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On RR couplings, singularity structures and all order α' contact interactions to BPS string amplitudes

Ehsan Hatefi

Centre for Research in String Theory, School of Physics and Astronomy, Queen Mary University of London, Mile End Road, London E1 4NS, U.K.
National Institute for Theoretical Physics, School of Physics and Mandelstam Institute for Theoretical Physics, University of the Witwatersrand, Wits, 2050, South Africa
Institute des Hautes Etudes Scientifiques Bures-sur-Yvette, F-91440, France

E-mail: ehsan.hatefi@cern.ch

Abstract: We evaluate five point world-sheet string theory amplitudes of one transverse scalar field, two world volume gauge fields (and two transverse scalars, a gauge field) in the presence of a closed string Ramond-Ramond vertex operator in its symmetric picture. We carry out all the entire S-matrix elements of five point mixed RR-scalars/gauge fields $\langle C^{-1}\phi^0 A^{-1} A^0\rangle$, $\langle C^{-1}\phi^{-1} A^0 A^0\rangle$, $\langle C^{-1} A^0 \phi^{-1}\phi^0\rangle$ and $\langle C^{-1} A^{-1}\phi^0\phi^0\rangle$ in detail and start comparing all order α' contact interactions and singularities in both transverse and world volume directions. We explore the presence of various new couplings in string theory effective actions and find out their all order α' higher derivative corrections in both type IIA and IIB. Ultimately we make various remarks for the singularities and contact terms whose RR momenta are embedded in transverse directions. α' corrections to some of Myers terms are also addressed.

Keywords: Brane Dynamics in Gauge Theories, Superstrings and Heterotic Strings, D-branes, Conformal Field Models in String Theory

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1 Introduction

The fundamental objects in string theory or the so called D-branes have been playing a key ingredient in various research topics on theoretical high energy physics as well as in super string theory [1–4].

Indeed either a BPS or non-BPS D_p-brane includes $(p+1)$-dimensional world volume fields which must be thought of a hypersurface like in a ten dimensional flat space time. We need to take into account some special boundary conditions to them, namely either Neumann or Dirichlet, depending on whether we apply those boundary conditions through transverse or its world volume fields [5, 6]. Note that recently some remarks for brane-anti brane have also been mentioned in [7].

To have more complete picture of the effective actions of string theory and what has been carried out up to now, we just point out to various papers that are important to the author. Myers in [8] did explore the form of a bosonic action which holds for multiple D_p-brane configurations and the generalization of Myers action with its all order α' corrections (using the mixed open-closed scattering amplitudes) has been done in [9].

Having performed [9], some new couplings were obtained. These new couplings are not inside Effective field theory (EFT) and their importance has played the fundamental role not only in performing the ADM reduction of IIB and exploring dS brane world-volume solutions [10] but also in deriving N^3 entropy of $M5$ branes. These couplings could have some specific role in super gravity solutions as well where the particular emphasis is paid on the near-extremal black-branes to actually get to n^3 entropy growth analysis [11].

A remarkable paper [12] on supersymmetrized version of that action was given. A part of the supersymmetric action is known, in fact it involves symmetric traces of the non-abelian fields and what needs exploration is further terms which do not belong to the category that we are looking for in this paper. Whereas the effective action for a bosonic brane given by [13] and naturally its supersymmetric one was written down by [14–18]. One could read off a review of all the DBI, Wess-Zumino and Chern-Simons action just for BPS branes from [19]. On the other hand, to reveal more about three standard ways of effective field theory of the D-brane action (which contain Taylor expansion-Myers Terms and Pull back), and to learn more about all sorts of higher derivative corrections of non-BPS and BPS branes , we advise the section five of [20].

It is also important to have some tools to actually deal with the mixed open-closed higher point functions of string amplitudes, where one can refer to some of the pioneer works on either effective actions or scattering amplitudes that are involved with several D_p-brane configurations as well as their string applications [21–33].

The paper is constructed as follows. In the next section we just introduce vertex operators with all details and notations and then we try to work out Type II super string computations with all order α' D-brane S-matrix of a Ramond-Ramond (RR) in symmetric picture, a scalar field in zero picture with two world volume gauge fields on different pictures where we try to address the entire S-matrix and explain the whole techniques that are involved in that particular amplitude.[1]

Afterwards we start comparing all the contact interactions and singularity structures of $\langle V_C V_\phi V_A V_A\rangle$ S-matrix in two different pictures in the presence of a symmetric RR vertex operator. Basically we compare both all order α' contact interactions and all the singularity structures of $\langle C^{-1}\phi^0 A^{-1} A^0\rangle$ with $\langle C^{-1}\phi^{-1} A^0 A^0\rangle$, where the superscripts refer to the chosen picture of each string operator. Although we regenerate all $t, s, u, (t+s+u)$-channel poles in effective field theory, we also find out some new contact interaction and singularities in the $\langle C^{-1}\phi^0 A^{-1} A^0\rangle$ S-matrix and for the first time , we explore their all order α' couplings in effective field theory as well.[2]

[1]We may wonder whether it is possible to apply T-duality to $\langle V_C V_A V_A V_A\rangle$ S-matrix of [19] to get to $\langle V_C V_\phi V_A V_A\rangle$ S-matrix. Indeed as it has been explored there are various terms in the S-matrix of $\langle V_C V_\phi V_A V_A\rangle$, that carry momentum of RR in transverse direction that cannot be obtained by T-duality transformation in flat ten dimensions of space-time. In fact the appearance of RR makes things subtle or complicated as argued in [35] and [36] accordingly.

[2]There is the possibility that some of the terms derived in different pictures of the vertex operators, might be related via Bianchi identities of the bulk. This would imply that some of the contact interactions might be redundant but not all. In some of the specific examples , some of the assumed contact terms seems to be reproduced by a specific combination of pull-back and Taylor expansion of the CS terms. One might use some of the new terms to eliminate either the pull-back or the Taylor expansion. Nevertheless,

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It is also worth reporting some sort of new singularities and new sort of Myers terms that appear in this particular picture of $\langle C^{-1}\phi^0 A^{-1} A^0\rangle$ S-matrix where those new terms are actually the terms that carry momentum of RR in transverse direction and do involve $p.\xi$ terms inside the S-matrix elements.

Note that these $p.\xi$ terms are derived by direct analysis of $\langle C^{-1}\phi^0 A^{-1} A^0\rangle$, due to non zero correlation function of RR field by the first term of scalar field's vertex operator in zero picture, that is, all $\langle e^{ip.x(z)}\partial_i x^i(x_1)\rangle$ terms are indeed non-zero. Therefore since scalar field's polarization is in the bulk , one expects to be concerned about all $p.\xi$ terms and p^i, p^j terms whose momenta of RR are carried in transverse directions. It is worth pointing out the following fact as follows. Recently, it is shown in [34] that, if one does not know all the Bianchi identities of RR in the bulk, then certainly there will be no chance to explore all the bulk singularities of non-BPS branes.

We perform full comparisons at each order of α' for all contact interactions as well, and that leads to finding out new couplings that can be derived by just S-matrix analysis not by any other tools to our knowledge.

The profound relation of open-closed string plays the crucial role in matching out all the singularities of string theory with EFT, as it has been shown that all order α' higher derivative corrections to SYM couplings produce all massless poles at $(t+s+u)$-channel poles through a RR coupling with various BPS open strings. It has also been emphasised that, this phenomenon could have played the major role for finding the universality conjecture on α' corrections of string theory [37].

We carry out the same analysis (this time for an RR, two scalars and a gauge field) in type IIA and IIB super string theory for both $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ and $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ S-matrices where we seem to find out the same $t, s, u, (t+s+u)$-singularity structures in the presence of an RR, even number of scalar fields. However, we claim that various new contact interactions appear in the S-matrix by considering both scalar fields in zero picture. Indeed we derive these new couplings, show that these couplings can just be discovered from $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ S-matrix and explore their all order α' corrections in effective field theory side. Finally we conclude by mentioning various remarks about these S-matrices in the conclusion section.

2 Type II Super string computations with all order α' D-brane couplings

In this section we would like to carry out the Conformal Field Theory (CFT) technique to be able to explore not only all the singularities but also all the infinite contact interactions of the mixture of a closed string RR (in its symmetric picture) and various BPS open string fields. Indeed our calculation makes sense at the level of a world-sheet five point mixed closed-open string amplitude which must be done on the upper half-complex plane. We find the entire S-matrix elements which hold on both world-volume and transverse component of D-branes.

One might be interested in seeing various efforts that have been performed on both BPS and non-BPS amplitudes [38–49].

we believe that not all the new couplings are redundant.

In order to find out the effective action of string theory one needs to deal with or calculate the scattering amplitudes and naturally the first step to do so, is to fix a particular picture of the vertices. Namely, the sum of the superghost charges must have been (-2) for disk amplitudes.

In our notations we use $\mu, \nu = 0, 1, \ldots, 9$ for the whole spacetime, while $a, b, c = 0, 1, \ldots, p$ for world volume space and $i, j = p+1, \ldots, 9$ for transverse directions. Here we would like to insist on the calculations in the presence of symmetric picture of RR but for the completeness we point out all the different vertex operators in various pictures as follows:

$$
\begin{aligned}
V_\phi^{(0)}(x) &= \xi_i\Big(\partial X^i(x) + \alpha' ik{\cdot}\psi\psi^i(x)\Big)e^{\alpha' ik\cdot X(x)},\\
V_\phi^{(-1)}(y) &= \xi.\psi(y)e^{-\phi(y)}e^{\alpha' ik\cdot X(y)},\\
V_A^{(0)}(x) &= \xi_a\Big(\partial X^a(x) + \alpha' iq{\cdot}\psi\psi^a(x)\Big)e^{\alpha' iq\cdot X(x)},\\
V_A^{(-1)}(y) &= \xi_a\psi^a(y)e^{-\phi(y)}e^{\alpha' iq\cdot X(y)}\\
V_C^{(-\frac{1}{2},-\frac{1}{2})}(z,\bar{z}) &= (P_-\not{H}_{(n)}M_p)^{\alpha\beta}e^{-\phi(z)/2}S_\alpha(z)e^{i\frac{\alpha'}{2}p\cdot X(z)}e^{-\phi(\bar{z})/2}S_\beta(\bar{z})e^{i\frac{\alpha'}{2}p\cdot D\cdot X(\bar{z})},\\
V_C^{(-\frac{3}{2},-\frac{1}{2})}(z,\bar{z}) &= (P_-\not{C}_{(n-1)}M_p)^{\alpha\beta}e^{-3\phi(z)/2}S_\alpha(z)e^{i\frac{\alpha'}{2}p\cdot X(z)}e^{-\phi(\bar{z})/2}S_\beta(\bar{z})e^{i\frac{\alpha'}{2}p\cdot D\cdot X(\bar{z})},
\end{aligned}
\tag{2.1}
$$

To our knowledge the vertex of RR in asymmetric picture has been first shown by an interesting paper on open string theory [50] and then it was argued with some more details in [51, 52] where the following kinematic relations are also considered

$$
k^2 = q^2 = p^2 = 0 \quad q.\xi = 0,
$$

We also apply Doubling trick to make use of holomorphic components of world sheet fields as well, that is,

$$
\tilde{X}^\mu(\bar{z}) \to D^\mu_\nu X^\nu(\bar{z}), \quad \tilde{\psi}^\mu(\bar{z}) \to D^\mu_\nu \psi^\nu(\bar{z}), \quad \tilde{\phi}(\bar{z}) \to \phi(\bar{z}), \quad \text{and} \quad \tilde{S}_\alpha(\bar{z}) \to M_\alpha{}^\beta S_\beta(\bar{z}),
$$

where

$$
D = \begin{pmatrix} -1_{9-p} & 0 \\ 0 & 1_{p+1} \end{pmatrix}, \quad \text{and} \quad M_p = \begin{cases} \dfrac{\pm i}{(p+1)!}\gamma^{a_1}\gamma^{a_2}\ldots\gamma^{a_{p+1}}\epsilon_{a_1\ldots a_{p+1}} & \text{for } p \text{ even} \\ \dfrac{\pm 1}{(p+1)!}\gamma^{a_1}\gamma^{a_2}\ldots\gamma^{a_{p+1}}\gamma_{11}\epsilon_{a_1\ldots a_{p+1}} & \text{for } p \text{ odd} \end{cases}
$$

Although all the details of spinor part have been verified in [20], we just clarify the definitions of projector and RR's field strength as follows

$$
(P_-\not{H}_{(n)})^{\alpha\beta} = C^{\alpha\delta}(P_-\not{H}_{(n)})_\delta{}^\beta, \qquad P_- = \frac{1}{2}(1-\gamma^{11}) \tag{2.2}
$$

and

$$
\not{H}_{(n)} = \frac{a_n}{n!}H_{\mu_1\ldots\mu_n}\gamma^{\mu_1}\ldots\gamma^{\mu_n},
$$

where for IIA and IIB we use $n = 2, 4, a_n = i$ and $n = 1, 3, a_n = 1$ appropriately.

Here we just work out with the holomorphic parts of correlations but the interested reader can easily find out all the tricks in the appendix part of [20] as well.

2.1 All order α' S-matrix element of $\langle C^{-1}\phi^0 A^{-1} A^0\rangle$

The complete form of the S-matrix element of a closed string RR (in its symmetric picture) n-form field strength and a transverse scalar field in zero picture and two world volume gauge fields $\langle C^{-1}\phi^0 A^{-1} A^0\rangle$ can be found by the following correlation functions

$$\mathcal{A}^{\langle C^{-1}\phi^0 A^{-1} A^0\rangle} \sim \int dx_1 dx_2 dx_3 dz d\bar{z}\, \langle V_\phi^{(0)}(x_1) V_A^{(-1)}(x_2) V_A^{(0)}(x_3) V_{RR}^{(-\frac{1}{2},-\frac{1}{2})}(z,\bar{z})\rangle, \tag{2.3}$$

We just look for a special ordering. Setting the Wick theorem, the amplitude is written down as follows

$$\begin{aligned}\mathcal{A}^{\langle C^{-1}\phi^0 A^{-1} A^0\rangle} \sim \int dx_1 dx_2 dx_3 dx_4 dx_5\, (P_- H\!\!\!/_{(n)} M_p)^{\alpha\beta} \xi_{1i}\xi_{2a}\xi_{3b} x_{45}^{-1/4}(x_{24}x_{25})^{-1/2} \\ \times (I_1 + I_2 + I_3 + I_4) \mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3),\end{aligned} \tag{2.4}$$

where $x_{ij} = x_i - x_j$, $x_4 = z$, $x_5 = \bar{z}$ and

$$\begin{aligned}
I_1 &= \langle :\partial X^i(x_1) e^{\alpha' ik_1.X(x_1)} : e^{\alpha' ik_2.X(x_2)} : \partial X^b(x_3) e^{\alpha' ik_3.X(x_3)} : e^{i\frac{\alpha'}{2}p.X(x_4)} : e^{i\frac{\alpha'}{2}p.D.X(x_5)} :\rangle \\
&\quad \times \langle : S_\alpha(x_4) : S_\beta(x_5) : \psi^a(x_2) :\rangle, \\
I_2 &= \langle :\partial X^i(x_1) e^{\alpha' ik_1.X(x_1)} : e^{\alpha' ik_2.X(x_2)} : e^{\alpha' ik_3.X(x_3)} : e^{i\frac{\alpha'}{2}p.X(x_4)} : e^{i\frac{\alpha'}{2}p.D.X(x_5)} :\rangle \\
&\quad \times \langle : S_\alpha(x_4) : S_\beta(x_5) :: \psi^a(x_2) : \alpha' ik_3.\psi\psi^b(x_3)\rangle, \\
I_3 &= \langle : e^{\alpha' ik_1.X(x_1)} : e^{\alpha' ik_2.X(x_2)} : \partial X^b(x_3) e^{\alpha' ik_3.X(x_3)} : e^{i\frac{\alpha'}{2}p.X(x_4)} : e^{i\frac{\alpha'}{2}p.D.X(x_5)} :\rangle \\
&\quad \times \langle : S_\alpha(x_4) : S_\beta(x_5) : \alpha' ik_1.\psi\psi^i(x_1) : \psi^a(x_2) :\rangle, \\
I_4 &= \langle : e^{\alpha' ik_1.X(x_1)} : e^{\alpha' ik_2.X(x_2)} : e^{\alpha' ik_3.X(x_3)} : e^{i\frac{\alpha'}{2}p.X(x_4)} : e^{i\frac{\alpha'}{2}p.D.X(x_5)} :\rangle \\
&\quad \times \langle : S_\alpha(x_4) : S_\beta(x_5) : \alpha' ik_1\cdot\psi\psi^i(x_1) :: \psi^a(x_2) : \alpha' ik_3\cdot\psi\psi^b(x_3) :\rangle.
\end{aligned} \tag{2.5}$$

We actually use the standard propagators, as follows

$$\begin{aligned}
\langle X^\mu(z) X^\nu(w)\rangle &= -\frac{\alpha'}{2}\eta^{\mu\nu}\log(z-w)\,, \\
\langle \psi^\mu(z)\psi^\nu(w)\rangle &= -\frac{\alpha'}{2}\eta^{\mu\nu}(z-w)^{-1}\,, \\
\langle \phi(z)\phi(w)\rangle &= -\log(z-w)
\end{aligned} \tag{2.6}$$

We also need to take into account the Wick's theorem to be able to investigate all the bosonic correlators. To see further details , the section 3 of [53] is strongly suggested.

Let us just address the most complicated fermionic correlation function of two spin operators/ two different currents and a fermion field, where all the possible contractions have to be considered.

Once again we use $x_4 = z$, $x_5 = \bar{z}$. Note that unlike the open string correlator where integration is on the real line x_4, x_5 are integrated on the upper half plane. It is only for the purposes of the Wick contractions that we can forget the complex conjugation of one variable to another, in order to simplify things.

$$
\begin{aligned}
I_6^{bcaid} &= \langle : S_\alpha(x_4) : S_\beta(x_5) : \psi^d\psi^i(x_1) :: \psi^a(x_2) : \psi^c\psi^b(x_3)\rangle \\
&= \bigg\{(\Gamma^{bcaid}C^{-1})_{\alpha\beta} + \alpha' r_1 \frac{\mathrm{Re}[x_{14}x_{25}]}{x_{12}x_{45}} + \alpha' r_2 \frac{\mathrm{Re}[x_{14}x_{35}]}{x_{13}x_{45}} + \alpha' r_3 \frac{\mathrm{Re}[x_{24}x_{35}]}{x_{23}x_{45}} \\
&\quad + \alpha'^2 r_4 \left(\frac{\mathrm{Re}[x_{14}x_{35}]}{x_{13}x_{45}}\right)\left(\frac{\mathrm{Re}[x_{24}x_{35}]}{x_{23}x_{45}}\right)\bigg\} 2^{-5/2} x_{45}^{5/4}(x_{14}x_{15}x_{34}x_{35})^{-1}(x_{24}x_{25})^{-1/2},
\end{aligned} \tag{2.7}
$$

so that

$$
\begin{aligned}
r_1 &= \Big(\eta^{da}(\Gamma^{bci}C^{-1})_{\alpha\beta}\Big), \\
r_2 &= \Big(-\eta^{cd}(\Gamma^{bai}C^{-1})_{\alpha\beta} + \eta^{db}(\Gamma^{cai}C^{-1})_{\alpha\beta}\Big), \\
r_3 &= \Big(-\eta^{ac}(\Gamma^{bid}C^{-1})_{\alpha\beta} + \eta^{ab}(\Gamma^{cid}C^{-1})_{\alpha\beta}\Big), \\
r_4 &= \Big((\eta^{cd}\eta^{ab} - \eta^{bd}\eta^{ac})(\gamma^i C^{-1})_{\alpha\beta}\Big)
\end{aligned} \tag{2.8}
$$

Replacing the above correlators and performing some simple algebraic computations, one can further simplify the amplitude and write it down in a closed form as follows

$$
\begin{aligned}
\mathcal{A}^{\langle C^{-1}\phi^0 A^{-1} A^0\rangle} \sim & \int dx_1 dx_2 dx_3 dx_4 dx_5 (P_- H\!\!\!/_{(n)} M_p)^{\alpha\beta} I \xi_{1i}\xi_{2a}\xi_{3b} x_{45}^{-1/4}(x_{24}x_{25})^{-1/2} \\
& \times\Big(I_7^a(a_1^i a_2^b) + a_1^i a_3^{ba} + a_2^b a_4^{ai} - \alpha'^2 k_{1d}k_{3c} I_6^{bcaid}\Big)\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3),
\end{aligned} \tag{2.9}
$$

where

$$
\begin{aligned}
I &= |x_{12}|^{\alpha'^2 k_1.k_2}|x_{13}|^{\alpha'^2 k_1.k_3}|x_{14}x_{15}|^{\frac{\alpha'^2}{2}k_1.p}|x_{23}|^{\alpha'^2 k_2.k_3} \\
&\qquad \times |x_{24}x_{25}|^{\frac{\alpha'^2}{2}k_2.p}|x_{34}x_{35}|^{\frac{\alpha'^2}{2}k_3.p}|x_{45}|^{\frac{\alpha'^2}{4}p.D.p}, \\
a_1^i &= ip^i\left(\frac{x_{54}}{x_{14}x_{15}}\right), \\
a_2^b &= ik_1^b\left(\frac{x_{14}}{x_{13}x_{34}} + \frac{x_{15}}{x_{35}x_{13}}\right) + ik_2^b\left(\frac{x_{24}}{x_{34}x_{23}} + \frac{x_{25}}{x_{35}x_{23}}\right), \\
a_3^{ba} &= \bigg\{(\Gamma^{bca}C^{-1})_{\alpha\beta} + (-\alpha'\eta^{ac}(\gamma^b C^{-1})_{\alpha\beta} + \alpha'\eta^{ab}(\gamma^c C^{-1})_{\alpha\beta})\frac{\mathrm{Re}[x_{24}x_{35}]}{x_{23}x_{45}}\bigg\} \\
&\qquad \times \alpha' i k_{3c} 2^{-3/2} x_{45}^{1/4}(x_{34}x_{35})^{-1}(x_{24}x_{25})^{-1/2} \\
a_4^{ai} &= \alpha' i k_{1d} 2^{-3/2} x_{45}^{1/4}(x_{24}x_{25})^{-1/2}(x_{14}x_{15})^{-1} \\
&\qquad \times\bigg\{(\Gamma^{aid}C^{-1})_{\alpha\beta} + \alpha'\eta^{ad}(\gamma^i C^{-1})_{\alpha\beta}\frac{\mathrm{Re}[x_{14}x_{25}]}{x_{12}x_{45}}\bigg\}, \\
I_7^a &= \langle : S_\alpha(x_4) : S_\beta(x_5) : \psi^a(x_2) :\rangle = 2^{-1/2} x_{45}^{-3/4}(x_{24}x_{25})^{-1/2}(\gamma^a C^{-1})_{\alpha\beta}.
\end{aligned} \tag{2.10}
$$

Now one could use the SL(2,R) invariance of the S-matrix and to remove the V_{CKG} we do gauge fixing over the position of open strings at zero, one and infinity. By doing gauge fixing as $(x_1 = 0, x_2 = 1, x_3 = \infty)$, one needs to address the following integration on the upper half plane over the position of RR

$$\int d^2z|1-z|^a|z|^b(z-\bar{z})^c(z+\bar{z})^d$$

where a, b, c are the combinations of the following Mandelstam variables

$$s = \frac{-\alpha'}{2}(k_1+k_3)^2, \quad t = \frac{-\alpha'}{2}(k_1+k_2)^2, \quad u = \frac{-\alpha'}{2}(k_2+k_3)^2$$

and the results of the integrations for $d = 0, 1$ and $d = 2$ were obtained accordingly in [54] and [20].

Therefore, the final form of the S-matrix in this particular picture to all orders in α' is obtained as follows

$$\mathcal{A}^{\langle C^{-1}\phi^0 A^{-1}A^0\rangle} = \mathcal{A}_1+\mathcal{A}_2+\mathcal{A}_3+\mathcal{A}_4+\mathcal{A}_5+\mathcal{A}_6+\mathcal{A}_7 \tag{2.11}$$

where

$$\begin{aligned}
\mathcal{A}_1 &\sim 2^{-1/2}\xi_{1i}\xi_{2a}\xi_{3b}\Big[-k_{3c}k_{1d}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{bcaid}) \\
&\qquad\qquad +k_{3c}p^i\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{bca})\Big]4(-t-s-u)L_1, \\
\mathcal{A}_2 &\sim 2^{-1/2}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{aid})\xi_{1i}\xi_{2a}k_{1d}\Big\{-2k_1.\xi_3(ut)+2k_2.\xi_3(st)\Big\}L_2 \\
\mathcal{A}_5 &\sim 2^{-1/2}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^i)\xi_{1i}\Big\{\xi_3.\xi_2(2ts)+2k_1.\xi_3(2k_3.\xi_2)t \\
&\qquad\qquad -4uk_1.\xi_2(k_1.\xi_3)+4sk_2.\xi_3k_1.\xi_2\Big\}L_1 \\
\mathcal{A}_4 &\sim -2^{-1/2}(st)L_2\Big\{\xi_{3b}\xi_{1i}\xi_{2a}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{bai})u+2k_3.\xi_2k_{1d}\xi_{1i}\xi_{3b}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{bid}) \\
&\qquad\qquad -\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{cid})k_{1d}k_{3c}\xi_{1i}(2\xi_2.\xi_3)\Big\} \\
\mathcal{A}_3 &\sim -2^{-1/2}\xi_{1i}k_{3c}\Big\{-2k_1.\xi_2\xi_{3b}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{bci})(us) \\
&\qquad\qquad +2k_1.\xi_3\xi_{2a}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{cai})(ut)\Big\}L_2 \\
\mathcal{A}_6 &\sim 2^{-1/2}(st)p^i\xi_{1i}\Big\{2k_3.\xi_2\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^b)\xi_{3b}-2\xi_3.\xi_2\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^c)k_{3c}\Big\}L_2 \\
\mathcal{A}_7 &\sim 2^{-1/2}p^i\xi_{1i}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^a)\xi_{2a}\Big\{2k_1.\xi_3(ut)-2\xi_3.k_2(st)\Big\}L_2
\end{aligned} \tag{2.12}$$

where the functions L_1, L_2, are

$$\begin{aligned}
L_1 &= (2)^{-2(t+s+u)-1}\pi\frac{\Gamma(-u+\frac{1}{2})\Gamma(-s+\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u)}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)}, \\
L_2 &= (2)^{-2(t+s+u)}\pi\frac{\Gamma(-u)\Gamma(-s)\Gamma(-t)\Gamma(-t-s-u+\frac{1}{2})}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)},
\end{aligned} \tag{2.13}$$

As we have expected by interchanging $\xi_{2a} \to k_{2a}$ and also $\xi_{3b} \to k_{3b}$, the whole S-matrix vanishes, which means that the amplitude does satisfy all the associated Ward identities

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and the amplitude is non zero for various p, n cases. Notice also we are dealing with all massless BPS strings so the expansion is low energy expansion. This S-matrix does have all t, s, u and particularly $(t+s+u)$ channel poles and in particular it has some extra singularities that are precisely carrying momentum of RR in the bulk direction where we will show that these terms cannot be derived in the other picture ($\langle C^{-1}\phi^{-1}A^0A^0\rangle$) S-matrix and we argue about them in the next section.

More significantly, in $\langle C^{-1}\phi^0 A^{-1}A^0\rangle$ S-matrix we discover the new form of contact interactions to all orders in α' that cannot be found in the other picture. For the precise definitions of the expansions and more kinematical definitions and identities, one needs to look at [19, 35].

Note that by sending $t, s, u \to 0$, one finds the expansion of the functions L_1, utL_2 as follows

$$\begin{aligned}
L_1 &= -2^{-1}\pi^{5/2}\Bigg(\sum_{n=0}^{\infty} c_n(s+t+u)^n + \frac{\sum_{n,m=0}^{\infty} c_{n,m}[s^n t^m + s^m t^n]}{(t+s+u)} \\
&\qquad + \sum_{p,n,m=0}^{\infty} f_{p,n,m}(s+t+u)^p[(s+t)^n(st)^m]\Bigg) \\
utL_2 &= -\pi^{3/2}\sum_{n=-1}^{\infty} b_n\frac{1}{s}(u+t)^{n+1} + \sum_{p,n,m=0}^{\infty} e_{p,n,m}s^p(tu)^n(t+u)^m.
\end{aligned} \tag{2.14}$$

with the following coefficients

$$\begin{aligned}
&b_{-1} = 1, && b_0 = 0, && b_1 = \frac{1}{6}\pi^2, \\
&b_2 = 2\zeta(3), && c_0 = 0, && c_1 = -\frac{\pi^2}{6}, \\
&e_{2,0,0} = e_{0,1,0} = 2\zeta(3), && e_{1,0,0} = \frac{1}{6}\pi^2, && \\
&e_{1,0,2} = \frac{19}{60}\pi^4, && e_{1,0,1} = e_{0,0,2} = 6\zeta(3), && \\
&e_{0,0,1} = \frac{1}{3}\pi^2, && e_{3,0,0} = \frac{19}{360}\pi^4, && \\
&e_{0,0,3} = e_{2,0,1} = \frac{19}{90}\pi^4, && e_{1,1,0} = e_{0,1,1} = \frac{1}{30}\pi^4, && \\
&c_2 = -2\zeta(3), && c_{1,1} = \frac{\pi^2}{6}, && c_{0,0} = \frac{1}{2}, \\
&c_{3,1} = c_{1,3} = \frac{2}{15}\pi^4, && c_{2,2} = \frac{1}{5}\pi^4, && \\
&c_{1,0} = c_{0,1} = 0, && c_{3,0} = c_{0,3} = 0\,, && \\
&c_{2,0} = c_{0,2} = \frac{\pi^2}{6}, && c_{1,2} = c_{2,1} = -4\zeta(3), && \\
&f_{0,1,0} = \frac{\pi^2}{3}, && f_{0,2,0} = -f_{1,1,0} = -6\zeta(3), && \\
&f_{0,0,1} = -2\zeta(3), && c_{4,0} = c_{0,4} = \frac{1}{15}\pi^4. &&
\end{aligned} \tag{2.15}$$

Meanwhile the result of the S-matrix in different picture of scalar field, that is, $\langle C^{-1}\phi^{-1}A^0A^0\rangle$ S-matrix was derived in [35] to be as follows

$$\mathcal{A}^{\langle C^{-1}\phi^{-1}A^0A^0\rangle} \sim \int dx_1dx_2dx_3dzd\bar{z}\,\langle V_\phi^{(-1)}(x_1)V_A^{(0)}(x_2)V_A^{(0)}(x_3)V_{RR}^{(-\frac{1}{2},-\frac{1}{2})}(z,\bar{z})\rangle,$$
$$\mathcal{A}^{\langle C^{-1}\phi^{-1}A^0A^0\rangle} = \mathcal{A}_1+\mathcal{A}_2+\mathcal{A}_3+\mathcal{A}_4+\mathcal{A}_5 \tag{2.16}$$

where

$$\begin{aligned}
\mathcal{A}_1 &\sim -2^{-1/2}\xi_{1i}\xi_{2a}\xi_{3b}\Big[k_{3d}k_{2c}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{bdaci})\Big]4(-t-s-u)L_1,\\
\mathcal{A}_2 &\sim 2^{-1/2}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{bdi})\xi_{1i}\xi_{3b}k_{3d}\Big\{2k_1.\xi_2(us)-2k_3.\xi_2(st)\Big\}L_2\\
\mathcal{A}_3 &\sim -2^{-1/2}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{aci})\xi_{1i}\xi_{2a}k_{2c}\Big\{-2k_2.\xi_3(st)+2k_1.\xi_3(ut)\Big\}L_2 \qquad (2.17)\\
\mathcal{A}_4 &\sim -2^{-1/2}(st)L_2\Big\{\xi_{3b}\xi_{1i}\xi_{2a}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{bai})u+2k_2.\xi_3k_{3d}\xi_{1i}\xi_{2a}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{dai})\\
&\qquad +2k_3.\xi_2k_{2c}\xi_{1i}\xi_{3b}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{bci})-\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{dci})k_{3d}k_{2c}\xi_{1i}(2\xi_2.\xi_3)\Big\}\\
\mathcal{A}_5 &\sim 2^{-1/2}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^i)\xi_{1i}\Big\{\xi_3.\xi_2(2ts)+2k_1.\xi_3(2k_3.\xi_2)t\\
&\qquad -4uk_1.\xi_2(k_1.\xi_3)+4sk_2.\xi_3k_1.\xi_2\Big\}L_1
\end{aligned}$$

Let us first compare the results of the same S-matrix in different pictures and then start producing all the singularity structures as well as new contact interactions.

3 Comparison on singularity structures of $\langle C^{-1}\phi^0A^{-1}A^0\rangle$ with $\langle C^{-1}\phi^{-1}A^0A^0\rangle$

First of all note that both $\mathcal{A}_5$'s in two different pictures are exactly matched. The first term $\mathcal{A}_3$ of $\langle C^{-1}\phi^0A^{-1}A^0\rangle$ is exactly the first term $\mathcal{A}_2$ of $\langle C^{-1}\phi^{-1}A^0A^0\rangle$. Now if we add the 2nd term $\mathcal{A}_2$ of $\langle C^{-1}\phi^{-1}A^0A^0\rangle$ with the 3rd term $\mathcal{A}_4$ of $\langle C^{-1}\phi^{-1}A^0A^0\rangle$ and apply momentum conservation along the world volume of brane we get

$$-2^{-1/2}stL_2(2k_3.\xi_2)\xi_{1i}\xi_{3b}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{bci})(-k_{1c}-p_c)$$

Now if we use the identity that has been found in [7], that is,

$$p_c\epsilon^{a_0\ldots a_{p-2}bc}=0 \tag{3.1}$$

then we get to know the fact that the first term of above equation precisely produces the 2nd term $\mathcal{A}_4$ of $\langle C^{-1}\phi^0A^{-1}A^0\rangle$.

One can see the derivation of (3.1) in various equations of [7]. For example, it is shown in equation (9) of [7] that, in order to get to the same result of three point function of one RR and a scalar field in both $\langle C^{-1}\phi^{-1}\rangle$ and $\langle C^{-2}\phi^0\rangle$ S-matrix, the equation (9) of [7] or (3.1) must hold. Another example to prove that (3.1) holds is as follows. It is shown in section five of [7] that, to get to the same result of four point function of $\langle C^{-1}T^0\phi^{-1}\rangle$ and $\langle C^{-2}T^0\phi^0\rangle$ S-matrix, the equation (3.1) must hold (see the footnote of 19 in page 14 of [7]). It has also been discovered that the amplitude of $\langle C^{-1}A^{-1}T^0T^0\rangle$ satisfies Ward identity associated to the gauge field if and only if the above identity (3.1) holds.

Likewise if we add the 1st term $\mathcal{A}_3$ with the 2nd term $\mathcal{A}_4$ of $\langle C^{-1}\phi^{-1}A^0A^0\rangle$ and also apply momentum conservation we find the following elements

$$-2^{-1/2}stL_2(2k_2.\xi_3)\xi_{1i}\xi_{2a}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{aci})(k_{1c}+p_c)$$

Once more one needs to apply the equation $p_c\epsilon^{a_0\cdots a_{p-2}ac}=0$ so that the first term of above precisely produces the 2nd term $\mathcal{A}_2$ of $\langle C^{-1}\phi^0A^{-1}A^0\rangle$.

Simultaneously if we add the first term $\mathcal{A}_2$ with the 2nd term $\mathcal{A}_3$ of $\langle C^{-1}\phi^0A^{-1}A^0\rangle$ with keeping in mind momentum conservation and $p_c\epsilon^{a_0\cdots a_{p-2}ac}=0$, we then precisely produce the 2nd term $\mathcal{A}_3$ of $\langle C^{-1}\phi^{-1}A^0A^0\rangle$.

Finally the last term $\mathcal{A}_4$ of $\langle C^{-1}\phi^{-1}A^0A^0\rangle$ is exactly equivalent with the last term $\mathcal{A}_4$ of $\langle C^{-1}\phi^0A^{-1}A^0\rangle$.[3]

Therefore the upshot is that we can precisely produce all the singularities of $\langle C^{-1}\phi^{-1}A^0A^0\rangle$ by $\langle C^{-1}\phi^0A^{-1}A^0\rangle$ S-matrix (as we will show later on), however, we have also some extra contact interactions and other singularities of $\langle C^{-1}\phi^0A^{-1}A^0\rangle$ S-matrix (in the zero picture of scalar field in the presence of a symmetric RR) that are absent in $\langle C^{-1}\phi^{-1}A^0A^0\rangle$ S-matrix and we will argue about them in a moment.

In fact from the direct calculations we observe the facts that at pole levels the whole $\mathcal{A}_6$ and $\mathcal{A}_7$ of $\langle C^{-1}\phi^0A^{-1}A^0\rangle$ S-matrix are extra terms that cannot be derived from direct computations of $\langle C^{-1}\phi^{-1}A^0A^0\rangle$ S-matrix.

Moreover, the 2nd contact interaction $\mathcal{A}_1$ of $\langle C^{-1}\phi^0A^{-1}A^0\rangle$ is also extra term that cannot be derived from direct computations of $\langle C^{-1}\phi^{-1}A^0A^0\rangle$ S-matrix on upper half plane, as we further elaborate on this coupling in the other section. Let us first produce the different singularity structures.

We do have massless scalar poles in t,s and $(t+s+u)$ channels as well as u channel gauge field poles. Here we just produce the s-channel scalar poles and finally by interchanging $2\leftrightarrow 3$ and exchanging the respected momenta and polarisations we can produce t- channel poles as well. If we replace the desired expansion of utL_2, we then obtain all s-channel poles of string amplitude as follows (normalization constant is $(2\pi)^{1/2}m_p$)

$$\frac{(2\pi\alpha')^2}{p!}\mu_p\xi_{1i}\xi_{2a}k_{2c}\epsilon^{a_0\cdots a_{p-2}ac}H^i_{a_0\cdots a_{p-2}}\sum_{n=-1}^{\infty}\frac{1}{s}b_n(u+t)^{n+1}(2k_1.\xi_3)\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3) \tag{3.2}$$

In order to produce all these massless s-channel scalars, one has to consider a field theory sub amplitude as

$$\mathcal{A}=V^i_\alpha(C_{p-1},A_2,\phi)G^{ij}_{\alpha\beta}(\phi)V^j_\beta(\phi,A_3,\phi_1) \tag{3.3}$$

where by taking into account the kinetic term of scalar fields $\frac{(2\pi\alpha')^2}{2}D^a\phi^iD_a\phi_i$ one obtains the following vertex as well as scalar propagator

$$\begin{aligned}V^j_\beta(\phi,A_3,\phi_1)&=-2ik_1.\xi_3(2\pi\alpha')^2T_p\xi_1^j\mathrm{Tr}\,(\lambda_3\lambda_1\lambda_\beta)\\(G^\phi)^{ij}_{\alpha\beta}&=-\frac{i\delta^{ij}\delta^{\alpha\beta}}{(2\pi\alpha')^2T_ps}\end{aligned} \tag{3.4}$$

[3]Notice to momentum conservation and $p_c\epsilon^{a_0\cdots a_{p-2}ac}=0$.

Now we need to consider the mixed Chern-Simons coupling and the so called Taylor expended of scalar field as follows

$$S_1 = i(2\pi\alpha')^2\mu_p \int d^{p+1}\sigma \frac{1}{(p-1)!}(\varepsilon^v)^{a_0\cdots a_p}\mathrm{Tr}\,\left(F_{a_0a_1}\phi^i\right)\partial_i C^{(p-1)}_{a_2\cdots a_p} \tag{3.5}$$

to actually derive the following vertex operator of an RR, an on-shell gauge field and an off-shell scalar field as

$$V^i_\alpha(C_{p-1}, A_2, \phi) = \frac{i(2\pi\alpha')^2\mu_p}{(p)!}(\varepsilon^v)^{a_0\cdots a_p} H^i{}_{a_2\cdots a_p}\xi_{2a_1}k_{2a_0}\mathrm{Tr}\,(\lambda_2\lambda_\alpha) \tag{3.6}$$

where $V^j_\beta(\phi, A_3, \phi_1)$ is derived from the kinetic term of the scalar field and in particular it has no correction, hence to be able to produce all s-channel poles we need to propose the higher derivative corrections to (3.5) as follows

$$\begin{aligned} S_2 = \sum_{n=-1}^{\infty} b_n(\alpha')^{(n+1)}\mu_p \int d^{p+1}\sigma \frac{1}{(p-1)!}(\varepsilon^v)^{a_0\cdots a_p} \\ \times\partial_i C_{p-1}\wedge D_{a_1}\ldots D_{a_{n+1}}FD^{a_1}\ldots D^{a_{n+1}}\phi^i \end{aligned} \tag{3.7}$$

Having taken (3.7), we were able to derive all order vertex operator of (3.6) as

$$V^i_\alpha(C_{p-1}, A_2, \phi) = \frac{i(2\pi\alpha')^2\mu_p}{(p)!}(\varepsilon^v)^{a_0\cdots a_p} H^i{}_{a_2\cdots a_p}\xi_{2a_1}k_{2a_0}\mathrm{Tr}\,(\lambda_2\lambda_\alpha)\sum_{n=-1}^{\infty} b_n(\alpha' k_2.k)^{n+1} \tag{3.8}$$

Now if we replace (3.8) and (3.4) inside (3.3), then one is able to precisely regenerate all order s-channel singularities of string amplitude (3.2) in the effective field theory as well.

All the u-channel gauge field poles of the string amplitude are given as

$$\begin{aligned} \mu_p(2\pi\alpha')^2\frac{1}{(p)!u}\epsilon^{a_0\cdots a_{p-2}bd}\xi_{1i}k_{1d}H^i{}_{a_0\cdots a_{p-2}} \\ \times\sum_{n=-1}^{\infty} b_n(s+t)^{n+1}(2k_3.\xi_2\xi_{3b} - 2k_2.\xi_3\xi_{2b} + 2\xi_3.\xi_2 k_{2b}) \end{aligned} \tag{3.9}$$

Note that these u-channel gauge field poles can be reconstructed in the effective field theory by the following field theory sub amplitude

$$\mathcal{A} = V^a_\alpha(C_{p-1}, \phi_1, A)G^{ab}_{\alpha\beta}(A)V^b_\beta(A, A_2, A_3), \tag{3.10}$$

where the vertices are

$$\begin{aligned} V^a_\alpha(C_{p-1}, \phi_1, A) &= \frac{i(2\pi\alpha')^2\mu_p}{(p)!}(\varepsilon^v)^{a_0\cdots a_{p-1}a}H^i{}_{a_1\cdots a_{p-1}}\xi_{1i}k_{a_0}\mathrm{Tr}\,(\lambda_1\lambda_\alpha)\sum_{n=-1}^{\infty} b_n(t+s)^{n+1}, \\ V^b_\beta(A, A_2, A_3) &= -iT_p(2\pi\alpha')^2\mathrm{Tr}\,(\lambda_2\lambda_3\lambda_\beta)\Big[2k_2.\xi_3\xi^b_2 - 2k_3.\xi_2\xi^b_3 + \xi_3.\xi_2(k_3-k_2)^b\Big], \\ G^{ab}_{\alpha\beta}(A) &= \frac{i\delta_{\alpha\beta}\delta^{ab}}{(2\pi\alpha')^2 T_p u},, \end{aligned} \tag{3.11}$$

where all order corrections to $V^a_\alpha(C_{p-1},\phi_1,A)$ have been derived from (3.7). By replacing these vertices into the field theory amplitude (3.10), one exactly produces all u-channel gauge field poles that appeared in (3.9).

For the completeness we just produce all the $(s+t+u)$- channel singularities of the S-matrix in the field theory as well. To do so, first we replace the part of the expansion of L_1 (which has poles) inside $\mathcal{A}_5$ so that one gets all the poles in string amplitude as

$$8\pi^3\mu_p\frac{\epsilon^{a_0\cdots a_p}\xi_{1i}H^i_{a_0\cdots a_p}}{(p+1)!(s+t+u)}\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3)\sum_{n,m=0}^{\infty}c_{n,m}(s^mt^n+s^nt^m)$$
$$\times\Big[2st\xi_2.\xi_3+4tk_1.\xi_3k_3.\xi_2+4sk_1.\xi_2k_2.\xi_3-4uk_1.\xi_2k_1.\xi_3\Big] \quad (3.12)$$

In order to produce these poles, one has to consider the following sub amplitude in field theory side

$$V^i_\alpha(C_{p+1},\phi)G^{ij}_{\alpha\beta}(\phi)V^j_\beta(\phi,\phi_1,A_2,A_3) \quad (3.13)$$

where the scalar propagator can be found by taking the kinetic term of scalar fields $(\frac{(2\pi\alpha')^2}{2}D^a\phi^iD_a\phi_i)$ and the vertex of $V^i_\alpha(C_{p+1},\phi)$ is obtained by taking the following effective action through Taylor expansion of scalar field

$$(2\pi\alpha')i\mu_p\int d^{p+1}\sigma\frac{1}{(p+1)!}(\varepsilon^v)^{a_0\cdots a_p}\,\mathrm{Tr}\,\left(\phi^i\right)\,\partial_iC^{(p+1)}_{a_0\cdots a_p}$$

so that

$$\begin{aligned}G^{ij}_{\alpha\beta}(\phi)&=\frac{-i\delta_{\alpha\beta}\delta^{ij}}{T_p(2\pi\alpha')^2k^2}=\frac{-i\delta_{\alpha\beta}\delta^{ij}}{T_p(2\pi\alpha')^2(t+s+u)},\\ V^i_\alpha(C_{p+1},\phi)&=i(2\pi\alpha')\mu_p\frac{1}{(p+1)!}(\varepsilon^v)^{a_0\cdots a_p}H^i_{a_0\cdots a_p}\mathrm{Tr}\,(\lambda_\alpha).\end{aligned} \quad (3.14)$$

To be able to produce all scalar massless poles of the string amplitude to all orders in α', one needs to know all order vertex operator of two scalar two gauge field couplings $V^j_\beta(\phi,\phi_1,A_2,A_3)$. This vertex operator can be found by employing all order α' SYM couplings [35] as follows

$$(2\pi\alpha')^4\frac{1}{2\pi^2}T_p\,(\alpha')^{n+m}\sum_{m,n=0}^{\infty}(\mathcal{L}^{nm}_1+\mathcal{L}^{nm}_2+\mathcal{L}^{nm}_3), \quad (3.15)$$
$$\begin{aligned}\mathcal{L}^{nm}_1&=-\mathrm{Tr}\,\Big(a_{n,m}\mathcal{D}_{nm}[D_a\phi^iD^b\phi_iF^{ac}F_{bc}]+b_{n,m}\mathcal{D}'_{nm}[D_a\phi^iF^{ac}D^b\phi_iF_{bc}]+\mathrm{h.c.}\Big),\\ \mathcal{L}^{nm}_2&=-\mathrm{Tr}\,\Big(a_{n,m}\mathcal{D}_{nm}[D_a\phi^iD^b\phi_iF_{bc}F^{ac}]+b_{n,m}\mathcal{D}'_{nm}[D_a\phi^iF_{bc}D^b\phi_iF^{ac}]+\mathrm{h.c.}\Big),\\ \mathcal{L}^{nm}_3&=\frac{1}{2}\mathrm{Tr}\,\Big(a_{n,m}\mathcal{D}_{nm}[D_a\phi^iD^a\phi_iF^{bc}F_{bc}]+b_{n,m}\mathcal{D}'_{nm}[D_a\phi^iF_{bc}D^a\phi_iF^{bc}]+\mathrm{h.c.}\Big),\end{aligned}$$

where the following definitions for all higher derivative operators have been considered [19]

$$\begin{aligned}\mathcal{D}_{nm}(EFGH)&\equiv D_{b_1}\cdots D_{b_m}D_{a_1}\cdots D_{a_n}EFD^{a_1}\cdots D^{a_n}GD^{b_1}\cdots D^{b_m}H,\\ \mathcal{D}'_{nm}(EFGH)&\equiv D_{b_1}\cdots D_{b_m}D_{a_1}\cdots D_{a_n}ED^{a_1}\cdots D^{a_n}FGD^{b_1}\cdots D^{b_m}H.\end{aligned}$$

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Since the off-shell scalar field is abelin, one needs to consider just two permutations of $\mathrm{Tr}\,(\lambda_1\lambda_\beta\lambda_2\lambda_3), \mathrm{Tr}\,(\lambda_\beta\lambda_1\lambda_2\lambda_3)$ to be able to derive all order vertex of $V^j_\beta(\phi,\phi_1,A_2,A_3)$ from the above corrections (3.15) as below

$$V^j_\beta(\phi,\phi_1,A_2,A_3) = \xi^j_1\frac{I_8}{2\pi^2}(\alpha')^{n+m}(a_{n,m}+b_{n,m})\Big((k_3\cdot k_1)^m(k_1\cdot k_2)^n+(k_3\cdot k)^m(k_2\cdot k)^n$$
$$+(k_1\cdot k_3)^n(k_1\cdot k_2)^m+(k\cdot k_3)^n(k\cdot k_2)^m\Big), \tag{3.16}$$

with the following definition for I_8

$$I_8 = (2\pi\alpha')^4 T_p \mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3\lambda_\beta)\Big[\frac{st}{2}\xi_2.\xi_3+tk_1.\xi_3k_3.\xi_2+sk_1.\xi_2k_2.\xi_3-uk_1.\xi_2k_1.\xi_3\Big]$$

where k is off-shell scalar field's momentum, and some of the coefficients $a_{n,m}$ and $b_{n,m}$ ($b_{n,m}$ is symmetric [19]) are

$$\begin{aligned}
a_{0,0} &= -\frac{\pi^2}{6}, & b_{0,0} &= -\frac{\pi^2}{12},\\
a_{1,0} &= 2\zeta(3), & a_{0,1} &= 0,\\
b_{0,1} &= -\zeta(3), & a_{1,1} &= a_{0,2} = -7\pi^4/90,\\
a_{2,2} &= (-83\pi^6-7560\zeta(3)^2)/945, & b_{2,2} &= -(23\pi^6-15120\zeta(3)^2)/1890,\\
a_{1,3} &= -62\pi^6/945, & &\\
a_{2,0} &= -4\pi^4/90, & b_{1,1} &= -\pi^4/180,\\
b_{0,2} &= -\pi^4/45, & a_{0,4} &= -31\pi^6/945,\\
a_{4,0} &= -16\pi^6/945, & a_{1,2} &= a_{2,1} = 8\zeta(5)+4\pi^2\zeta(3)/3,\\
a_{0,3} &= 0, & a_{3,0} &= 8\zeta(5),\\
b_{1,3} &= -(12\pi^6-7560\zeta(3)^2)/1890, & &\\
a_{3,1} &= (-52\pi^6-7560\zeta(3)^2)/945, & b_{0,3} &= -4\zeta(5),\\
b_{1,2} &= -8\zeta(5)+2\pi^2\zeta(3)/3, & b_{0,4} &= -16\pi^6/1890.
\end{aligned} \tag{3.17}$$

They are computed in [19]. Now if we use momentum conservation, we get $k_3\cdot k = k_2.k_1 - (k^2)/2$ and $k_2\cdot k = k_1.k_3-(k^2)/2$, whereas k^2 in (3.16) is cancelled with the k^2 in the denominator of the propagator. Since we just want to produce singularities, we are ignoring those contact terms and considering (3.16) and (3.14) inside (3.13), one explores the sub amplitude in field theory as follows

$$16\pi\mu_p\frac{\epsilon^{a_0\cdots a_p}\xi_{1i}H^i_{a_0\cdots a_p}}{(p+1)!(s+t+u)}\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3)\sum_{n,m=0}^{\infty}(a_{n,m}+b_{n,m})[s^mt^n+s^nt^m]$$
$$\Big[2st\xi_2.\xi_3+4tk_1.\xi_3k_3.\xi_2+4sk_1.\xi_2k_2.\xi_3-4uk_1.\xi_2k_1.\xi_3\Big] \tag{3.18}$$

Now we show that all the poles in field theory of (3.18) can be matched with string amplitude poles that appeared in (3.12). After omitting the common factors of both string and field theory we compare string amplitude with sub amplitude in field theory for various cases of n, m. For $n = m = 0$, the amplitude (3.18) does carry the following factor

$$-4(a_{0,0} + b_{0,0}) = -4\left(\frac{-\pi^2}{6} + \frac{-\pi^2}{12}\right) = \pi^2$$

where the corresponding term for the string amplitude carries $(2\pi^2 c_{0,0})$ which is exactly equivalent to the factor of π^2 in field theory sub amplitude. At α' order, (3.18) carries the following coefficient

$$-(a_{1,0} + a_{0,1} + b_{1,0} + b_{0,1})(s + t) = 0$$

where the corresponding term for the string amplitude is now proportional to $\pi^2(c_{1,0} + c_{0,1})(s+t)$ which is zero as appeared in the field theory sub amplitude. At $(\alpha')^2$ order, (3.18) has the following numerical factor

$$-4(a_{1,1} + b_{1,1})st - 2(a_{0,2} + a_{2,0} + b_{0,2} + b_{2,0})[s^2 + t^2] = \frac{\pi^4}{3}(st) + \frac{\pi^4}{3}(s^2 + t^2)$$

where the corresponding term for the string amplitude is now proportional to $\pi^2[c_{1,1}(2st) + (c_{2,0} + c_{0,2})(s^2 + t^2)]$, which is exactly equivalent to the factor of field theory sub amplitude. The comparisons at orders of $(\alpha')^3, (\alpha')^4$ are also done in [35]. Hence, one can keep comparing to all orders and show that indeed all singularities of $(t + s + u)$ channels of string amplitude can be precisely reconstructed by the above field theory sub amplitudes.

Before further analysis let us compare all order contact interactions on two different pictures, start finding new coupling in the string theory effective action and also explore its all order α' corrections.

4 Comparison on contact interactions to all α' orders

If we look at the whole S-matrix elements in two different pictures and apply momentum conservation to the 1st term $\mathcal{A}_1$ of $\langle C^{-1}\phi^0 A^{-1} A^0\rangle$ we obtain the following elements

$$\begin{aligned}-2^{3/2}(-t - s - u)L_1\xi_{1i}\xi_{2a}\xi_{3b}k_{3c}(-k_{3d} - k_{2d} - p_d)\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{bcaid})\\ = 2^{3/2}\xi_{1i}\xi_{2a}\xi_{3b}k_{3c}(k_{2d})\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{bcaid})(-t - s - u)L_1\end{aligned}$$

which is exactly $\mathcal{A}_1$ of $\langle C^{-1}\phi^{-1} A^0 A^0\rangle$, where we have also used $p_d\epsilon^{a_0\cdots a_{p-4}bcad} = 0$, moreover without any further attempts, one reveals that the first term $\mathcal{A}_4$ of $\langle C^{-1}\phi^0 A^{-1} A^0\rangle$ is exactly the 1st term $\mathcal{A}_4$ of $\langle C^{-1}\phi^{-1} A^0 A^0\rangle$, however, in below we show that there is some other contact interaction that can just be found by $\langle C^{-1}\phi^0 A^{-1} A^0\rangle$ S-matrix.

5 Other contact interaction to all orders in α'

From the direct computations we observed the fact that at the level of contact interactions there is an extra contact term inside the S-matrix of $\langle C^{-1}\phi^0 A^{-1} A^0\rangle$ that has been overlooked from the direct calculations of $\langle C^{-1}\phi^{-1} A^0 A^0\rangle$ S-matrix. Indeed the 2nd contact interaction $\mathcal{A}_1$ of $\langle C^{-1}\phi^0 A^{-1} A^0\rangle$ is extra term that is not only needed into the entire amplitude but also cannot be derived from direct computations of $\langle C^{-1}\phi^{-1} A^0 A^0\rangle$ on upper half plane.

Hence the following coupling is extra contact interaction to all orders in α', which must have been appeared in S-matrix because it stands correctly on the field theory side to all orders as well. Thus let us first write it down

$$4\pi^{1/2}\mu_p\xi_{1i}\xi_{2a}\xi_{3b}k_{3c}p^i\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{bca})(-t-s-u)L_1 \tag{5.1}$$

where we normalized the S-matrix by a coefficient of $(2\pi)^{1/2}\mu_p$.

Having taken the expansion of L_1 inside (5.1), we first produce the leading term of string amplitude by the EFT coupling, in fact it can be produced by mixing Chern-Simons coupling and Taylor expansion as follows

$$S_3 = \frac{i(2\pi\alpha')^3}{2}\mu_p\int d^{p+1}\sigma \quad \mathrm{Tr}\,(\partial_i C_{(p-3)}\wedge F\wedge F\Phi^i) \tag{5.2}$$

Therefore one can explore the next order to the above coupling, which is α'^4. Indeed all order α' corrections to the above coupling can be discovered by applying the proper higher derivative corrections on the above coupling and the coefficients can just be fixed by taking the elements in the expansion of L_1, so that all order corrections to above couplings are

$$\begin{aligned}(s+t+u)^{n+1}H\phi AA &= \left(\frac{\alpha'}{2}\right)^{n+1}H(D_aD^a)^{n+1}(\phi AA),\\ (st)^m H\phi AA &= (\alpha')^{2m}HD_{a_1}\cdots D_{a_{2m}}\Phi\partial^{a_1}\cdots\partial^{a_m}A\partial^{a_{m+1}}\cdots\partial^{a_{2m}}A,\\ (s+t)^n H\phi AA &= (\alpha')^n HD_{a_1}\cdots D_{a_n}\Phi\partial^{a_1}\cdots\partial^{a_n}(AA),\\ (s)^n t^m H\phi AA &= (\alpha')^{n+m}HD_{a_1}\cdots D_{a_n}D_{a_1}\cdots D_{a_m}\Phi\partial^{a_1}\cdots\partial^{a_m}A\partial^{a_1}\cdots\partial^{a_n}A.\end{aligned}$$

Note that in above couplings, inside the covariant derivative terms the connections or commutator terms do not appear and to get them, one needs to compute higher point amplitudes like $\langle C\phi AAA\rangle$.

Therefore, we argue that for higher point function of string theory amplitudes, involving the mixed RR, a scalar and two gauge fields, there is a subtle issue as follows.

Indeed to be able to get to all the corrected and all order contact interactions as well as singularities of the mixed string theory amplitudes, one should consider the scalar field in zero picture as it has just been clarified in detail by the comparisons of $\langle C^{-1}\phi^0 A^{-1} A^0\rangle$ with $\langle C^{-1}\phi^{-1} A^0 A^0\rangle$ S-matrix.

It would be nice to generalize this idea to see what happens for the mixed amplitudes of closed string RR, two scalar fields and one gauge field which we carry it out in the next section.

It would be even nicer if we could do it on asymmetric picture of RR ,that is, find out $\langle C^{-2}\phi^0 A^0 A^0\rangle$ to actually generalize the rules and symmetries of string theory, where we leave it for the future works, although partial results for simpler systems, like for brane anti brane have already been announced in [7]. Let us now generate all the other singularity structures of $\langle C^{-1}\phi^0 A^{-1} A^0\rangle$ in the effective field theory.

6 Other singularities of $\langle C^{-1}\phi^0 A^{-1} A^0\rangle$

Having produced some of the singularities and contact interactions, we are now ready to derive some other singularities of $\langle C^{-1}\phi^0 A^{-1} A^0\rangle$ S-matrix. These singularities do exist for this five point world-sheet S-matrix which includes a symmetric RR, a scalar field in the zero picture and two gauge fields. In fact by direct calculations we have shown that besides having some other contact interactions, even at pole levels the whole $\mathcal{A}_6$ and $\mathcal{A}_7$ of $\langle C^{-1}\phi^0 A^{-1} A^0\rangle$ are also extra singularities that cannot be derived from the direct computations of $\langle C^{-1}\phi^{-1} A^0 A^0\rangle$ S-matrix. Let us write them down as follows

$$\begin{aligned}\mathcal{A}_7 &\sim 2^{-1/2} p^i \xi_{1i} \mathrm{Tr}\,(P_- H\!\!\!/_{(n)} M_p \gamma^a)\xi_{2a}(2k_1.\xi_3)utL_2\\ \mathcal{A}_6 &\sim 2^{-1/2} stL_2 p^i \xi_{1i} \mathrm{Tr}\,(P_- H\!\!\!/_{(n)} M_p \gamma^b)\Big(2k_3.\xi_2\xi_{3b} - 2\xi_3.\xi_2 k_{3b} - 2\xi_3.k_2\xi_{2b}\Big)\end{aligned} \tag{6.1}$$

First we try to produce all these new s-channel poles of $\mathcal{A}_7$. By considering the desired expansion, one gets all new s-channel poles (note that normalization constant is $(2\pi)^{1/2}m_p$) of $\langle C^{-1}\phi^0 A^{-1} A^0\rangle$ S-matrix as

$$\frac{(2\pi\alpha')^2}{p!}\mu_p p.\xi_1 \xi_{2a} 2k_1.\xi_3 \epsilon^{a_0\cdots a_{p-1}a} H_{a_0\cdots a_{p-1}} \sum_{n=-1}^{\infty} \frac{1}{s} b_n (u+t)^{n+1} \mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3) \tag{6.2}$$

In order to produce all these new massless s-channel scalars, one has to apply the following field theory sub amplitude

$$\mathcal{A} = V^i_\alpha(C_{p-1}, A_2, \phi) G^{ij}_{\alpha\beta}(\phi) V^j_\beta(\phi, A_3, \phi_1) \tag{6.3}$$

where to follow the related vertices, the kinetic term of scalar fields $\frac{(2\pi\alpha')^2}{2} D^a\phi^i D_a\phi_i$ has to be taken into account, so that we obtain

$$\begin{aligned} V^j_\beta(\phi, A_3, \phi_1) &= -2ik_1.\xi_3 (2\pi\alpha')^2 T_p \xi_1^j \mathrm{Tr}\,(\lambda_3\lambda_1\lambda_\beta)\\ (G^\phi)^{ij}_{\alpha\beta} &= -\frac{i\delta^{ij}\delta^{\alpha\beta}}{(2\pi\alpha')^2 T_p s}\end{aligned} \tag{6.4}$$

Now one needs to re-consider the mixed Chern-Simons coupling and Taylor expended of scalar field, where the extremely important point has to be pointed out as follows. This turn, we take integration by parts and employ the momentum of external gauge field directly to RR (p-1) form potential to be able to produce the necessary field strength of RR whereas the total derivative terms are indeed zero at infinity, hence we find out the following effective action

$$S_4 = i(2\pi\alpha')^2 \mu_p \int d^{p+1}\sigma \frac{1}{(p-1)!} (\varepsilon^v)^{a_0\cdots a_p} \partial_i H_{a_0\cdots a_{p-1}} \mathrm{Tr}\,\left(A_{a_p}\phi^i\right) \tag{6.5}$$

Having set the above action, we obtain the following vertex in the effective field theory

$$V^i_\alpha(C_{p-1}, A_2, \phi) = p^i \frac{i(2\pi\alpha')^2\mu_p}{(p)!} (\varepsilon^v)^{a_0\cdots a_p} H_{a_0\cdots a_{p-1}} \xi_{2a_p} \mathrm{Tr}\,(\lambda_2\lambda_\alpha) \tag{6.6}$$

Note that $V^j_\beta(\phi, A_3, \phi_1)$ was derived from the kinetic term of the scalar field and it has no correction, that is why to produce all the singularities we need to propose all the higher derivative corrections to the new action of (6.5) as follows

$$S_5 = \sum_{n=-1}^{\infty} b_n(\alpha')^{(n+1)}\mu_p \int d^{p+1}\sigma \frac{1}{(p-1)!}(\varepsilon^v)^{a_0\cdots a_p} \\ \times\partial_i H_{a_0\cdots a_{p-1}} D_{a_1}\dots D_{a_{n+1}} A_{a_p} D^{a_1}\dots D^{a_{n+1}}\phi^i \tag{6.7}$$

now we are allowed to actually reveal all order vertex operator of $V^i_\alpha(C_{p-1}, A_2, \phi)$ as

$$V^i_\alpha(C_{p-1}, A_2, \phi) = p^i \frac{i(2\pi\alpha')^2\mu_p}{(p)!}(\varepsilon^v)^{a_0\cdots a_p} H_{a_0\cdots a_{p-1}}\xi_{2a_p}\mathrm{Tr}\,(\lambda_2\lambda_\alpha)\sum_{n=-1}^{\infty} b_n(t+u)^{n+1} \tag{6.8}$$

Replacing (6.8) and (6.4) to (6.3), we are then able to precisely regenerate all order new s-channel singularities (6.2) in the field theory side too. Finally let us reconstruct all new u-channel singularities.

Having replaced the desired expansion, we get all new u-channel poles (normalisation constant is $(2\pi)^{1/2}m_p$) of string amplitude as follows

$$\frac{(2\pi\alpha')^2}{p!}\mu_p p.\xi_1\epsilon^{a_0\cdots a_{p-1}b} H_{a_0\cdots a_{p-1}}\sum_{n=-1}^{\infty}\frac{1}{u}b_n(s+t)^{n+1}\Big(2k_3.\xi_2\xi_{3b} - 2\xi_3.\xi_2 k_{3b} - 2\xi_3.k_2\xi_{2b}\Big) \tag{6.9}$$

All these u-channel gauge poles are also produced by considering the following sub amplitude in the field theory

$$\mathcal{A} = V^a_\alpha(C_{p-1}, \phi_1, A) G^{ab}_{\alpha\beta}(A) V^b_\beta(A, A_2, A_3) \tag{6.10}$$

Here we consider the mixed Chern-Simons coupling and Taylor expended of scalar field, and not only this time we take integration by parts but also we do apply the momentum of external gauge field directly to RR potential to be able to produce the necessary field strength of RR, keeping in mind the above remarks, we obtain the following vertex

$$V^a_\alpha(C_{p-1}, \phi_1, A) = p^i \frac{i(2\pi\alpha')^2\mu_p}{(p)!}(\varepsilon^v)^{a_0\cdots a_{p-1}a} H_{a_0\cdots a_{p-1}}\xi_{1i}\mathrm{Tr}\,(\lambda_1\lambda_\alpha) \tag{6.11}$$

where $V^b_\beta(A, A_2, A_3)$ has no correction, so the only way of obtaining all the poles is to actually impose all infinite higher derivative corrections to the mixed Chern-Simons Taylor expansion of scalar field, so that now we can derive the generalization of above vertex to all orders as

$$V^a_\alpha(C_{p-1}, \phi_1, A) = p^i \frac{i(2\pi\alpha')^2\mu_p}{(p)!}(\varepsilon^v)^{a_0\cdots a_{p-1}a} H_{a_0\cdots a_{p-1}}\xi_{1i}\mathrm{Tr}\,(\lambda_1\lambda_\alpha)\sum_{n=-1}^{\infty} b_n(t+s)^{n+1} \tag{6.12}$$

Now by taking into account (6.12), the known $V^b_\beta(A, A_2, A_3)$ and gauge field propagator $G^{ab}_{\alpha\beta}(A) = \frac{i\delta_{\alpha\beta}\delta^{ab}}{(2\pi\alpha')^2 T_p u}$ inside the sub amplitude (6.10) we are then able to precisely reconstruct all order new u-channel singularities in the effective field theory side as well.

In the next section we further generalize our knowledge by dealing with the mixed RR scalars/ gauge field S-matrices to see what happens to the S-matrix in the presence of two scalar fields (in different pictures), a gauge field and a symmetric RR field strength.

6.1 All order S-matrix of $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$

In this section we would like to see what is going on for the mixed higher point function of a symmetric RR, two transverse scalar fields (in two different pictures) and a gauge field. We do the whole details to get to the entire S-matrix to all orders in α' so the $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ S-matrix is shown by

$$\mathcal{A}^{\langle C^{-1}A^0\phi^{-1}\phi^0\rangle} \sim \int dx_1dx_2dx_3dzd\bar{z}\,\langle V_A^{(0)}(x_1)V_\phi^{(-1)}(x_2)V_\phi^{(0)}(x_3)V_{RR}^{(-\frac{1}{2},-\frac{1}{2})}(z,\bar{z})\rangle, \quad (6.13)$$

Further simplification can be done to get to the closed form of S-matrix as follows

$$\mathcal{A}^{\langle C^{-1}A^0\phi^{-1}\phi^0\rangle} \sim \int dx_1dx_2dx_3dx_4dx_5\,(P_-H\!\!\!/_{(n)}M_p)^{\alpha\beta}\xi_{1a}\xi_{2i}\xi_{3j}x_{45}^{-1/4}(x_{24}x_{25})^{-1/2}$$
$$\times(I_1+I_2+I_3+I_4)\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3), \quad (6.14)$$

where $x_{ij}=x_i-x_j$, $x_4=z$,$x_5=\bar{z}$, and also

$$\begin{aligned}
I_1 &= \langle :\partial X^a(x_1)e^{\alpha' ik_1.X(x_1)}:e^{\alpha' ik_2.X(x_2)}:\partial X^j(x_3)e^{\alpha' ik_3.X(x_3)}:e^{i\frac{\alpha'}{2}p.X(x_4)}:e^{i\frac{\alpha'}{2}p.D.X(x_5)}:\rangle\\
&\quad\times\langle :S_\alpha(x_4):S_\beta(x_5):\psi^i(x_2):\rangle,\\
I_2 &= \langle :\partial X^a(x_1)e^{\alpha' ik_1.X(x_1)}:e^{\alpha' ik_2.X(x_2)}:e^{\alpha' ik_3.X(x_3)}:e^{i\frac{\alpha'}{2}p.X(x_4)}:e^{i\frac{\alpha'}{2}p.D.X(x_5)}:\rangle\\
&\quad\times\langle :S_\alpha(x_4):S_\beta(x_5)::\psi^i(x_2):\alpha' ik_{3c}\psi^c\psi^j(x_3)\rangle,\\
I_3 &= \langle :e^{\alpha' ik_1.X(x_1)}:e^{\alpha' ik_2.X(x_2)}:\partial X^j(x_3)e^{\alpha' ik_3.X(x_3)}:e^{i\frac{\alpha'}{2}p.X(x_4)}:e^{i\frac{\alpha'}{2}p.D.X(x_5)}:\rangle\\
&\quad\times\langle :S_\alpha(x_4):S_\beta(x_5):\alpha' ik_{1b}\psi^b\psi^a(x_1):\psi^i(x_2):\rangle,\\
I_4 &= \langle :e^{\alpha' ik_1.X(x_1)}:e^{\alpha' ik_2.X(x_2)}:e^{\alpha' ik_3.X(x_3)}:e^{i\frac{\alpha'}{2}p.X(x_4)}:e^{i\frac{\alpha'}{2}p.D.X(x_5)}:\rangle\\
&\quad\times\langle :S_\alpha(x_4):S_\beta(x_5):\alpha' ik_{1b}\psi^b\psi^a(x_1):\psi^i(x_2):\alpha' ik_{3c}\psi^c\psi^j(x_3):\rangle. \quad (6.15)
\end{aligned}$$

If we work with all possible contractions, then one finds out the compact form of the following fermionic correlation function as follows

$$\begin{aligned}
I_6^{jciab} &= \langle :S_\alpha(x_4):S_\beta(x_5):\psi^b\psi^a(x_1)::\psi^i(x_2):\psi^c\psi^j(x_3)\rangle\\
&= \bigg\{(\Gamma^{jciab}C^{-1})_{\alpha\beta}+\alpha' r_1\frac{\mathrm{Re}[x_{14}x_{35}]}{x_{13}x_{45}}+\alpha' r_2\frac{\mathrm{Re}[x_{24}x_{35}]}{x_{23}x_{45}} \qquad (6.16)\\
&\quad+\alpha'^2 r_3\left(\frac{\mathrm{Re}[x_{14}x_{35}]}{x_{13}x_{45}}\right)\left(\frac{\mathrm{Re}[x_{24}x_{35}]}{x_{23}x_{45}}\right)\bigg\}2^{-5/2}x_{45}^{5/4}(x_{14}x_{15}x_{34}x_{35})^{-1}(x_{24}x_{25})^{-1/2},
\end{aligned}$$

where

$$\begin{aligned}
r_1 &= \Big(-\eta^{bc}(\Gamma^{jia}C^{-1})_{\alpha\beta}+\eta^{ac}(\Gamma^{jib}C^{-1})_{\alpha\beta}\Big),\\
r_2 &= \Big(\eta^{ij}(\Gamma^{cab}C^{-1})_{\alpha\beta}\Big),\\
r_3 &= \Big(\eta^{bc}\eta^{ij}(\gamma^a C^{-1})_{\alpha\beta}-\eta^{ac}\eta^{ij}(\gamma^b C^{-1})_{\alpha\beta}\Big) \qquad (6.17)
\end{aligned}$$

Substituting the closed form of the correlators into the amplitude we now claim the final answer for the S-matrix can be written down by

$$\mathcal{A}^{\langle C^{-1}A^0\phi^{-1}\phi^0\rangle}\sim\int dx_1dx_2dx_3dx_4dx_5(P_-H\!\!\!/_{(n)}M_p)^{\alpha\beta}I\xi_{1a}\xi_{2i}\xi_{3j}x_{45}^{-1/4}(x_{24}x_{25})^{-1/2} \quad (6.18)$$
$$\times\Big(I_7^i(a_1^a a_3^j)+a_1^a a_2^{ji}+a_3^j a_4^{ia}-\alpha'^2 k_{1b}k_{3c}I_6^{jciab}\Big)\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3),$$

where

$$\begin{aligned}
I &= |x_{12}|^{\alpha'^2 k_1.k_2}|x_{13}|^{\alpha'^2 k_1.k_3}|x_{14}x_{15}|^{\frac{\alpha'^2}{2}k_1.p}|x_{23}|^{\alpha'^2 k_2.k_3}\\
&\qquad\qquad\qquad \times|x_{24}x_{25}|^{\frac{\alpha'^2}{2}k_2.p}|x_{34}x_{35}|^{\frac{\alpha'^2}{2}k_3.p}|x_{45}|^{\frac{\alpha'^2}{4}p.D.p},\\
a_1^a &= ik_2^a\left(\frac{x_{42}}{x_{14}x_{12}}+\frac{x_{52}}{x_{15}x_{12}}\right)+ik_3^a\left(\frac{x_{43}}{x_{14}x_{13}}+\frac{x_{53}}{x_{15}x_{13}}\right)\\
a_3^j &= ip^j\left(\frac{x_{54}}{x_{34}x_{35}}\right)\\
a_2^{ji} &= \left\{(\Gamma^{jci}C^{-1})_{\alpha\beta}+(\alpha'\eta^{ij}(\gamma^c C^{-1})_{\alpha\beta})\frac{\mathrm{Re}[x_{24}x_{35}]}{x_{23}x_{45}}\right\}\\
&\qquad\qquad\qquad \times\alpha' ik_{3c}2^{-3/2}x_{45}^{1/4}(x_{34}x_{35})^{-1}(x_{24}x_{25})^{-1/2}\\
a_4^{ia} &= \alpha' ik_{1b}2^{-3/2}x_{45}^{1/4}(x_{24}x_{25})^{-1/2}(x_{14}x_{15})^{-1}\Big\{(\Gamma^{iab}C^{-1})_{\alpha\beta}\Big\},\\
I_7^i &= \langle :S_\alpha(x_4):S_\beta(x_5):\psi^i(x_2):\rangle = 2^{-1/2}x_{45}^{-3/4}(x_{24}x_{25})^{-1/2}(\gamma^i C^{-1})_{\alpha\beta}.
\end{aligned}$$

It now becomes clear that the S-matrix of (6.18) is SL(2,R) invariant and after gauge fixing over the position of open strings one needs to come over the integrals on upper half complex plane on the location of RR. By evaluating those integrals one eventually writes down the complete form of the S-matrix to all orders as follows

$$\mathcal{A}^{\langle C^{-1}A^0\phi^{-1}\phi^0\rangle} = \mathcal{A}_1+\mathcal{A}_2+\mathcal{A}_3+\mathcal{A}_4+\mathcal{A}_5+\mathcal{A}_6 \tag{6.19}$$

where

$$\begin{aligned}
\mathcal{A}_1 &\sim 2^{-1/2}\xi_{1a}\xi_{2i}\xi_{3j}p^j\,\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^i)\Big[-2k_3^a(ut)+2k_2^a(us)\Big]L_2\\
\mathcal{A}_2 &\sim 2^{-1/2}k_{3c}\Big\{-2k_2.\xi_1\xi_{2i}\xi_{3j}(us)L_2\,\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{jci})\\
&\qquad +2k_3.\xi_1\xi_{2i}\xi_{3j}(ut)L_2\,\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{jci})\\
&\qquad +4t\xi_2.\xi_3k_3.\xi_1L_1\,\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^c)-4s\xi_2.\xi_3k_2.\xi_1L_1\,\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^c)\Big\}\\
\mathcal{A}_3 &\sim 2^{-1/2}k_{1b}\xi_{1a}\xi_{2i}\xi_{3j}4(-u-s-t)L_1\Big(\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{iab})p^j-k_{3c}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{jciab})\Big)\\
\mathcal{A}_4 &\sim 2^{-1/2}(ut)L_2\Big\{-s\xi_{1a}\xi_{2i}\xi_{3j}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{jia})\\
&\qquad -2k_3.\xi_1k_{1b}\xi_{2i}\xi_{3j}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{jib})\Big\}\\
\mathcal{A}_5 &\sim 2^{1/2}(st)L_2\xi_2.\xi_3\xi_{1a}k_{1b}k_{3c}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{cab})\\
\mathcal{A}_6 &\sim 2^{1/2}\xi_3.\xi_2\Big(ts\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^a)\xi_{1a}+2tk_3.\xi_1\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^b)k_{1b}\Big)L_1
\end{aligned} \tag{6.20}$$

where the functions L_1, L_2 are given in (2.13).

On the other hand if we actually consider both scalar fields in zero picture in the presence of a symmetric RR, then we get the whole S-matrix as

$$\mathcal{A}^{\langle C^{-1}A^{-1}\phi^0\phi^0\rangle} \sim \int dx_1dx_2dx_3dzd\bar{z}\,\langle V_A^{(-1)}(x_1)V_\phi^{(0)}(x_2)V_\phi^{(0)}(x_3)V_{RR}^{(-\frac{1}{2},-\frac{1}{2})}(z,\bar{z})\rangle,$$

Having done all integrals, one could find the final answer (for further details, look at [9]) for the entire S-matrix of a symmetric RR with both transverse scalars in zero picture and a gauge field as follows

$$\mathcal{A}^{\langle C^{-1}A^{-1}\phi^0\phi^0\rangle} = \mathcal{A}_1+\mathcal{A}_2+\mathcal{A}_3+\mathcal{A}_4+\mathcal{A}_5+\mathcal{A}_6+\mathcal{A}_7+\mathcal{A}_8+\mathcal{A}_9+\mathcal{A}_{10} \tag{6.21}$$

where

$$\begin{aligned}
\mathcal{A}_1 &\sim -2^{-1/2}\xi_{1a}\xi_{2i}\xi_{3j}\Big[k_{3c}k_{2b}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{jciba}) - k_{2b}p^j\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{iba})\\
&\qquad -k_{3c}p^i\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{jca}) + p^ip^j\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^a)\Big]4(-s-t-u)L_1,\\
\mathcal{A}_2 &\sim 2^{-1/2}\Big\{-2\xi_1.k_2k_{3c}\xi_{3j}\xi_{2i}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{jci})\Big\}(us)L_2\\
\mathcal{A}_3 &\sim 2^{-1/2}\Big\{\xi_{1a}\xi_{2i}\xi_{3j}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{jia})\Big\}(-ust)L_2\\
\mathcal{A}_4 &\sim 2^{-1/2}\Big\{2k_3.\xi_1k_{2b}\xi_{3j}\xi_{2i}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{jib})\Big\}(ut)L_2\\
\mathcal{A}_5 &\sim 2^{-1/2}\Big\{2\xi_3.\xi_2k_{2b}k_{3c}\xi_{1a}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{cba})\Big\}(st)L_2\\
\mathcal{A}_6 &\sim 2^{1/2}(us)L_2\Big\{p^j\xi_1.k_2\xi_{2i}\xi_{3j}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^i)\Big\}\\
\mathcal{A}_7 &\sim -2^{-1/2}(ut)L_2\Big\{2k_3.\xi_1p^i\xi_{3j}\xi_{2i}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^j)\Big\}\\
\mathcal{A}_8 &\sim 2^{1/2}L_1\Big\{2k_2.\xi_1k_{3c}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^c)(-s\xi_2.\xi_3)\Big\}\\
\mathcal{A}_9 &\sim 2^{1/2}L_1\Big\{2k_3.\xi_1k_{2b}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^b)(-t\xi_2.\xi_3)\Big\}\\
\mathcal{A}_{10} &\sim 2^{1/2}L_1\Big\{\xi_{1a}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^a)(ts\xi_3.\xi_2)\Big\}
\end{aligned} \tag{6.22}$$

where the functions L_1, L_2 are already appeared in (2.13).

It is worth highlighting the point that, this S-matrix also satisfies Ward identity, that is, by substituting $\xi_{1a}\to k_{1a}$, the entire amplitude vanishes and the amplitude holds for various p,n cases. Let us do the comparisons $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ with $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ S-matrix at both level of singularity structures and contact interactions, find out various new couplings and in particular find out their corrections and eventually get to the conclusion.

7 Comparison on singularity structure of $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ with $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$

In this section we are going to compare all the singularities of $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ with $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ S-matrix. The first term $\mathcal{A}_6$ of $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ is exactly equivalent to $\mathcal{A}_{10}$ of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$, likewise the last term $\mathcal{A}_2$ of $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ is the same as $\mathcal{A}_8$ of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ S-matrix.

Now if we add the second term $\mathcal{A}_6$ of $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ with the third term $\mathcal{A}_2$ of $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ and make use of momentum conservation along the world volume of branes, we obtain

$$2^{1/2}L_1(2tk_3.\xi_1)\xi_2.\xi_3\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^b)(-k_{2b}-p_b)$$

Now by applying the following equation $p_b\epsilon^{a_0\cdots a_{p-1}b}=0$, we then realize the fact that the first term in above equation precisely produces the $\mathcal{A}_9$ term of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$.

Meanwhile $\mathcal{A}_5$ of $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ can be written down as

$$2^{1/2}(st)L_2\xi_{1a}\xi_2.\xi_3k_{3c}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{cab})(-k_{3b}-k_{2b}-p_b)$$

where the first term has no contribution to S-matrix. Because of the antisymmetric property of ϵ and the fact that it is symmetric with respect to k_3 so the result for the first term is zero. More evidently the third term in above equation has no contribution because $p_b\epsilon^{a_0\ldots a_{p-3}cab}=0$ and the second term precisely produces $\mathcal{A}_5$ of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$.

The same so happens to the other terms, namely if we add the 2nd terms of $\mathcal{A}_2$ and $\mathcal{A}_4$ of $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ and apply the momentum conservation, then we are able to precisely produce $\mathcal{A}_4$ of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ which is related to all s-channel poles.

Indeed without any further details the first term $\mathcal{A}_2$ of $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ is exactly $\mathcal{A}_2$ term of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ so that all t-channel poles are then reproduced in both pictures. By considering the 2nd term $\mathcal{A}_1$ of $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ we are then able to generate $\mathcal{A}_6$ term of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$.

Finally to be able to produce all the second kind of s-channel poles one has to subtract the first term of $\mathcal{A}_1$ of $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ from $\mathcal{A}_7$ term of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ such that upon considering the following identity

$$\xi_2\xi_{3j}\epsilon^{a_0\ldots a_p}(-p^jH^i_{a_0\ldots a_p}+p^iH^j_{a_0\ldots a_p})=0$$

we believe that the first term of $\mathcal{A}_1$ of $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ is exactly the same $\mathcal{A}_7$ term of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$.

Henceforth, we could precisely produce all the singularities of this five point function in two different pictures. However, note that we have some extra contact interactions in $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ amplitude while they are absent in $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ S-matrix. These extra contact interactions are needed by symmetries of string theory amplitudes as we point out/hint them in a moment.

For the completeness we first would like to produce all the singularities. This amplitude has u-channel gauge poles that can be read off from the string amplitude as follows

$$\mu_p(2\pi\alpha')^2 2k_{2a}k_{3a_{p-1}}\xi_2.\xi_3\frac{1}{(p-2)!u}\epsilon^{a_0\cdots a_{p-1}a}H_{a_0\cdots a_{p-3}}\xi_{1a_{p-2}}\sum_{n=-1}^{\infty}b_n\left(\frac{\alpha'}{2}\right)^{n+1}(s+t)^{n+1} \tag{7.1}$$

where these u-channel poles should be produced by the following sub amplitude in the effective field theory

$$\mathcal{A}=V^a_\alpha(C_{p-3},A_1,A)G^{ab}_{\alpha\beta}(A)V^b_\beta(A,\phi_2,\phi_3), \tag{7.2}$$

Considering the kinetic terms of scalars $iT_p\frac{(2\pi\alpha')^2}{2}\mathrm{Tr}\,(D^a\phi^iD_a\phi_i)$ and gauge fields we obtain the following vertices

$$\begin{aligned}V^b_\beta(A,\phi_2,\phi_3)&=i\lambda^2T_p\xi_2.\xi_3(k_2-k_3)^b\mathrm{Tr}\,(\lambda_2\lambda_3\lambda_\beta)\\ G^{ab}_{\alpha\beta}(A)&=\frac{-i}{\lambda^2T_p}\frac{\delta^{ab}\delta_{\alpha\beta}}{k^2}\quad,\end{aligned} \tag{7.3}$$

The kinetic terms have no corrections so we need to apply all higher derivative corrections to Chern-Simons couplings as follows

$$S_6 = i(2\pi\alpha')^2\mu_p \int d^{p+1}\sigma \quad \sum_{n=-1}^{\infty} b_n(\alpha')^{n+1} \quad C_{(p-3)} \wedge D_{a_0\cdots a_n}F \wedge D^{a_0\cdots a_n}F \tag{7.4}$$

Now if one considers S_6, then one is able to obtain the following vertex operator to all orders in α' as follows

$$\begin{aligned} V^a_\alpha(C_{p-3}, A_1, A) &= \frac{\lambda^2\mu_p}{(p-2)!}(\epsilon)^{a_0\cdots a_{p-1}a}(H^{(p-2)})_{a_0\cdots a_{p-3}}\xi_{1a_{p-2}}k_{a_{p-1}} \\ &\qquad \times \mathrm{Tr}\,(\lambda_1\lambda_\alpha)\sum_{n=-1}^{\infty} b_n(\alpha' k_1.k)^{n+1} \end{aligned} \tag{7.5}$$

Replacing above vertices (7.5) and (7.3) into (7.2), we are then able to exactly produce all u-channel gauge poles in the field theory side.

On the other hand, if we employ all order α' SYM couplings as appeared in (3.15), and also apply a following sub amplitude of field theory

$$\mathcal{A} = V^a_\alpha(C_{p-1}, A)G^{ab}_{\alpha\beta}(A)V^b_\beta(A, A_1, \phi_2, \phi_3)$$

then we will be able to produce all $(t+s+u)$ gauge field poles. Note that this task has been completely done in section four of [9] and in order to avoid rewriting the old contents of the paper, we refer the interested reader to that section four of [9].

Let us reconstruct all t-channel poles and finally by interchanging $1 \leftrightarrow 2$ for all the momenta, the polarisations and t to s, we are able to produce all s-channel poles as well. All the t-channel poles of the string amplitude are given by

$$\frac{16\xi_{2i}\xi_{3j}k_2.\xi_1\pi^2\mu_p}{t(p+1)!}\left\{2p^j\epsilon^{a_0\cdots a_p}H^i_{a_0\cdots a_p} - 2(p+1)k_{3a}\epsilon^{a_0\cdots a_{p-1}a}H^{ij}_{a_0\cdots a_{p-1}}\right\}\sum_{n=-1}^{\infty} b_n(\alpha' k_3.k)^n \tag{7.6}$$

These t-channel poles can be regenerated in the field theory side, and to do so one needs to take into account the following sub amplitude and vertices in the field theory as

$$\begin{aligned} \mathcal{A} &= V^i_\alpha(C_{p+1}, \phi_3, \phi)G^{ij}_{\alpha\beta}(\phi)V^j_\beta(\phi, A_1, \phi_2) \\ V^j_\beta(\phi, A_1, \phi_2) &= -2i(2\pi\alpha')^2T_pk_2.\xi_1\xi^j_2\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_\beta) \\ G^{ij}_{\alpha\beta}(\phi) &= \frac{-i}{(2\pi\alpha')^2T_p}\frac{\delta^{ij}\delta_{\alpha\beta}}{t} \end{aligned} \tag{7.7}$$

Consider the Taylor expansion of the two scalar fields as

$$S_7 = \frac{(2\pi\alpha')^2\mu_p}{2}\int d^{p+1}\sigma\frac{1}{(p+1)!}\epsilon^{a_0\cdots a_p}\mathrm{Tr}\,\left(\Phi^j\Phi^i\right)\partial_j\partial_i C^{(p+1)}_{a_0\cdots a_p}$$

and then work out with pull-back and both mixing term involving Taylor and pull-back as follows

$$S_8 = \frac{(2\pi\alpha')^2\mu_p}{2}\int d^{p+1}\sigma\frac{1}{(p+1)!}\epsilon^{a_0\cdots a_p}\bigg[p(p+1)\,\mathrm{Tr}\,\left(D_{a_0}\Phi^i\,D_{a_1}\Phi^j\right)C^{(p+1)}_{ija_2\cdots a_p}$$

$$+2(p+1)\mathrm{Tr}\,\left(\Phi^j D_{a_0}\Phi^i\right)\partial_j C^{(p+1)}_{ia_1\cdots a_p}\bigg] \quad (7.8)$$

where one needs to also add the following Myers terms

$$S_9 = \frac{i}{4}(2\pi\alpha')^2\mu_p\int d^{p+1}\sigma\frac{1}{(p-1)!}\epsilon^{a_0\cdots a_p}\,\mathrm{Tr}\,\left(F_{a_0a_1}[\Phi^j,\Phi^i]\right)C^{(p+1)}_{ija_2\cdots a_p}. \quad (7.9)$$

with S_8 and take all the integrations by parts to actually get to the following action

$$S_{10} = \frac{(2\pi\alpha')^2}{2}\mu_p\int d^{p+1}\sigma\frac{1}{(p+1)!}\epsilon^{a_0\cdots a_p}\left[(p+1)\mathrm{Tr}\,\left(D_{a_0}\Phi^j\Phi^i\right)H^{(p+2)}_{ija_1\cdots a_p}\right]$$

Eventually in order to produce the first t-channel pole, one must consider the summation of the Taylor expansion and S_{10} as follows

$$\frac{\mu_p(2\pi\alpha')^2}{2(p+1)!}\int d^{p+1}\sigma\epsilon^{a_0\cdots a_p}\left[\mathrm{Tr}\left(\Phi^j\Phi^i\right)\partial_j H^{(p+2)}_{ia_0\cdots a_p}+(p+1)\mathrm{Tr}\,\left(D_{a_0}\Phi^j\Phi^i\right)H^{(p+2)}_{ija_1\cdots a_p}\right] \quad (7.10)$$

From (7.10) we now look for the vertex of $V^i_\alpha(C_{p+1},\phi_3,\phi)$ as follows

$$V^i_\alpha(C_{p+1},\phi_3,\phi) = \frac{\mu_p(2\pi\alpha')^2}{(p+1)!}\mathrm{Tr}\,(\lambda_3\lambda_\alpha)\epsilon^{a_0\cdots a_p}\left[p^j\xi_{3j}H^i_{a_0\cdots a_p}+(p+1)H^{ij}_{a_1\cdots a_p}k_{3a_0}\xi_{3j}\right] \quad (7.11)$$

However, to produce all the other t-channel poles, one needs to apply all order higher derivative corrections to (7.10) as below

$$\frac{\mu_p(2\pi\alpha')^2}{2(p+1)!}\int d^{p+1}\sigma\epsilon^{a_0\cdots a_p}\sum_{n=-1}^{\infty}b_n(\alpha')^n\bigg[\mathrm{Tr}\left(D_{a_1\ldots a_n}\Phi^j D^{a_1\ldots a_n}\Phi^i\right)\partial_j H^{(p+2)}_{ia_0\cdots a_p}$$

$$+(p+1)\mathrm{Tr}\,\left(D_{a_0}D_{a_1\ldots a_n}\Phi^j D^{a_1\ldots a_n}\Phi^i\right)H^{(p+2)}_{ija_1\cdots a_p}\bigg] \quad (7.12)$$

to indeed obtain the following vertex to all orders in α' as follows

$$V^i_\alpha(C_{p+1},\phi_3,\phi) = \frac{\mu_p(2\pi\alpha')^2}{(p+1)!}\mathrm{Tr}\,(\lambda_3\lambda_\alpha)\epsilon^{a_0\cdots a_p}\sum_{n=-1}^{\infty}b_n(\alpha' k_3.k)^n$$

$$\times\left[p^j\xi_{3j}H^i_{a_0\cdots a_p}+(p+1)H^{ij}_{a_1\cdots a_p}k_{3a_0}\xi_{3j}\right] \quad (7.13)$$

Now if we replace (7.13) inside (7.7) then we are exactly able to regenerate all order t-channel singularities in the field theory side as well.

Note that all of the new couplings that we have discovered, can just be derived with scattering computations not by any duality transformation. Because the coefficients of these couplings can just be fixed without any ambiguity by S-matrix analysis. We now turn to contact interaction terms.

8 Comparison on contact interactions

If we look at the precise computations of the S-matrices in two different pictures, we then realize the fact that the first term $\mathcal{A}_4$ of $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ is exactly the term that has been shown up in $\mathcal{A}_3$ of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$.

As we can readily observe, we have just left with two contact terms in $\mathcal{A}_3$ of $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ while in $\mathcal{A}_1$ of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ we do have four different terms, so let us keep comparing.

Now if we apply the momentum conservation to the 2nd term $\mathcal{A}_3$ of $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ and apply the Bianchi equation that we have already got, that is, $p_b\epsilon^{a_0\cdots a_{p-3}cba} = 0$ then we are able to precisely produce the first term $\mathcal{A}_1$ of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$.

Eventually we apply momentum conservation to the only remaining term of $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ which is its first $\mathcal{A}_3$ term and do subtract it from the second and third terms $\mathcal{A}_1$ of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ such that upon holding the following equation, we are able to generate the second and third term $\mathcal{A}_1$ of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$.

$$\xi_{2i}\xi_{3j}\xi_{1a}k_{3b}\epsilon^{a_0\cdots a_{p-2}ab}(p^jH^i_{a_0\cdots a_{p-2}} - p^iH^j_{a_0\cdots a_{p-2}}) = 0$$

Once more $p_b\epsilon^{a_0\cdots a_{p-2}ab} = 0$, whereas up to a sign the third term $\mathcal{A}_1$ of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ is also produced.

However, note to the important point that there is no chance to actually produce even the leading order α' of the fourth contact interaction $\mathcal{A}_1$ of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$. The reason is that, there is no left over term inside $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ S-matrix to be compared with that fourth term $\mathcal{A}_1$ of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ S-matrix. Therefore, let us further elaborate on the needed contact interactions of this string amplitude.

9 The needed contact interaction for $\langle C^{-1}\phi^{-1}\phi^0A^0\rangle$

As we have seen above, we were able to produce all the first three contact terms $\mathcal{A}_1$ of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ to all orders, however, we have evidently observed that indeed there is no chance to produce the fourth term contact interaction $\mathcal{A}_1$ of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ by direct computations of $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$.

In fact we claim that this extra contact interaction must be appeared in the entire S-matrix as it plays the crucial role in all order α' contact interaction terms in both type IIA and IIB super string theory. Let us first write it down and then we try to construct its all order α' higher derivative couplings.

Hence we figure out the following term inside $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ S-matrix

$$-4\pi^{1/2}\mu_p\xi_{1a}\xi_{2i}\xi_{3j}p^ip^j\mathrm{Tr}\,(P_-H\!\!\!/_{(n)}M_p\gamma^a)(-t-s-u)L_1 \tag{9.1}$$

is indeed needed. We normalized the S-matrix by a coefficient of $(2\pi)^{1/2}\mu_p$ and considered the expansion of L_1 (with the aforementioned coefficients) given in (2.14). Thus the first leading term of L_1 can be produced by Chern-Simons coupling and Taylor expanded of both scalar fields through closed string RR as follows

$$S_{11} = \frac{i(2\pi\alpha')^3}{2}\mu_p\int d^{p+1}\sigma \quad \mathrm{Tr}\,(\partial_j\partial_iC_{(p-1)}\wedge F\Phi^i\Phi^j) \tag{9.2}$$

Therefore one explores the next order term which is α'^4 and indeed all order α' corrections to the above coupling with exact coefficients can be discovered by applying the proper higher derivative corrections. For example the $(st)^m HA\phi\phi$ and $(s+t)^n HA\phi\phi$ contact terms of the S-matrix (inside the expansion of L_1) can be shown to be matched to all orders by the following couplings

$$\begin{aligned}(s+t)^n HA\Phi\Phi &= (\alpha')^n H\partial_{a_1}\cdots\partial_{a_n} AD^{a_1}\cdots D^{a_n}(\Phi\Phi),\\ (st)^m HA\Phi\Phi &= (\alpha')^{2m} H\partial_{a_1}\cdots\partial_{a_{2m}} AD^{a_1}\cdots D^{a_m}\Phi D^{a_{m+1}}\cdots D^{a_{2m}}\Phi\end{aligned}$$

Note that the first correction to the above coupling (9.2) and the other new coupling in (5.2) is of α'^4 order.

It is also worth keeping in mind the fact that by expanding the string amplitude of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$, we could also explore new couplings at leading order as follows.

Let us write down the explicit form of the string amplitude, indeed if we extract the related trace, consider the expansion of stL_2 inside $\mathcal{A}_5$ of $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ S-matrix, we then obtain the following elements of string amplitude

$$\begin{aligned}&-2\xi_3.\xi_2 k_{2b}k_{3c}\xi_{1a}\pi^2\mu_p\frac{16}{(p-2)!}\epsilon^{a_0\cdots a_{p-3}cba}H_{a_0\cdots a_{p-3}}\\ &\qquad\times\left(\sum_{n=-1}^{\infty} b_n\left(\frac{1}{u}(t+s)^{n+1}\right)+\sum_{p,n,m=0}^{\infty} e_{p,n,m}u^p(st)^n(s+t)^m\right)\end{aligned}\tag{9.3}$$

where we have already produced all the u-channel poles, now to obtain the new couplings, we need to focus on the second term in (9.3) as

$$-2\xi_3.\xi_2 k_{2b}k_{3c}\xi_{1a}\pi^2\mu_p\frac{16}{(p-2)!}\epsilon^{a_0\cdots a_{p-3}cba}H_{a_0\cdots a_{p-3}}\sum_{p,n,m=0}^{\infty} e_{p,n,m}u^p(st)^n(s+t)^m\tag{9.4}$$

where (9.4) satisfies the Ward identity associated to the gauge field, which means that by replacing ξ_{1a} to k_{1a}, apply the momentum conservation and taking the following identity for RR

$$p^a\epsilon^{a_0\cdots a_{p-3}cba}=0$$

the amplitude vanishes. Thus we understand that (9.4) has to be reconstructed by new coupling and the structure of this new coupling is shown by

$$\int_{\sum_{p+1}} d^{p+1}\sigma\quad \mathrm{Tr}\left(C_{p-3}\wedge F\wedge D\phi^i\wedge D\phi_i\right)\tag{9.5}$$

Note that (9.5) is considered by the fact that it has to cover up the whole world volume space and more crucially it has to be antisymmetric with respect to interchanging the momenta of both scalar fields. We now apply $e_{1,0,0}=\frac{\pi^2}{6}$ and $e_{0,0,1}=\frac{\pi^2}{3}$ to (9.4) to be able to start constructing new couplings at order of α'^3.

Indeed if we replace $e_{1,0,0} = \frac{\pi^2}{6}$ to (9.4) and consider the above remarks, then one can show that, this term of S-matrix can be generated by the following new coupling as follows

$$S_{12} = \frac{(2\pi\alpha')^3\mu_p\pi}{12} \int d^{p+1}\sigma \frac{1}{(p-3)!}(\varepsilon^v)^{a_0\cdots a_p}\left(\frac{\alpha'}{2}\right) \times C^{(p-3)}_{a_0\cdots a_{p-4}} \mathrm{Tr}\left(F_{a_{p-3}a_{p-2}}(D^a D_a)\Big[D_{a_{p-1}}\phi^i D_{a_p}\phi_i\Big]\right) \tag{9.6}$$

Notice that, if we do the same for $e_{0,0,1} = \frac{\pi^2}{3}$, namely if we replace $e_{0,0,1} = \frac{\pi^2}{3}$ into (9.4) then one gets to know that, this particular term of S-matrix can be obtained by the following new coupling

$$S_{13} = \frac{(2\pi\alpha')^3\mu_p\pi}{6} \int d^{p+1}\sigma\,(\alpha')\,\mathrm{Tr}\left(C_{p-3}\wedge D^{b_1}F\wedge D_{b_1}\Big[D\phi^i\wedge D\phi_i\Big]\right) \tag{9.7}$$

where these couplings are of α'^3 order.

Hence the above couplings (9.2), more crucially (9.6) and (9.7) are needed in order to consider the symmetries of the S-matrix with respect to interchanging of the scalar fields. We can also investigate the closed form of the corrections to all orders in α'. So to produce the whole (9.4), one applies the proper higher derivative corrections to (9.5) so that the closed form of the string corrections can be found as follows

$$\begin{aligned} S_{14} = \frac{\lambda^3\mu_p}{2\pi}\int d^{p+1}\sigma \sum_{p,n,m=0}^{\infty} e_{p,n,m}\,(\alpha')^{2n+m}\left(\frac{\alpha'}{2}\right)^p \\ \times\mathrm{Tr}\Big(C_{p-3}\wedge D^{b_1}\cdots D^{b_m}D^{a_1}\cdots D^{a_{2n}}F\wedge \\ (D^aD_a)^p D_{b_1}\cdots D_{b_m}\Big[D_{a_1}\cdots D_{a_n}D\phi^i\wedge D_{a_{n+1}}\cdots D_{a_{2n}}D\phi_i\Big]\Big) \end{aligned} \tag{9.8}$$

Note that these new couplings of (9.6), (9.7) and (9.8) can not be derived by the standard effective field theory ways of Taylor, Myers terms nor by pull-back formalism. Indeed not only the structure of the above new couplings but also their coefficients can just be explored by this S-matrix analysis.

Note that there is no Ward identity for the amplitudes of scalar fields in the presence of RR, thus we argue that for two scalars and a gauge field in the presence of RR, there is a subtle issue. Indeed to be able to get to the corrected all order contact interactions of higher point functions of string theory amplitudes, one needs to consider both scalar fields in zero picture as we have clarified in detail in the above S-matrix.

It would be nice to generalize this conjecture to even number of scalars in the presence of a closed string RR or even it would be nicer to check it for the non-BPS amplitudes where the first non-trivial amplitude to be carried out is $\langle C^{-1}T^0\phi^{-1}\phi^0\rangle$ to be compared with $\langle C^{-1}T^{-1}\phi^0\phi^0\rangle$ S-matrix. It would be even more significant if we could carry out these S-matrices on asymmetric picture of RR ($\langle C^{-2}T^0\phi^0\phi^0\rangle$). It is more crucial to actually deal with the higher point mixed RR- scalar field massless strings to actually generalize the rules and symmetries of string theory amplitudes. We hope to answer these higher point functions of string amplitudes and the other issues in future works. Although an interesting proposal for picture changing operator has been appeared in [55], however, we find it complicated to be applied to the real string amplitudes, nevertheless, it would be great to find the deep connections behind those topics as well.

10 Conclusion

In this paper, we have evaluated the five point world-sheet string theory amplitudes of the mixed RR, scalar and gauge fields, namely we have carried out with entire details the whole $\langle C^{-1}\phi^0 A^{-1}A^0\rangle$, $\langle C^{-1}\phi^{-1}A^0A^0\rangle$, $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ and $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ S-matrices.

We have regenerated all $t, s, u, (t+s+u)$- channel poles in effective field theory. We also found out new contact interactions as well as some new singularities that appear in$\langle C^{-1}\phi^0 A^{-1}A^0\rangle$ S-matrix where those new terms were actually the terms that carry momentum of RR in transverse direction and involved $p.\xi$ terms inside the S-matrix elements. These $p.\xi$ terms are needed in the entire form of S-matrix , due to non zero correlation function of RR field by the first term of scalar field vertex operator in zero picture. Indeed all $\langle e^{ip.x(z)}\partial_i x^i(x_1)\rangle$ terms are non-zero so we have reconstructed the S-matrices such that by considering all the scalar fields in zero pictures in the presence of RR, we were able to produce all $p.\xi$ terms as well as p^i, p^j terms (inside the S-matrices) whose momenta of RR are carried in transverse directions.

By comparing $\langle C^{-1}\phi^0 A^{-1}A^0\rangle$ with $\langle C^{-1}\phi^{-1}A^0A^0\rangle$ S-matrix we found a coupling inside the $\langle C^{-1}\phi^0 A^{-1}A^0\rangle$ S-matrix as follows

$$S_3 = \frac{i(2\pi\alpha')^3}{2}\mu_p \int d^{p+1}\sigma \quad \mathrm{Tr}\left(\partial_i C_{(p-3)} \wedge F \wedge F\Phi^i\right)$$

where this coupling can be explained by the effective field theory ways as, the mixed Chern-Simons and Taylor expansion of scalar field was needed. We then generalized its all order higher derivative corrections. We produced all the new singularities of this S-matrix in section six of this paper as well.

We also compared $\langle C^{-1}A^0\phi^{-1}\phi^0\rangle$ with $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ S-matrix for all order α' contact interactions as well as singularities in both transverse and world volume directions of the S-matrices and that leads to finding out various new couplings in string theory effective actions. First we found the following coupling

$$S_{11} = \frac{i(2\pi\alpha')^3}{2}\mu_p \int d^{p+1}\sigma \quad \mathrm{Tr}\left(\partial_j\partial_i C_{(p-1)} \wedge F\Phi^i\Phi^j\right)$$

and claimed that this coupling can be verified just by $\langle C^{-1}A^{-1}\phi^0\phi^0\rangle$ S-matrix where from field theory we employed the Taylor expansion of scalar fields and then we generalized its all order corrections.

Basically, we claim that various new contact interactions appear in the S-matrix by considering both scalar fields in zero picture. Indeed we derived the following new couplings

$$S_{12} = \frac{(2\pi\alpha')^3\mu_p\pi}{12}\int d^{p+1}\sigma \frac{1}{(p-3)!}(\varepsilon^v)^{a_0\cdots a_p}\left(\frac{\alpha'}{2}\right) \times C^{(p-3)}_{a_0\cdots a_{p-4}}\mathrm{Tr}\left(F_{a_{p-3}a_{p-2}}(D^aD_a)\Big[D_{a_{p-1}}\phi^i D_{a_p}\phi_i\Big]\right)$$

as well as

$$S_{13} = \frac{(2\pi\alpha')^3 \mu_p \pi}{6} \int d^{p+1}\sigma \left(\alpha'\right) \mathrm{Tr}\left(C_{p-3} \wedge D^{b_1} F \wedge D_{b_1}\left[D\phi^i \wedge D\phi_i\right]\right)$$

These couplings are needed in order to consider the symmetries of the S-matrix with respect to interchanging of the scalar fields and their all order α' corrections generalized in (9.8).

Note that these two above couplings can not be derived by the standard effective field theory ways of Taylor, Myers terms nor by pull-back formalism. Indeed not only the structures of the above new couplings but also their coefficients can just be explored by $\langle C^{-1} A^{-1} \phi^0 \phi^0 \rangle$ S-matrix analysis and not by any other tools.

Note that there is no Ward identity for the amplitudes of scalar fields in the presence of RR, thus we argue that for two scalars and a gauge field in the presence of RR, there was a subtle issue. Indeed to be able to get to new couplings as well as the corrected all order contact interactions of higher point functions of string theory amplitudes, one needs to consider both scalar fields in zero picture as we have clarified in detail in this paper. Eventually we have made use of Myers terms and the terms whose RR momenta are embedded in transverse directions, to be able to derive all the singularity structures of an RR, two scalars and a gauge field amplitude.

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