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# RESEARCH

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# Control for Markov sampled-data systems with event-driven transmitter

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# Abstract

In this paper, feedback control problem is considered for networked systems with discrete, infinite distributed delays and sampled-data. A Markov chain is used to characterize the random sampled measurement process of the networked control systems. In addition, an event-driven transmitter is introduced to transmit the control signal according to the measurement sampling period. Based on Lyapunov functional and the matrix analysis techniques, several sufficient conditions are given to ensure the asymptotical stability in the mean square of the addressed control systems. Furthermore, a novel output feedback controller is proposed with both sampling and event-driven transmitter-induced delay indexes. Finally, a simulation example is provided to illustrate the effectiveness of the theoretical results and the proposed method.

Keywords: stabilization; event-trigger; time-delay; sampled-data

# 1 Introduction

In the traditional feedback control systems, the connections between system components are established by point-to-point cables. Compared to the traditional point-to-point systems, networked control systems are real-time control systems where sensors, actuators and controllers are interconnected by a shared digital communication network. Networked control systems (NCSs) offer many advantages such as lower cost, simpler installation, easier maintenance, and resource sharing [1]. And NCSs have great applications in aircrafts and spacecrafts control, robotics, process control and vehicles [2]. Therefore, the field of NCSs has been becoming a hot research topic [3–5].

Since digital microprocessors are quickly becoming indispensable in practical applications, control designing problems of systems tend to be implemented on digital platforms [6, 7]. The periodic sampling leads to conservativeness in the usage of computational resource and bandwidth, because the constant sampling period is chosen to guarantee stability in the worst case [8]. For reducing the usage of computational resource and limited bandwidth, the nonuniform sampler was employed in the implementations of NCSs. Recently, several initial attempts have been proposed to study the stability of NCSs with nonuniformly sampled systems [9, 10].

On the other hand, time delays widely exist in practical systems due to the unreliable communication channel [11, 12]. It is well known that time delay makes the analysis and synthesis of NCSs more complex and important. And time delay is also the major cause



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for NCS performance deterioration and potential system instability [13]. Discrete time delay is common [14], Liou and Ray proposed the synthesis of a stochastic regulator in the presence of randomly varying delays from the controller to actuator [15]. Distributed time delay  $\sum_{r=1}^{+\infty} \mu_r x(k-r)$  is another important delay, which has recently drawn much research interest when modeling a realistic complex system [16].

With the very interesting results reported in [17, 18], it is seen that, in some cases, the activities of sensors/actuators are even triggered by events characterizing stochastic processes, *e.g.*, Markov process [19, 20]. Experimental result [21] shows that the event-triggered control scheme can efficiently reduce the number of control task executions so that communication resources can be saved significantly while retaining satisfactory closed-loop performance.

In practical engineering, discrete and distributed time delays always appear simultaneously in the systems, and the measurement, communication and control updates need the nonuniform sampler. It is therefore essential and challenging to investigate the control for Markov sampled-data systems with event-driven transmitter, which has great potential in practical applications. Therefore, for the mixed time-delay NCSs, an interesting problem is to find a co-design method of the event-triggered control scheme in this paper. In consequence of the above discussion, the networked-based feedback control problem with event-driven transmitter is investigated for NCSs. The main contributions of this paper are the following ones: (1) we consider nonuniform sample data, discrete and distributed timedelays, and present criteria for ensuring stochastic stability of the closed-loop networked system; (2) A novel output feedback controller incorporating both Markov-based sampling,  $\sum_{r=1}^{+\infty} \mu_r x(k - r)$  and event-driven transmitter-induced delay indexes is proposed.

## 2 System description

Consider the following networked control system:

$$\begin{cases} x(k+1) = Ax(k) + Bx(k-d(k)) + C \sum_{r=1}^{+\infty} \mu_r x(k-r) + Du(k), \\ y(\theta_i) = Ex(\theta_i), \quad \theta_i \in \{kh\}, k, i \in N, \end{cases}$$
(1)

where  $x(k) \in \mathbb{R}^n$  is the system state,  $u(k) \in \mathbb{R}^q$  is the control input, and  $y(\theta_i) \in \mathbb{R}^q$  is the measurement sampled at arbitrary instants  $\theta_i$  ( $0 = \theta_0 < \theta_1 < \cdots < \theta_i < \cdots$ ). *A*, *B*, *C*, *D* and *E* are known real matrices with appropriate dimensions. The positive integer d(k) denotes the time-varying delay satisfying  $d_1 \le d(k) \le d_2$ ,  $k \in \mathbb{N}^+$ , where  $d_1$  and  $d_2$  are known positive integers. The constants  $\mu_r \ge 0$  (r = 1, 2, ...) satisfy  $\sum_{r=1}^{+\infty} r\mu_r < +\infty$  and  $\overline{\mu} \triangleq \sum_{r=1}^{+\infty} \mu_r < +\infty$ .

The schematic diagram for system (1) is shown in Figure 1. System (1) is derived by discretizing the original continuous-time system under the sampling period *h*. The measurement cannot be sampled any instant *kh*. Then the previous measurement y(k-1) is applied in the controller side, and the measurement received by the controller can be expressed as follows:

$$y(k) = Ex(k - \tau_k), \quad \theta_i \le k < \theta_{i+1}, \tag{2}$$

where the sampling-induced delay index  $\tau_k \triangleq k - \theta_i$  is still a random variable, and  $\tau_k$  is assumed to be governed by a Markov chain. Moreover, the transition probability matrix



### $\Pi$ is defined as follows:

$$\Pi = \begin{bmatrix} \pi_0 & 1 - \pi_0 & 0 & \cdots & 0 \\ \pi_1 & 0 & 1 - \pi_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \pi_{N-1} & 0 & 0 & \cdots & 1 - \pi_{N-1} \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix},$$
(3)

where  $\pi_i \in \{0, 1\}, i \in \{0, 1, \dots, N-1\}$ .

### 3 Observer-based output feedback control

Suppose that the system output is available and will be sampled before it is transmitted to the controller through network. Then there is the sampling-induced delay in the sensor-to-controller channel. Certainly, in the controller-to-actuator channel, there is the event-driven induced delay. Therefore, both sampling-induced delay and event-driven induced delay indexes are taken into account to design the output feedback controller:

$$\begin{cases} \hat{x}(k+1) = A\hat{x}(k) + B\hat{x}(k-d(k)) + Du(k) + C\sum_{r=1}^{+\infty} \mu_r \hat{x}(k-r) \\ + L_{\tau_{k-1},\tau_k}(y(k-\tau_k) - \hat{y}(k)), \\ \hat{y}(k) = E\hat{x}(k), \quad \hat{x}(0) = 0, \\ u(k) = K_{\tau_{k-1},\tau_k} \hat{x}(k-\tau_{k-1}), \end{cases}$$
(4)

where  $\hat{x}(k) \in \mathbb{R}^n$ ,  $\hat{y}(k) \in \mathbb{R}^q$  are the state and output of the controller, respectively.  $K_{\tau_{k-1},\tau_k} \in \mathbb{R}^{m \times n}$  is the controller gain, and  $L_{\tau_{k-1},\tau_k} \in \mathbb{R}^{n \times q}$  is the observer gain needed to be designed, where

$$\tau_{k-1} = \begin{cases} k-1-\theta_i & \text{if } \theta_i < k < \theta_{i+1}, \\ k-1-\theta_{i-1} & \text{if } k = \theta_i. \end{cases}$$
(5)

Define the following augmented state vector X(k),  $\hat{X}(k)$ , X(k - d(k)),  $\hat{X}(k - d(k))$ , and the error vector  $\boldsymbol{e}(k) = X(k) - \hat{X}(k)$ ,  $\boldsymbol{e}(k - d(k)) = X(k - d(k)) - \hat{X}(k - d(k))$  and

$$X(k) = \begin{pmatrix} x^T(k) & x^T(k-1) & \cdots & x^T(k-N) \end{pmatrix}^T,$$
$$\hat{X}(k) = \begin{pmatrix} \hat{x}^T(k) & \hat{x}^T(k-1) & \cdots & \hat{x}^T(k-N) \end{pmatrix}^T.$$

The augmented system of (1) and (4) can be represented as

$$X(k+1) = \bar{A}X(k) + \bar{D}\bar{K}_{\tau_{k-1},\tau_k}R_{\tau_{k-1}}\hat{X}(k) + \bar{B}X(k-d(k)) + \bar{C}\sum_{r=1}^{+\infty}\mu_r X(k-r),$$

$$\hat{X}(k+1) = (\bar{A} - \bar{L}_{\tau_{k-1},\tau_k}\bar{E})\hat{X}(k) + \bar{B}\hat{X}(k-d(k)) + \bar{C}\sum_{r=1}^{+\infty}\mu_r\hat{X}(k-r)$$

$$+ \bar{L}_{\tau_{k-1},\tau_k}\bar{E}R_{\tau_k}X(k) + \bar{D}\bar{K}_{\tau_{k-1},\tau_k}R_{\tau_{k-1}}\hat{X}(k),$$
(6)

where

$$\begin{split} \bar{A} &= \begin{bmatrix} A & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix}, \qquad \bar{B} = \begin{bmatrix} B & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix}, \qquad \bar{D} = \begin{bmatrix} D \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \\ \bar{C} &= \begin{bmatrix} C & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix}, \qquad \bar{E} = \begin{bmatrix} E^T \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T, \qquad \bar{K}_{\tau_{k-1},\tau_k} = \begin{bmatrix} K_{\tau_{k-1},\tau_k} \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T, \\ \bar{L}_{\tau_{k-1},\tau_k} &= \begin{bmatrix} L_{\tau_{k-1},\tau_k} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \qquad R_{\tau_k} = \begin{bmatrix} 0 & \cdots & 0 & I_{\tau_k} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}. \end{split}$$

The closed-loop system based on the augmented systems (6) and (7) with  $Z(k) = [X^T(k)\mathbf{e}^T(k)]^T$  can be further expressed as follows:

$$Z(k+1) = \hat{A}Z(k) + \hat{B}Z(k-d(k)) + \hat{C}\sum_{r=1}^{+\infty}\mu_r Z(k-r),$$
(8)

where

$$\hat{A} = \begin{bmatrix} A^* & -\bar{D}\bar{K}_{\tau_{k-1},\tau_k}R_{\tau_{k-1}}\\ \bar{L}_{\tau_{k-1},\tau_k}\bar{E} & \bar{A} - \bar{L}_{\tau_{k-1},\tau_k}\bar{E} \end{bmatrix}, \qquad \hat{B} = \begin{bmatrix} \bar{B} & 0\\ 0 & \bar{B} \end{bmatrix}^T, A^* = \bar{A} + \bar{D}\bar{K}_{\tau_{k-1},\tau_k}R_{\tau_{k-1}} - \bar{L}_{\tau_{k-1},\tau_k}\bar{E}R_{\tau_k}, \qquad \hat{C} = \begin{bmatrix} \bar{C} & 0\\ 0 & \bar{C} \end{bmatrix}^T.$$

# 4 Main results

In the following, the main results of this paper will be presented, which can be used to study the asymptotical stability in the mean square of system (8).

**Theorem 1** Considering the discrete system (1), the closed system (8) with controller (4) is asymptotically stable in the mean square if there exist matrices  $P_i > 0$ ,  $\Omega > 0$ , W > 0,  $K_{i,j}$ 

*and*  $L_{i,j}$  (*i*, *j* = 0, 1, ..., *N*) *satisfying* 

$$\begin{bmatrix} \Phi & \hat{A}^T \bar{P}_i \hat{B} & \hat{A}^T \bar{P}_i \hat{C} \\ * & \hat{B}^T \bar{P}_i \hat{B} - \Omega & \hat{B}^T \bar{P}_i \hat{C} \\ * & * & \hat{C}^T \bar{P}_i \hat{C} - \frac{1}{\bar{\mu}} W \end{bmatrix} < 0,$$

$$\tag{9}$$

where  $\Phi = \hat{A}^T \overline{P}_i \hat{A} - P_i + (1 + d_1 + d_2)\Omega + \overline{\mu} W$ .

*Proof* Construct the following Lyapunov-Krasovskii function:

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k),$$
(10)

where

$$V_{1}(k) = Z^{T}(k)P_{\tau_{k-1}}Z(k), \qquad V_{2}(k) = \sum_{r=k-d(k)}^{k-1} Z^{T}(r)\Omega Z(r),$$
$$V_{3}(k) = \sum_{s=k-d_{2}+1}^{k-d_{1}} \sum_{r=s}^{k-1} Z^{T}(r)\Omega Z(r), \qquad V_{4}(k) = \sum_{r=1}^{+\infty} \mu_{r} \sum_{s=k-r}^{k-1} Z^{T}(s) WZ(s),$$

where  $P_{\tau_{k-1}} > 0$ ,  $\Omega > 0$ , W > 0. Then let  $\tau_{k-1} = i$  and  $\tau_k = j$ . Calculating the difference of V(k) along system (8) and taking the mathematical expectation  $\mathbb{E}\{\Delta V_i(k)\} = \mathbb{E}\{V_i(k+1)|k\} - V_i(k)$ , we have

$$\mathbb{E}\left\{\Delta V_{1}(k)\right\} = \mathbb{E}\left\{V_{1}(k+1)|k\right\} - V_{1}(k) = W_{P}^{T}\bar{P}_{i}W_{P} - Z^{T}(k)P_{i}Z(k),\tag{11}$$

where  $\bar{P}_i = \sum_{j=0}^N \pi_{ij} P_j$ ,  $W_P = \hat{A}Z(k) + \hat{B}Z(k - d(k)) + \hat{C} \sum_{r=1}^{+\infty} \mu_r Z(k - r)$ .

$$\mathbb{E}\left\{\Delta V_{2}(k)\right\} = \sum_{r=k+1-d(k)}^{k} Z^{T}(r)\Omega Z(r) - \sum_{r=k-d(k)}^{k-1} Z^{T}(r)\Omega Z(r)$$

$$\leq \sum_{r=k-d_{2}+1}^{k-d_{1}} Z^{T}(r)\Omega Z(r) - Z^{T}(k-d(k))\Omega Z(k-d(k))$$

$$+ Z^{T}(k)\Omega Z(k), \qquad (12)$$

$$\mathbb{E}\left\{\Delta V_{3}(k)\right\} = \sum_{s=k+2-d_{2}}^{k+1-d_{1}} \sum_{r=s}^{k} Z^{T}(r)\Omega Z(r) - \sum_{s=k+1-d_{2}}^{k-d_{1}} \sum_{r=s}^{k-1} Z^{T}(r)\Omega Z(r)$$

$$= (d_{2} - d_{1})Z^{T}(k)\Omega Z(k) - \sum_{s=k-d_{2}+1}^{k-d_{1}} Z^{T}(s)\Omega Z(s), \qquad (13)$$

$$\mathbb{E}\left\{\Delta V_{4}(k)\right\} = -\frac{1}{\bar{\mu}} \left(\sum_{r=1}^{+\infty} \mu_{r} Z(k-r)\right)^{T} W\left(\sum_{r=1}^{+\infty} \mu_{r} Z(k-r)\right)$$

$$+ \bar{\mu} Z^{T}(k) W Z(k). \qquad (14)$$

A combination of (11)-(14) leads to

$$\mathbb{E}\{\Delta V(k)\} \le \xi^T(k)\Psi\xi(k),\tag{15}$$

where

$$\begin{split} \Psi &= \begin{bmatrix} \Phi & \hat{A}^T \bar{P}_i \hat{B} & \hat{A}^T \bar{P}_i \hat{C} \\ * & \hat{B}^T \bar{P}_i \hat{B} - \Omega & \hat{B}^T \bar{P}_i \hat{C} \\ * & * & \hat{C}^T \bar{P}_i \hat{C} - \frac{1}{\bar{\mu}} W \end{bmatrix}, \\ \xi(k) &= \begin{bmatrix} Z^T(k) & Z^T(k-d(k)) & \sum_{r=1}^{+\infty} \mu_r Z^T(k-r) \end{bmatrix}^T. \end{split}$$

Hence, by inequality (9) in Theorem 1, we can get  $\mathbb{E}\{\Delta V(k)\} \le 0$ . From Lyapunov stability theory, we can conclude that closed system (8) with controller (4) is asymptotically stable in the mean square. The proof of Theorem 1 has been completed.

Next, we present the results on the solvability of the control problem based on Theorem 1, where the cone complementarity linearization approach is introduced to deal with the constraint. The main result is concluded in the following theorem by using the Schur complement method and letting  $G_0 = \bar{P}_0^{-1}$ ,  $G_{i+1} = \bar{P}_{i+1}^{-1}$ .

**Remark 1** In [4], Markov-based sample data was reflected on the networked control systems, in which the sampling-induced delay index was modeled by a Markov chain. Different from [4], our model includes discrete time-delays, distributed time-delays  $(\sum_{r=1}^{+\infty} \mu_r x(k-r))$  and Markov-based sample data.

**Remark 2** Unlike the method in [4], Theorem 1 gives the sufficient and necessary condition for the stability, which helps to reduce the conservatism. It should be pointed out that Theorem 1 can be easily applied to stability analysis by LMIs conditions for systems with time-varying and distributed delays in this paper.

**Theorem 2** There exists output controller (4) such that closed-loop system (8) is asymptotically stable in the mean square if there exist positive matrices  $P_i$ ,  $G_0$ ,  $G_{i+1}$ ,  $\Omega$  and W such that the following conditions hold for all  $i \in \{0, 1, 2, ..., N - 1\}$ :

$$\begin{bmatrix} -\Delta_{i} & H_{i,0} & H_{i,i+1} \\ * & -G_{0} & 0 \\ * & * & -G_{i+1} \end{bmatrix} < 0, \qquad \begin{bmatrix} -\Delta_{N} & H_{N,0} \\ * & -G_{0} \end{bmatrix} < 0, \tag{16}$$

where  $\triangle_i = \text{diag}\{P_i - \bar{\mu}W - (1 + d_2 - d_1)\Omega, \Omega, \frac{1}{\bar{\mu}}W\}, G_0 = \bar{P}_0^{-1}, G_{i+1} = \bar{P}_{i+1}^{-1}, G_N = \bar{P}_N^{-1} and$ 

$$H_{i,j} = \begin{bmatrix} \bar{A} + \bar{D}\bar{K}_{i,j}R_i - \bar{L}_{i,j}\bar{E}R_j & -\bar{D}\bar{K}_{i,j}R_i \\ \bar{L}_{i,j}\bar{E}R_0 & \bar{A} - \bar{L}_{i,j}\bar{E}R_0 \\ \bar{B} & 0 \\ 0 & \bar{B} \\ \bar{C} & 0 \\ 0 & \bar{C} \end{bmatrix}$$

Based on the cone complementarity linearization method [22], the controller and observer gains can be obtained by the following nonlinear minimization problem subject to (16):

$$\begin{cases} \operatorname{Min}\{\operatorname{Tr}(\sum_{i=1}^{N}(P_{i}G_{i}))\},\\ \begin{bmatrix} P_{i} & I\\ * & G_{i} \end{bmatrix} \geq 0, \quad i \in \{1, 2, \dots, N\} \end{cases}$$

**Remark 3** According to [23], the computational complexity of the controller design by solving LMIs is defined by  $T(F) = O(F^3)$  [4], where F is the total number of scalar decision variables. From (16) and the minimization problem, we can see that F satisfies  $F = N(4n^2 + 2n) + (2N - 1)(nq + nm)$  for Theorem 2. Therefore, the computational complexity of Theorem 2 is  $O(N^3n^6)$  for the system dimension n and the transition matrix dimension N.

### **5** Numerical examples

In this section, the simulation results are presented to illustrate the theoretical results derived in this paper.

**Example 1** Consider the networked system (1) with N = 2. And other networked system parameters are given as follows:

$$A = \begin{bmatrix} 0.2 & 0.4 & 0.4 \\ 0.2 & -0.8 & 0.1 \\ 0.3 & 1 & 0.5 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0.4 & 0.6 \\ 1.2 & 2 & 0.9 \\ 0.2 & 0.5 & 1 \end{bmatrix}, \qquad D = \begin{bmatrix} 1 \\ 0.8 \\ 0.5 \end{bmatrix}, \qquad C = \begin{bmatrix} 3 & 1 & 1.1 \\ -0.4 & 1 & 1.2 \\ 0.8 & 0.4 & 0.8 \end{bmatrix}, \qquad E = \begin{bmatrix} 1.2 \\ 0.3 \\ 0.6 \end{bmatrix}^T, \qquad \Pi = \begin{bmatrix} 0.4 & 0.6 & 0 \\ 0.7 & 0 & 0.3 \\ 1 & 0 & 0 \end{bmatrix}, \qquad d(k) = 1 + \frac{1 + (-1)^k}{2}, \qquad \mu_r = 2^{-(r+1)}.$$

It is easy to verify that  $d_1 = 1$ ,  $d_2 = 2$ , and  $\bar{\mu} = \frac{1}{2}$ . By using Matlab Toolbox and applying Theorem 2 to this example, the following feasible controller and observer gain matrices are obtained for controller (4) as follows:

$K_{00} = [0.3771]$	-0.4372	-0.2837],	$K_{01} = [0.2839]$	-0.4604	-0.3302],
<i>K</i> <sub>10</sub> = [0.3764	-0.4371	-0.2835],	$K_{12} = [0.2839]$	-0.4598	-0.3290],
<i>K</i> <sub>20</sub> = [0.3700	-0.4379	-0.2838],	$L_{00} = [-0.8111$	0.1966	0.5295] <sup><i>T</i></sup> ,
$L_{01} = [-1.3290$	0.2894	0.8304] <sup><i>T</i></sup> ,	$L_{10} = [-0.8143]$	0.1986	0.5326] <sup><i>T</i></sup> ,
$L_{12} = [-1.3280]$	0.2901	0.8314] <sup><i>T</i></sup> ,	$L_{20} = [-0.8590$	0.2127	$0.5658]^{T}$ .

The sampling-induced delay index  $\tau_k$  is a random variable,  $\tau_k$  is assumed to be governed by a Markov chain. And the transition probability matrix is  $\Pi$ . Based on the experiment, the channel-induced delay is described in Figure 2. For the initial states  $x(0) = [0.2244 - 0.0447 - 0.4180]^T$  and  $\hat{x}(0) = [-0.7205 \ 0.0183 - 1.6126]^T$ , the trajectories of x







and  $\hat{x}$  are shown in Figures 3 and 4, respectively. We can conclude that, based on the event-driven transmitter, system (8) is asymptotically stable in mean square.

### 6 Conclusions

In this paper, we have presented a theoretical framework to analyze network-based output feedback control for Markov sampled-data systems with mixed delays. At first, the networked control systems model is constructed including nonuniform sample data, discrete and distributed time-delays. Furthermore, the novel output feedback controller incorporating both Markov-based sampling and event-driven transmitter-induced delay indexes is proposed.

Based on the results obtained in this paper, a new form of controller  $u(k) = \sum_r k_r x(k-r)$  will be developed and most efforts will be made to solve different  $k_r$  in our subsequent

# work. And we will consider other approaches to give the sufficient and necessary conditions to ensure stability of the closed-loop system.

### **Competing interests**

The authors declare that they have no competing interests.

### Authors' contributions

WC analyzed the control problem and the numerical example, LL designed sketch, and RL gave some suggestions to revise the paper. All authors read and approved the final manuscript.

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