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Research Article

Orthogonal Space-Time Block Codes in Vehicular Environments: Optimum Receiver Design and Performance Analysis

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We consider orthogonal space-time block codes (OSTBC) in vehicular environments, where the channels are nonidentically distributed. It is shown that the nonidentical channel statistics lead to nonidentical channel estimation errors, which consequently affect the performance and even the existing receiver structure of OSTBC. We show that the conventional symbol-by-symbol (SBS) decoder of OSTBC is suboptimum in vehicular environments. A new optimum decoder is derived, which can be simplified to a new SBS decoder under certain conditions. To the best of our knowledge, our work here is the first to consider the optimum decoder for OSTBC in vehicular environments. Performance analysis and simulations are provided, which show that our new decoder substantially outperforms the conventional decoder.

1. Introduction

Recently, wireless vehicular communications, for example, vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communications, have attracted more and more attention [1–5], as they show substantial potential to enhance the traffic safety [2], efficiency, and information availability [3]. Several standards are being developed for vehicular communications, such as IEEE 802.11p—wireless access of vehicular environments (WAVE), or IEEE 802.20, which is designed for high-speed mobility situations, for example, for a high-speed train.

Vehicular communication brings forward several challenges, for example, the high mobility and the variation of the vehicular environment requires a robust communication link. Fortunately, the size of a vehicle allows it to be equipped with several antennas and to make use of multiple-input multiple-output (MIMO) systems. The well-known orthogonal space-time block code (OSTBC) [6] is, therefore, a suitable technique in vehicular communication [1], since it provides robust transmissions with very simple decoding schemes.

OSTBC has already been included in several IEEE standards, for example, Alamouti's code [7] in IEEE 802.11b and IEEE 802.11n. The receiver structure and the performance of OSTBC have been extensively studied in many works with both perfect and estimated channel state information (CSI) at the receiver; see [8–10] and the references therein. These works, however, are based on the assumption that the channels are independent and identically distributed (i.i.d.), but this assumption is not expected to hold in vehicular environments. In a vehicular environment, both the transmit and receive antennas are amounted at heights of 1–3 meters [3]. The surrounding reflectors of the signals consist of nearby vehicles and roadside buildings, which can be very close to one antenna but far from the others. The link distances are also instantly variable from less than 1-2 meters to several tens of meters. Therefore, the channels are more likely to be non-identically distributed.

The issue of OSTBC over non-identical channels first appeared in cooperative diversity scenarios [11–13], where the distributed nodes normally experience non-identical statistics. The performance of OSTBC over non-identical channels was also implicitly discussed in [14–16], as the issue of non-identical channels can be viewed as a special case of the correlated channels. More recently, we have investigated the receiver structure and the performance of OSTBC over non-identical channels with both coherent detection [17] and differential detection [18]. However, all the existing works on OSTBC over non-identical channels make the ideal assumption that the CSI is perfectly known at the receiver. But, the rapidly variable environments and the Doppler shift caused by the moving vehicles make the channel estimation problem nontrivial in vehicular environments.

Generally, non-identical channels will result in non-identical channel estimation errors. These estimation errors will consequently affect the performance of the current systems, and even the structure of the existing receiver. Therefore, in this paper we will consider the OSTBC in vehicular environments with non-identical channels. We show that the conventional symbol-by-symbol (SBS) decoder [19] for OSTBC is no longer optimum in vehicular communications. The optimum decoder is obtained, which can be simplified to a new SBS decoder under certain conditions. To the best of our knowledge, our work here is the first to consider the optimum decoder for OSTBC over non-identical channels with channel estimation. Our analytical and simulation results show that our new decoder provides a much better performance compared to the conventional SBS decoder in vehicular environments.

The rest of the paper is organized as follows. In Section 2, we describe the system model. Section 3 examines the structure of the optimum and the SBS decoder. Performance analysis is given in Section 4. Sections 5 and 6 are numerical examples and conclusion, respectively.

2. System Model

We consider a V2V or V2I communication system, where the transmitter has M_T antennas and the receiver has N_R antennas. The transmit/receive antennas can be colocated in one vehicle/infrastructure, or distributed in several. If the antennas are not colocated, we assume the synchronization is perfect. The space-time block code $\mathbf{S}$ is a $P \times M_T$ matrix, where each row of $\mathbf{S}$ is transmitted through M_T transmit antennas at one time, and the transmission covers P symbol periods. It has a linear complex orthogonal design, and can be represented as [20]

$$\mathbf{S} = \sum_{k=1}^{K} \left(s_k \mathbf{A}_k + s_k^* \mathbf{B}_k \right). \tag{1}$$

Here, $\mathbf{A}_k$ and $\mathbf{B}_k$ are $P \times M_T$ matrices with constant complex entries, and K is the number of information symbols transmitted in one block. Therefore, each entry of $\mathbf{S}$ is a linear combination of the symbols s_k, $k = 1, \dots, K$, and their conjugates s_k^*, where each s_k is from a certain complex signal constellation. The rate of the OSTBC is defined as K/P.

For OSTBC, we have [6]

$$\mathbf{S}^H \mathbf{S} = \operatorname{diag}\left[\sum_{k=1}^{K} \lambda_{1,k} |s_k|^2, \dots, \sum_{k=1}^{K} \lambda_{M_T,k} |s_k|^2 \right] = \mathbf{D}, \tag{2}$$

where $\{\lambda_{i,k}\}_{i=1}^{M_T}$ are nonnegative numbers. For an arbitrary signal constellation, it requires that

$$\begin{aligned} \mathbf{A}_k^H \mathbf{A}_l + \mathbf{B}_l^H \mathbf{B}_k &= \delta_{k,l} \operatorname{diag}[\lambda_{1,k}, \dots, \lambda_{M_T,k}], \\ \mathbf{A}_k^H \mathbf{B}_l + \mathbf{A}_l^H \mathbf{B}_k &= 0. \end{aligned} \tag{3}$$

We assume here M-ary phase-shift keying (MPSK) modulation and a constant transmitted energy per information bit as E_b. Therefore, the total energy assigned to one block is $E_b K \log_2 M$. From the orthogonality condition (2), it can be seen that the total energy for one block is given by $\sum_m^{M_T} \sum_k^K \lambda_{m,k} |s_k|^2$. Thus, the transmitted energy per MPSK symbol is given by

$$E_s = \frac{E_b K \log_2 M}{\sum_m^{M_T} \sum_k^K \lambda_{m,k}}. \tag{4}$$

The received signal at tth block is a $P \times N_R$ matrix, which is given by

$$\mathbf{R}(t) = \mathbf{S}(t)\mathbf{H}(t) + \mathbf{N}(t). \tag{5}$$

Here, $\mathbf{N}(t)$ is a $P \times N_R$ noise matrix, whose entries are i.i.d., complex, Gaussian random variables with means zero and variances $N_o/2$ per dimension. $\mathbf{H}(t)$ is a $M_T \times N_R$ channel matrix, where each entry h_{mn} is the channel gain of the link from mth transmit antenna to nth receive antenna. We assume h_{mn} is a circularly complex Gaussian random variable with mean zero and variance $2\sigma_{mn}^2$. It is also assumed that the channels are all block-wise constant. The autocorrelation function of each channel is given as $E[h_{mn}(t) h_{mn}^*(t')] = 2\sigma_{mn}^2 R(t - t')$, where $R(t - t') = J_o(2\pi f_d T_b (t - t'))$ for Jakes' model [21], and it is identical for all channels.

In order to coherently detect the code matrix $\mathbf{S}(t)$ in (5), the channel matrix must be estimated first. In this paper, we apply pilot-symbol assisted modulation (PASM) [22], such that a pilot block is inserted into the data stream every L_f blocks. During the pilot block, each transmit antenna transmits a known pilot symbol at its own designated time slot. The receiver estimates the channel matrix $\mathbf{H}(t)$ based on the information set $\Lambda(t)$, which contains the $2L_p$ received pilot blocks nearest in time to the tth block.

Without loss of generality, we consider the component $h_{mn}(t)$ of the channel matrix $\mathbf{H}(t)$ and let $\mathbf{p}_{mn}$ be the column vector storing the $2L_p$ nearest received pilot symbols from the mth transmit antenna to the nth receive antenna. Using the result from [22], it can be shown that the minimum mean square error estimate (MMSE) of $h_{mn}(t)$ is given by

$$\hat{h}_{mn}(t) = \mathbf{d}_{mn}^H(t) \mathbf{p}_{mn}, \tag{6}$$

where

$$\mathbf{d}_{mn}(t) = \mathbf{G}^{-1} \mathbf{v}_{mn}(t) \tag{7}$$

represents a Weiner filter, with $\mathbf{G} = (1/2)E[\mathbf{p}_{mn}\mathbf{p}_{mn}^H]$ being the autocorrelation matrix of the received pilot samples $\mathbf{p}_{mn}$, and $\mathbf{v}_{mn}(t) = (1/2)E[h_{mn}^*(t)\mathbf{p}_{mn}]$ being the correlation of $h_{mn}(t)$ and $\mathbf{p}_{mn}$.

The channel estimation error, defined as $e_{mn}(t) = h_{mn}(t) - \widehat{h}_{mn}(t)$, is a Gaussian random variable with mean zero and variance $2v_{mn}^2(t) = \sigma_{mn}^2 - \mathbf{v}_{mn}^H(t)\mathbf{G}^{-1}\mathbf{v}_{mn}(t)$ [22]. Note that $e_{mn}(t)$ is independent of $\widehat{h}_{mn}(t)$. Therefore, given the information set $\Lambda(t)$, each $h_{mn}(t)$ is a conditional Gaussian with mean $\widehat{h}_{mn}(t)$ and variance $2v_{mn}^2(t)$. It is obvious that if the statistics of the channel gains on the different links are different, the variances of channel estimation are different in general.

3. Optimum and Symbol-by-Symbol Decoders

One important advantage of OSTBC is that the ML decoder can reduce to an SBS decoder, which greatly reduces the decoding complex. This conventional SBS decoder is optimum when channels are identical with perfect CSI [6] or with imperfect CSI [10]. It is also an optimum receiver in the case of non-identical channels with perfect CSI [17]. However, in the vehicular environments where the channels are non-identical and the CSI is imperfect, the conventional receiver is no longer optimum. Therefore, we need to investigate the structure of optimum decoder first.

For ML decoding, we compute the likelihood $p(\mathbf{R}(t), \Lambda(t) \mid \mathbf{S}(t))$ for each possible value of the signal block $\mathbf{S}(t)$. Since, we have

$$p(\mathbf{R}(t), \Lambda(t)\mathbf{S}(t)) = p(\mathbf{R}(t)\mathbf{S}(t), \Lambda(t))p(\Lambda(t)\mathbf{S}(t)), \tag{8}$$

and the information set $\Lambda(t)$ is independent of $\mathbf{S}(t)$, the ML decoding rule simplifies to

$$\widehat{\mathbf{S}}(t) = \arg\max_{\mathbf{S}(t)} p(\mathbf{R}(t) \mid \mathbf{S}(t), \Lambda(t)), \tag{9}$$

where $\mathbf{R}(t)$ is conditionally Gaussian with mean $\mathbf{S}(t)\widehat{\mathbf{H}}(t)$, given $\mathbf{S}(t)$ and $\Lambda(t)$.

The column vectors of $\mathbf{R}(t)$ are independent of one another and each has covariance matrix of

$$\mathbf{C}_n(t) = \mathbf{S}(t)\mathbf{V}_n(t)\mathbf{S}^H(t) + N_o\mathbf{I}_{p\times p}, \quad n = 1, \ldots, N_R, \tag{10}$$

where

$$\mathbf{V}_n(t) = \operatorname{diag}[2v_{mn}^2(t)]_{m=1}^{M_T}, \quad n = 1, \ldots, N_R. \tag{11}$$

The probability density function of the received signal is now given by

$$\begin{aligned} &p(\mathbf{R}(t) \mid \mathbf{S}(t), \Lambda(t)) \\ &= \left(\prod_{n=1}^{N_R} \det(\pi\mathbf{C}_n(t))\right)^{-1} \\ &\cdot \exp\left(-\sum_{n=1}^{N_R}\left(\mathbf{r}_n(t) - \mathbf{S}(t)\widehat{\mathbf{h}}_n(t)\right)^H \mathbf{C}_n^{-1}(t)\left(\mathbf{r}_n(t) - \mathbf{S}(t)\widehat{\mathbf{h}}_n(t)\right)\right). \end{aligned} \tag{12}$$

Therefore, the ML block-by-block receiver becomes

$$\begin{aligned} \widehat{\mathbf{S}}(t) = \arg\min_{\mathbf{S}(t)} \Bigg(&\sum_{n=1}^{N_R}\left(\mathbf{r}_n(t) - \mathbf{S}(t)\widehat{\mathbf{h}}_n(t)\right)^H \\ &\times \mathbf{C}_n^{-1}(t)\left(\mathbf{r}_n(t) - \mathbf{S}(t)\widehat{\mathbf{h}}_n(t)\right)\Bigg). \end{aligned} \tag{13}$$

As we will show later, depending on whether the non-identical channels are associated with transmit antennas or receiver antennas, there are different effects on the OSTBC. For the sake of illustration, we will consider two typical cases in the following sections.

Case 1. Channels gains from different transmit antennas to a common receive antenna are identically distributed, but the gains associated with different receive antennas are non-identically distributed. Therefore, the variance of $h_{mn}(t)$ reduces to $2\sigma_{on}^2$, and the variance of estimation error reduces to $2v_{on}^2(t)$.

Case 2. Channels gains from a common transmit antenna to different receive antennas are identically distributed, but the gains associated with different transmit antennas are non-identically distributed. Therefore, the variance of $h_{mn}(t)$ reduces to $2\sigma_{mo}^2$, and the variance of estimation error reduces to $2v_{mo}^2(t)$.

Other more complex cases can be viewed as the combination of these two cases. Here, notice that the variances of channel gains are constant, but the variances of the estimation errors depend on the position of the code block.

3.1. Case 1: Channels Associated with One Common Receive Antenna Are Identically Distributed. In this case, since $2v_{mn}^2 = 2v_{on}^2$ for all m, we have

$$\mathbf{V}_n(t) = 2v_{on}^2\mathbf{I}_{N_T\times N_T}, \quad n = 1, \ldots, N_R. \tag{14}$$

If the STBC employed satisfies

$$\mathbf{S}(t)\mathbf{S}^H(t) = \beta\mathbf{I}_{P\times P}, \tag{15}$$

where β is a constant, then the $\mathbf{C}_n(t)$'s become constants proportional to an identity matrix. Therefore, the ML receiver (13) simplifies to

$$\widehat{\mathbf{S}}(t) = \arg\min_{\mathbf{S}(t)} \left\| \widetilde{\mathbf{R}}(t) - \mathbf{S}(t)\widetilde{\widehat{\mathbf{H}}}(t) \right\|^2, \tag{16}$$

where

$$\begin{aligned} \widetilde{\mathbf{R}}(t) &= \left[\sqrt{\frac{1}{2v_{on}^2\beta + N_o}}\mathbf{r}_n(t)\right]_{n=1}^{N_R} \\ &= \mathbf{R}(t)\operatorname{diag}\left[\sqrt{\frac{1}{2v_{on}^2\beta + N_o}}\right]_{n=1}^{N_R}, \\ \widetilde{\widehat{\mathbf{H}}}(t) &= \left[\sqrt{\frac{1}{2v_{on}^2\beta + N_o}}\widehat{\mathbf{h}}_n(t)\right]_{n=1}^{N_R} \\ &= \widehat{\mathbf{H}}(t)\operatorname{diag}\left[\sqrt{\frac{1}{2v_{on}^2\beta + N_o}}\right]_{n=1}^{N_R}. \end{aligned} \tag{17}$$

Applying (3) to (16), the receiver can be further simplified to an SBS detector, given by

$$\hat{s}_k(t) = \arg \max_{k'=1\cdots K} \Re\left[z_{k'}(t)s_{k'}^*(t)\right], \tag{18}$$

where

$$z_{k'}(t) = \mathrm{Tr}\left[\tilde{\mathbf{R}}^H(t)\mathbf{B}_{k'}\tilde{\hat{\mathbf{H}}}(t) + \tilde{\hat{\mathbf{H}}}^H(t)\mathbf{A}_{k'}^H\tilde{\mathbf{R}}(t)\right]. \tag{19}$$

Therefore, in Case 1, the ML decoding can also be achieved by a SBS decoder, under the condition that the received signal matrix $\mathbf{R}(t)$ and the estimated channel matrix $\hat{\mathbf{H}}(t)$ are properly weighted column by column, according to the variances of the channel estimation errors.

3.2. Case 2: Channels Associated with a Common Transmit Antenna Are Identically Distributed. In Case 2, since the channels are identically distributed with a common transmit antenna, each column vector of $\mathbf{R}(t)$ has the same covariance matrix

$$\mathbf{C}(t) = \mathbf{S}(t)\mathbf{V}(t)\mathbf{S}^H(t) + N_o\mathbf{I}_{p\times p}, \tag{20}$$

where

$$\mathbf{V}(t) = \mathrm{diag}[2\nu_{mo}^2]_{m=1}^{N_T}. \tag{21}$$

It can easily be seen that $\mathbf{C}^{-1}(t)$ is not a diagonal matrix, because of the non-identical $2\nu_{mo}^2$'s.

Since, the values of $2\nu_{mo}^2$'s do not depend on the decoder structure at the receiver side, the ML decoder

$$\begin{aligned}\hat{\mathbf{S}}(t) = \arg \min_{\mathbf{S}(t)} \Bigg(\sum_{n=1}^{N_R}\left(\mathbf{r}_n(t) - \mathbf{S}(t)\hat{\mathbf{h}}_n(t)\right)^H \\ \times\mathbf{C}^{-1}(t)\left(\mathbf{r}_n(t) - \mathbf{S}(t)\hat{\mathbf{h}}_n(t)\right)\Bigg)\end{aligned} \tag{22}$$

cannot reduce to a SBS decoder, no matter how the receiver structure is designed. Fortunately, the most practical OSTBC used in actual communication systems is Alamouti's code [7], which only requires two transmit antennas. In such cases, the ML decoder in Case 2 only requires an affordable decoding complexity of M^2, where M is the order of the modulation.

4. Performance Analysis

In this section, we will examine the bit error performance of the new optimum SBS decoder proposed for Case 1. For the sake of simplicity, we drop the block index t hereafter, but note that the results obtained do depend on the positions of blocks.

4.1. Conditional Bit Error Probability. With PSK modulation, that is, $s_k = \sqrt{E_s}e^{j\phi_k}$, the decoding rule (18) is equivalent to

$$\hat{s}_k = \arg \max_{k'=1\cdots K} \Re\left[z_{k'}e^{-j\phi_{k'}}\right], \tag{23}$$

where

$$z_{k'} = \mathrm{Tr}\left[\tilde{\mathbf{R}}^H\mathbf{B}_{k'}\tilde{\hat{\mathbf{H}}} + \tilde{\hat{\mathbf{H}}}^H\mathbf{A}_{k'}^H\tilde{\mathbf{R}}\right] = x_{k'} + \mu_{k'}, \tag{24}$$

$$\begin{aligned}x_{k'} = \sum_{k=1}^{K}\Big[&s_k^*\mathrm{Tr}\left[\tilde{\mathbf{H}}^H\mathbf{A}_k^H\mathbf{B}_{k'}\tilde{\hat{\mathbf{H}}} + \tilde{\hat{\mathbf{H}}}^H\mathbf{A}_{k'}^H\mathbf{B}_k\tilde{\mathbf{H}}\right] \\ &+s_k\mathrm{Tr}\left[\tilde{\hat{\mathbf{H}}}^H\mathbf{A}_{k'}^H\mathbf{A}_k\tilde{\mathbf{H}} + \tilde{\mathbf{H}}^H\mathbf{B}_k^H\mathbf{B}_{k'}\tilde{\hat{\mathbf{H}}}\right]\Big],\end{aligned} \tag{25}$$

$$\mu_{k'} = \mathrm{Tr}\left[\tilde{\mathbf{N}}^H\mathbf{B}_{k'}\tilde{\hat{\mathbf{H}}} + \tilde{\hat{\mathbf{H}}}^H\mathbf{A}_{k'}^H\tilde{\mathbf{N}}\right]. \tag{26}$$

For equally likely symbols, we can assume $s_{k'} = \sqrt{E_s}$ without loss of generality, thus the BEP depends on the probability $P_\alpha(e) = P(\Re[z_{k'}e^{-j\alpha}] < 0 \mid s_{k'} = \sqrt{E_s})$, where α is some angle depending on modulation order [23]. For BPSK modulation, the BEP is obviously given by $P_b = P_{\alpha=0}(e)$. For QPSK modulation with Gray mapping, the BEP is given by $P_b = P_{\alpha=\pi/4}(e)$ [23].

Conditioning on the information set Λ and $s_{k'}$, and substituting (3) into (25), we can see that $x_{k'}$ is a Gaussian random variable, which is given by

$$(x_{k'} \mid s_{k'}, \Lambda) \sim CN\left(s_{k'}\sum_{n=1}^{N_R}\frac{\mathcal{H}}{\mathcal{V}_n}, E_s\sum_{m=1}^{M_T}\sum_{n=1}^{N_R}\frac{2\nu_{on}^2\,|\,\xi_{mn}\,|^2}{\mathcal{V}_n^2}\right), \tag{27}$$

where

$$\begin{aligned}\mathcal{H} &= \sum_{m=1}^{M_T}\lambda_{m,k'}\left|\hat{h}_{mn}\right|^2, \\ \mathcal{V}_n &= 2\nu_{on}^2\beta + N_o,\end{aligned} \tag{28}$$

$$\begin{aligned}\xi_{mn} = \sum_{k=1}^{K}\sum_{i=1}^{M_T}\Big(&\left(\mathbf{a}_{k,m}^H\mathbf{b}_{k',i} + \mathbf{b}_{k,m}^H\mathbf{b}_{k',i}\right)\hat{h}_{mn} \\ &+\left(\mathbf{a}_{k',i}^H\mathbf{b}_{k,m} + \mathbf{a}_{k',i}^H\mathbf{a}_{k,m}\right)\hat{h}_{mn}^*\Big).\end{aligned} \tag{29}$$

Here, $\mathbf{a}_{k,i}$ and $\mathbf{b}_{k,i}$ are the ith column vectors of matrices $\mathbf{A}_k$ and $\mathbf{B}_k$, respectively. Similarly, the noise term $\mu_{k'}$ in (26) is also a conditional Gaussian random variable, which is given by

$$\left(\mu_{k'} \mid s_{k'}, \Lambda\right) \sim CN\left(0, \frac{N_o}{2}\sum_{n=1}^{N_R}\frac{\mathcal{H}}{\mathcal{V}_n^2}\right). \tag{30}$$

Therefore, conditioning on the information set Λ, the probability $P_\alpha(e)$ is given by

$$\begin{aligned}&P_\alpha(e \mid \Lambda) \\ &=Q\left(\sqrt{\frac{E_s\left(\sum_{n=1}^{N_R}(\mathcal{H}/\mathcal{V}_n)\right)^2\cos^2\alpha}{E_s\sum_{m=1}^{M_T}\sum_{n=1}^{N_R}\left(\nu_{on}^2|\xi_{mn}|^2/\mathcal{V}_n^2\right)+(N_o/2)\sum_{n=1}^{N_R}(\mathcal{H}/\mathcal{V}_n^2)}}\right).\end{aligned} \tag{31}$$

In the conditional probability above, since both the denominator and the numerator contains the estimated channel gains $\{\hat{h}_{mn}\}$, it is difficult to average (31) over $\{\hat{h}_{mn}\}$ directly and obtain the exact BEP. Therefore, in the following section we will first investigate the exact BEP in a special case, and then introduce the performance bounds and approximations in general situations.

4.2. Exact Bit Error Probability for the Special Case of Perfect CSI. If the CSI is perfect, such that $2v_{mn}^2 = 0$ for all m and n, one has $\hat{h}_{mn} = h_{mn}$. The conditional probabilities (31) can be simplified to

$$P_\alpha(e \mid \Lambda) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{E_s\cos^2\alpha}{N_o\sin^2\theta}\sum_{n=1}^{N_R}\sum_{m=1}^{N_T}\lambda_{m,k'}\left|h_{mn}\right|^2\right)d\theta, \tag{32}$$

where we use the Craigs alternative form of the Q-function [24]. Observing that the channel gains $\{h_{mn}\}$ are independent of one another, we can average over them one by one with the help of the following lemma [25, equation (7.76)].

Lemma 1. *If x is a real Gaussian random variable with mean m_x and variance σ_x^2, we have*

$$E\left[\exp\left(wx^2\right)\right] = \frac{\exp\left(wm_x^2/\left(1-2w\sigma_x^2\right)\right)}{\sqrt{1-2w\sigma_x^2}}, \tag{33}$$

where w is any complex constant with real part less than $1/2\sigma_x^2$.

Applying Lemma 1 to the conditional BEP (32), we obtain the exact error probability, which is given by

$$P_\alpha(e) = \frac{1}{\pi}\int_0^{\pi/2}\prod_{m=1}^{M_T}\prod_{n=1}^{N_R}\left(1+\frac{2\sigma_{mn}^2E_s\lambda_{m,k'}\cos^2\alpha}{N_o\sin^2\theta}\right)^{-1}d\theta. \tag{34}$$

4.3. Bounds and Approximations of Bit Error Probability with Imperfect CSI. If the channels are estimated, as we mentioned above, the exact average BEP is difficult to obtain. Therefore, performance approximations and bounds need to be applied. In the following section, we will use Alamouti's code [7] as an example to show how to analyze the average BEP. The method used in this paper can similarly be extended to other OSTBC's.

Using Alamouti's code [7], the code matrix and $\mathbf{A}_k$ and $\mathbf{B}_k$ are given by

$$\mathbf{S} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}$$
$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\quad \mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\quad \mathbf{B}_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},\quad \mathbf{B}_2 = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}, \tag{35}$$

respectively. Thus, $\lambda_{i,k} = 1$ for all i and k.

Substituting (35) into (29), we have

$$\begin{aligned}\begin{bmatrix} \xi_{11} & \xi_{12} \\ \xi_{21} & \xi_{22} \end{bmatrix} &= \begin{bmatrix} \hat{h}_{11} & \hat{h}_{12} \\ -\hat{h}_{21} & -\hat{h}_{22} \end{bmatrix} + \begin{bmatrix} \hat{h}_{11}^* & \hat{h}_{12}^* \\ \hat{h}_{21}^* & \hat{h}_{22}^* \end{bmatrix} \\ &= \begin{bmatrix} 2\Re\left[\hat{h}_{11}\right] & 2\Re\left[\hat{h}_{12}\right] \\ -2\Im\left[\hat{h}_{21}\right] & -2\Im\left[\hat{h}_{22}\right] \end{bmatrix}.\end{aligned} \tag{36}$$

Since the channel gain h_{mn} is circularly Gaussian, it is easy to see that $\hat{h}_{mn}$ is also circularly Gaussian, and thus we make the approximation that

$$E_s\sum_{m=1}^{M_T}\sum_{n=1}^{N_R}\frac{v_{on}^2\left|\xi_{mn}\right|^2}{\mathcal{V}_n^2} \approx 2E_s\sum_{m=1}^{M_T}\sum_{n=1}^{N_R}\frac{v_{on}^2\left|\hat{h}_{mn}\right|^2}{\mathcal{V}_n^2}. \tag{37}$$

This approximation is justified on the grounds that the two terms have the same means, which means that it can give a close approximation to the final BEP, when averaging the conditional BEP over all possible values of the estimated channel gains.

Applying the above approximation, we first rewrite (31) as

$$P_\alpha(e \mid \Lambda) = Q\left(\sqrt{\frac{E_s\left(\sum_{n=1}^{N_R}\sum_{m=1}^{M_T}\left|\hat{h}_{mn}\right|^2/\mathcal{V}_n\right)^2\cos^2\alpha}{2E_s\mathcal{I}+(N_o/2)\mathcal{N}}}\right), \tag{38}$$

where the terms $\mathcal{I} = \sum_{m=1}^{M_T}\sum_{n=1}^{N_R}(v_{on}^2/\mathcal{V}_n)(|\hat{h}_{mn}|^2/\mathcal{V}_n)$ and $\mathcal{N} = \sum_{n=1}^{N_R}\sum_{m=1}^{M_T}(1/\mathcal{V}_n)(|\hat{h}_{mn}|^2/\mathcal{V}_n)$ can be upper and lower bounded as

$$\begin{aligned}&\sum_{m=1}^{M_T}\sum_{n=1}^{N_R}\frac{\left|\hat{h}_{mn}\right|^2}{\mathcal{V}_n}\max_{n=1,\ldots,N_T}\left[\frac{v_{on}^2}{\mathcal{V}_n}\right] \\ &\qquad \geq \mathcal{I} \geq \sum_{m=1}^{M_T}\sum_{n=1}^{N_R}\frac{\left|\hat{h}_{mn}\right|^2}{\mathcal{V}_n}\min_{n=1,\ldots,N_T}\left[\frac{v_{on}^2}{\mathcal{V}_n}\right], \\ &\sum_{n=1}^{N_R}\sum_{m=1}^{M_T}\frac{\left|\hat{h}_{mn}\right|^2}{\mathcal{V}_n}\max_{n=1,\ldots,N_T}\left[\frac{1}{\mathcal{V}_n}\right] \\ &\qquad \geq \mathcal{N} \geq \sum_{n=1}^{N_R}\sum_{m=1}^{M_T}\frac{\left|\hat{h}_{mn}\right|^2}{\mathcal{V}_n}\min_{n=1,\ldots,N_T}\left[\frac{1}{\mathcal{V}_n}\right].\end{aligned} \tag{39}$$

Consequently, the conditional probability can be bounded as

$$P_\alpha(e \mid \Lambda) \geq Q\left(\sqrt{\frac{E_s\left(\sum_{n=1}^{N_R}\sum_{m=1}^{M_T}\left(\left|\hat{h}_{mn}\right|^2/\mathcal{V}_n\right)\right)\cos^2\alpha}{2E_s\min\left[v_{on}^2/\mathcal{V}_n\right]+(N_o/2)\min\left[1/\mathcal{V}_n\right]}}\right),$$
$$P_\alpha(e \mid \Lambda) \leq Q\left(\sqrt{\frac{E_s\left(\sum_{n=1}^{N_R}\sum_{m=1}^{M_T}\left(\left|\hat{h}_{mn}\right|^2/\mathcal{V}_n\right)\right)\cos^2\alpha}{2E_s\max\left[v_{on}^2/\mathcal{V}_n\right]+(N_o/2)\max\left[1/\mathcal{V}_n\right]}}\right). \quad (40)$$

Since the random variables $\{\hat{h}_{mn}\}$ in the denominator have been cancelled with the common terms in the numerator, now it is possible to average over the estimated channels.

Observing that the estimated channel gains $\{\hat{h}_{mn}\}$ are also independent Gaussian random variables with means zero and variances $\{2\sigma_{mn}^2 - 2v_{mn}^2\}$, we can average the above inequalities following the same steps from (32) to (34), and obtain

$$P_\alpha(e) \geq \frac{1}{\pi}\int_0^{\pi/2}\prod_{m=1}^{M_T}\prod_{n=1}^{N_R}\left(1+\frac{(2\sigma_{mn}^2-2v_{mn}^2)\mu_l}{\mathcal{V}_n\sin^2\theta}\right)^{-1}d\theta, \quad (41)$$

$$P_\alpha(e) \leq \frac{1}{\pi}\int_0^{\pi/2}\prod_{m=1}^{M_T}\prod_{n=1}^{N_R}\left(1+\frac{(2\sigma_{mn}^2-2v_{mn}^2)\mu_u}{\mathcal{V}_n\sin^2\theta}\right)^{-1}d\theta, \quad (42)$$

where

$$\mu_l = \frac{E_s\cos^2\alpha}{4E_s\min\left[v_{on}^2/\mathcal{V}_n\right]+N_o\min\left[1/\mathcal{V}_n\right]},$$
$$\mu_u = \frac{E_s\cos^2\alpha}{4E_s\max\left[v_{on}^2/\mathcal{V}_n\right]+N_o\max\left[1/\mathcal{V}_n\right]}. \quad (43)$$

In order to obtain a more accurate approximation to the error performance, we propose two more approximations, namely, the geometric approximation and the arithmetic approximation. The terms $\mathcal{L}$ and $\mathcal{N}$ can be closely approximated as

$$\sum_{m=1}^{M_T}\sum_{n=1}^{N_R}\frac{\left|\hat{h}_{mn}\right|^2}{\mathcal{V}_n}\left[\frac{v_{on}^2}{\mathcal{V}_n}\right]_a \approx \mathcal{L} \approx \sum_{m=1}^{M_T}\sum_{n=1}^{N_R}\frac{\left|\hat{h}_{mn}\right|^2}{\mathcal{V}_n}\left[\frac{v_{on}^2}{\mathcal{V}_n}\right]_g,$$
$$\sum_{n=1}^{N_R}\sum_{m=1}^{M_T}\frac{\left|\hat{h}_{mn}\right|^2}{\mathcal{V}_n}\left[\frac{1}{\mathcal{V}_n}\right]_a \approx \mathcal{N} \approx \sum_{n=1}^{N_R}\sum_{m=1}^{M_T}\frac{\left|\hat{h}_{mn}\right|^2}{\mathcal{V}_n}\left[\frac{1}{\mathcal{V}_n}\right]_g, \quad (44)$$

respectively. Here,

$$\left[\frac{v_{on}^2}{\mathcal{V}_n}\right]_g = \left(\prod_{n=1}^{N_R}\frac{v_{on}^2}{\mathcal{V}_n}\right)^{1/N_R}, \qquad \left[\frac{v_{on}^2}{\mathcal{V}_n}\right]_a = \frac{1}{N_R}\sum_{n=1}^{N_R}\frac{v_{on}^2}{\mathcal{V}_n},$$
$$\left[\frac{1}{\mathcal{V}_n}\right]_g = \left(\prod_{n=1}^{N_R}\frac{1}{\mathcal{V}_n}\right)^{1/N_R}, \qquad \left[\frac{1}{\mathcal{V}_n}\right]_a = \frac{1}{N_R}\sum_{n=1}^{N_R}\frac{1}{\mathcal{V}_n} \quad (45)$$

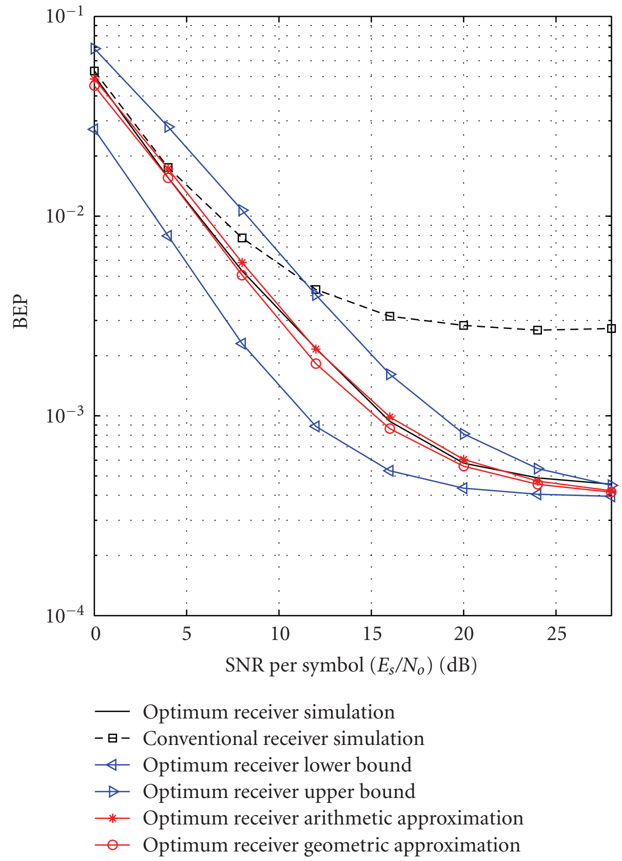

FIGURE 1: Case 1: BEP results for the conventional and the optimum SBS receivers, 2Tx and 2Rx Alamouti's code with QPSK modulation, $f_dT_b = 0.1$, channels variances of 0.5 and 5, respectively.

denote, respectively, the geometric and arithmetic means of all $(v_{on}^2/\mathcal{V}_n)$'s and $(1/\mathcal{V}_n)$'s. Following the same steps as above, the approximations of the probability are given by

$$P_\alpha(e) \approx \frac{1}{\pi}\int_0^{\pi/2}\prod_{m=1}^{M_T}\prod_{n=1}^{N_R}\left(1+\frac{(2\sigma_{mn}^2-2v_{mn}^2)\mu_g}{\mathcal{V}_n\sin^2\theta}\right)^{-1}d\theta, \quad (46)$$

$$P_\alpha(e) \approx \frac{1}{\pi}\int_0^{\pi/2}\prod_{m=1}^{M_T}\prod_{n=1}^{N_R}\left(1+\frac{(2\sigma_{mn}^2-2v_{mn}^2)\mu_a}{\mathcal{V}_n\sin^2\theta}\right)^{-1}d\theta, \quad (47)$$

where

$$\mu_g = \frac{E_s\cos^2\alpha}{4E_s\left[v_{on}^2/\mathcal{V}_n\right]_g+N_o\left[1/\mathcal{V}_n\right]_g},$$
$$\mu_a = \frac{E_s\cos^2\alpha}{4E_s\left[v_{on}^2/\mathcal{V}_n\right]_a+N_o\left[1/\mathcal{V}_n\right]_a}. \quad (48)$$

Note that if the channel estimation errors approach to zero, the two bounds (41) and (42), and the two approximations (46) and (47) all converge to the exact BEP result (34) for the special case of perfect CSI. This further validates our derivations.

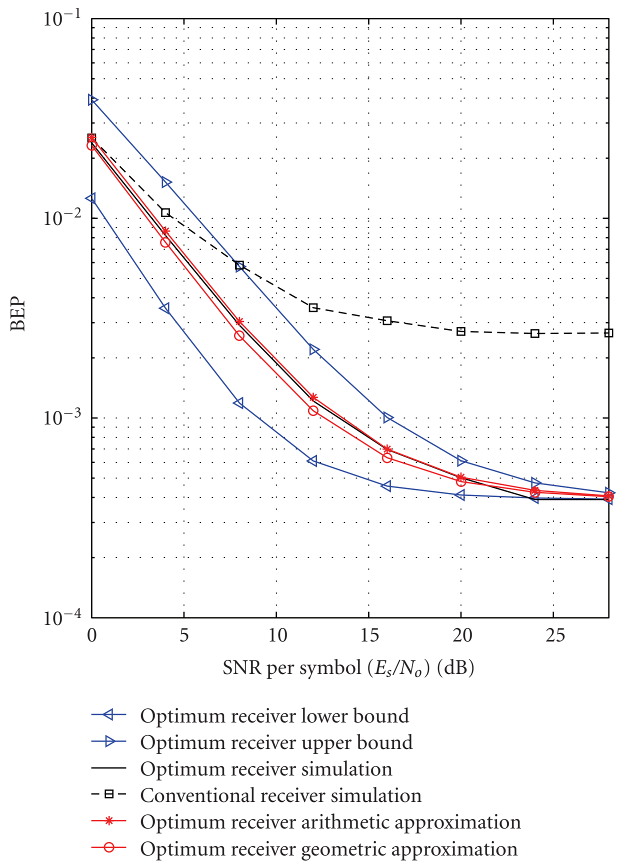

FIGURE 2: Case 1: BEP results for the conventional and the optimum SBS receivers, 2Tx and 2Rx Alamouti's code with QPSK modulation, $f_d T_b = 0.1$, channel variances are 0.9 and 9, respectively.

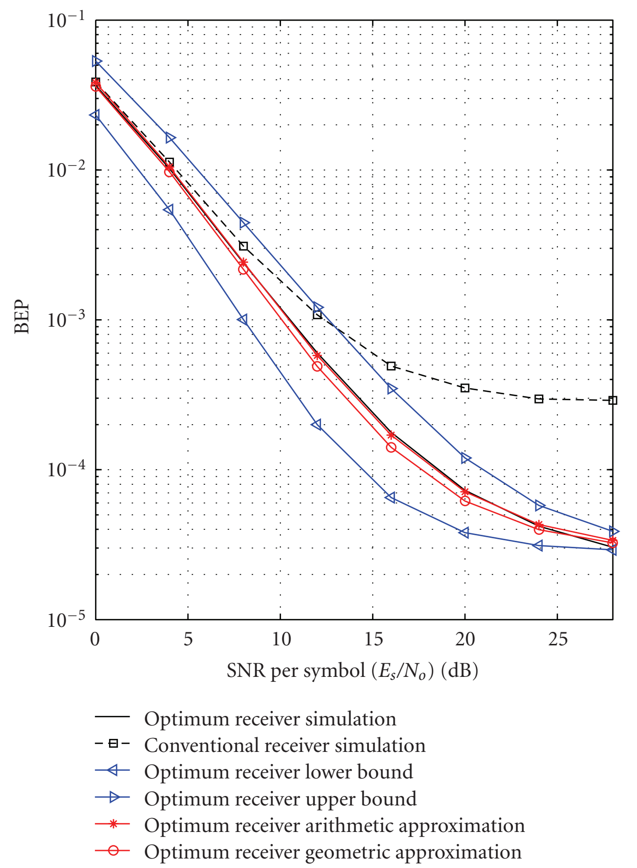

FIGURE 3: Case 1: BEP results for the conventional and the optimum SBS receivers, 2Tx and 2Rx Alamouti's code with QPSK modulation, $f_d T_b = 0.06$, channel variances are 0.5 and 5, respectively.

Since we omitted the block index t here, the BEP results obtained above are based on the tth block. The average BEP for all the blocks can be calculated by averaging over the L_f blocks within two adjacent pilot blocks.

5. Numerical Examples

In the numerical examples, we consider a vehicular communication system with 2 transmit and 2 receive antennas. The Alamouti's code is applied with QPSK modulation. As we mentioned in Section 2, since the channels are blockwise constant, we use the block fade rate $f_d T_b$ for the BEP computation and simulation. Pilot blocks are inserted after every 9 data blocks, and the 4 nearest pilot blocks are used to estimate the channel using PSAM.

In Figure 1, we consider Case 1, where the variances of the channel gains related to the first and second receive antennas are 0.5 and 5, respectively. The block fade rate is set to 0.1. The simulation results show that our optimum receiver provides a large performance gain compared to the conventional receiver. The irreducible error floor caused by the channel fading is also greatly reduced by the optimum receiver.

The analytical lower (41) and upper (42) bounds in Figure 1 show the same trend as the exact BEP curve, such that they decrease in parallel with the increase of SNR. The three curves converge in the high SNR region. Furthermore, both the geometric (46) and arithmetic (47) approximations can closely approximate the exact BEP performance in all SNR regions, with the latter being a closer approximation, the difference being no larger than 0.5 dB.

In Figures 2 and 3, we change the channel variances and the block fade rate, and similar observations can be made. Notice that in Case 1, the performance gain enjoyed by the optimum SBS receiver comes with little overhead, as it only requires linear processing of the received signal and the estimated channel matrices.

Considering Case 2, we plot the simulation results of the conventional SBS decoder and the proposed optimum decoder (22) in Figure 4. The block fade rate is set to 0.1 and the variances of the the channels associated with the first and the second transmit antennas are set to $(9, 1)$, $(5, 1)$, and $(2, 1)$, respectively. All the simulation results show that the optimum decoder can provide a better performance than the conventional SBS decoder. If the difference between the channel variances is larger, the performance gain is

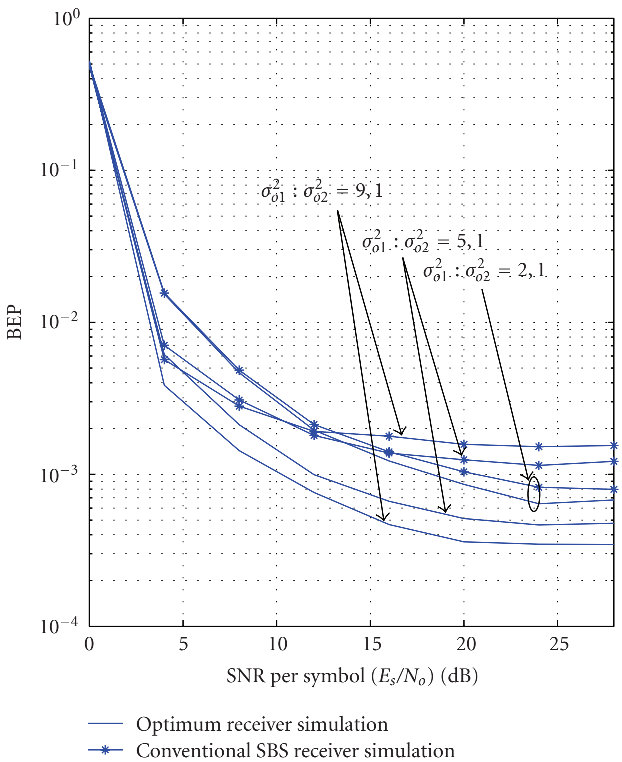

FIGURE 4: Case 2: BEP results for the conventional SBS and the optimum receivers, 2Tx and 2Rx Alamouti's code with QPSK modulation, $f_d T_b = 0.1$.

also greater. However, since the optimum decoder has a higher decoding complexity of M^2, compared with the linear decoding complexity of M for the conventional SBS decoder, it is possible to tradeoff between the performance and the complexity. The simulation results show that if the ratio of the channel variances is smaller than 2 : 1, the conventional SBS decoder can be safely applied.

6. Conclusion

This paper considers OSTBC in a vehicular environment, where the channels are non-identical and the CSI is not perfect. It is shown that the conventional SBS decoder is not optimum in this situation. Two typical cases are considered for the case of non-identical channels with channel estimation.

In Case 1, where the non-identical channels are associated with a common receive antenna, the optimum decoder is derived. We show that this optimum decoder can be simplified to an SBS decoder, under the condition that the received signal and the estimated channel matrices are properly weighted. In Case 2, where the non-identical channels are associated with a common transmit antenna, we also derive the optimum decoder. But it is shown that no matter how the receiver structure is designed, the optimum decoder cannot be simplified to a SBS decoder.

The performance of the optimum decoder is also investigated. The upper/lower bounds and close approximations of the BEP performance are obtained for Case 1. Both the analytical and the simulation results show that our optimum decoder substantially outperforms the conventional SBS decoder.

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