:

$$\begin{aligned}\text{First class constraints} &: \mathcal{A} = \mathcal{B}^\mu = 0 \\ \text{Second class constraints} &: D_\mu = 0\end{aligned}$$

- First class approach:

$$\text{First class constraints} : \mathcal{A} = \mathcal{B}^\mu = \mathcal{C}_{\mu\nu} = \mathcal{D}_m = 0$$

In this section we extend these conditions to the nondegenerate super-Poincaré case.

We extend $\mathcal{ABCD}$ constraints to the nondegenerate super-Poincaré space as

$$\begin{aligned}\mathcal{A} &= \frac{1}{2}P_m P_n \eta^{mn} + \Omega^\mu D_\mu + \frac{1}{2}\Sigma^{mn} S_{mn} \\ \mathcal{B}^\mu &= (D\gamma^m)^\mu P_m - iS_{mn}(\gamma^{mn}\Omega)^\mu = (D\not{P})^\mu - i(\not{S}\Omega)^\mu \\ \mathcal{C}_{\mu\nu} &= D_\mu D_\nu + \frac{1}{2i}S_{mn}P_l(\gamma^{mnl})_{\mu\nu} = \frac{1}{2}D_{[\mu}D_{\nu]} + \frac{1}{4i}(\not{S}\not{P})_{[\mu\nu]} \\ \mathcal{D}_m &= (D\gamma_m \partial_\sigma D) + \frac{4}{i}\Sigma_{mn}S^{nl}P_l \quad .\end{aligned} \tag{3.6}$$

Although $\mathcal{D}_m$ constraint includes trilinear term, Σ^{mn} commutes with other generators except S.

The $\mathcal{ABCD}$ algebra is computed analogously to the Virasoro constraints by using the metric relation and antisymmetricity of the structure constant. The $\mathcal{ABCD}$ constraints in the nondegenerate super-Poincaré space given in (3.6) satisfy the following algebra;

$$\begin{aligned}[\mathcal{A}(1), \mathcal{A}(2)] &= i\left(\mathcal{A}(1) + \mathcal{A}(2)\right)\partial_\sigma \delta(2-1) \\ [\mathcal{A}(1), \mathcal{B}^\mu(2)] &= i\left(\mathcal{B}^\mu(1) + \mathcal{B}^\mu(2)\right)\partial_\sigma \delta(2-1) \\ [\mathcal{A}(1), \mathcal{C}_{\mu\nu}(2)] &= i\left(\mathcal{C}_{\mu\nu}(1) + \mathcal{C}_{\mu\nu}(2)\right)\partial_\sigma \delta(2-1) \\ [\mathcal{A}(1), \mathcal{D}_m(2)] &= i\left(\mathcal{D}_m(1) + 2\mathcal{D}_m(2)\right)\partial_\sigma \delta(2-1) \\ [\mathcal{B}^\mu(1), \mathcal{B}^\nu(2)] &= i\frac{1}{2}\left(\mathcal{C}_{\rho\lambda}(1) + \mathcal{C}_{\rho\lambda}(2)\right)(\gamma_m)^{\mu\rho}(\gamma^m)^{\nu\lambda}\partial_\sigma \delta(2-1) \\ &\quad + \left[(\gamma^m)^{\mu\nu}\left(4P_m\mathcal{A} + 2(\mathcal{B}\gamma_m\Omega) + \frac{i}{2}\mathcal{D}_m\right)\right. \\ &\quad \left. -4\mathcal{B}^{(\mu}\Omega^{\nu)} + i\mathcal{C}_{\rho\lambda}(\gamma^m)^{\mu\rho}(\gamma^n)^{\nu\lambda}\Sigma_{mn}\right]\delta(2-1) \\ [\mathcal{B}^\mu(1), \mathcal{C}_{\nu\rho}(2)] &= \left[4\mathcal{A}\delta^\mu_{[\nu}D_{\rho]} + \frac{1}{8}\mathcal{B}^\lambda(\gamma_{mn})_\lambda{}^\mu(\gamma^{mn}\not{P})_{[\nu\rho]}\right. \\ &\quad \left. + \mathcal{C}_{\sigma[\nu}c^{\mu\sigma}_{\rho]} + S_{mn}\tilde{c}^{mn;\mu}_{\nu;\rho} - (\partial_\sigma S_{mn})\frac{1}{2}(D\gamma_l)^\mu(\gamma^{mnl})_{\nu\rho}\right]\delta(2-1) \\ &\quad + S_{mn}\left(\frac{1}{2}(D\gamma_l)^\mu(\gamma^{mnl})_{\nu\rho} - (\gamma^{mn})^\mu{}_{[\nu}D_{\rho]}\right)(1)\partial_\sigma\delta(2-1)\end{aligned}$$

$$
\begin{aligned}
[\mathcal{B}^\mu(1),\mathcal{D}_m(2)] &= \left[-4\mathcal{A}(D\gamma_m)^\mu + \mathcal{C}_{\rho\nu}(\gamma^n)^{\nu\mu}(\gamma_n\gamma_m\Omega)^\rho\right](1)\partial_\sigma\delta(2-1)\\
&\quad + \Big[8\mathcal{A}\partial_\sigma(D\gamma_m)^\mu - 2(\mathcal{B}\gamma^{nl})^\mu\Sigma_{mn}P_l\\
&\quad +4\partial_\sigma\mathcal{C}_{\nu\rho}\left((\gamma_m)^{\mu\nu}\Omega^\rho + \frac{1}{2}(\gamma^n)^{\mu\nu}(\gamma_m\gamma_n\Omega)^\rho\right)\\
&\quad +2\mathcal{D}^n(\gamma_n\gamma_m\Omega)^\mu + 2\mathcal{D}^l(\gamma_{lm}\Omega)^\mu + c^\mu{}_m{}^{nl}S_{nl}\Big]\,\delta(2-1)\\
[\mathcal{C}_{\mu\nu}(1),\mathcal{C}_{\rho\lambda}(2)] &= -\frac{i}{16}(\not{S}\gamma^l)_{[\mu\nu]}(\not{S}\gamma_l)_{[\rho\lambda]}(1)\partial_\sigma\delta(2-1)\\
&\quad + [\mathcal{C}_{\sigma\eta}c^{\sigma\eta} + S_{mn}\tilde{c}^{mn}_{\mu\nu;\rho\lambda}]\delta(2-1)\\
[\mathcal{C}_{\mu\nu}(1),\mathcal{D}_m(2)] &= -4\mathcal{C}_{\mu\rho}(\not{P}\gamma_m)_\nu{}^\rho(1)\partial_\sigma\delta(2-1)\\
&\quad + \big[-4(\mathcal{B}\gamma_m)_{[\mu}\partial_\sigma D_{\nu]} + 8(\partial_\sigma\mathcal{C}_{\mu\nu})P_m - 8(\partial_\sigma\mathcal{C}_{\mu\rho})(\gamma\not{P})_\nu{}^\rho\\
&\quad -2\mathcal{C}_{\rho[\mu}(\gamma^{nl})^\rho{}_{\nu]}\Sigma_{mn}P_l - \frac{1}{2}\mathcal{D}^l(\gamma_{lm}\not{P})_{\mu\nu} + c^{nl}_{\mu\nu m}S_{nl}\Big]\,\delta(2-1)\\
[\mathcal{D}_m(1),\mathcal{D}_n(2)] &= -2\mathcal{C}_{\mu\nu}(\not{P}\gamma_{mn})^{\mu\nu}(1)\partial_\sigma{}^2(2-1)\\
&\quad + \left(2\mathcal{C}_{\mu\nu}\partial_\sigma(\not{P}\gamma_{mn})^{\mu\nu} - 8\mathcal{D}_{(m}P_{n)} + 8\mathcal{D}_lP^l\eta_{mn}\right)\partial_\sigma\delta(2-1)\\
&\quad - 4\Big[\partial_\sigma(\mathcal{B}\gamma_{mn}\partial_\sigma D) + 2(\partial_\sigma\mathcal{B})\gamma_{mn}(\partial_\sigma D) - (\partial_\sigma\mathcal{C}_{\mu\nu})(\not{P}\gamma_{mn})^{\mu\nu}\\
&\quad -2\mathcal{D}^k\Sigma^{m[k}P_{n]} - \partial_\sigma(\mathcal{D}_{[m}P_{n]}) + c^{lk}_{mn}S_{lk}\Big]\,\delta(2-1)
\end{aligned}
\tag{3.7}
$$

where $c^\mu{}_m{}^{nl}$, $c_{\mu\nu m}{}^{nl}$ and $c_{mn}{}^{lk}$ are coefficient functions, for example

$$
\begin{aligned}
[\mathcal{B},\mathcal{C}] &\ni\\
c^{\mu\sigma}_\rho &= 4\delta^\mu_\rho\Omega^\sigma - 2(\gamma^m)^{\mu\sigma}(\gamma_m\Omega)_\rho + \frac{1}{2}(\gamma_{mn})^\sigma{}_\rho(\gamma^{mn}\Omega)^\mu\\
\tilde{c}^{mn;\mu}_{\nu;\rho} &= -\frac{1}{2i}(\gamma^{mnl})_{\sigma[\nu}P_l(c^{\mu\sigma}_{\rho]} - 2\delta^\sigma_\rho\Omega^\mu) - 2\Sigma^{mn}\delta^\mu_{[\nu}D_{\rho]}\\
&\quad -\frac{1}{4}(\gamma^{mn}\not{\Sigma})^\mu{}_{[\nu}D_{\rho]} + \frac{1}{2}(D\gamma^k)\Sigma_{kl}(\gamma^{mnl}_{\nu\rho}) + (\gamma^{mn})^\mu{}_{[\nu}\partial_\sigma D_{\rho]}\\
[\mathcal{C},\mathcal{C}] &\ni\\
c^{\sigma\eta}_{\mu\nu;\rho\lambda} &= 2\delta^\eta_{[\lambda|}\delta^\sigma_{[\mu}\not{P}_{\nu]|\rho]} + \frac{1}{8}\delta^\sigma_{[\mu}(\gamma_{mn})^\eta{}_{\nu]}(\gamma^{mn}\not{P})_{[\rho\lambda]} - \frac{1}{8}\delta^\sigma_{[\rho}(\gamma_{mn})^\eta{}_{\lambda]}(\gamma^{mn}\not{P})_{[\mu\nu]}\\
\tilde{c}^{mn}_{\mu\nu;\rho\lambda} &= \frac{i}{4}c^{\sigma\eta}_{\mu\nu;\rho\lambda}(\gamma^{mn}\not{P})_{[\sigma\eta]} - iD_{[\mu}(\gamma_l\Omega)_{\nu]}(\gamma^{mnl})_{\rho\lambda} + iD_{[\rho}(\gamma_l\Omega)_{\lambda]}(\gamma^{mnl})_{\mu\nu}\\
&\quad -\frac{i}{4}(\gamma^{ml}\not{P})_{[\mu\nu]}(\gamma^n{}_l\not{P})_{[\rho\lambda]} - \frac{i}{8}(\gamma^{mnl})_{\mu\nu}\left((\gamma^k\not{S})_{[\rho\lambda]}\Sigma_{lk} - (\gamma_l\partial_\sigma\not{S})_{[\rho\lambda]}\right)
\end{aligned}
$$

Since coefficients of S_{mn} do not vanish even on the constrained surface, consistent space is coset space G/H where G is nondegenerate super-Poincaré group and H is Lorentz group. Subgroup H is gauged and the coset space is defined by

$$
S_{mn} = 0 \quad . \tag{3.8}
$$

$\mathcal{ABCD}$ constraints in (3.6) are reducible up to $S=0$ constraint ($\approx$)

$$
D_\mu\mathcal{B}^\nu \approx \mathcal{C}_{\mu\rho}\not{P}^{\rho\nu} \quad , \quad (\mathcal{B}\not{P})_\mu \approx 2\mathcal{A}D_\mu - 2\mathcal{C}_{\mu\rho}\Omega^\rho \quad \text{etc.}
$$

The type II theory has two sets of constraints (3.6), $(\mathcal{ABCDA'B'C'D'}) = 0$. $\mathcal{A'B'C'D'}$ are constructed by bilinears of right currents $\mathring{\triangleright}_{M'}$. They satisfy the same algebra (3.7) with opposite sign, and $\mathcal{ABCD}$ commute with $\mathcal{A'B'C'D'}$.

Next the second class approach is also examined. The consistency condition is the closure of algebra between first and second class constraints $D_\mu = 0$;

$$\begin{aligned}
[\mathcal{A}(1), D_\mu(2)] &= iD_\mu(1)\partial_\sigma\delta(2-1) \ \approx\ 0 \\
\{\mathcal{B}^\mu(1), D_\nu(2)\} &= \Bigg[4\mathcal{A}\delta^\mu_\nu - \left(4\Omega^\rho\delta^\mu_\nu - (\gamma^m)^{\mu\rho}(\gamma_m\Omega)_\nu + \frac{1}{2}(\gamma_{mn}\Omega)^\mu(\gamma_{mn})^\rho{}_\nu\right) D_\rho \\
&\quad + \left(2\Sigma^{nm}\delta^\mu_\nu + \frac{1}{4}(\gamma^{nm}\not{\Sigma})^\mu{}_\nu\right) S_{mn}\Bigg]\delta(2-1) \\
&\quad -\not{S}^\mu{}_\nu(1)\partial_\sigma\delta(2-1) \ \approx\ 0
\end{aligned}$$

They vanish up to second class and first class constraints $D_\mu = \mathcal{A} = \mathcal{B}^\mu = S_{mn} = 0$ ($\approx$).

Therefore the constraint sets for the type II Green-Schwarz superstring in nondegenerate super-Poincaré space are summarized as;

- Second class approach:

$$\begin{aligned}
\text{First class constraints}: &\quad \mathcal{A}, \mathcal{B}^\mu = \mathcal{A}', \mathcal{B}^{\mu'} = S_{mn} = S_{m'n'} = 0 \\
\text{Second class constraints}: &\quad D_\mu = D_{\mu'} = 0
\end{aligned}$$

- First class approach:

$$\text{First class constraints}: \quad \mathcal{A}, \mathcal{B}^\mu, \mathcal{C}_{\mu\nu}, \mathcal{D}_m = \mathcal{A}', \mathcal{B}^{\mu'}, \mathcal{C}_{\mu'\nu'}, \mathcal{D}_{m'} = S_{mn} = S_{m'n'} = 0$$

It is important that all the κ-symmetric Virasoro constraints are written by bilinears of $\mathring{\triangleright}$ with arbitrary coefficients a's

$$\mathcal{B}^\mu a_\mu = \frac{1}{2}\mathring{\triangleright}_M\rho^{MN}\mathring{\triangleright}_N\,,\ \mathcal{C}_{\mu\nu}\check{a}^{\mu\nu} = \frac{1}{2}\mathring{\triangleright}_M\check{\rho}^{MN}\mathring{\triangleright}_N\,,\ \mathcal{D}_m\tilde{a}^m = \frac{1}{2}\mathring{\triangleright}_M\tilde{\rho}^{MN}\mathring{\triangleright}_N \tag{3.9}$$

$$\rho^{MN} = \begin{array}{c} S \\ D \\ P \\ \Omega \\ \Sigma \end{array}\left(\begin{array}{ccccc} & & & -i(\gamma^{mn})^\mu{}_\nu a_\mu & \\ & & (\gamma^n)^{\mu\nu}a_\nu & & \\ & (\gamma^m)^{\mu\nu}a_\mu & & & \\ -i(\gamma^{nl})^\nu{}_\mu a_\nu & & & & \\ & & & & \end{array}\right)$$

$$\check{\rho}^{MN} = \begin{array}{c} S \\ D \\ P \\ \Omega \\ \Sigma \end{array}\left(\begin{array}{ccccc} & & \frac{1}{2i}(\gamma^{mnl})_{\mu\nu}\check{a}^{\mu\nu} & & \\ & \check{a}^{[\mu\nu]} & & & \\ \frac{1}{2i}(\gamma^{nlm})_{\mu\nu}\check{a}^{\mu\nu} & & & & \\ & & & & \\ & & & & \end{array}\right)$$

$$\tilde{\rho}^{MN} = \begin{array}{c} S \\ D \\ P \\ \Omega \\ \Sigma \end{array}\left(\begin{array}{ccccc} & & \frac{4}{i}\tilde{a}^k\Sigma_{km}\delta^l_n & & \\ & \tilde{a}^m(\gamma_n)^{(\mu\nu)}\partial_\sigma & & & \\ \frac{4}{i}\tilde{a}^k\Sigma_{kn}\delta^m_l & & & & \\ & & & & \\ & & & & \end{array}\right)$$

ρ's are nilpotent matrices; $\rho^5 = \check{\rho}^3 = \tilde{\rho}^3 = 0$ by lowering index with η_{MN}. They satisfy $\rho^2 = \check{\rho}$, where this relation gives $\{\mathcal{B},\mathcal{B}\} \approx \mathcal{C}\partial_\sigma\delta$. Sum of $\mathcal{ABCD}$ $\mathcal{A}'\mathcal{B}'\mathcal{C}'\mathcal{D}'$ becomes a manifestly T-dual bilinear as

$$\Xi(a) = \frac{1}{2}\mathring{\triangleright}_M \Xi^{MN}(a)\mathring{\triangleright}_N \ , \quad \Xi^{MN}(a) = [\eta + \rho(a) + \check{\rho}(\check{a}) + \tilde{\rho}(\tilde{a})]^{MN} \quad . \tag{3.10}$$

Ξ is covariant under the T-duality rotation; $\Xi(a) \to \Xi(a') = M\Xi(a)M^T$ under $\mathring{\triangleright} \to M\mathring{\triangleright}$ with $M\eta M^T = \eta$, since Ξ^{MN} is upper triangular matrix. A set of κ-symmetric Virasoro constraints is consistent to T-duality symmetry.

4 Manifestly T-dual formulation of type II superstring curved background

We present a manifestly T-dual formulation of the low energy effective gravity theory for the type II superstring. The procedure was given in [24], and we extend this to a type II superspace. The space is spanned by the nondegenerate super-Poincaré algebra, and curved background fields are expanded by this basis. The vielbein includes R-R field strength as well as all background fields. Torsions include supercurvature tensors. It turns out that all superconnections are solved in terms of a prepotential which is included in the vielbein.

4.1 Vielbein, torsion and Bianchi identity

Covariant derivatives in curved background are given with vielbein $E_{\underline{A}}{}^{\underline{M}}$ as

$$\triangleright_{\underline{A}} = E_{\underline{A}}{}^{\underline{M}}(Z^{\underline{N}})\mathring{\triangleright}_{\underline{M}} \tag{4.1}$$

which satisfy the following algebra

$$[\triangleright_{\underline{A}}(1), \triangleright_{\underline{B}}(2)\} = -iT_{\underline{AB}}{}^{\underline{C}}\triangleright_{\underline{C}}\delta(2-1) - i\eta_{\underline{AB}}\partial_\sigma\delta(2-1) \quad . \tag{4.2}$$

Vectors in this space, $\hat{\Lambda}_i = \Lambda_i{}^{\underline{M}}\mathring{\triangleright}_{\underline{M}}$ with $i = 1, 2$, satisfy the following supersymmetric Lie bracket in a manifestly T-dual formulation

$$\begin{aligned} \left[\hat{\Lambda}_1, \hat{\Lambda}_2\right]_{\mathrm{T}} &= -i\hat{\Lambda}_{12} \\ \Lambda_{12}{}^{\underline{M}} &= \Lambda_{[1}{}^{\underline{N}}(D_{\underline{N}}\Lambda_{2]}{}^{\underline{M}}) + \Lambda_1{}^{\underline{N}}(D^{\underline{M}}\Lambda_{2\underline{N}}) + \Lambda_1{}^{\underline{N}}\Lambda_2{}^{\underline{L}}f_{\underline{NL}}{}^{\underline{M}} \quad . \end{aligned} \tag{4.3}$$

There is an ambiguity from Schwinger term proportional to $\partial_\sigma\delta$ in (4.2). The vielbein field $\hat{E}_{\underline{A}} = E_{\underline{A}}{}^{\underline{M}}\mathring{\triangleright}_{\underline{M}}$ has gauge symmetry generated by the above bracket as

$$\begin{aligned} \delta_\Lambda\hat{E}_{\underline{A}} &= \left[\hat{E}_{\underline{A}}, \hat{\Lambda}\right]_{\mathrm{T}} \\ \delta_\Lambda E_{\underline{A}}{}^{\underline{M}} &= E_{\underline{A}}{}^{\underline{N}}(D_{\underline{N}}\Lambda^{\underline{M}}) - \Lambda^{\underline{N}}(D_{\underline{N}}E_{\underline{A}}{}^{\underline{M}}) + E_{\underline{A}}{}^{\underline{N}}(D^{\underline{M}}\Lambda_{\underline{N}}) + E_{\underline{A}}{}^{\underline{N}}\Lambda^{\underline{L}}f_{\underline{NL}}{}^{\underline{M}} \quad . \end{aligned} \tag{4.4}$$

The general coordinate transformation parameter is $\lambda^m + \lambda^{m'}$, the gauge transformation of B field is $\lambda^m - \lambda^{m'}$, the local supersymmetry parameters are λ^μ, $\lambda^{\mu'}$ and the two local Lorentz parameters are λ^{mn} and $\lambda^{m'n'}$. Other parameters are unphysical whose symmetries are fixed by dimensional reduction constraints in terms of $\tilde{\Sigma}$ and $\tilde{\Omega}$.

Manifestly T-dual formulation allows to impose orthonormal condition on vector parts of vielbein $e_{\underline{a}}{}^{\underline{m}} = \begin{pmatrix} e_a{}^m & e_a{}^n B_{nm} \\ 0 & e_m{}^a \end{pmatrix}$ for $\mathring{\triangleright}_{\underline{m}} = (P_m,\ \partial_\sigma x^m)$ which satisfies $e\eta e^T = \eta$ with $\eta_{MN} = \eta_{AB} = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$. The vielbein field is an element of O(d,d) symmetry which is continuous symmetry of zero-mode part of whole string T-duality. For supersymmetric case it is enlarged to

$$E_{\underline{A}}{}^{\underline{M}} \eta_{\underline{MN}} E_{\underline{B}}{}^{\underline{N}} = \eta_{\underline{AB}} \ \ , \ \ \eta_{\underline{MN}} = \eta_{\underline{AB}} \ \ , \tag{4.5}$$

which is $\mathrm{OSp}(\mathrm{d}^2,\mathrm{d}^2|2^{\mathrm{d}/2},2^{\mathrm{d}/2})$ for d dimensional space. Both flat currents and curved currents satisfy algebras with the same Schwinger term. Torsion with lower indices is totally graded antisymmetric, and is expressed from (3.4) and (4.5) as

$$T_{\underline{ABC}} \equiv T_{\underline{AB}}{}^{\underline{D}} \eta_{\underline{DC}} = \frac{1}{3!} T_{[\underline{ABC})} = \frac{1}{2} (D_{[\underline{A}} E_{\underline{B}}{}^{\underline{M}}) E_{\underline{C})\underline{M}} + E_{\underline{A}}{}^{\underline{M}} E_{\underline{B}}{}^{\underline{N}} E_{\underline{C}}{}^{\underline{L}} f_{\underline{MNL}} \tag{4.6}$$

with $D_{\underline{A}} = E_{\underline{A}}{}^{\underline{M}} D_{\underline{M}}$. Graded antisymmetricity requires suitable sign factor which are omitted in this expression, with $\mathcal{O}_{[AB)} = \mathcal{O}_{AB} - (-)^{AB} \mathcal{O}_{BA}$. Bianchi identity gives the following totally graded antisymmetric tensor to vanish

$$\mathcal{I}_{\underline{ABCD}} = \frac{1}{4!} \mathcal{I}_{[\underline{ABCD})} = (D_{[\underline{A}} T_{\underline{BCD})}) + \frac{3}{4} T_{[\underline{AB}}{}^{\underline{E}} T_{\underline{CD})\underline{E}} = 0 \quad . \tag{4.7}$$

We divide the coset space G/H as follows;

$$\begin{cases} \mathrm{G} = (\underline{S},\ \underline{D},\ \underline{P},\ \underline{\Omega},\ \underline{\Sigma}) \\ \mathrm{H} = (\underline{S}) \\ \tilde{\mathrm{H}} = (\underline{\Sigma}) \\ \mathrm{K} = (\underline{D},\ \underline{P},\ \underline{\Omega}) \end{cases} \tag{4.8}$$

The torsion constraints with Lorentz lower indices are imposed in such a way that local Lorentz symmetries satisfy the same algebra to the flat algebra

$$T_{\underline{SAB}} = f_{\underline{SAB}} \quad . \tag{4.9}$$

The algebras among covariant derivatives in K, H, $\tilde{\mathrm{H}}$ are given as follows with $\nabla_I \in$K$=(\underline{D},\underline{P},\underline{\Omega})$

$$\begin{cases} [\nabla_I, \nabla_J\} = -iR_{IJ}{}^{\underline{S}} \nabla_{\underline{S}} - iT_{IJ}{}^K \nabla_K - i f_{IJ\underline{S}} \nabla_{\underline{\Sigma}} \\ [\nabla_{\underline{S}}, \nabla_{\underline{S}}] = -i f_{\underline{SS\Sigma}} \nabla_{\underline{S}}\,,\ [\nabla_{\underline{S}}, \nabla_{\underline{\Sigma}}] = -i f_{\underline{S\Sigma S}} \nabla_{\underline{\Sigma}}\,,\ [\nabla_{\underline{S}}, \nabla_I] = -i f_{\underline{S}I}{}^J \nabla_J \\ [\nabla_{\underline{\Sigma}}, \nabla_I] = -iT_{\underline{\Sigma}I\underline{\Sigma}} \nabla_{\underline{S}} - iR_I{}^{J\underline{S}} \nabla_J\,,\ [\nabla_{\underline{\Sigma}}, \nabla_{\underline{\Sigma}}] = -iT_{\underline{\Sigma\Sigma\Sigma}} \nabla_{\underline{S}} - iT_{\underline{\Sigma\Sigma}}{}^I \nabla_I \end{cases} \ . \tag{4.10}$$

Nontrivial torsions are obtained only from $[\nabla_I, \nabla_J\}$. Torsions with Lorentz upper index are curvature tensors. Supercurvatures $R_{IJ}{}^{\underline{S}}$ are written in terms of superconnections $\omega_I{}^{\underline{S}} = E_I{}^{\underline{S}}$ as

$$
\begin{aligned}
R_{IJ}{}^{\underline{S}} = T_{IJ\underline{\Sigma}} = & -(D_I E_J{}^{\underline{M}})\omega_{\underline{M}}{}^{\underline{S}} - (D_J \omega_{\underline{M}}{}^{\underline{S}})E_I{}^{\underline{M}} + (D_\Sigma E_{\underline{I}}{}^{\underline{M}})E_{J\underline{M}} \\
& -(\omega_I{}^{\underline{S}} E_J{}^{\underline{M}} \omega^{\underline{NS}} + E_I{}^{\underline{M}} E_J{}^{\underline{N}} r^{\underline{S}}{}_{\underline{\Sigma}} + E_I{}^{\underline{N}} \omega_J{}^{\underline{S}} \omega^{\underline{MS}}) f_{\underline{SMN}} \\
& -(E_I{}^{\underline{\alpha}} E_J{}^{\underline{\beta}} \omega^{\underline{PS}} + E_I{}^{\underline{P}} E_J{}^{\underline{\alpha}} \omega^{\underline{\beta S}} + E_I{}^{\underline{\beta}} E_J{}^{\underline{P}} \omega^{\underline{\alpha S}}) f_{\underline{\alpha\beta P}} \quad .
\end{aligned}
\tag{4.11}
$$

Some curvature tensors are related by Bianchi identities, and $R_{mn}{}^{\underline{S}} = \frac{1}{2}(\gamma_{mn})^\mu{}_\nu R_\mu{}^{\nu\underline{S}}$. Torsions with two and three $\underline{\Sigma}$ lower indices, $T_{\underline{\Sigma\Sigma}I}$ and $T_{\underline{\Sigma\Sigma\Sigma}}$, are unphysical.

4.2 Torsion constraints from κ-symmetric Virasoro

Let us obtain conditions on torsions from the Green-Schwarz superstring in curved space. Virasoro constraints in curved space are (3.1) by replacing $\mathring{\triangleright}_{\underline{M}}$ with $\triangleright_{\underline{A}}$ as

$$
\begin{aligned}
\mathcal{H}_\tau &= \frac{1}{2} \triangleright_{\underline{A}} \hat{\delta}^{\underline{AB}} \triangleright_{\underline{B}} \\
\mathcal{H}_\sigma &= \frac{1}{2} \triangleright_{\underline{A}} \eta^{\underline{AB}} \triangleright_{\underline{B}} = \frac{1}{2} \mathring{\triangleright}_{\underline{M}} \eta^{\underline{MN}} \mathring{\triangleright}_{\underline{N}} \quad .
\end{aligned}
\tag{4.12}
$$

They satisfy the same Virasoro algebra as in (3.3) from totally antisymmetricity of torsion and the relations of metric η and $\hat{\delta}$ without introducing new torsion constraints. The "section condition" is the σ-diffeomorphism constraint $\mathcal{H}_\sigma = 0$ which guarantees geometry generated by the bracket (4.3) from (3.4).

Natural extension of $\mathcal{ABCD}$ and $\mathcal{A}'\mathcal{B}'\mathcal{C}'\mathcal{D}'$ constraints in (3.6) and (3.9) to the ones in curved space are obtained by replacing $\mathring{\triangleright}_{\underline{M}}$ with $\triangleright_{\underline{A}}$ with the same matrices ρ's in (3.9). It is obtained by replacing $\mathring{\triangleright} = E^{-1}\triangleright$ in $\mathcal{ABCD}$ constraints from the relation in (3.10). Sets of constrains, which determine curved background fields of the type II Green-Schwarz superstring, are the followings;

- Second class approach:

$$
\begin{aligned}
&\text{First class constraints} \quad : \mathcal{A}, \mathcal{B}^\alpha = \mathcal{A}', \mathcal{B}^{\alpha'} = \triangleright_{\underline{S}} = 0 \\
&\text{Second class constraints} : \triangleright_{D_{\underline{\alpha}}} = 0
\end{aligned}
$$

- First class approach:

$$
\text{First class constraints} : \mathcal{A}, \mathcal{B}^\alpha, \mathcal{C}_{\alpha\beta}, \mathcal{D}_a = \mathcal{A}', \mathcal{B}^{\alpha'}, \mathcal{C}_{\alpha'\beta'}, \mathcal{D}_{a'} = \triangleright_{\underline{S}} = 0 \quad .
$$

Both notations of the fermionic covariant derivative, $\triangleright_{D_{\underline{\alpha}}} = \triangleright_{\underline{\alpha}}$, are used.

We analyze both two approaches of fermionic constraints analogously to [32, 33]. Let us examine the second class approach. Commutator of the second class constraints and the diffeomorphism constraint is trivial, $[\triangleright_{\underline{\alpha}}, \int \mathcal{H}_\sigma] = i\partial_\sigma \triangleright_{\underline{\alpha}} \approx 0$. Commutator of $\triangleright_{\underline{\alpha}}$ with $\mathcal{H}_\tau$ gives following conditions

$$
\begin{aligned}
[\triangleright_{\underline{\alpha}}, \mathcal{H}_\tau] &= -iT_{\underline{\alpha BC}}(\eta^{\underline{BD}} \triangleright_{\underline{D}})(\hat{\delta}^{\underline{CE}} \triangleright_{\underline{E}})\delta(2-1) + i(-)^{\underline{\alpha}} \triangleright_{\underline{\alpha}}(1)\partial_\sigma \delta(2-1) \\
&\approx 0 \\
&\Leftrightarrow \; T_{\underline{\alpha}BC'} = 0 \quad \text{for } {}_{B,C=(S,D_\alpha,P)}
\end{aligned}
\tag{4.13}
$$

These constraints $T_{\underline{\alpha}BC'} = 0$ reflect holomorphicity.

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Further constraints are obtained from anticommutator with the κ-symmetry generator

$$\begin{aligned}\{\triangleright_{\underline{\alpha}}, \mathcal{B}^{\underline{\beta}}\} &\approx 0 \\ &\Leftrightarrow \{\triangleright_\alpha, \triangleright_\beta\} \approx f_{\alpha\beta c}\triangleright_c\,,\ \ \{\triangleright_{\alpha'}, \triangleright_{\beta'}\} \approx -f_{\alpha\beta c}\triangleright_{c'}\,,\ \ \{\triangleright_\alpha, \triangleright_{\beta'}\} \approx 0 \\ &\Rightarrow T_{\alpha\beta c} = f_{\alpha\beta c},\ T_{\alpha\beta X} = 0 \text{ for } {}_{X=(S,S',D_\alpha,D'_\alpha,P')}\ ,\quad \text{same for } ' \text{ indices}\end{aligned} \tag{4.14}$$

where $\approx$ allows terms including $\triangleright_{\underline{\alpha}}$ and $\triangleright_S$. Bianchi identity and totally graded antisymmetricity restrict torsion as $T_{\alpha\beta}{}^{\underline{\gamma}} = 0 = T_{\alpha a}{}^{\underline{\gamma}}$. Torsion constraints are

$$\begin{cases} T_{\underline{\alpha}BC'} = 0 \ \text{ for } {}_{B,C=(S,D_\alpha,P)} \\ T_{\alpha\beta c} = f_{\alpha\beta c}\,,\ T_{\alpha\beta X} = 0 \text{ for } {}_{X=(S,S',D_\alpha,D_{\alpha'},P',\Omega,\Omega')} \\ T_{\alpha b Y} = 0 \text{ for } {}_{Y=(S,S',D_{\alpha'},P,P',\Omega,\Omega')} \\ \ \text{same for } ' \text{ indices}\,. \end{cases} \tag{4.15}$$

In the first class approach commutators among $\mathcal{ABCD}$ constraints lead to the same conditions to the above up to $\triangleright_S$ terms. Commutators $[\mathcal{B}^{\underline{\alpha}}, \mathcal{H}_\tau]$ and $\{\mathcal{B}^{\underline{\alpha}}, \mathcal{B}^{\underline{\beta}}\}$ lead to the same torsion condition (4.15).

4.3 Prepotential

The consistency condition of the nondegenerate super-Poincaré space leads to Lorentz constraint, (3.8), so it is possible to choose a local Lorentz frame as

$$\triangleright_{S_{\underline{ab}}} = S_{\underline{ab}} \quad . \tag{4.16}$$

This space is a coset space G/H where H is Lorentz, then H coordinates can be removed without modifying algebra of covariant derivatives supplying H constraints. This is because the symmetry generator satisfies the symmetry algebra regardless of the presence or absence of H-coordinates, and the algebra of symmetry generators and covariant derivatives have same structure constant with opposite sign. Covariant derivatives include Σ terms which can be removed with supplying S constraints, so the coefficient of Σ in vielbein can be chosen to zero. Vielbein fields can be parameterized as

$$E_{\underline{A}}{}^{\underline{M}} = \begin{array}{c} {\scriptstyle S_{\underline{ab}}} \\ {\scriptstyle D_{\underline{\alpha}}} \\ {\scriptstyle P_{\underline{a}}} \\ {\scriptstyle \Omega^{\underline{\alpha}}} \\ {\scriptstyle \Sigma^{\underline{ab}}} \end{array} \begin{pmatrix} \mathbf{1}_{\underline{ab}}{}^{\underline{mn}} & 0 & 0 & 0 & 0 \\ \omega_{\underline{\alpha}}{}^{\underline{mn}} & e_{\underline{\alpha}}{}^{\underline{\mu}} & B_{\underline{\alpha}}{}^{m} & B_{\underline{\alpha\mu}} & 0 \\ \omega_{\underline{a}}{}^{\underline{mn}} & \psi_{\underline{a}}{}^{\underline{\mu}} & e_{\underline{a}}{}^{\underline{m}} & -\check{B}_{\underline{a\mu}} & 0 \\ \omega^{\underline{\alpha mn}} & F^{\underline{\alpha\mu}} & -\check{\psi}^{\underline{\alpha m}} & e^{\underline{\alpha}}{}_{\underline{\mu}} & 0 \\ r^{\underline{abmn}} & -\check{\omega}^{\underline{ab\mu}} & -\check{\omega}^{\underline{abm}} & -\check{\omega}^{\underline{ab}}{}_{\underline{\mu}} & \mathbf{1}^{\underline{ab}}{}_{\underline{mn}} \end{pmatrix} \quad . \tag{4.17}$$

The orthonormal condition (4.5) requires that "checked variables" are transposed variables plus more many terms, for example, $\check{B}_{a\mu} = (B_{\mu a})^T + \psi_a{}^\alpha B_{\alpha\mu} - \frac{1}{2}\psi_a{}^\alpha B_{b\alpha}(B_\mu{}^b)^T$. In linearized level $\check{B}_{\underline{\alpha\mu}} = B_{\underline{\mu\alpha}}$, $\check{B}_{\underline{a\mu}} = B_{\underline{\mu a}}$, $\check{e}^{\underline{\alpha}}{}_{\underline{\mu}} = e_{\underline{\mu}}{}^{\underline{\alpha}}$, $\check{\psi}^{\underline{\alpha m}} = \psi^{\underline{m\alpha}}$, $\check{\omega}^{\underline{ab\mu}} = \omega^{\underline{\mu ab}}$, $\check{\omega}^{\underline{abm}} = \omega^{\underline{mab}}$, $\check{\omega}^{\underline{ab}}{}_{\underline{\mu}} = \omega_{\underline{\mu}}{}^{\underline{ab}}$.

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Vielbein fields are classified by canonical dimensions as:

type II string field	vielbein field (dim.)
B fields	$B_{\underline{\alpha\mu}}\ (-1),\ B_{\underline{a\mu}}\ \left(-\frac{1}{2}\right),\ e_{[\underline{am}]}\ (0)$
gravity field	$e_{(\underline{am})},\ e_{\underline{\alpha}}{}^{\underline{\mu}}\ (0)$
superconnections	$\omega_{\underline{\alpha}}{}^{\underline{mn}}\ \left(\frac{1}{2}\right),\ \omega_{\underline{a}}{}^{\underline{mn}}\ (1),\ \omega^{\underline{\alpha mn}}\ \left(\frac{3}{2}\right)$
gravitino	$\psi_{\underline{a}}{}^{\underline{\mu}}\ \left(\frac{1}{2}\right)$
$\mathrm{R}-\mathrm{R}$ field strength	$F^{\alpha\mu'}, F^{\alpha'\mu}\ (1)$
covariance compensator	$r^{\underline{abmn}}\ (2)$

(4.18)

Three kinds of superconnections as well as other fields are solved by a set of torsion constraints (4.15) resulted from the κ-symmetry.

In linearized level we have

$$\begin{aligned} E_{\underline{A}}{}^{\underline{M}} &= \delta_{\underline{A}}^{\underline{M}} + H_{\underline{A}}{}^{\underline{B}}\delta_{\underline{B}}^{\underline{M}} \\ T_{\underline{ABC}} &= f_{\underline{ABC}} + \frac{1}{2}D_{[\underline{A}}H_{\underline{BC})} + \frac{1}{2}H_{[\underline{A}}{}^{\underline{M}}f_{\underline{BC})\underline{M}} \\ \mathcal{I}_{\underline{ABCD}} &= D_{[\underline{A}}T_{\underline{BCD})} + \frac{3}{2}T_{[\underline{AB}}{}^{\underline{E}}f_{\underline{CD})\underline{E}} = 0 \quad . \end{aligned}$$

Vielbein fields are solved in linearized level;

dim	torsion constraint $\Rightarrow$	field
$-\frac{1}{2}$	$T_{\alpha\beta\gamma}=0$	$B_{a(\alpha}(\gamma^a)_{\beta\gamma)} = D_{(\alpha}B_{\beta\gamma)}$
	$T_{\alpha\beta\gamma'}=0$	$B_{a\alpha'} = \frac{1}{8}D_\alpha B_{\beta\gamma'}(\gamma_a)^{\alpha\beta} + \frac{1}{16}(D_{\gamma'}B_{\alpha\beta})(\gamma_a)^{\alpha\beta}$
0	$T_{\alpha\beta c}=f_{\alpha\beta c}$	$e_{(\alpha}{}^{\gamma}(\gamma_c)_{\beta)\gamma} + e_c{}^m(\gamma_m)_{\alpha\beta} = -D_{(\alpha}B_{\beta)c} - D_cB_{\alpha\beta}$
	$T_{\alpha\beta c'}=0$	$e_{a'}{}^m = \frac{1}{8}D_\alpha B_{\beta a'}(\gamma^m)^{\alpha\beta} + \frac{1}{16}D_{a'}B_{\alpha\beta}(\gamma^m)^{\alpha\beta}$
$\frac{1}{2}$	$T_{\alpha bc}=0$	$\omega_{\alpha bc} = -D_\alpha e_{bc} - D_{[b}B_{c]\alpha} - \psi_{[b}{}^{\beta}(\gamma_{c]})_{\alpha\beta}$
	$T_{\alpha\beta}{}^{\gamma}=0$	$\omega_{(\alpha}{}^{bc}(\gamma_{bc})^{\gamma}{}_{\beta)} = -D_{(\alpha}e_{\beta)}{}^{\gamma} + \psi^{a\gamma}(\gamma_a)_{\alpha\beta}$
	$T_{\alpha bc'}=0$	$\psi_{c'}{}^{\gamma}(\gamma_b)_{\alpha\gamma} = D_\alpha e_{bc'} + D_{[b}B_{c']\alpha}$
1	$T_{abc}=0$	$\omega_{abc} = -\frac{1}{3!}D_{[a}e_{bc]}$
	$T_{a\beta}{}^{\gamma}=0$	$\frac{1}{2}\omega_a{}^{cd}(\gamma_{cd})^{\gamma}{}_{\beta} = -F^{\alpha\gamma}(\gamma_a)_{\beta\alpha} - D_ae_\beta{}^\gamma + D_\beta\psi_a{}^\gamma$
$\frac{3}{2}$	$T_\alpha{}^{\beta\gamma}=0$	$\omega^{(\beta\|bc}(\gamma_{bc})^{\|\gamma)}{}_\alpha = -4D_\alpha F^{\beta\gamma} - 2D^{(\beta}e^{\gamma)}{}_\alpha$

(4.19)

where only part of relations are listed and similar relations for the primed (right) sector are also hold.

Constraints are solved dimension by dimension. Gauges are chosen so that all of $E_{\underline{A}}{}^{\underline{M}}$ are fixed in terms of $E_{\underline{D}}{}^{\Omega} = E_{\underline{DD}} = B_{\underline{\alpha\beta}}$. The dimension -1 field, $E_{\underline{DD}}$, is a prepotential superfield.

Anti-hermiticity of derivative operators is defined by $\int e^{\phi}\Psi(i\triangleright\chi) = -\int e^{\phi}(i\triangleright\Psi)\chi$ with $\triangleright = \frac{1}{i}\partial + \cdots$, where the integral measure contains dilaton ϕ which is the only density available. This leads to a constraint $\tilde{T}_{\underline{A}}$ as

$$\tilde{T}_{\underline{A}} = e^{\phi}\overleftarrow{\triangleright}_{\underline{A}}e^{-\phi} = 0$$
$$\tilde{T}_{\underline{\alpha}} = 0 \;\Rightarrow\; \lambda_{\underline{\alpha}} = (D_{\underline{\alpha}}\phi) = \frac{i}{4}\omega_{\underline{\beta}}{}^{\underline{bc}}(\gamma_{\underline{bc}})^{\underline{\beta}}{}_{\underline{\alpha}} - (D_{\underline{\mu}}e_{\underline{\alpha}}{}^{\underline{\mu}}) - \partial_{\underline{m}}B_{\underline{\alpha}}{}^{\underline{m}} - B_{\underline{\alpha}}{}^{\underline{m}}(\partial_{\underline{m}}\phi) \quad .$$

Dilaton ϕ and dilatino $\lambda_{\underline{\alpha}} = D_{\underline{\alpha}}\phi$ are introduced through $\tilde{T}_{\underline{\alpha}} = 0$. Equating this constraint together with torsion constraints with dimension 1/2 in (4.19), $T_{\alpha\beta}{}^{\beta} = T_{\beta bc}(\gamma^{bc})^{\beta}{}_{\alpha} = 0$, fixes the trace of $E_{\underline{D\Omega}} = e_{\underline{\alpha}}{}^{\underline{\beta}}$ from $D_{\alpha}\phi$ terms. Analogously $\tilde{T}_{\underline{a}} = 0$ constraint together with the trace of torsion constraints with dimension 1, $T_{a\beta}{}^{\beta} = 0$, gives an equation for $D_{\underline{\alpha}}D_{\underline{\beta}}\phi$ that fixes ϕ as $D_{\underline{\alpha}}D_{\underline{\beta}}E_{\underline{DD}} + D_{\underline{\alpha}}D_{\underline{\beta}}V$ with some prepotential V as an homogeneous solution [34].

In order to obtain the usual d-dimensional gravity theory, dimensional reduction constraints and the section condition must be imposed. Dimensional reduction constraints are written in terms of $\tilde{\triangleright}_{\underline{M}}$ so that the local geometry generated by covariant derivatives is not modified. The section condition, $\mathcal{H}_{\sigma} = 0$, is imposed in the doubled space with coordinates $(Z^{M}, Z^{M'})$. Simple separation of double coordinates into momenta and σ derivative of coordinates exists. In this formalism we do not fix the representation of R-R field strength as bi-spinor of double Lorentz groups, instead we leave bi-product of two chiral spinors of each Lorentz group. Then dimensional reduction conditions relate two Lorentz groups and two chiral spinors. For cases when these conditions mix chirality under T-duality symmetry transformation IIA and IIB spinors are interchanged.

5 AdS$^5\times$ S$_5$ superspace

In this section AdS$^5\times$S$_5$ superspace is explained in a manifestly T-dual formulation, showing the algebra, torsions, curvatures and dimensional reduction conditions. The current algebra for a type IIB superstring in AdS$^5\times$S$_5$ background was given in [20, 21] in which Σ currents were included while Lorentz generators were absent by a constraint $S = 0$. In this section we rederive AdS$^5\times$S$_5$ algebra preserving Lorentz generator manifestly and performing current redefinition in such a way that torsions of the algebra become totally graded antisymmetric. Then dimensional reduction constraints and section conditions in our manifestly T-dual formulation are given reducing the AdS$^5\times$S$_5$ algebra in conventional space with usual coordinates $x^m, \theta_1^{\mu}, \theta_2^{\mu}$.

A superstring in AdS$_5\times$S^5 is described by a coset G/H with G=PSU(2,2|4) and H=SO(5)$\times$SO(4,1). Wick rotations change the coset into G/H with G=GL(4|4) and H=[Sp(4)GL(1)]^2. The Sp(4) invariant metric, which is antisymmetric, is denoted by $\hat{\epsilon}$. The currents and coordinates have GL(4|4) index, $\mathrm{A} = (\mathrm{a}, \bar{\mathrm{a}})$ and $\mathrm{a}, \bar{\mathrm{a}} = 1, \cdots, 4$;

$$\frac{\mathrm{G}}{\mathrm{H}} \ni g_{\mathrm{M}}{}^{\mathrm{A}} \quad , \quad g \to g_G\, g\, g_H \quad , \quad (g^{-1})_{\mathrm{A}}{}^{\mathrm{M}}\partial_{\sigma}g_{\mathrm{M}}{}^{\mathrm{B}} = J_{\mathrm{A}}{}^{\mathrm{B}} \quad , \quad \partial_{\mathrm{A}}{}^{\mathrm{M}}g_{\mathrm{M}}{}^{\mathrm{B}} = D_{\mathrm{A}}{}^{\mathrm{B}}$$

where $\partial_{\mathrm{A}}{}^{\mathrm{M}}$ is canonical conjugate of $g_{\mathrm{M}}{}^{\mathrm{A}}$. Symmetrization, antisymmetrization and traceless antisymmetrization of indices are (ab)=ab+ba, [ab]=ab-ba and ⟨ab⟩=[ab]-trace part respectively. Linear combinations of two GL(4|4) matrices, $D_{\rm AB}$ and $J_{\rm AB}$, make 64×2 nondegenerate super-AdS currents, $(S, D_\pm, P_\pm, \Omega_\pm, \Sigma, \Delta, \Delta', E, E')$. This model is rather conventional description so that one Lorentz S is involved. Although there appear Σ_+ and Σ_- in the algebra, only sum $\Sigma = \Sigma_+ + \Sigma_-$ has nondegenerate commutator with S. $\Omega_\pm$ is modified from the one in the flat space denoted by $\omega_\pm$ by the Ramond-Ramond field strengths as $\omega_\pm \to \Omega_\pm = \omega_\pm + \frac{1}{2}D_\mp$. Connection of this model and T-dual formalism is explained in the end of this section.

The nondegenerate super-AdS currents with GL(4|4) indies are as follows:

$\triangleright_{\underline{M}}$	number of generators	GL(4|4) currents
S	20	$S_{\rm ab} = \frac{1}{2}D_{(\rm ab)},\ S_{\bar{\rm a}\bar{\rm b}} = \frac{1}{2}D_{(\bar{\rm a}\bar{\rm b})}$
D_+	16	$(D_+)_{\rm a\bar{b}} = E^{1/4}D_{\rm a\bar{b}} + E^{-1/4}D_{\bar{\rm b}{\rm a}} + \frac{1}{2}(E^{1/4}J_{\rm a\bar{b}} + E^{-1/4}J_{\bar{\rm b}{\rm a}})$
D_-	16	$(D_-)_{\rm a\bar{b}} = E^{1/4}D_{\rm a\bar{b}} - E^{-1/4}D_{\bar{\rm b}{\rm a}} + \frac{1}{2}(-E^{1/4}J_{\rm a\bar{b}} + E^{-1/4}J_{\bar{\rm b}{\rm a}})$
P_+	10	$(P_+)_{\rm ab} = \frac{1}{2}(D_{\langle\rm ab\rangle} + J_{\langle\rm ab\rangle}),\ (P_+)_{\rm ab} = \frac{1}{2}(D_{\langle\bar{\rm a}\bar{\rm b}\rangle} + J_{\langle\bar{\rm a}\bar{\rm b}\rangle})$
P_-	10	$(P_-)_{\rm ab} = \frac{1}{2}(D_{\langle\rm ab\rangle} - J_{\langle\rm ab\rangle}),\ (P_-)_{\bar{\rm a}\bar{\rm b}} = \frac{1}{2}(D_{\langle\bar{\rm a}\bar{\rm b}\rangle} - J_{\langle\bar{\rm a}\bar{\rm b}\rangle})$
Ω_+	16	$(\Omega_+)_{\rm a\bar{b}} = (\omega_+)_{\rm a\bar{b}} + \frac{1}{2}(D_-)_{\rm a\bar{b}}$
Ω_-	16	$(\Omega_-)_{\rm a\bar{b}} = (\omega_-)_{\rm a\bar{b}} + \frac{1}{2}(D_+)_{\rm a\bar{b}}$
		$(\omega_+)_{\rm a\bar{b}} = E^{1/4}J_{\rm a\bar{b}} - E^{-1/4}\bar{J}_{\bar{\rm b}{\rm a}}$
		$(\omega_-)_{\rm a\bar{b}} = -E^{1/4}J_{\rm a\bar{b}} - E^{-1/4}\bar{J}_{\bar{\rm b}{\rm a}}$
Σ_+	20	$(\Sigma_+)_{\rm ab} = \sigma_{\rm ab} + \frac{1}{2}S_{\rm ab}\,,\ (\Sigma_+)_{\bar{\rm a}\bar{\rm b}} = \sigma_{\bar{\rm a}\bar{\rm b}} + \frac{1}{2}S_{\bar{\rm a}\bar{\rm b}}$
Σ_-	20	$(\Sigma_-)_{\rm ab} = \sigma_{\rm ab} - \frac{1}{2}S_{\rm ab}\,,\ (\Sigma_-)_{\bar{\rm a}\bar{\rm b}} = \sigma_{\bar{\rm a}\bar{\rm b}} - \frac{1}{2}S_{\bar{\rm a}\bar{\rm b}}$
		$\sigma_{\rm ab} = \frac{1}{2}J_{(\rm ab)},\ \sigma_{\bar{\rm a}\bar{\rm b}} = \frac{1}{2}J_{(\bar{\rm a}\bar{\rm b})}$
$\Delta,\ \Delta'$	2	${\rm Str}D = {\rm tr}D_{\rm AdS} - {\rm tr}D_S,\ {\rm Tr}D = {\rm tr}D_{\rm AdS} + {\rm tr}D_S$
$E,\ E'$	2	$E = {\rm Sdet}g = \dfrac{{\rm det}g_{\rm AdS}}{{\rm det}g_S},\ E' = {\rm Det}g = {\rm det}g_{\rm AdS}\cdot{\rm det}g_S$

(5.1)

Algebra of currents in type II AdS^5×S$_5$ is given as follows:

- $++$ *commutators*

$$
\begin{aligned}
\{(D_+)_{\rm a\bar{b}}(1), (D_+)_{\rm c\bar{d}}(2)\} &= -2\left(\hat{\epsilon}_{\rm ac}(P_+)_{\bar{\rm b}\bar{\rm d}} - \hat{\epsilon}_{\bar{\rm b}\bar{\rm d}}(P_+)_{\rm ac} + \frac{1}{4}\hat{\epsilon}_{\rm ac}\hat{\epsilon}_{\bar{\rm b}\bar{\rm d}}\Delta\right)\delta(2-1) \\
[(D_+)_{\rm a\bar{b}}(1), (P_+)_{\rm cd}(2)] &= \hat{\epsilon}_{\rm a\langle c}\left(\omega_+ + \frac{1}{2}D_-\right)_{\rm d\rangle\bar{b}}\delta(2-1) = \hat{\epsilon}_{\rm a\langle c}(\Omega_+)_{\rm d\rangle\bar{b}}\delta(2-1) \\
\{(D_+)_{\rm a\bar{b}}(1), (\Omega_+)_{\rm c\bar{d}}(2)\} &= -2\left(\hat{\epsilon}_{\rm ac}(\Sigma_+)_{\bar{\rm b}\bar{\rm d}} + \hat{\epsilon}_{\bar{\rm b}\bar{\rm d}}(\Sigma_+)_{\rm ac}\right)\delta(2-1) + 2\hat{\epsilon}_{\rm ac}\hat{\epsilon}_{\bar{\rm b}\bar{\rm d}}\partial_\sigma\delta(2-1) \\
[(P_+)_{\rm ab}(1), (P_+)_{\rm cd}(2)] &= \hat{\epsilon}_{\langle\rm a|\langle c}(\Sigma_+)_{\rm d\rangle|b\rangle}\delta(2-1) + \hat{\epsilon}_{\rm a\langle c}\hat{\epsilon}_{\rm d\rangle b}\partial_\sigma\delta(2-1) \\
[(P_+)_{\rm ab}(1), (\Omega_+)_{\rm c\bar{d}}(2)] &= \frac{1}{2}\hat{\epsilon}_{\rm c\langle a}(\Omega_- - D_+)_{\rm b\rangle|\bar{d}}\delta(2-1)
\end{aligned}
$$

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$$\{(\Omega_+)_{\mathrm{a}\bar{\mathrm{b}}}(1),(\Omega_+)_{\mathrm{c}\bar{\mathrm{d}}}(2)\} = \left(\hat{\epsilon}_{\mathrm{ac}}\left(P_+-\frac{1}{2}P_-\right)_{\bar{\mathrm{b}}\bar{\mathrm{d}}} - \hat{\epsilon}_{\bar{\mathrm{b}}\bar{\mathrm{d}}}\left(P_+-\frac{1}{2}P_-\right)_{\mathrm{ac}} + \frac{1}{8}\hat{\epsilon}_{\mathrm{ac}}\hat{\epsilon}_{\bar{\mathrm{b}}\bar{\mathrm{d}}}\Delta\right)\delta(2-1)$$

This is summarized in the table of indices of structure constant $f_{AB}{}^{C} = \begin{array}{|c|c|}\hline & B \\ \hline A & C \\ \hline\end{array}$

	D_+	P_+	Ω_+
D_+	P_+	Ω_+	$\Sigma_+, \partial_\sigma\delta$
P_+	Ω_+	$\Sigma_+, \partial_\sigma\delta$	D_+, Ω_-
Ω_+	$\Sigma_+, \partial_\sigma\delta$	D_+, Ω_-	P_+, P_-

(5.2)

- $-\,-$ *commutators*

$$\begin{aligned}
\{(D_-)_{\mathrm{a}\bar{\mathrm{b}}}(1),(D_-)_{\mathrm{c}\bar{\mathrm{d}}}(2)\} &= 2\left(\hat{\epsilon}_{\mathrm{ac}}(P_-)_{\bar{\mathrm{b}}\bar{\mathrm{d}}} - \hat{\epsilon}_{\bar{b}\bar{d}}(P_-)_{\mathrm{ac}} + \frac{1}{4}\hat{\epsilon}_{\mathrm{ac}}\hat{\epsilon}_{\bar{\mathrm{b}}\bar{\mathrm{d}}}\Delta\right)\delta(2-1)\\
[(D_-)_{\mathrm{a}\bar{\mathrm{b}}}(1),(P_-)_{\mathrm{cd}}(2)] &= \hat{\epsilon}_{\mathrm{a}\langle\mathrm{c}}\left(\omega_- + \frac{1}{2}D_+\right)_{\mathrm{d}\rangle\bar{\mathrm{b}}}\delta(2-1) = \hat{\epsilon}_{\mathrm{a}\langle\mathrm{c}}(\Omega_-)_{\mathrm{d}\rangle\bar{\mathrm{b}}}\delta(2-1)\\
\{(D_-)_{\mathrm{a}\bar{\mathrm{b}}}(1),(\Omega_-)_{\mathrm{c}\bar{\mathrm{d}}}(2)\} &= 2\left(\hat{\epsilon}_{\mathrm{ac}}(\Sigma_-)_{\bar{\mathrm{b}}\bar{\mathrm{d}}} + \hat{\epsilon}_{\bar{\mathrm{b}}\bar{\mathrm{d}}}(\Sigma_-)_{\mathrm{ac}}\right)\delta(2-1) + 2\hat{\epsilon}_{\mathrm{ac}}\hat{\epsilon}_{\bar{\mathrm{b}}\bar{\mathrm{d}}}\partial_\sigma\delta(2-1)\\
[(P_-)_{\mathrm{ab}}(1),(P_-)_{\mathrm{cd}}(2)] &= -\hat{\epsilon}_{\langle\mathrm{a}|\langle\mathrm{c}}(\Sigma_-)_{\mathrm{d}\rangle|\mathrm{b}\rangle}\delta(2-1) - \hat{\epsilon}_{\mathrm{a}\langle\mathrm{c}}\hat{\epsilon}_{\mathrm{d}\rangle\mathrm{b}}\partial_\sigma\delta(2-1)\\
[(P_-)_{\mathrm{ab}}(1),(\Omega_-)_{\mathrm{c}\bar{\mathrm{d}}}(2)] &= \frac{1}{2}\hat{\epsilon}_{\mathrm{c}\langle\mathrm{a}}(\Omega_+ - D_-)_{\mathrm{b}\rangle|\bar{\mathrm{d}}}\delta(2-1)\\
\{(\Omega_-)_{\mathrm{a}\bar{\mathrm{b}}}(1),(\Omega_-)_{\mathrm{c}\bar{\mathrm{d}}}(2)\} &= \left(-\hat{\epsilon}_{\mathrm{ac}}\left(P_- - \frac{1}{2}P_+\right)_{\bar{\mathrm{b}}\bar{\mathrm{d}}} + \hat{\epsilon}_{\bar{\mathrm{b}}\bar{\mathrm{d}}}\left(P_- - \frac{1}{2}P_+\right)_{\mathrm{ac}} - \frac{1}{8}\hat{\epsilon}_{\mathrm{ac}}\hat{\epsilon}_{\bar{\mathrm{b}}\bar{\mathrm{d}}}\Delta\right)\delta(2-1)
\end{aligned}$$

The structure constant among the $-$ currents is summarized as

	D_-	P_-	Ω_-
D_-	P_-	Ω_-	$\Sigma_-, \partial_\sigma\delta$
P_-	Ω_-	$\Sigma_-, \partial_\sigma\delta$	D_-, Ω_+
Ω_-	$\Sigma_-, \partial_\sigma\delta$	D_-, Ω_+	P_+, P_-

(5.3)

- $+\,-$ *commutators*

$$\begin{aligned}
\{(D_+)_{\mathrm{a}\bar{\mathrm{b}}}(1),(D_-)_{\mathrm{c}\bar{\mathrm{d}}}(2)\} &= -2\left(\hat{\epsilon}_{\mathrm{ac}}S_{\bar{\mathrm{b}}\bar{\mathrm{d}}} - \hat{\epsilon}_{\bar{\mathrm{b}}\bar{\mathrm{d}}}S_{\mathrm{ac}}\right)\delta(2-1)\\
[(D_+)_{\mathrm{a}\bar{\mathrm{b}}}(1),(P_-)_{\mathrm{cd}}(2)] &= \hat{\epsilon}_{\mathrm{a}\langle\mathrm{c}}(D_-)_{\mathrm{d}\rangle\bar{\mathrm{b}}}\delta(2-1)\\
\{(D_+)_{\mathrm{a}\bar{\mathrm{b}}}(1),(\Omega_-)_{\mathrm{c}\bar{\mathrm{d}}}(2)\} &= -\left(\hat{\epsilon}_{\mathrm{ac}}(P_-)_{\bar{\mathrm{b}}\bar{\mathrm{d}}} - \hat{\epsilon}_{\bar{\mathrm{b}}\bar{\mathrm{d}}}(P_-)_{\mathrm{ac}} + \frac{1}{4}\hat{\epsilon}_{\mathrm{ac}}\hat{\epsilon}_{\bar{\mathrm{b}}\bar{\mathrm{d}}}\Delta\right)\delta(2-1)\\
[(P_+)_{\mathrm{ab}}(1),(P_-)_{\mathrm{cd}}(2)] &= \frac{1}{2}\hat{\epsilon}_{\langle\mathrm{a}|\langle\mathrm{c}}S_{\mathrm{d}\rangle|\mathrm{b}\rangle}\delta(2-1)\\
[(P_+)_{\mathrm{ab}}(1),(\Omega_-)_{\mathrm{c}\bar{\mathrm{d}}}(2)] &= -\frac{1}{2}\hat{\epsilon}_{\mathrm{c}\langle\mathrm{a}}(D_-)_{\mathrm{b}\rangle|\bar{\mathrm{d}}}\delta(2-1)\\
\{(\Omega_+)_{\mathrm{a}\bar{\mathrm{b}}}(1),(\Omega_-)_{\mathrm{c}\bar{\mathrm{d}}}(2)\} &= \frac{1}{2}\left(\hat{\epsilon}_{\mathrm{ac}}S_{\bar{\mathrm{b}}\bar{\mathrm{d}}} - \hat{\epsilon}_{\bar{\mathrm{b}}\bar{\mathrm{d}}}S_{\mathrm{ac}}\right)\delta(2-1)\\
[(P_+)_{\mathrm{ab}}(1),(D_-)_{\mathrm{c}\bar{\mathrm{d}}}(2)] &= -\frac{1}{2}\hat{\epsilon}_{\mathrm{c}\langle\mathrm{a}}(D_+)_{\mathrm{b}\rangle\bar{\mathrm{d}}}\delta(2-1)\\
[(\Omega_+)_{\mathrm{a}\bar{\mathrm{b}}}(1),(P_-)_{\mathrm{cd}}(2)] &= \frac{1}{2}\hat{\epsilon}_{\mathrm{a}\langle\mathrm{c}}(D_+)_{\mathrm{d}\rangle\bar{\mathrm{b}}}\delta(2-1)
\end{aligned}$$

The structure constant between the + current and − currents is summarized as

$$\begin{array}{|c|c|c|c|} \hline & D_- & P_- & \Omega_- \\ \hline D_+ & S & D_- & P_- \\ \hline P_+ & D_+ & S & D_- \\ \hline \Omega_+ & P_+ & D_+ & S \\ \hline \end{array} \tag{5.4}$$

Non-vanishing torsions are

$$\begin{aligned} &\text{dim } 0 : T_{DDP} = f_{DDP} \ , \quad T_{PPS} = f_{PPS} \ , \quad T_{D\Omega S} = f_{D\Omega S} \\ &\text{dim } 1 : T_{P\Omega D'}, \ T_{DD'}{}^S = R_{DD'}{}^S \\ &\text{dim } 2 : T_{\Omega\Omega P}, \ T_{\Omega\Omega P'} \ , \ T_{PP'}{}^S = R_{PP'}{}^S \\ &\text{dim } 3 : T_{\Omega\Omega'}{}^S = R_{\Omega\Omega'}{}^S \end{aligned}$$

These torsions are consistent with torsion constraints to (4.13), (4.15) and (4.9).

Let us examine the current algebra $\mathrm{AdS}_5\times\mathrm{S}^5$ by translating GL(4|4) indices into space-time indices, then write down Bianchi identities

$$\begin{aligned} \text{dim. } 2 \ [D_\alpha, D_{\beta'}, P_a] = 0 \ &\Rightarrow \ R_{\alpha\beta'}{}^{ab} = T_{\beta'}{}^{\beta[a}(\gamma^{b]})_{\alpha\beta} \\ [D_\alpha, D_\beta, P_{c'}] = 0 \ &\Rightarrow \ R_{aa'}{}^{bc}(\gamma^a)_{\alpha\beta} = T_{a'(\alpha|}{}^{\gamma'} R_{\gamma'|\beta)}{}^{bc} \\ \text{dim. } 3 \ [D_\alpha, P_b, \Omega^{\gamma'}] = 0 \ &\Rightarrow \ R^{\beta\gamma';cd}(\gamma_b)_{\alpha\beta} = T^{c'\gamma'}{}_\alpha R_{c'b}{}^{cd} + T_b{}^{\gamma'\gamma} R_{\gamma'\alpha}{}^{cd} \end{aligned}$$

Torsions with dimension 1 are written in terms of R-R field strength $F^{\alpha\beta'}$ with dimension 1 as

$$T_{a\alpha'}{}^\beta = (\gamma_a)_{\alpha'\beta'} F^{\beta\beta'} \ , \ T_{a'\alpha}{}^{\beta'} = (\gamma_a)_{\alpha\beta} F^{\alpha\beta'} \ .$$

The R-R field strength of the $\mathrm{AdS}_5\times\mathrm{S}^5$ space is given by $F^{\alpha\beta'} = \frac{1}{r_{\mathrm{AdS}}}(\Gamma_5)^{\alpha\beta'}$ with the AdS radius, r_{AdS}. Prepotential $B_{\alpha\beta'}$ has terms such as $\frac{1}{g}F$, $F\theta^2 x$ and $F\theta^4$ where g is string coupling.

This description is in conventional space with coordinates (x^m, θ^μ, $\theta^{\mu'}$), so we must reduce dimensions from the doubled space. Dimensional reduction is imposed using symmetry generators $\tilde{\triangleright}_M$ without modifying local algebras of covariant derivatives. We impose set of dimensional reduction constraints

$$\tilde{P} - \tilde{P}' = \tilde{S} - (-\tilde{S}') = 0 \, , \ \tilde{\Omega} = \tilde{\Omega}' = \tilde{\Sigma} = \tilde{\Sigma}' = 0 \quad , \tag{5.5}$$

which reduce into one set of nondegenerate superspace. Physical translation generator is $\tilde{P}_m + \tilde{P}'_{m'} \to \tilde{P}_m$, while usual coordinate is $x^m + x'^m \to x^m$. Physical Lorentz generator is $\tilde{S}_{mn} + (-\tilde{S}'_{m'n'}) \to \tilde{S}_{mn}$. Lorentz coordinates are suppressed by the gauge fixing condition $u^{mn} = u^{m'n'} = 0$ corresponding to first class constraints $S = S' = 0$. Type IIB fermionic coordinates are chosen as follows, resulting the conventional Lorentz symmetry generator;

$$\begin{aligned} [\tilde{S}_{mn}, \theta^\mu] &= \frac{i}{2}(\gamma_{mn})^\mu{}_\nu \theta^\nu \ , \ [\tilde{S}_{mn}, \theta^{\mu'}] = \frac{i}{2}(\gamma_{mn})^\mu{}_\nu \theta^{\nu'} \ , \ \Gamma_{11}\theta = \theta \, , \ \Gamma_{11}\theta' = \theta' \\ \tilde{S}_{mn} &= ix_{[m}\partial_{n]} - i\theta^\mu \partial_\nu (\gamma_{mn})^\nu{}_\mu - i\theta^{\mu'} \partial_{\nu'} (\gamma_{mn})^\nu{}_\mu \quad . \end{aligned}$$

6 Conclusions

We present a superspace formulation of type II superstring background with manifest T-duality symmetry by following the procedure given in [24]. The nondegenerate super-Poincaré algebra is given in (2.6) to define the superspace. The κ-symmetric Virasoro constraints, $\mathcal{ABCD}$ constraints, in the nondegenerate super-Poincaré currents are also given in (3.6) and (3.7). Then we double whole set of nondegenerate super-Poincaré algebra to construct type II theory with manifest T-duality.

The vielbein superfield $E_{\underline{AM}}$ is introduced in (4.17) which contains all fields. Torsion constraints are obtained from κ-symmetric Virasoro constraints in (4.15). These torsion constraints are solved, and all superconnections and vielbein fields are written in terms of a prepotential $E_{\underline{\alpha\beta}}$.

This formulation is a double superspace field theory with two sets of coordinates in (2.3). Gauge fixing for coset constraints $S = 0$ allows to gauge away u Lorentz coordinates. Dimensional reduction constraints $\tilde{\Omega} = \partial_\varphi + \ldots = 0$ and $\tilde{\Sigma} = \partial_v + \ldots = 0$ allow to remove φ and v. The space has the O(d,d) T-duality symmetry as well as supersymmetry. Further dimensional reductions of Lorenz and translation generators and gauge fixing of the section condition are imposed to obtain the usual coordinate space.

We leave interesting aspects such as R-R gauge fields, D-branes, interchanging IIA and IIB and M-theory, for future problem. The manifestly T-dual formulation in other gauge fixing may allow to explore further, such as exotic branes and low energy effective theory of F-theory.

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