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## On composite two Higgs doublet models

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**ABSTRACT:** We investigate the issue of anomalous contribution to the  $T$  parameter and to Flavor Changing Neutral Currents in models with two Higgs doublets arising as composite pseudo Nambu-Goldstone modes. The non linear Lagrangians of several models are explicitly derived and the anomalous contributions to  $T$  are identified. The breaking patterns  $SU(5) \rightarrow SU(4) \times U(1)$  and  $SU(5) \rightarrow SU(4)$ , are analyzed first and we show how anomalous contributions to  $T$  arise in both models. Apart from that, the embedding of the Standard Model fermions in a  $\mathbf{10}$  of  $SU(5)$  avoids at the same time large corrections to the  $Zb\bar{b}$  coupling and Flavor Changing Neutral Current transitions. Finally, we propose a model based on the breaking  $SO(9)/SO(8)$  that is free from anomalous contributions to  $T$  and in which the problems of the  $Zb\bar{b}$  coupling and of Flavor Changing Neutral Currents can be simultaneously solved.

**KEYWORDS:** Higgs Physics, Beyond Standard Model, Technicolor and Composite Models

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**1 Introduction**

The discovery of a new resonance, with mass of about 125 GeV and properties compatible with those of a Higgs particle [1, 2] opens up a new era for particle physics, since the exploration of the Electroweak Symmetry Breaking (EWSB) sector is just at its beginning. A major priority is now the determination of the Higgs properties, i.e. production cross sections and branching ratios. This might shed light on the mechanism of EWSB and remove the veil off any new physics that may control the tera-scale.

If the Higgs has a non trivial substructure, deviations from the Standard Model (SM) couplings may be observed, while underlying symmetries can protect its mass from dangerous quantum corrections. This simple solution to the gauge hierarchy problem is naturally realized if the Higgs fields are pseudo-Goldstone bosons (PNGB) arising from a spontaneous breaking  $G \rightarrow H$  due to some strong dynamics at some scale  $f$  [3–6]. Once the strong sector is integrated out, the residual effective theory can be described in terms of a non-linear sigma model with the Higgs belonging to the coset space  $G/H$ . Interestingly, some

of these models can be considered as 4-dimensional duals of 5-dimensional gauge-Higgs unified theories realized in  $\text{AdS}_5$  space. However, the most important phenomenology can be extracted from effective theories with fairly general considerations about the strong sector and in terms of an expansion in the parameter  $\epsilon \equiv v/f$ , with  $v$  the Fermi scale.

Models with minimal [7–9] and non-minimal [10–12] scalar sectors have been studied in the literature. Let us notice that several complications are introduced once non-minimal models are considered. The most important issue is to protect the ratio between the vector boson masses. LEP precision measurements at the  $Z$  pole strongly constraint this ratio, quantified in term of the oblique electroweak parameter  $T$ . Two Higgs doublet models are notorious for breaking the custodial symmetry that protect the  $T$  parameter. The situation is even worse in composite models, which are inherently non-renormalizable implying a less constrained Lagrangian.

In this paper, we address the  $T$  parameter and other issues for models with two Higgs doublets (2HDM) arising as composite PNCB. Once the embedding of the SM fermions is defined for each case, the Glashow-Weinberg prescription [13] for natural FCNC of the Higgs couplings has to be checked and we exhibit two models where it is automatically realized. Compositeness is also a source of violations of gauge coupling universality and the models are also selected by their ability to preserve the (relative) agreement of the SM prediction for the  $Z\bar{b}_L b_L$  couplings with experiment. This is ensured by a rule on the embedding of the  $b$ -quark in the models [15].

The general Lagrangian formalism for the PNCB's was formulated in [17], but in the Minimal Composite Higgs Model (MCHM) based on  $\text{SO}(5)/\text{SO}(4)$  [7] the non-linear realization of the symmetry has also been conveniently formulated in terms of a  $\text{SO}(5)$  vector reminiscent of a linear realization of the breaking. Because of its utility, we generalize here this approach to the various cosets related to 2HDM and write in a compact form the PNCB Lagrangian, gauge couplings, Higgs potential and Yukawa couplings in terms of the new variables linearly transforming in fundamental representations. Then, our analysis of the custodial symmetry and tree-level contributions to  $T$  for the different cosets defining 2HDM's is simplified by the use of these coordinates. As pointed out in [11] and as also discussed below, in the  $\text{Sp}(6)/\text{Sp}(4) \times \text{SU}(2)$  case with a specific embedding of the SM gauge symmetries, there are no contributions to the  $T$  parameter. Here, we also describe another  $T$ -safe model based on the coset  $\text{SO}(9)/\text{SO}(8)$ .

For concreteness, we first focus on the symmetry breaking patterns  $\text{SU}(5) \rightarrow \text{SU}(4) \times H'$  with  $H' = \text{U}(1)$  or nothing. In the former case, eight pseudo Goldstone modes are delivered, fitting into two SM doublets. In the latter scenario, the breaking leads to an additional SM singlet. We find how the Higgs couplings to  $W$  bosons and fermions are modified with respect to those of the SM Higgs boson, so that different production and decay rates are expected. However, in both cases, only by aligning the vacuum expectation values (vev) of the two Higgs the contribution to  $T$  could vanish. The lowest dimensional representations of  $\text{SU}(5)$ ,  $\mathbf{5}$  and  $\mathbf{10}$ , provide a natural embedding for the SM fermions. We perform a systematic study of all the possibilities, clearly differentiating between cases that can be successfully embedded into a 5d completion and those in which this is not so straightforward. We find that the embedding of the SM fermions in the  $\mathbf{10}$  can lead to

a simultaneous resolution of the usual problems of anomalous  $Z\bar{b}_L b_L$  [15] and the flavor problems [16]. The contribution to the (Coleman-Weinberg) PNGB potential from these fermions is displayed and, in general, does not have the symmetry to align the Higgs vev's.

Finally, we propose a  $SO(9)/SO(8)$  model, where the two Higgs couplings to the gauge bosons are much like in the MCHM and, indeed, there are no tree-level contribution to the  $T$  parameter. Another interesting feature of this model is an embedding of the SM symmetries in  $SO(8)$ , which preserve an extra  $U(1)$  that requires the SM fermions to couple to only one Higgs doublet. This avoids FCNC problems from the Higgs sector since the other PNGB doublet becomes inert. The  $Z\bar{b}_L b_L$  issue is also coped with when all the fermions are embedded in the spinorial representations. We also survey the different C2HDM that have been studied in the literature in our framework and compare and contrast them with the new models presented here.

The paper is organized as follows. In sections 2–3 we state the problem and set the notation studying the coset and discussing the gauge interactions for  $SU(5)/SU(4) \times U(1)$  and  $SU(5)/SU(4)$ . In section 4 we discuss in detail the possible embeddings for the SM fermions in  $SU(5)$  representations and their phenomenological consequences. In section 5 we study the scalar potential in the  $SU(5)$  case. In section 6 we introduce the gauge interactions and the tree level contribution to the  $T$  parameter for different cosets. In section 7 we analyze the  $SO(9)/SO(8)$  gauge and fermion couplings to the Higgs sector. Finally we conclude with some general observations.

## 2 Non linear realization of the Higgs fields

The general formulation of non linear representations (n.l.r.) is given in CCWZ. Here we introduce an alternative formulation which has also been used in models with a single composite Higgs doublet like those quoted above.

We begin with the study the breaking patterns  $SU(5)/SU(4)$  and  $SU(5)/SU(4) \times U(1)$ . The coset space of the two cases is quite similar, although they contain a different number of Goldstone modes. The Goldstone modes can be parametrized as the matrix

$$\Pi = \left( \begin{array}{c|c} \frac{1}{\sqrt{20}}\phi_0 \mathbf{1}_4 & \begin{matrix} \Phi \\ \tilde{\Phi} \end{matrix} \\ \hline \Phi^\dagger \quad \tilde{\Phi}^\dagger & -\frac{4}{\sqrt{20}}\phi_0 \end{array} \right), \tag{2.1}$$

where  $\Phi$  and  $\tilde{\Phi}$  are written in terms of two complex  $SU(2)_L$  doublets  $\phi_1, \phi_2$  as

$$\Phi \equiv \frac{\phi_1 + i\phi_2}{\sqrt{2}} \quad \text{and} \quad \tilde{\Phi} \equiv \frac{\tilde{\phi}_1 + i\tilde{\phi}_2}{\sqrt{2}}, \tag{2.2}$$

with  $\tilde{\phi}_i = i\sigma_2 \phi_i^*$  (see appendix A for the conventions on the generators of the group). Each  $(\phi_i, \tilde{\phi}_i)$  transforms as a  $(\mathbf{2}, \mathbf{2})$  of  $SU(2)_L \times SU(2)_R$ , while the 9<sup>th</sup> Goldstone boson, the singlet  $\phi_0$ , is present only in the  $SU(5)/SU(4)$  coset. Correspondingly, the Goldstone boson matrix for the symmetric coset  $SU(5)/SU(4) \times U(1)$  is obtained for  $\phi_0 = 0$ . The matrix  $\Pi$  parametrizes the elements of the cosets in terms of the PNGB fields.

To write the n.l.r. of the Higgs fields, it is convenient to choose a specific direction for the vacuum  $\Sigma_0$ . The breaking  $SU(5) \rightarrow SU(4)$  can be parametrized either through the fundamental or the symmetric representation of  $SU(5)$ ,

$$\mathbf{5} : \Sigma_0^T = (0 \ 0 \ 0 \ 0 \ 1) \quad \mathbf{15} : \Sigma_0 = \text{diagonal}(0 \ 0 \ 0 \ 0 \ 1) \quad [SU(5)/SU(4)],$$

while  $SU(5) \rightarrow SU(4) \times U(1)$  can be parametrized through the adjoint representation,

$$\mathbf{24} : \Sigma_0 = \frac{1}{\sqrt{20}} \text{diagonal}(1 \ 1 \ 1 \ 1 \ -4) \quad [SU(5)/SU(4) \times U(1)].$$

The expressions for the n.l.r. in terms of the Goldstone bosons in the two cases are given by

$$\begin{aligned} \mathbf{5} : \Sigma &= e^{\frac{i\Pi}{f}} \Sigma_0 \\ \mathbf{15} : \Sigma &= e^{\frac{i\Pi}{f}} \Sigma_0 e^{\frac{i\Pi^T}{f}} \\ \mathbf{24} : \Sigma &= e^{\frac{i\Pi}{f}} \Sigma_0 e^{\frac{-i\Pi}{f}} \end{aligned} \quad (2.3)$$

where the appropriate vacuum  $\Sigma_0$  must be used and  $f$  is the scale at which the breaking occurs.

The n.l.r. is obtained in terms of a 5-components unit vector  $u$  as follows. In the  $SU(5)/SU(4)$  case,  $u$  reads

$$u = e^{-i\frac{3}{5}\frac{\phi_0}{f}} \begin{pmatrix} i\frac{\sin(\frac{\varphi}{f})}{\varphi} \begin{pmatrix} \Phi \\ \tilde{\Phi} \end{pmatrix} \\ \cos(\frac{\varphi}{f}) + i\phi_0\frac{\sin(\frac{\varphi}{f})}{\varphi} \end{pmatrix} = e^{\frac{i\Pi}{f}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = e^{\frac{i\Pi}{f}} u_0, \quad u^\dagger u = 1, \quad (2.4)$$

where  $\varphi = \sqrt{|\Phi|^2 + |\tilde{\Phi}|^2 + \phi_0^2}$  and

$$\Sigma = u \quad [\mathbf{5}], \quad \Sigma = uu^T \quad [\mathbf{15}] \quad (2.5)$$

In the  $SU(5)/SU(4) \times U(1)$  case, with  $\phi_0 = 0$ ,

$$u = \begin{pmatrix} i\frac{\sin(\frac{\varphi}{f})}{\varphi} \begin{pmatrix} \Phi \\ \tilde{\Phi} \end{pmatrix} \\ \cos(\frac{\varphi}{f}) \end{pmatrix} = e^{\frac{i\Pi}{f}} u_0, \quad u^\dagger u = 1, \quad (2.6)$$

where now  $\varphi = \sqrt{|\Phi|^2 + |\tilde{\Phi}|^2}$ . In this case the  $\Sigma$  matrix reads

$$\Sigma = -\frac{\sqrt{5}}{2} \left( uu^\dagger - \frac{\mathbf{1}_5}{5} \right). \quad (2.7)$$

Because  $\varphi$  is invariant under the preserved  $SU(4)$ ,  $u$  linearly transforms as a  $\mathbf{4} + \mathbf{1}$ , like the Goldstone fields in eq. (2.1). For the  $SU(5)/SU(4) \times U(1)$  case, the  $U(1)$  invariance is obtained by the multiplication of a phase defined by the transformation of the Goldstone

bosons:  $u \rightarrow g u e^{i\theta}$ , with  $\theta = \theta(g, \phi^i)$ . This corresponds to the transformation of  $e^{\frac{i\Pi}{f}}$  defined in CCWZ once the SU(4) invariance of vacuum is taken into account. This phase disappears in the realization  $\Sigma$  as defined in eq. (2.7).

It is useful to formulate the Lagrangian for the PNCB fields and their interactions<sup>1</sup> in terms of  $u$ . First note that  $\Sigma\Sigma^\dagger$  are projectors along the vector  $u$  for the **5** and the **15** vacua. It implies that the insertions of the  $\Sigma$  field in the different terms of the effective Lagrangian are quite limited. In practice one has only to introduce the minimal number of  $u$ 's needed to ensure the SU(5) invariance. This is equivalent to the insertion of the unitary matrix  $U = e^{\frac{i\Pi}{f}}$  in the CCWZ formalism. However, the breaking by a **15** preserves an additional parity symmetry:  $u \rightarrow -u$ . Therefore the  $u$ 's must come by pairs, hence as  $\Sigma = uu^T$ . For the **24** case, let us remark that the unit matrix in eq. (2.7) is irrelevant as it does not introduce any new operator in the effective Lagrangian. Therefore we henceforth replace eq. (2.7) by  $\Sigma = uu^\dagger$ , which is a projector,  $\Sigma^2 = \Sigma$ . Therefore, the effective Lagrangian should only have the minimal number of insertions of the  $\Sigma$  field as needed for the SU(5) invariance in each interaction, much like in the previous case. This drastically simplifies the analysis of flavour changing effects from Higgs couplings to fermions below.

We now turn to the analysis of the Goldstone boson Lagrangians. This can be done in terms of the  $u$ 's transforming in the fundamental representation of SU(5), the simplest n.l.r., from which the other ones can be built. Of course, the Lagrangians for the Goldstone bosons depend only on the specific coset as shown in CCWZ. We easily determine the explicit Lagrangians in terms of the  $u$ 's by just imposing the equivalence with the general CCWZ expression. There are two SU(5) invariants with two derivatives since  $u^\dagger u = 1$ :

$$\begin{aligned} \partial_\mu u^\dagger \partial^\mu u &= u_0^\dagger \partial_\mu U^\dagger (U U^\dagger) \partial^\mu U u_0 \\ u^\dagger \partial_\mu u \partial^\mu u^\dagger u &= u_0^\dagger U^\dagger \partial_\mu U u_0 u_0^\dagger \partial^\mu U^\dagger U u_0 \end{aligned} \tag{2.8}$$

where  $U = e^{\frac{i\Pi}{f}}$ . Notice the projections along the  $u_0$  direction. The element of the SU(5) algebra  $U^\dagger \partial_\mu U$  can be expanded as,  $i d_\mu^i T^i + i E_\mu^a T^a + (d/E)_\mu^0 T^0$ , where  $T^a$  are the generators of SU(4),  $T^0$  the one of the U(1) and  $T^i$  those of the coset. The component along the U(1) generator is  $d_\mu^0$  for SU(5)/SU(4), and  $E_\mu^0$  for SU(4) × U(1). The  $d_\mu$ 's transform linearly under local SU(4) and SU(4) × U(1), respectively, while the  $E_\mu$ 's transform as gauge fields [17, 18].

Replacing these expression in eq. (2.8) and projecting along  $u_0$ , one gets for the two invariants two different expressions in terms of  $d_\mu^i d^{\mu i}$  and  $E_\mu^0 E^{\mu 0}$ . However, since  $u_0$  is a SU(4) singlet, the correspondent  $E_\mu^a E^{\mu a}$  terms are absent in both the global invariants in eq. (2.8), while the terms  $d_\mu^i$  transforming as **4** +  $\bar{\mathbf{4}}$  are absent from the quartic one. Then, the CCWZ Lagrangian,  $\mathcal{L}_{\text{PNCB}} = \Sigma_i d_\mu^i d^{\mu i}$  is given in each case by the combination that eliminates or consistently normalizes the singlet term, resulting in local SU(4) invariant two-derivative PNCB Lagrangians. They are as follows:

$$\begin{aligned} \mathcal{L}_{\text{PNCB}} &= f^2 (\partial^\mu u^\dagger \partial_\mu u - u^\dagger \partial_\mu u \partial^\mu u^\dagger u) \quad [\text{SU}(5)/\text{SU}(4) \times \text{U}(1)] \\ &= f^2 (\partial^\mu u^\dagger \partial_\mu u - \frac{3}{8} u^\dagger \partial_\mu u \partial^\mu u^\dagger u) \quad [\text{SU}(5)/\text{SU}(4)] \end{aligned} \tag{2.9}$$

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<sup>1</sup>For a review of the more usual CCWZ approach in the context of EWSB see, e.g., [18].

Vertex	2HDM	C2HDM
$hW^+W^-/ZZ$	$2\frac{m_{W,Z}^2}{v}$	$2\frac{m_{W,Z}^2}{v}\sqrt{1-\epsilon^2}$
$h^2WW/ZZ$	$\frac{m_{W,Z}^2}{v^2}$	$\frac{m_{W,Z}^2}{v^2}(1-2\epsilon^2)$
$H^2WW/ZZ$	$\frac{m_{W,Z}^2}{v^2}$	$\frac{m_{W,Z}^2}{v^2}r_\epsilon\sqrt{1-\epsilon^2}$
$A^2WW$	$\frac{m_W^2}{v^2}$	$\frac{m_W^2}{v^2}r_\epsilon\sqrt{1-\epsilon^2}$
$A^2ZZ$	$\frac{m_Z^2}{2v^2}$	$\frac{1}{2}\frac{m_Z^2}{v^2}(r_\epsilon\sqrt{1-\epsilon^2}-2\epsilon^2r_\epsilon^2)$
$H^+H^-W^+W^-$	$2\frac{m_W^2}{v^2}$	$2\frac{m_W^2}{v^2}(r_\epsilon\sqrt{1-\epsilon^2}-2\epsilon^2r_\epsilon^2)$
$H^+H^-ZZ$	$2\frac{m_Z^2}{v^2}\left(1-2\frac{m_W^2}{m_Z^2}\right)^2$	$2\left[\frac{m_Z^2}{v^2}r_\epsilon\sqrt{1-\epsilon^2}-4\frac{m_W^2}{v^2}\left(1-\frac{m_W^2}{m_Z^2}\right)r_\epsilon^2\right]$
$H^-W^+A_\mu(H+iA)$	$-\frac{ge}{2}$	$-\frac{ge}{2}r_\epsilon^2$
$H^-W^+ZH$	$\frac{g'e}{2}$	$\frac{g'e}{2}r_\epsilon^2$
$H^-W^+ZA$	$i\frac{g'e}{2}$	$i\frac{g'e}{2}r_\epsilon^2\left[1-2\kappa\epsilon^2\left(1+\frac{g^2}{g'^2}\right)\right]$
$H^+H^+W^-W^-$	$\times$	$\kappa\frac{4m_W^2}{v^2}\epsilon^2r_\epsilon^2$

**Table 1.** Gauge interaction for the Higgs fields in the C2HDM case following from eq. (3.1), as opposed to the usual renormalizable 2HDM case. The Higgs fields definition is given in eq. (3.2). We used the shorthanded notation  $\epsilon = v/f$  and  $r_\epsilon = \epsilon/\arcsin(\epsilon)$ .

Notice that in the first lagrangian in (2.9) there is only one invariant under local SU(4), while the second Lagrangian differs by the presence of a second one,  $d^0d^0 = (5/8)u^\dagger\partial_\mu u\partial^\mu u^\dagger u$ , since with our normalization of  $T^0$ ,  $T^0u_0 = -\sqrt{8/5}$ .

It is worth remarking that the use of the  $u$  variables and the projection into the  $u_0$  automatically eliminates most of the  $E_\mu$ 's, towards the construction of the locally invariant Lagrangians. This procedure is generalized in section 6 to other cosets. If useful, the corresponding expressions for the  $\Sigma$ 's can be obtained by using their expressions in terms of the  $u$ 's. In the next section we show that the anomalous contribution to the T-term is generally encoded in the quartic term in the  $u$ 's of the Lagrangian.

### 3 Gauge interactions

The gauge Lagrangian for the Goldstone bosons in the effective theory can be written from eq. (2.9) introducing covariant derivatives for the  $u$  variables,  $\partial u_i \rightarrow \partial u_i - i(\hat{W}u)_i$ , with  $\hat{W}_\mu = gW_\mu^+T_L^- + gW_\mu^-T_L^+ + eA_\mu(T_L^3 + T_R^3) + g\sec\theta_W Z_\mu(T_L^3 - \tan^2\theta_W T_R^3)$ . Therefore the coupling of the SM gauge bosons to the PNCB fields contains two algebraically different terms,

$$u^\dagger\hat{W}_\mu\hat{W}_\nu u - \kappa u^\dagger\hat{W}_\mu u u^\dagger\hat{W}_\nu u, \quad (3.1)$$



and only the first one, quadratic in  $u$ , is analogous to the SM.

The gauge interactions following from the previous expression are given in table (1), with Higgs fields defined as

$$\phi_1 = \begin{pmatrix} G^+ \\ h + iG^0 \end{pmatrix} \quad \phi_2 = \begin{pmatrix} H^+ \\ H + iA \end{pmatrix}. \tag{3.2}$$

and  $G^{0,+}$  the electroweak Goldstone bosons. It is interesting to note that different couplings between the Higgs boson and the W and Z vectors could be important to explain the cross-section and branching ratios of the observed excess around 125 GeV [20].

In momentum space, the gauge Lagrangian reads

$$\mathcal{L}_{\text{gauge}} = \frac{1}{2} p_{\mu\nu}^T \left[ \Pi_0(q^2) \text{tr}(\hat{W}_\mu \hat{W}_\nu) + \Pi_1(q^2) (u^\dagger \hat{W}_\mu \hat{W}_\nu u - \kappa u^\dagger \hat{W}_\mu u u^\dagger \hat{W}_\nu u) \right] \tag{3.3}$$

where  $\kappa = 1$  or  $3/8$  as in eq. (2.9) and  $\Pi_0(q^2)$  and  $\Pi_1(q^2)$  are form factors. Of course, this Lagrangian is not invariant under  $SU(2)_L \times SU(2)_R \subset SU(4)$  because only the SM subgroup is gauged. In 2HDM's, only the alignment between the two doublets in the vacuum ensures a custodial  $SU(2)$  to protect the  $T$  parameter.

Indeed, in both the cases under consideration the quartic term spoils the custodial symmetry already at tree level. To investigate this crucial point, let us keep in eq. (3.3) only the components of  $u$  with electric charge  $T_L^3 + T_R^3 = 0$ , namely,  $u_2$  and  $u_3$  with the conventions defined in appendix A. The embedding of the  $SU(2)_L \times U(1)_Y$  generators in  $SU(2)_L \times SU(2)_R \subset SU(4)$  is uniquely defined by  $\mathbf{4} = (\mathbf{2}, \mathbf{2})$ , so that the masses of the gauge bosons are obtained from eq. (3.3) as follows:

$$\mathcal{L}_0 = f^2 (\langle |u_2|^2 \rangle + \langle |u_3|^2 \rangle) \frac{g^2}{2} \left( W_\mu^+ W_\mu^- + \frac{1}{2 \cos^2 \theta_W} \left( 1 - \kappa \frac{(\langle |u_2|^2 \rangle - \langle |u_3|^2 \rangle)^2}{\langle |u_2|^2 \rangle + \langle |u_3|^2 \rangle} \right) Z_\mu Z_\mu \right). \tag{3.4}$$

The additional contribution to the Z mass only vanishes for  $\langle |u_2| \rangle = \langle |u_3| \rangle$ , which is precisely the condition for the alignment of the two doublets in the vacuum and the conservation of the custodial symmetry. Since the PNGB potential is generated at loop level, where  $SU(2)_L \times SU(2)_R \subset SU(4)$  is explicitly broken, it cannot generically guarantee such a degeneracy for the solution, so that a large contribution to the  $T$  parameter should be present already at tree level. This would impose an unnatural hierarchy between the two relevant scales,  $f \gg v$  (slightly less for  $\kappa = 3/8$ ).

In principle: *i*) the custodial symmetry could be effective in a region of the parameter space for the solution of the equations of motion, and *ii*) the contributions to the  $T$  parameter (both at tree and loop level) from the scalar sector can be partially compensated by fermionic contributions [19]. Nevertheless, the mechanism for this approximate cancellation must be natural.

The conditions to avoid Higgs misalignments in composite 2HDM have been exhaustively discussed in [11], in particular, by the use of additional discrete symmetries. They were able to build a model with one Higgs coupled to the SM fermions and an inert Higgs that does not contribute to the gauge boson masses. The natural discrete symmetry for



SU(5) is charge conjugation ( $\phi_1 \rightarrow \phi_1$  and  $\phi_2 \rightarrow -\phi_2$ , see eq. (2.2)). To investigate this point, as much as the mostly important flavour issues, we now turn to a study of the SM fermion embedding and their couplings to the two Higgs bosons.

#### 4 Fermion embeddings

As usual in Composite Higgs models, the coset structure does not fix the embedding of the fermions in  $G$  representations. Moreover, different choices will in principle generate different terms in the Coleman-Weinberg potential. However, some general considerations allow to make the model theoretically consistent and phenomenologically viable. Some of the important issues are:

1. LEP has measured the  $Zb_L\bar{b}_L$  coupling with high precision, finding results in agreement with the SM prediction at nearly 0.25% level.<sup>2</sup> As shown in [15], a custodial symmetry can protect this coupling from large contributions due to composite states. The argument boils down to the conclusion that the left handed quark doublet (at least the third generation) should be embedded in a  $(\mathbf{2}, \mathbf{2})_{\mathbf{2/3}}$  representation of  $SU(2)_L \times SU(2)_R \times U(1)_X$  in order to protect this coupling from getting too large corrections.
2. If the SM fermions can be embedded into  $G$  representations in a unique way, then the problem of FCNC is considerably reduced [10, 16]. There are cases where this can be achieved by the addition of discrete symmetries [11]. In the SO(9)/SO(8) model described below this is done by a  $U(1) \in SO(8)$ .
3. In 5d models of gauge-Higgs unification that are perturbative dual to the strongly coupled 4d model, where the PNCB are the 5-th component of gauge bosons, the fermions of both chiralities should be in the same representation of the gauge group (global group, from the 4d point of view).<sup>3</sup> However, this is not a stringent consideration for the purely 4d model, and we will actually explore some of the advantages of using LH and RH fermions in different representations.

In terms of the SM fermions, the Lagrangian in momentum space reads [7]

$$\begin{aligned}
 \mathcal{L}_{\text{fermion}} = & \bar{q}_L \not{p} [\Pi_0^q + \Pi_1^q g_{qL}(u_i)] q_L + \\
 & + \bar{t}_R \not{p} [\Pi_0^u + \Pi_1^u g_{tR}(u_i)] t_R + \bar{b}_R \not{p} [\Pi_0^d + \Pi_1^d g_{bR}(u_i)] b_R + \\
 & + f \bar{t}_R [M_1^u g_{tq}(u_i)] q_L + f \bar{b}_R [M_1^d g_{bq}(u_i)] q_L
 \end{aligned}
 \tag{4.1}$$

where the  $\Pi_i(Q^2)$  and  $M_i Q^2$  are form factors controlled by the strong sector and the  $g(u_i)$  functions of the Higgs fields. These polynomials are defined by the possible insertions of

<sup>2</sup>Recent 2 loop calculations indicates that the agreement between SM and experimental values of the  $Zb_L\bar{b}_L$  coupling may not be as good as previously expected [14].

<sup>3</sup>In principle, this requirement can be circumvented by introducing extra fermion multiplets and mass terms in the IR brane. However, this is a potential source of FCNC, so that we discard this assemblage in our discussion.

the n.l.r.  $u$ 's, or  $\Sigma$ 's in the fermion bilinear terms  $\bar{\Psi}_A^i g^{ij}(u) \Psi_B^j$ , with  $A, B = L, R$  that are SU(5) invariants. The Lagrangian is obtained by restricting the fermion multiplets to their SM components.

For simplicity, in what follows we will limit ourselves to the lower dimensional representations of SU(5), i.e. **5** and **10**, although higher dimensional representations in principle can be used. In terms of  $SU(2)_L \times SU(2)_R$  they read

$$\begin{aligned} \mathbf{5} &= (\mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{1}) \\ \mathbf{10} &= (\mathbf{2}, \mathbf{2}) + (\mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}) \end{aligned} \tag{4.2}$$

Let us first discuss the case in which both the left (LH) and right (RH) fermions are embedded in the fundamental representation. To accommodate hypercharge, each RH singlet must be embedded in an independent **5**, with appropriate X-charge. This implies that, by  $U(1)_X$  invariance,  $q_L$  must be associated to two **5**, respectively  $\Psi_L^t$  and  $\Psi_L^b$ . In a 5D picture, this is achieved decoupling the additional degrees of freedom through a mass term in the brane [8, 10]. However, since  $b_L$  does not belong to  $(\mathbf{2}, \mathbf{2})_{2/3}$ , the  $Z b_L b_L$  vertex is not protected. In models with only one Higgs doublet, this is not necessarily a shortcoming since the breaking is proportional to the relatively small bottom Yukawa coupling [10]. In 2HDM this is a cause for concern for large  $\tan \beta$ .

Turning to the case in which both the LH and RH fermions are embedded in a **10**, the RH singlets must be in a unique  $(\mathbf{1}, \mathbf{3}) \in \mathbf{10}_{2/3}$ , with  $T_{3R} = 0$  for  $u_R$  and  $T_{3R} = -1$  for  $d_R$ . As a consequence, also  $q_L$  is embedded in a unique representation and  $Z b_L b_L$  is protected. This is the SU(5) counterpart of the choice of the real **10** for the fermions in the SO(5)/SO(4) model.

In both cases, there is only one possible embedding of the SM fermions in the  $SU(2)_L \times SU(2)_R$  representations. Another important feature: for all  $\bar{\Psi}_A^i g^{ij}(u) \Psi_B^j$ , we find  $g^{ij}(u) \propto uu^\dagger = \Sigma$ . Indeed,  $\Sigma$  is a projector,  $\Sigma^2 = \Sigma$ , and  $\Sigma \Psi_{L,R}^i \Sigma = 0$  for the SM in a **10**. The expressions for the  $g$  functions, as defined in eq. (4.1), are collected in table 2 for both SU(5)/SU(4) and SU(5)/SU(4)  $\times$  U(1), choosing the appropriate  $u$  in eqs. (2.4)–(2.6). A crucial attribute of the mass terms is that the up quarks get masses only from one combination of the two Higgs, while the down quarks couple to the orthogonal one. Therefore there is no flavour violation from the Higgs couplings.

In all the previous cases, a 5-dimensional gauge-Higgs unification can be consistently constructed. However, from a purely 4d perspective, the embedding of LH and RH fermions in different representations make sense, so that we will now briefly comment also on this possibility. Since we are confining ourselves only to the **5** and **10**, only for SU(5)/SU(4) the embedding in different representations is possible. The simplest possibilities are given by:

- (a)  $(\Psi_L, \Psi_R) \in (\mathbf{5}, \mathbf{10})$ , with invariant  $\mathbf{5}_\Sigma^T \bar{\mathbf{10}}_R \mathbf{5}_L$ . Note that left doublet can be embedded in the  $(\mathbf{2}, \mathbf{2})_{2/3}$ , protecting  $Z b_L \bar{b}_L$ .
- (b)  $(\Psi_L, \Psi_R) \in (\mathbf{10}, \bar{\mathbf{10}})$ , with invariant  $\epsilon^{ijklmn} (\mathbf{10}_R)_{ij} (\mathbf{10}_L)_{km} (\mathbf{5}_\Sigma)_n$ . Since  $b_L = (\mathbf{10}_L)_{54}$  and  $b_R = (\bar{\mathbf{10}}_R)_{34/43}$ , a mass term  $\bar{b}_R b_L$  is forbidden and the bottom mass can only be generated at higher order, explaining in a natural way the bottom/top mass gap.

	<b>5</b>	<b>10</b>
$g_{qL}$	$ u_3 ^2 +  u_4 ^2,  u_1 ^2 +  u_2 ^2$	$\begin{pmatrix}  u_5 ^2 +  u_3 ^2 & u_4^* u_3 \\ u_4 u_3^* &  u_5 ^2 +  u_4 ^2 \end{pmatrix}$
$g_{tR}$	$ u_5 ^2$	$ u_1 ^2 +  u_2 ^2 +  u_3 ^2 +  u_4 ^2$
$g_{bR}$	$ u_5 ^2$	$ u_3 ^2 +  u_4 ^2$
$g_{tq}$	$u_5 \begin{pmatrix} u_3 \\ u_4 \end{pmatrix}^\dagger$	$u_5^* \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}^T (i\sigma_2)$
$g_{bq}$	$u_5 \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}^\dagger$	$u_5^* \begin{pmatrix} u_3 \\ u_4 \end{pmatrix}^T (i\sigma_2)$

**Table 2.** Couplings between the SM fermions and the  $u$  variables, as defined in eq. (4.1), according to the SU(5) representation in which they are embedded. The result can be applied both to SU(5)/SU(4) and SU(5)/SU(4)  $\times$  U(1) choosing the appropriate expression for  $u$ . In the case of the **5**, the first value for  $g_{qL}$  refers to  $q_L^t$ , the second to  $q_L^b$ .

However in both this cases there is no direct couplings between the fermions and the  $u_5$ .

- (c)  $(\Psi_L, \Psi_R) \in (\mathbf{5}, \bar{\mathbf{5}})$ , which leads to a phenomenology similar to the  $(\mathbf{5}, \mathbf{5})$  case up to a phase in the Yukawa interactions.

Let us comment on how the flavor problem is solved in our case. As pointed out in [11], the problem is twofold: on the one hand, one has to face the usual 2HDM problem of ensuring Minimal Flavor Violation (MFV) in the Yukawa sector; on the other hand, there could be more invariant Yukawa couplings of the fermions to the Higgs n.l.r. and a priori there is no reason to have alignment between the different flavor matrices defining different couplings [16]. In the SU(5) models under consideration, the flavour mixing is solely given by the mass matrices so that MFV is realized because

- (i) the representations considered, **5** and **10**, are small enough to allow only one embedding of the up and down quarks, as displayed in eq. (4.2);
- (ii) all the fermions only couple to one  $\Sigma$ , so that their couplings to the Higgs are flavour diagonal.

Hence, there is no tree-level flavour changing from the Higgs sector. Unfortunately, the  $t$ -quark, instrumental in EWSB, couples to  $u_3$ , which is a combination of the real part of a Higgs field and the imaginary part of the other, pushing for the misalignment for their vev's. To point out this serious problem, we briefly discuss the PNGB potential in the next section.

## 5 Scalar potential

The Higgs fields arise as Goldstone modes, so the shift symmetry forbids a tree level potential. However, the global symmetry is explicitly broken once the SM group is gauged and the fermions are coupled to the Higgs system, so that the Goldstone modes get a Coleman-Weinberg potential. The potential can be computed with standard techniques, and may be written in general as

$$V = V_{\text{gauge}} + V_{\text{fermion}}, \quad (5.1)$$

In terms of the  $u$  variables, and to lowest order in  $g^2$  and  $g'^2$ , the gauge part of the potential for the neutral components reads

$$V_{\text{gauge}} = \frac{3}{2} \int \frac{d^2 Q}{16\pi^2} \frac{\Pi_1}{\Pi_0} \left( \frac{3g^2 + g'^2}{4} (|u_2|^2 + |u_3|^2) + D^A D^A \right) \quad (5.2)$$

with  $D^A = g_A u^\dagger T^A u$ . This term is the additional contribution arising from the terms quartic in  $u$  in the gauge part of the Lagrangian, as given in eq. (2.9), and closely resembles the supersymmetric case (without  $B\mu$  term).

The fermionic contribution  $V_{\text{fermion}}$  in eq. (5.1) can be simplified in the reasonable limit in which  $b_R$  has a relatively small coupling to the strong sector. With this assumption, and to lowest order in the ratios of structure functions  $\Pi_1^a/\Pi_0^a$ , defined in eq. (4.1)) the potential for the neutral components results:

$$V_{\text{fermion}}^{(\mathbf{10})} = \int \frac{d^4 Q}{16\pi^2} \left[ \left( \rho^{q_3} + \frac{1}{2} \rho^{t_R} \right) (|u_2|^2 + |u_3|^2) + \left( \rho^{q_3} - \frac{f^2}{Q^2} Y_t^2 \right) |u_3|^2 + \frac{1}{2} \rho^{t_R} (|u_2|^2 + |u_3|^2)^2 + \frac{f^2}{Q^2} Y_t^2 (|u_2|^2 + |u_3|^2) |u_3|^2 + \dots \right] \quad (5.3)$$

for fermions embedded in a **10**, and

$$V_{\text{fermion}}^{(\mathbf{5})} = \int \frac{d^4 Q}{16\pi^2} \left[ \rho^{q_3} (1 - |u_2|^2 - |u_3|^2) + \rho^{t_R} (|u_2|^2 + |u_3|^2) - \frac{f^2}{Q^2} Y_t^2 |u_3|^2 (1 - |u_2|^2 - |u_3|^2) + \rho^{t_R} |u_2|^2 |u_3|^2 + \dots \right] \quad (5.4)$$

for fermions embedded in a **5**, where the integrands are the ratios,

$$\rho^f(Q^2) = \frac{\Pi_1^f}{\Pi_0^f}, \quad (f = t_R, b_R, q_3) \quad \text{and} \quad Y_t(Q^2) = \frac{|M_1^t|^2}{\Pi_0^{q_3} \Pi_0^{t_R}} \quad (5.5)$$

Of course, a quantitative discussion of the Higgs vev and the spectrum is pointless in presence of so many unknown form factors. However, some comments are now in order:

1. To return to the Higgs alignment issue, notice that the important negative contribution in the potential corresponds to the  $t$ -quark Yukawa coupling to  $u_3$ . Hence it drives the vacuum through the  $u_3$  direction which breaks the custodial symmetry as

already stressed. The  $b$ -quark coupling to  $u_2$ , neglected in (5.3), (5.4), is too small to compete. As already quoted above, Higgs alignment can be enforced in composite 2HDM by discrete symmetries [11] that allow to restrict the couplings of SM fermions to a single Higgs in the case of the  $SU(4)/SU(2) \times SU(2) \times U(1)$  model. In our case, the interesting property of the models is precisely the automatic uniqueness of these couplings and also the fact that quarks of different charges interact with different Higgs fields. Therefore, we cannot impose a symmetry that, at most, would kill our single Yukawa coupling. We have to rely on a partial compensation mechanism.

2. Since the custodial symmetry is essentially broken by the Higgs coupling to  $q_L$  (see appendix A) that explicitly breaks  $SU(2)_R$ , one could try to improve the situation introducing custodial fermions in the  $(\mathbf{2}, \mathbf{2})$  of  $SU(2)_L \times SU(2)_R$  [21] to alleviate the constraint from the  $T$ -parameter. Interestingly, they might also contribute through loops to increase  $BR(h \rightarrow \gamma\gamma)$  [22], in agreement with the recent observations at the LHC. They naturally provide an interesting phenomenological set up, detailed study of which is beyond the mandate of this paper.
3. For  $SU(5)/SU(4)$ , the  $\varphi_0$  component of  $u$  is a main concern too, since it is the PNGB associated to a abelian symmetry and, as such, an axion-like scalar subject to strict experimental bounds. It always appears in the potential as  $|u_5|^2 = 1 - \sin^2\left(\frac{\varphi}{f}\right) \frac{|\Phi|^2 + |\tilde{\Phi}|^2}{\varphi^2}$ . The fact that  $\varphi_0$  only appears as a function of  $\varphi$  implies that there is always a massless particle in the spectrum. Indeed, if we consider the simplified case in which  $u$  has only one non vanishing component, since  $V = V(|u|^2)$ , at the minimum the mass matrix in the  $(u, \varphi_0)$  basis has the form

$$M^2 = \begin{pmatrix} 1 & \frac{\partial |u|^2}{\partial \eta} \\ \frac{\partial |u|^2}{\partial \eta} & \left(\frac{\partial |u|^2}{\partial \eta}\right)^2 \end{pmatrix} \partial^2 V / \partial |u|^2$$

and has a zero eigenvalue. The result trivially generalizes to more  $u$  components. This problem is independent of the choice of fermion embeddings. It may be solved introducing two or more right handed neutrinos. Consider  $N_R^{1,2}$ , two neutrino that couple to two different strong sector operators, having representations  $N_R^{1,2} = 5 + \zeta^{1,2}\bar{5}$ . In this case a potential for the singlet field is generated at the leading order by the coupling  $N_R^1 \bar{N}_R^2 \Sigma + h.c.$ . This might lead to interesting phenomenological consequences<sup>4</sup> that are beyond the scope of the present paper.

## 6 Other composite 2HDM's: non-linear realizations, PNGB Lagrangian and $T$ parameter

As pointed out in the previous section, the presence of a term quartic in  $u$  in the metric generates a contribution to the  $T$  parameter already at tree level. This despite the presence of a large residual global symmetry  $H \subset G$  containing the custodial symmetry as a subgroup.

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<sup>4</sup>In particular, the breaking of lepton number by the strong sector should be treated carefully [12]. We thank J. Serra for calling our attention on this point.

In this section we display the PNGB Lagrangian for the different cosets leading to 2HDM's. We consider cosets  $G/H_1 \times H_2$  with  $H_2 = \emptyset, U(1), SU(2)$  and we generalize the approach in section 2. As already noticed above for  $G = SU(5)$ , it is useful to introduce as coset coordinates  $p$  orthogonal unit vectors  $u^\alpha$  ( $\alpha = 1, \dots, p$ ) in the fundamental representation  $\mathbf{N}$  of  $G$  whose scalar products satisfy:

$$u^\dagger_\alpha u^\beta = \delta^\beta_\alpha \quad \text{tr } u^\dagger u \equiv \sum_\alpha u^\dagger_\alpha u^\alpha = p \quad (6.1)$$

where  $p = 1$  for  $H_2 = \emptyset, U(1)$  and  $p = 2$  for  $H_2 = O(2), SU(2)$ . They are functions of the PNGB fields,  $\xi^A$  defined by

$$u(\xi) = e^{i\Pi(\xi)} u(0) \quad \Pi(\xi) = \sum_A \xi_A T^A \quad (6.2)$$

where  $\xi_A$  are the PNGB fields and  $T^A$  are the coset charges, while  $u(0)$  corresponds to the vev('s) of one or more n.l.r. of the Higgs in fundamental representation(s) needed for the breaking.  $G$  acts as  $u \rightarrow g u h_2^\dagger(g, \xi)$ . In particular, as already displayed above for  $G = SU(5)$ , n.l.r. transforming as larger irreducible representations are obtained with  $\Sigma = uu^\dagger$  for the adjoint representation,  $\Sigma = uu^T$  for the symmetric, etc. Notice the contraction of the  $p$  indices associated to the action of  $H_2$ .

For  $G = SU(5)$  the results are already given in (2.9). For the other cosets associated to C2HDM's, we just display the PNGB Lagrangians that are obtained along the same lines. For simplicity, the Lagrangians are arbitrarily normalized.<sup>5</sup> There are two invariants with two derivatives, one quadratic, and the other quartic in the  $u$ 's. By replacing  $U^\dagger \partial_\mu U = i d_\mu^i T^i + i E_\mu^a T^a$  as explained in section 2, we recall that by  $H_1$  invariance of  $u(0)$ , the quadratic term becomes  $\sum d_\mu^i{}^2 + c \sum' E_\mu^a{}^2$ , with  $\sum'$  restricted to the  $H_2$  charges, while only the last term is present in the quartic term (with another coefficient). By the appropriate combination of the two invariants, we thus obtain the CCWZ Lagrangian,  $\sum d_\mu^i{}^2$ , in terms of the  $u$ 's.

First, consider  $SO(6)/SO(4) \times SO(2) \equiv SU(4)/SU(2) \times SU(2) \times U(1)$  and let us first consider the construction in terms of two  $\mathbf{4}$  representations of  $SU(4)$  satisfying the previous rules, namely,  $u_i^\alpha$  ( $\alpha = 1, 2; i = 1, \dots, 4$ ),  $H_2 = SU(2)$ . Then,

$$\mathcal{L}_{\text{PNGB}} = f^2 \text{tr} \left( \partial^\mu u^\dagger \partial_\mu u - u^\dagger \partial_\mu u \partial^\mu u^\dagger u \right) \quad (SU(4)/SU(2)^2 \times U(1)) \quad (6.3)$$

But, this can be also written in terms of two (real) vectors of  $SO(6)$ , denoted  $v_a^\alpha$  ( $\alpha = 1, 2; a = 1, \dots, 6$ ),  $H_2 = SO(2)$  to obtain,

$$\mathcal{L}_{\text{PNGB}} = f^2 \text{tr} \left( \partial^\mu v^T \partial_\mu v - v^T \partial_\mu v \partial^\mu v^T v \right) \quad (SO(6)/SO(4) \times SO(2)) \quad (6.4)$$

which, up to a normalization, is the same as (6.3) if one replaces  $v_a^\alpha = \bar{u}^\alpha \Gamma_a u^\alpha$ , where  $\Gamma_a$  are  $SO(6)$  Dirac matrices.

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<sup>5</sup>Actually the normalization can be fixed the end, having fitted to data the expressions for the gauge boson masses and normalized the Higgs boson kinetic term.

The  $\text{Sp}(6)/\text{Sp}(4) \times \text{SU}(2)$  model has some interesting properties. With  $H_1 = \text{Sp}(4)$  and  $H_2 = \text{SU}(2)$ , the n.l.r. can be displayed in terms of two 6-vectors such that  $u_2 = \Omega u_1^*$  leading to a triplet of quaternions as discussed in appendix D. With the procedure described above we get

$$\mathcal{L} = f^2 \text{tr} \left( \partial^\mu u^\dagger \partial_\mu u - \partial^\mu u^\dagger u u^\dagger \partial_\mu u \right) \quad (\text{Sp}(6)/\text{Sp}(4) \times \text{SU}(2)) \quad (6.5)$$

The  $\text{SO}(9)/\text{SO}(8)$  manifold is the 8-sphere, resulting in the corresponding metric for the Lagrangian for the real 9-vector  $u$ ,

$$\mathcal{L}_{\text{PNGB}} = f^2 \text{tr} \left( \partial^\mu u^T \partial_\mu u \right) \quad (\text{SO}(9)/\text{SO}(8)) \quad (6.6)$$

Here,  $H_1 = \text{SO}(8)$  and, since there is no  $H_2$ , the quadratic terms already contains only the PNGB part.

The next step is to define the embedding of the SM charges inside  $H_1 \times H_2$ . Then, substituting the standard derivatives with the SM covariant derivatives,  $\partial u_i \rightarrow \partial u_i - i(\hat{W}u)_i$ , with  $\hat{W}_\mu = gW_\mu^+ T_L^- + gW_\mu^- T_L^+ + eA_\mu(T_L^3 + T_R^3) + g \sec \theta_W Z_\mu(T_L^3 - \tan^2 \theta_W T_R^3)$ , the gauge boson masses and possible contributions to the  $T$  parameter can be read from the previous expressions. All 2HDM's have  $\text{SU}(2)_L \times \text{SU}(2)_R$  as a subgroup of the linear symmetry of the coset with the Higgs fields transforming in two  $(\mathbf{2}, \mathbf{2})$ , so to define the wanted custodial symmetry. Let us take first  $\text{SU}(2)_L \times \text{SU}(2)_R \subset H_1$ . The mass matrix for the gauge bosons can be written,

$$\frac{\mathcal{M}_{AB}^2}{g_A g_B} = u^\dagger \{T^A, T^B\} u - \kappa u^\dagger T^A u u^\dagger T^B u \quad (6.7)$$

where  $\kappa$  is given in the different  $\mathcal{L}_{\text{PNGB}}$  above, and where only the neutral components are retained, so that  $(T_L^3 + T_R^3)u = 0$ . Since all the states are doublets or singlets of each  $\text{SU}(2)$ ,  $u^\dagger \{T^A, T^B\} u = 1/2 \delta^{AB} \sin^2(\varphi/f)$ . The first term is analogous to the SM gauge bosons mass term. The second term is easily calculated, and gives the additional contribution to the  $Z$ -mass in (3.4) and the corresponding anomalous contribution to the  $T$ -parameter. Although the calculation is straightforward, we justify the results from a group theoretical point of view in appendix B.

Up to the factor  $\kappa$  the result is the same for all models where the SM gauge symmetries are embedded in  $H_1$  and that are strongly constrained by  $T$  unless the Higgs are aligned. The exception are the models based on  $\text{SO}(9)/\text{SO}(8)$  which have only the quadratic part. In view of this success it deserves some limelight and it is further analysed in the next section.

The  $\text{Sp}(6)/\text{Sp}(4) \times \text{SU}(2)$  models are special because, according to the embedding of the SM they can have enough symmetry to preserve the custodial symmetry that protects the  $T$  parameter [11]. Indeed, with the Higgs bosons in the  $(\mathbf{2}, \mathbf{2})$ , one of the factors in  $\text{SU}(2)_L \times \text{SU}(2)_R$  must be in  $H_2 = \text{SU}(2)$  and the other in  $H_1 = \text{Sp}(4)$ . The latter can come from a maximal subgroup  $\text{SU}(2) \times \text{U}(1)$  of  $\text{Sp}(4)$  or from the  $\text{SU}(2) \times \text{SU}(2)$  one. Only the latter has sufficient symmetry to protect  $T$ , since the two Higgs vev's become invariant under the  $\text{SU}(2)_C$  defined by the sums of the charges in the three  $\text{SU}(2)$  factors. This is further discussed in appendix B and the final result is

$$M_W = M_Z \cos(\theta_W) = \frac{gf}{\sqrt{2}} \sin(h/f) \cos(h/f) \quad (6.8)$$



where  $h$  is the combined vev's of the two Higgs. Notice that higher order couplings of the Higgs sector to the gauge bosons are present at tree-level even if  $\rho = 1$  and, also, the factor  $\cos(h/f)$  with respect to the other models.

## 7 A composite two Higgs doublet model based on SO(9)/SO(8)

We turn now to a composite 2HDM based on the symmetry breaking pattern SO(9)/SO(8). As stressed in section 6, there is no tree-level contribution to the  $T$  parameter. The n.l.r. is a 9-vector

$$u = e^{\frac{i\pi}{f}} u_0 = \begin{pmatrix} \frac{\sin(\varphi/f)}{\varphi} \phi \\ \cos(\varphi/f) \end{pmatrix}, \tag{7.1}$$

where  $\varphi = \sqrt{\phi^T \phi}$ , and  $\phi$  the vector containing the eight NGBs belonging to a  $\mathbf{8}_V$  of SO(8).<sup>6</sup> The PNCB Lagrangian has the single term in eq. (6.6). We need to embed the SM gauge group in  $SU(2) \times SU(2) \subset SO(8)$  to include the necessary custodial symmetry, and the two Higgses in the  $\mathbf{8}_V$  transform as two  $(\mathbf{2}, \mathbf{2})$ . The selection of these embeddings are given in appendix C. There are two inequivalent choices that basically differ by a U(1) factor that differentiates between the Higgs doublets:

$$\begin{aligned} \mathbf{8}_V &= (\mathbf{2}, \mathbf{2}) + (\mathbf{2}, \mathbf{2}) \in SU(2) \times SU(2) \subset SO(8) \\ \mathbf{8}_V &= (\mathbf{2}, \mathbf{2})_{+1} + (\mathbf{2}, \mathbf{2})_{-1} \in SU(2) \times SU(2) \times U(1) \subset SO(8) \end{aligned}$$

In the first case, the fermions can couple to both Higgs doublets, which is a source of flavour changing. We do not investigated possible discrete symmetries and concentrate on the second embedding where the Higgs doublets have opposite U(1) charges.

To guarantee a correct electroweak charge assignment for the Higgs doublets, the additional U(1) should not take part in the definition of the hypercharge, still given by  $Y = T_L^3 + T_R^3$ . With these assignments, replacing the SM covariant derivative in eq. (6.6), one obtains the masses of the gauge bosons

$$\mathcal{L}_m = \frac{1}{2} g^2 f^2 \sin^2(\varphi/f) \left( W_\mu^+ W_\mu^- + \frac{1}{2 \cos \theta_W} Z_\mu Z_\mu \right), \tag{7.2}$$

so that there are no tree-level restriction on  $v/f$ , independently of the alignment between the two Higgs doublet in the vacuum.

Let us turn now to the fermions. Once the NGBs are assigned to the  $\mathbf{8}_V$  of SO(8) decomposing under  $SU(2) \times SU(2) \times U(1)$  as  $(\mathbf{2}, \mathbf{2})_{+1} + (\mathbf{2}, \mathbf{2})_{-1}$ , the other two eight-dimensional representations of SO(8) transform under the  $SU(2) \times SU(2) \times U(1)$  subgroup as:  $\mathbf{8}_s = (\mathbf{1}, \mathbf{1})_{+2} + (\mathbf{1}, \mathbf{1})_{-2} + (\mathbf{3}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{3})_0$  and  $\mathbf{8}_c = (\mathbf{2}, \mathbf{2})_{-1} + (\mathbf{2}, \mathbf{2})_{+1}$ . The assignments for  $U(1)_X$  are parallel to those in section 4. Barring larger representations, there are two choices for the fermions, which are made explicit below together with, in brackets, their embeddings in  $SO(9) \times U(1)_X$ :

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<sup>6</sup>We use here the conventions for the SO(8) representations and subgroups from [24]

1.  $t_L \in \mathbf{8}_V(\in \mathbf{9}_{2/3})$ ,  $b_L \in \mathbf{8}_V(\in \mathbf{9}_{-1/3})$ ,  $b_R \in \mathbf{1}(\in \mathbf{1}_{-1/3})$ ,  $t_R \in \mathbf{1}(\in \mathbf{1}_{2/3})$ . A brane mass term allows for the reconstruction of the  $q_L$  doublet as sketched in section 4. The Yukawa couplings are the singlets in the products  $\mathbf{8}_V \times \mathbf{8}_V \times \mathbf{1} \in \mathbf{9} \times \mathbf{9} \times \mathbf{1}$ .
2.  $q_L \in \mathbf{8}_c(\in \mathbf{9}_{2/3})$ ,  $t_R, b_R \in \mathbf{8}_s(\in \mathbf{8}_{s2/3})$  so that all fermions transform under  $SO(9) \times U(1)_X$  as a  $\mathbf{16} = \mathbf{8}_s + \mathbf{8}_c$  of  $SO(8)$ , both  $t_R$  and  $b_R$  belonging to the  $(\mathbf{1}, \mathbf{3})$  of  $SU(2) \times SU(2)$ . The Yukawa invariants are in the products  $\mathbf{8}_V \times \mathbf{8}_s \times \mathbf{8}_c \in \mathbf{9} \times \mathbf{16} \times \mathbf{16}$ .

The latter seems more attractive, in particular because it is naturally consistent with gauge-Higgs unification scenarios. In both cases, the fermions couple only to the Higgs doublet with opposite  $U(1) \subset SO(8)$  charge, so that the other one is a so-called inert Higgs.<sup>7</sup> Therefore the FCNC effects are suppressed because each fermion couples to only one and the same Higgs doublet and each fermion can be uniquely embedded in  $SO(8) \times U(1)$  representations. The model satisfies all the items in our fermion shopping list at the beginning of section 4.

## 8 Conclusions

In this paper we have studied some composite Higgs models with extended scalar sectors. A detailed analysis has been carried out for the cosets  $SU(5)/SU(4) \times U(1)$ , and  $SO(9)/SO(8)$ , which deliver two SM doublets or, for  $SU(5)/SU(4)$ , two SM doublets and one singlet.

In both the  $SU(5)$  cases, a contribution to the  $T$  parameter is present already at tree level unless there is an unexpected alignment of vevs in the scalar potential. A survey of various C2HDM's shows that this is naturally expected in other examples of this class of models, unless the symmetries defining the coset are large enough so that any element in the coset is invariant under an  $SU(2)$ , so defining the custodial symmetry. We observe that two specific cosets,  $Sp(6)/Sp(4) \times SU(2)$  and  $SO(9)/SO(8)$  exhibit such a group theoretical property, hence no invariants contributing to  $T$  can be written. This ensures an effective custodial symmetry at tree level independent from the scalar potential.

Turning to fermions, we analyzed the embedding of the matter fields in the smallest  $SU(5)$  and  $SO(9)$  representations, paying attention to the effects on the  $Zb\bar{b}$  vertex and on the presence of Higgs-mediated FCNC. In the  $SU(5)$  cases, we find that  $Zb\bar{b}$  is protected only when the SM fermions are embedded in a  $\mathbf{10}$ , while Higgs-mediated FCNC are always absent because the non linear Higgs has a single coupling to fermions. The up and down type quarks couple to different linear combinations of the Higgs doublets, so that the latter are both active. However the  $t$  quark coupling tends to misalign the Higgs vev's and increases the contribution to  $T$ . Therefore, these  $SU(5)$  models do need a compensation mechanism to be phenomenologically viable.

In the  $SO(9)$  case an additional abelian charge embedded in  $SO(8)$  can be used to ensure the fermions to be coupled to just one of the doublets, making the other one inert. Moreover, when the left handed quark doublet is embedded in the spinorial representation, also the  $Zb\bar{b}$  vertex is protected.

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<sup>7</sup>For a recent discussion of inert Higgs phenomenology see [11].

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## A Embedding of the Standard Model fermions in SU(5) representations

The SM charges, embedded in the first four entries of the  $5 \times 5$  SU(5), are given by

$$T_L^i = 1_{2 \times 2} \times \tau^i, \quad T_R^3 = \tau^3 \times 1_{2 \times 2} \quad (\text{A.1})$$

with  $\tau^i$  the usual SU(2) generators. In terms of  $(T_L^3, T_R^3)$  charges, this choice corresponds to a SU(5) fundamental decomposed as

$$\mathbf{5} = \begin{pmatrix} (+, +) \\ (-, +) \\ (+, -) \\ (-, -) \\ (0, 0) \end{pmatrix}.$$

The SU(5) fundamental decomposes as  $\mathbf{5} = (\mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{1})$  under  $SU(2)_L \times SU(2)_R$ . For SU(5)/SU(4), the two LH quarks must be embedded in different bidoublets:

$$\begin{aligned} q_L^t &\in (\mathbf{2}, \mathbf{2})_{2/3} && \text{with } t_L = (+, -) \\ q_L^b &\in (\mathbf{2}, \mathbf{2})_{-1/3} && \text{with } b_L = (-, +) \\ u_R &\in (\mathbf{1}, \mathbf{1})_{2/3} \\ d_R &\in (\mathbf{1}, \mathbf{1})_{-1/3} \end{aligned}$$

where in the last column we specified the  $(T_L^3, T_R^3)$  assignment.

The specific embeddings are

$$\Psi_{q_L^t} = \begin{pmatrix} 0_{2 \times 1} \\ q_L \\ 0 \end{pmatrix}_{2/3} \quad \Psi_{q_L^b} = \begin{pmatrix} q_L \\ 0_{2 \times 1} \\ 0 \end{pmatrix}_{-1/3} \quad \Psi_{t_R} = \begin{pmatrix} 0_{2 \times 1} \\ 0_{2 \times 1} \\ t_R \end{pmatrix}_{2/3} \quad \Psi_{b_R} = \begin{pmatrix} 0_{2 \times 1} \\ 0_{2 \times 1} \\ b_R \end{pmatrix}_{-1/3} \quad (\text{A.2})$$

Turning to the SU(5) symmetric representation, it corresponds to  $\mathbf{10} = (\mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{3}) + (\mathbf{3}, \mathbf{1})$  under  $SU(2)_L \times SU(2)_R$ . To accommodate hypercharge, the  $U(1)_X$  charge has to be fixed to  $X = 2/3$ , with assignment

$$\begin{aligned} q_L &\in (\mathbf{2}, \mathbf{2})_{2/3} && \text{with } t_L = (+, -), b_L = (-, -) \\ u_R &\in (\mathbf{1}, \mathbf{3})_{2/3} && \text{with } t_R = (0, 0) \\ d_R &\in (\mathbf{1}, \mathbf{3})_{2/3} && \text{with } b_R = (0, -1) \end{aligned}$$

The explicit embeddings are

$$\begin{aligned}
 \Psi_{q_L} &= \frac{1}{2} \left( \begin{array}{c|c} 0_{4 \times 4} & 0_{2 \times 1} \\ \hline 0_{1 \times 2} & q_L^T \\ & 0 \end{array} \right), \\
 \Psi_{t_R} &= \frac{t_R}{2} \left( \begin{array}{c|c} 0_{2 \times 2} & i\sigma_2 \\ \hline i\sigma_2 & 0_{2 \times 2} \\ & 0_{2 \times 1} \\ & 0 \end{array} \right), \\
 \Psi_{d_R} &= \frac{b_R}{\sqrt{2}} \left( \begin{array}{c|c} 0_{2 \times 2} & 0_{2 \times 2} \\ \hline 0_{2 \times 2} & i\sigma_2 \\ & 0_{2 \times 1} \\ & 0 \end{array} \right). \tag{A.3}
 \end{aligned}$$

## B Algebraic analysis of the $T$ -parameter

In this appendix we give a more detailed analysis of the terms corresponding to anomalous  $T$  parameters. The so-called  $D$ -terms,  $D^A = u^\dagger T^A u$ , where  $T^A$  are the generators of a group  $G$  in the (arbitrary) representation of  $u$ , have been studied in the context of supersymmetry. For completeness, this appendix contains a the study of the anomalous  $T$  parameter defined in (6.7) as  $D^A D_A$ , based on [25], from where we borrow the following lemma.

*Given  $u$  defined by its little group  $H \subset G$ ,  $D_A = u^\dagger T^A u \neq 0$ , if and only if  $T^A \in G/H$  and commutes with the elements of  $H$ .*

This lemma has been very useful in the study of symmetry breaking in supersymmetric theories where the fields are complex, even when they transform in real representations if they belong to chiral supermultiplets. Here it is just a curiosity since the calculations are straightforward.

To apply it in our case, let us take  $G = \text{SU}(2) \times \text{SU}(2)$  and  $u \in (\mathbf{2}, \mathbf{2})$ , and write it as a vector  $u_i = x_i + y_i$ , ( $i = 1, \dots, 4$ ) such that the generators are real, antisymmetric and act on  $u$  as a 4-rotation,  $T^{ij} u_k = \delta^{jk} u_i - \delta^{ik} u_j$ . Notice that any  $u$  can be rotated in the plane  $u_1 = u_2 = 0$ , with little group  $H = \text{U}(1)$  generated by the (electric) charge,  $T^{12}$ . Then  $T^{34}$  is the only charge in the coset  $G/H$  that commutes with  $T^{12}$ . Indeed,  $u^\dagger T^{34} u = i(y_3 x_4 - y_4 x_3)$  is the only non-vanishing component of the  $D$ -terms and, in this basis,  $T^{34}$  is the charge corresponding to  $Z$  and the anomalous contribution to the  $Z$  mass is proportional to  $(D^{34})^2$ .

Now consider a critical  $u$ , such that  $x_i$  and  $y_i$  are aligned and can be rotated to  $u_1 = u_2 = u_3 = 0$ . Its little group is the vectorial (or custodial)  $H = \text{SU}(2)$  of rotations in this hyperplane. The coset  $G/H$  is given by the three axial charges that transform as a triplet under  $H$ . Therefore all  $D^{ij} = 0$  and there is no anomalous term in the  $Z$  mass. The conclusion is that the complex character of the  $u$  coordinates, related to  $H_2 = \text{U}(1)$  in the cosets defining the Higgs doublet as PngB, is at the origin of the anomalous  $T$  parameter in these cases, unless one can ensure the two Higgs alignment.

The case where  $\text{SU}(2) \times \text{SU}(2)$  is embedded in both  $H_1$  and  $H_2$ , is different since now the equivalent of the  $D$ -terms  $D^A = u^\dagger T^A u$ , are matrices. Let us concentrate on the relevant instance of  $\text{Sp}(6)/\text{Sp}(4) \times \text{SU}(2)$  and choose the embedding of the SM subgroup through

the  $SU(2)^2 \in Sp(4)$ , hence  $SU(2)^3 \in Sp(6)$ , so that  $u$  decomposes as  $(\mathbf{2}, \mathbf{1}, \mathbf{2}) + (\mathbf{1}, \mathbf{2}, \mathbf{2})$  while the bottom components are invariant. Due to the large symmetry, the two Higgs in  $u$  can be aligned so that the generic  $u$  encoding the two Higgs PNGB's has a little group the custodial group defined, with some convention, as the sum of the three  $SU(2)$  charges, defining the custodial  $SU(2)_C$ . Therefore, the  $T$  parameter must be non anomalous.

The two Higgs behave as  $(\mathbf{2}, \mathbf{2})$  if we choose  $SU(2)_L \in$  the sum of those in  $H_1 = Sp(4)$  and  $SU(2)_R$  as the remaining one in  $H_2$ , or vice-versa. We recall that the latter 'acts from the right'. After the gauging of the SM symmetries one immediately finds the masses of the gauge bosons by rotating the Higgs quaternions in  $u$  to the element  $u^T = (u_0 \sin \beta \mathbf{1}_2, u_0 \cos \beta \mathbf{1}_2, \cos(h/f) \mathbf{1}_2)$ ,  $u_0 = h \sin(h/f)$ , and  $h$  is the usual combination of the Higgs vev's. The result,

$$\frac{f^2}{2} \left( g^2 W_\mu^+ W^{-\mu} + (gW_\mu^0 - g'B_\mu)^2 \right) \sin^2(h/f) \cos^2(h/f), \tag{B.1}$$

where the term in  $\sin^4(h/f)$  comes from the quartic term in the PNGB lagrangian, is clearly consistent with the custodial symmetry, introducing a correction factor  $\cos^2(h/f)$  with respect to the results in the other 2HDM cosets.

### C Embedding of the Standard Model charges in $SO(9)$

In this appendix we describe the embeddings of the  $SO(9)/SO(8)$  coset (PNGB)  $\mathbf{8}_V = (\mathbf{2}, \mathbf{2}) + (\mathbf{2}, \mathbf{2}) \in SU(2) \times SU(2) \subset SO(8)$ . A systematic way to find them is to follow all the chains of maximal subgroups starting from those of  $SO(8)$  down to those that contain  $SU(2) \times SU(2)$  and discard all those that lead to the wrong decomposition for the  $\mathbf{8}_V$  of PNGB. We follow the conventions in [24] and the results are easily checked from the tables therein. We simplify the notations by writing  $SU(2)^n$  for the  $\times$  product of  $n$   $SU(2)$ 's.

Two of the so-defined chains, namely  $SO(8) \rightarrow SU(4) \times U(1) \rightarrow SU(2)^2 \times U(1)$  and  $SO(8) \rightarrow Sp(4) \times SU(2) \rightarrow SU(2)^2 \times U(1)$ , yield the decomposition:

$$\mathbf{8}_V = (\mathbf{2}, \mathbf{2})_{+1} + (\mathbf{2}, \mathbf{2})_{-1} \quad \mathbf{8}_c = (\mathbf{2}, \mathbf{2})_{-1} + (\mathbf{2}, \mathbf{2})_{+1} \tag{C.1}$$

$$\mathbf{8}_s = (\mathbf{1}, \mathbf{1})_{+2} + (\mathbf{1}, \mathbf{1})_{-2} + (\mathbf{3}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{3})_0 \tag{C.2}$$

where the  $U(1)$  charges are also displayed for each  $SU(2)^2$  representation.

Three other chains, namely,  $SO(8) \rightarrow SO(7) \rightarrow SU(4) \rightarrow SU(2)^2$ ,  $SO(8) \rightarrow SO(7) \rightarrow SU(2)^3 \rightarrow SU(2)^2$  and  $SO(8) \rightarrow SU(2)^4 \rightarrow SU(2)^2$ , have the same  $SU(2)^2$  decompositions, but since there is no  $U(1)$  factor, there is no distinction between the two  $(\mathbf{2}, \mathbf{2})$ 's or the two  $(\mathbf{1}, \mathbf{1})$ 's.

Finally, note that besides the quadratic invariant for each  $\mathbf{8}$ , there is the invariant  $\mathbf{8}_s \mathbf{8}_V \mathbf{8}_c$  that will be useful in the analysis of Yukawa couplings.

### D Analysis of the $Sp(6)/Sp(4) \times SU(2)$ coset

$Sp(6)$  is the group of unitary transformations such that  $\Omega \equiv 1_3 \times \sigma_2$  is invariant, i.e.,  $U\Omega U^T = \Omega$ . It is useful to work in terms of quaternions:  $Q = q^\alpha e_\alpha$ , with  $e_i = i\sigma_i$ ,  $e_4 = 1_2$ . Notice that:  $Q^\dagger Q = \sum q^{i2} 1_2$ ,  $\sigma_2 Q = Q^* \sigma_2$ . Then, the  $Sp(6)$  algebra can be written as the

$3 \times 3$  hermitian quaternion matrices with elements  $iQ_{ij}$  such that  $iQ_{ii}^4 = 0$ . The diagonal elements  $iQ_{ii}$ , ( $i = 1, 2, 3$ ) are then associated with the  $SU(2)^3$  sub algebra. Adding only  $iQ_{12}$  it is upgraded to  $Sp(4) \times SU(2)$ , while  $iQ_{13}$  and  $iQ_{23}$  define the coset  $Sp(6)/Sp(4) \times SU(2)$ , transforming as a  $(\mathbf{4}, \mathbf{2})$  of  $Sp(4) \times SU(2)$ . In particular, the associated PNGB matrix  $\Pi$  can be written as

$$\Pi = \left( \begin{array}{c|c} \mathbf{0}_4 & \begin{matrix} -i\mathbf{H}_1 \\ -i\mathbf{H}_2 \end{matrix} \\ \hline \begin{matrix} i\mathbf{H}_1^\dagger & i\mathbf{H}_2^\dagger \end{matrix} & \mathbf{0}_2 \end{array} \right) \quad (\text{D.1})$$

where the  $\mathbf{H}_i \equiv h_i^\alpha e_\alpha$  are the two Higgs doublets reshuffled into the familiar quaternion form.

We then define the vector with quaternion components  $u_i$ , ( $i = 1, 2, 3$ ) such that:

$$u = e^{i\Pi(\mathbf{H}_i)} u(0) = \frac{\sin(h/f)}{h} \begin{pmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \cot(h/f) \mathbf{1} \end{pmatrix} \quad u(0) = \begin{pmatrix} \mathbf{0}_2 \\ \mathbf{0}_2 \\ \mathbf{1}_2 \end{pmatrix} \quad (\text{D.2})$$

where  $h^2 = \text{tr} \sum_{i=1, \dots, 4} (h_1^2 + h_2^2)$  so that it transforms as defined in section 6, namely,  $u \rightarrow guh_2^\dagger(g, \mathbf{H}_i)$  with  $g \in G$  and  $h_2 \in H_2$ . Therefore  $u$  can be also viewed as two orthogonal 6-vectors  $u_\alpha$ ,  $\alpha = 1, 2$ , each one transforming as the  $\mathbf{6}$  of  $Sp(6)$ , with  $u_2 = i\Omega u_1^*$ . Notice that, as quaternions,  $\Omega u = u^* \sigma_2$ .

Another possible parametrization for the breaking  $Sp(6) \rightarrow Sp(4) \times SU(2)$  is given by the vacuum

$$\Sigma_0 = \begin{pmatrix} \mathbf{0}_2 & & \\ & \mathbf{0}_2 & \\ & & \sigma_2 \end{pmatrix}, \quad (\text{D.3})$$

in an antisymmetric  $\mathbf{14}$  of  $Sp(6)$ . Then the n.l.r. can be written in terms of the PNGB as  $\Sigma = U\Sigma_0 U^T = u\sigma_2 u^T$ .

Following the procedure described in sections 2 and 6 to combine the invariants to have equivalence with the CCWZ construction, the kinetic Lagrangian results

$$\mathcal{L} = \frac{f^2}{4} \left( \partial^\mu u^\dagger \partial_\mu u - \partial^\mu u^\dagger u u^\dagger \partial_\mu u \right), \quad (\text{D.4})$$

so that a tree-level contribution to  $T$  is expected from the second term.

Using the  $SU(2)^3$  symmetry both quaternions  $\mathbf{H}_1$  and  $\mathbf{H}_2$  can be rotated and aligned along their components  $e_4 = \mathbf{1}_2$ , which is invariant under a (diagonal)  $SU(2)$ , the custodial symmetry. The effectiveness of custodial symmetry can also be explicitly checked substituting the ordinary derivatives in D.4 with covariant ones:  $\partial_\mu u \rightarrow \partial_\mu u + iW_\mu^a (T_L^a u - u T_L^a) + iB_\mu Y u$ . In the notation of the previous sections, we are specializing to the case in which  $U(1)_Y \subset Sp(4)$  and  $SU(2)_L = H_2$ , so that the latter acts on  $u$  both from the left and from the right. It is immediate to write the covariant derivative for the other case.

Turning to the fermions, the smaller  $Sp(6)$  representations decompose under  $SU(2) \times Sp(4)$  as

$$\begin{aligned} \mathbf{14} &= (\mathbf{2}, \mathbf{4}) + (\mathbf{1}, \mathbf{5}) + (\mathbf{1}, \mathbf{1}) \\ \mathbf{21} &= (\mathbf{2}, \mathbf{4}) + (\mathbf{1}, \mathbf{10}) + (\mathbf{3}, \mathbf{1}) \end{aligned} \quad (\text{D.5})$$

Since there are several ways to embed the SM fermions, a severe problem of Flavor Changing Neutral Currents is present.

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