Ma and Yan *Advances in Difference Equations* 2013, 2013:362
http://www.advancesindifferenceequations.com/content/2013/1/362

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH **Open Access**

Delay-dependent robust H_∞ filter for T-S fuzzy time-delay systems with exponential stability

Yuechao Ma and Huijie Yan*

*Correspondence:
18716023220@163.com
College of Science, Yanshan University, 438 Hebeidajie Road, Haigang District, Qinhuangdao, Hebei 066004, P.R. China

Abstract

This paper deals with the problem of delay-dependent robust H_∞ filter for T-S fuzzy time-delay systems with exponential stability. The purpose is to design filter parameters such that the filtering error system is exponentially stable and satisfies a prescribed H_∞ performance. In terms of linear matrix inequalities (LMIS), some sufficient conditions for the solvability of this problem are presented. Thanks to the new filter, the obtained stability criterion is less conservative than the existing ones. Finally, three examples are provided to demonstrate the effectiveness and the superiority of the proposed design methods.

Keywords: T-S fuzzy systems; time-delay systems; H_∞ filter; fuzzy system models; exponentially stable; linear matrix inequality (LMI)

1 Introduction

In the past several decades, robust filtering problem has received extensive attention of people. The current study of robust filtering mainly concentrated on two aspects: Kalman filter and H_∞ filter. Among them, the research of H_∞ filter is wider, and many important and interesting results have been proposed in terms of all kinds of approaches (see, for example, [1–3]). Actual industrial system such as the power grid, chemical processes, nuclear reactor and others often contain time-delay, and time-delay is the main factor that leads to system performance degradation and instability. Therefore, the research of the filtering problem for time-delay systems has important theoretical significance and application value.

In recent years, the research of the filtering for time-delay systems has made abundant achievements. Delay-dependent robust H_∞ and $L_2 - L_\infty$ filtering for a class of uncertain nonlinear time-delay systems was studied in [4]. H_∞ filtering of time-delay T-S fuzzy systems based on piecewise Lyapunov-Krasovskii functional was investigated in [5]. A new fuzzy H_∞ filter design for nonlinear continuous-time dynamic systems with time-varying delays was reported in [6]. Robust H_∞ filtering for a class of uncertain Lurie time-delay singular systems was studied in [7]. Delay-dependent H_∞ filtering for singular Markovian jump time-delay systems was studied in [8].

T-S fuzzy system has wide application in the network, economy, environment and other fields, it has attracted more and more concern of the scholars (see, for example, [9–11]). H_∞ filter has come to play an important role in fuzzy model during the past years, so the filtering of fuzzy system is especially important. Delay-dependent nonfragile robust H_∞ filtering of T-S fuzzy time-delay systems was investigated in [12]. An improved H_∞ filter design for nonlinear system with time-delay via T-S fuzzy models was studied in [13]. Exponential H_∞ filter design for uncertain Takagi-Sugeno fuzzy systems with time-delay was reported in [14]. New results on H_∞ filtering for fuzzy systems with interval time-varying delays was studied in [15]. Delay-dependent non-fragile H_∞ filtering for uncertain fuzzy systems based on switching fuzzy model and piecewise Lyapunov function was studied in [16]. But at present, the problem of delay-dependent robust H_∞ filter for T-S fuzzy time-delay systems with exponential stability has rarely been reported.

For T-S fuzzy time-delay systems with exponential stability, this paper discusses the design methods of delay-dependent robust H_∞ filter. First of all, it gave a criterion of exponential stability, and then discussed the conditions and design methods of delay-dependent robust H_∞ filter. The result of designed filter is exponential stability for the augmented system via LMI. Thanks to the new filter, the obtained criterion is less conservative than the existing ones. Finally, some numerical examples are given to show the effectiveness and the superiority of the proposed design methods.

2 System description and preliminary lemma

Consider the following T-S fuzzy time-delay system, which is described by plant Rule i: IF $\varepsilon_1(t)$ is M_{i1} and $\varepsilon_2(t)$ is $M_{i2} \cdots \varepsilon_p(t)$ is M_{ip}, THEN

$$\begin{aligned}
\dot{x}(t) &= (A_{1i} + \Delta A_{1i})x(t) + (A_{2i} + \Delta A_{2i})x(t-d) + (B_i + \Delta B_i)\omega(t), \\
y(t) &= (C_{1i} + \Delta C_{1i})x(t) + (C_{2i} + \Delta C_{2i})x(t-d) + (D_i + \Delta D_i)\omega(t), \\
z(t) &= (L_i + \Delta L_i)x(t), \\
x(t) &= \phi(t), \quad t \in [-d, 0],
\end{aligned} \tag{1}$$

where $i \in R := \{1, 2, \dots, r\}$, r is the number of IF-THEN rules. $x(t) \in R^n$ is the input vector, $\omega(t) \in R^m$ is the disturbance vector of the system which belongs to $L_2[0, +\infty)$, $y(t) \in R^q$ is the measurable output vector, $z(t) \in R^p$ is the signal vector to be estimated, $\phi(t)$ is a compatible vector-valued initial function. $A_{1i}, A_{2i}, B_i, C_{1i}, C_{2i}, D_i, L_i$ are constant matrices with appropriate dimensions. $\varepsilon_j(t)$ and M_{ij} $(j = 1, 2, \dots, p)$ are the premise variables and the fuzzy sets. $d > 0$ is the constant time delay. ΔA_{1i}, ΔA_{2i}, ΔB_i, ΔC_{1i}, ΔC_{2i}, ΔD_i, ΔL_i are unknown matrices representing parametric uncertainties and are assumed to be of the form

$$\begin{bmatrix} \Delta A_{1i} & \Delta A_{2i} & \Delta B_i \\ \Delta C_{1i} & \Delta C_{2i} & \Delta D_i \end{bmatrix} = \begin{bmatrix} H_{1i} \\ H_{2i} \end{bmatrix} F_i [E_{1i} \quad E_{2i} \quad E_{3i}], \qquad \Delta L_i = H_{3i} F_i E_{4i}, \tag{2}$$

where H_{1i}, H_{2i}, H_{3i}, E_{1i}, E_{2i}, E_{3i}, E_{4i} are known real constant matrices with appropriate dimensions, and F_i is an unknown real time-varying matrix satisfying

$$F_i^T F_i \le I. \tag{3}$$

I is a unit matrix with appropriate dimensions. The parametric uncertainties ΔA_{1i}, ΔA_{2i}, ΔB_i, ΔC_{1i}, ΔC_{2i}, ΔD_i, ΔL_i are said to be admissible if both (2) and (3) hold.

Remark 1 When $\Delta D_i = \Delta E_i = \Delta L_i = 0$, system (1) was studied in [14]. The system in this paper is a class of fuzzy time-delay systems broader than others.

Let $\varepsilon(t) = [\varepsilon_1(t)\ \varepsilon_2(t)\ \cdots\ \varepsilon_p(t)]^T$, through the use of 'fuzzy blending', the fuzzy system (1) can be inferred as follows:

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{r} h_i\big(\varepsilon(t)\big)\big[(A_{1i} + \Delta A_{1i})x(t) + (A_{2i} + \Delta A_{2i})x(t-d) + (B_i + \Delta B_i)\omega(t)\big], \\
y(t) &= \sum_{i=1}^{r} h_i\big(\varepsilon(t)\big)\big[(C_{1i} + \Delta C_{1i})x(t) + (C_{2i} + \Delta C_{2i})x(t-d) + (D_i + \Delta D_i)\omega(t)\big], \\
z(t) &= \sum_{i=1}^{r} h_i\big(\varepsilon(t)\big)\big[(L_i + \Delta L_i)x(t)\big], \\
\beta_i\big(\varepsilon(t)\big) &= \prod_{j=1}^{p} M_{ij}\big(\varepsilon_j(t)\big), \qquad h_i\big(\varepsilon(t)\big) = \frac{\beta_i(\varepsilon(t))}{\sum_{i=1}^{r}\beta_i(\varepsilon(t))},
\end{aligned} \tag{4}$$

where $M_{ij}(\varepsilon_j(t))$ is the grade of membership of $\varepsilon_j(t)$ in M_{ij}. It is easy to see that $\beta_i(\varepsilon(t)) \geq 0$ and $\sum_{i=1}^{r}\beta_i(\varepsilon(t)) \geq 0$. Hence, we have $h_i(\varepsilon(t)) \geq 0$ and $\sum_{i=1}^{r} h_i(\varepsilon(t)) = 1$.

In this paper, we consider the following fuzzy filter:

$$\begin{aligned}
\dot{\hat{x}}(t) &= \sum_{i=1}^{r} h_i\big(\varepsilon(t)\big)\big[A_{fi}\hat{x}(t) + B_{fi}y(t) + C_{fi}\hat{x}(t-d)\big], \\
\hat{z}(t) &= \sum_{i=1}^{r} h_i\big(\varepsilon(t)\big)\big[L_{fi}\hat{x}(t)\big], \quad i = 1, 2, \ldots, r,
\end{aligned} \tag{5}$$

where $\hat{x}(t) \in R^k$ is the filter state vector, $\hat{z}(t)$ is the estimated vector, A_{fi}, B_{fi}, C_{fi}, L_{fi} with compatible dimensions are matrices to be determined.

Remark 2 When $C_{fi} = 0$, the fuzzy filter (5) was studied in [14] and [5]. This paper improves the function of the filter in [14] and [5], this makes the obtained result less conservative than the existing ones.

From (4) and (5), we obtain the filtering error system as follows:

$$\begin{aligned}
\dot{\tilde{x}}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\big(\varepsilon(t)\big)h_j\big(\varepsilon(t)\big)\big[A_{ij}\tilde{x}(t) + A_{d_{ij}}\tilde{x}(t-d) + B_{ij}\omega(t)\big], \\
e(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\big(\varepsilon(t)\big)h_j\big(\varepsilon(t)\big)\big[L_{ij}\tilde{x}(t)\big],
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
&e(t) = z(t) - \hat{z}(t), \qquad \tilde{x}(t) = \big[x^T(t) \quad \hat{x}^T(t)\big]^T, \\
&A_{ij} = \begin{bmatrix} A_{1i} + \Delta A_{1i} & 0 \\ B_{fj}(C_{1i} + \Delta C_{1i}) & A_{fj} \end{bmatrix}, \qquad A_{d_{ij}} = \begin{bmatrix} A_{2i} + \Delta A_{2i} & 0 \\ B_{fj}(C_{2i} + \Delta C_{2i}) & C_{fj} \end{bmatrix}, \\
&B_{ij} = \begin{bmatrix} B_i + \Delta B_i \\ B_{fj}(D_i + \Delta D_i) \end{bmatrix}, \qquad L_{ij} = [L_i + \Delta L_i \quad -L_{fj}].
\end{aligned}$$

Definition 1 The filtering error system (6) is said to be exponentially stable, if there exist scalars $\delta > 0$ and $\varepsilon > 0$ such that $\|x(t)\| \leq \delta \sup_{-d \leq \theta \leq 0} \|\phi(\theta)\| e^{-\varepsilon t}$.

System (6) can be abbreviated as the following form:

$$\begin{aligned} \dot{\tilde{x}}(t) &= \tilde{A}\tilde{x}(t) + \tilde{A}_d \tilde{x}(t-d) + \tilde{B}\omega(t), \\ e(t) &= \tilde{L}\tilde{x}(t), \end{aligned} \tag{7}$$

where

$$\tilde{A} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\big(\varepsilon(t)\big)h_j\big(\varepsilon(t)\big)A_{ij}, \qquad \tilde{A}_d = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\big(\varepsilon(t)\big)h_j\big(\varepsilon(t)\big)A_{d_{ij}},$$
$$\tilde{B} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\big(\varepsilon(t)\big)h_j\big(\varepsilon(t)\big)B_{ij}, \qquad \tilde{L} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\big(\varepsilon(t)\big)h_j\big(\varepsilon(t)\big)L_{ij}.$$

The robust H_∞ filtering problem to be addressed in this paper is formulated as follows: given the T-S fuzzy time-delay system (4) and a prescribed level of noise attenuation $\gamma > 0$, determine a filter with exponentially stable in the form of (5) such that the following requirements are satisfied:

(a) The filtering error system (6) is exponentially stable,
(b) Under zero initial conditions, (6) satisfies

$$\|e(t)\|_2 < \gamma \|\omega(t)\|_2 \tag{8}$$

for any nonzero $\omega(t) \in L_2[0, \infty)$ and all admissible uncertainties.

Lemma 1 [17] *Given a set of suited dimension real matrices E, F, H, Q is a symmetric matrix such that*

$$Q + HFE + E^T F^T H^T < 0$$

for all F satisfies $F^{\mathrm{T}} F \leq I$ if and only if there exists a scalar $\varepsilon > 0$ such that

$$Q + \varepsilon HH^T + \varepsilon^{-1} E^T E < 0.$$

Lemma 2 [18] *Suppose that $x(t) \in R^n$ is the vector function with a continuous derivative, if $\begin{bmatrix} U & M_1 & M_2 \\ * & Z_1 & Z_2 \\ * & * & Z_3 \end{bmatrix} \geq 0$, where $U, M_1, M_2, Z_1, Z_2, Z_3 \in R^{n\times n}$, such that the following integration is well defined, then*

$$-\int_{t-d}^{t} \dot{x}^T(s) U \dot{x}(s)\, ds \leq \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix}^T \begin{bmatrix} M_1^T + M_1 + dZ_1 & -M_1^T + M_2 + dZ_2 \\ * & -M_2^T - M_2 + dZ_3 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix}.$$

3 Main results

Theorem 1 *For prescribed scalar $d > 0$, the system* (6) *is exponentially stable, and* (8) *is satisfied if there exists symmetric positive definite matrix W, U and invertible matrix P*

such that

$$\sum = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & P^T\tilde{B} & \tilde{A}^T H^T & \tilde{L}^T \\ * & \Sigma_{22} & 0 & \tilde{A}_d^T H^T & 0 \\ * & * & -\gamma^2 I & \tilde{B}^T H^T & 0 \\ * & * & * & -(dU)^{-1} & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{9}$$

$$\Sigma_{11} = P^T\tilde{A} + \tilde{A}^T P + H^T W H + H^T\big(M_1^T + M_1 + dZ_1\big)H,$$

$$\Sigma_{12} = P^T\tilde{A}_d + H^T\big(-M_1^T + M_2 + dZ_2\big)H,$$

$$\Sigma_{22} = -H^T W H + H^T\big(-M_2^T - M_2 + dZ_3\big)H.$$

Proof First, we shall show the exponential stability of the system (6).

For any $t \geq d$, choose a Lyapunov functional candidate to be:

$$V(\tilde{x}_t) = \tilde{x}^T(t)P\tilde{x}(t) + \int_{t-d}^{t} \tilde{x}^T(s)H^T W H\tilde{x}(s)\,ds + \int_{-d}^{0}\int_{t+\theta}^{t} \dot{\tilde{x}}^T(s)H^T U H\dot{\tilde{x}}(s)\,ds\,d\theta, \tag{10}$$

where W, U are symmetric positive definite matrices, and P is an invertible matrix to be determined, $H = [I\ I]$, $\tilde{x}_t = \tilde{x}(t+\beta)$, $-d \leq \beta \leq 0$.

When $\omega(t) = 0$, through Lemma 2, we get

$$\dot{V}(\tilde{x}_t) \leq \begin{bmatrix} \tilde{x}(t) \\ \tilde{x}(t-d) \end{bmatrix}^T (\Gamma_1 + \Gamma_2) \begin{bmatrix} \tilde{x}(t) \\ \tilde{x}(t-d) \end{bmatrix},$$

where

$$\Gamma_1 = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ * & \Sigma_{22} \end{bmatrix}, \qquad \Gamma_2 = d\begin{bmatrix} \tilde{A}^T H^T \\ \tilde{A}_d^T H^T \end{bmatrix} U \begin{bmatrix} \tilde{A}^T H^T \\ \tilde{A}_d^T H^T \end{bmatrix}^T.$$

Now, applying the Schur complements, it is easy to see from (9) that there exists a scalar $\delta_0 > 0$ such that for any $t \geq d$,

$$\dot{V}(\tilde{x}_t) \leq -\delta_0 \|\tilde{x}(t)\|^2. \tag{11}$$

Now, by (10) and (11), we have

$$\begin{aligned} \frac{d}{dt}\big[e^{\varepsilon t}V(\tilde{x}_t)\big] &= e^{\varepsilon t}\big[\varepsilon V(t) + \dot{V}(\tilde{x}_t)\big] \\ &\leq e^{\varepsilon t}\left[(\varepsilon\delta_1 - \delta_0)\|\tilde{x}(t)\|^2 + \varepsilon\delta_2 \int_{t-d}^{t} \|\tilde{x}(s)\|^2\,ds\right], \end{aligned}$$

where $\delta_1 > 0$, $\delta_2 > 0$.

Integrating both sides from 0 to $T > 0$ gives

$$e^{\varepsilon T}V(\tilde{x}_T) - V(\tilde{x}_0) \leq (\varepsilon\delta_1 - \delta_0)\int_0^T e^{\varepsilon t}\|\tilde{x}(t)\|^2\,dt + \varepsilon\delta_2 \int_0^T e^{\varepsilon t}\,dt \int_{t-d}^{t} \|\tilde{x}(s)\|^2\,ds.$$

Since

$$\begin{aligned}\int_0^T e^{\varepsilon t}\,dt\int_{t-d}^t \|\tilde{x}(s)\|^2\,ds &= \int_{-d}^0 \|\tilde{x}(s)\|^2\,ds\int_0^{s+d} e^{\varepsilon t}\,dt + \int_0^{T-d}\|\tilde{x}(s)\|^2\,ds\int_s^{s+d} e^{\varepsilon t}\,dt \\ &\quad + \int_{T-d}^T \|\tilde{x}(s)\|^2\,ds\int_s^T e^{\varepsilon t}\,dt \\ &\le \int_{-d}^0 de^{\varepsilon(s+d)}\|\tilde{x}(s)\|^2\,ds + \int_0^{T-d} de^{t(s+d)}\|\tilde{x}(s)\|^2\,ds \\ &\quad + \int_{T-d}^T de^{\varepsilon(s+d)}\|\tilde{x}(s)\|^2\,ds \\ &= de^{\varepsilon d}\int_{-d}^T e^{\varepsilon s}\|\tilde{x}(s)\|^2\,ds.\end{aligned}$$

Let the scalar $\varepsilon > 0$ small enough such that $\varepsilon\delta_1 - \delta_0 + d\varepsilon\delta_2 e^{\varepsilon d} \le 0$. Then, we get that there exists a scalar $\kappa > 0$ such that

$$e^{\varepsilon T}V(\tilde{x}_T) \le V(\tilde{x}_0) + \left[\varepsilon\delta_1 - \delta_0 + d\varepsilon\delta_2 e^{\varepsilon d}\right]\int_0^T e^{\varepsilon t}\|\tilde{x}(t)\|^2\,dt \le \kappa \sup_{-d\le\theta\le 0}\|\phi(\theta)\|^2.$$

Taking into account that

$$V(\tilde{x}_T) \ge \lambda_{\min}(P)\|\tilde{x}(T)\|^2.$$

It is not difficult to see that, for any $T > 0$,

$$\|\tilde{x}(T)\| \le \sqrt{\frac{\kappa}{\lambda_{\min}(P)}}\sup_{-d\le\theta\le 0}\|\phi(\theta)\|e^{-\varepsilon T}.$$

Therefore, by Definition 1, the T-S fuzzy time-delay system (6) is exponentially stable.

Next, we show that for any nonzero $\omega(t) \in L_2[0,\infty)$, system (6) satisfies (8) under the zero initial condition. To this end, we introduce

$$J_T = \int_0^T \left[e(t)^T e(t) - \gamma^2\omega(t)^T\omega(t)\right]dt,$$

where the scalar $T > 0$. Consider the Lyapunov function of the augmented system (10) for $0 \ne \omega(t) \in L_2[0,\infty)$, we have

$$\dot{V}(\tilde{x}_t) \le \begin{bmatrix}\tilde{x}(t)\\ \tilde{x}(t-d)\\ w(t)\end{bmatrix}^T \left(\Gamma_1' + \Gamma_2'\right)\begin{bmatrix}\tilde{x}(t)\\ \tilde{x}(t-d)\\ w(t)\end{bmatrix},$$

where

$$\Gamma_1' = \begin{bmatrix}\Sigma_{11} & \Sigma_{12} & P^T\tilde{B}\\ * & \Sigma_{22} & 0\\ * & * & 0\end{bmatrix}, \qquad \Gamma_2' = d\begin{bmatrix}\tilde{A}^T H^T\\ \tilde{A}_d^T H^T\\ \tilde{B}^T H^T\end{bmatrix} U \begin{bmatrix}\tilde{A}^T H^T\\ \tilde{A}_d^T H^T\\ \tilde{B}^T H^T\end{bmatrix}^T.$$

It can be shown that for any nonzero $\omega(t) \in L_2[0,\infty)$ and $T > 0$,

$$\begin{aligned} J_T &= \int_0^T \big[\dot{V}(\tilde{x}_t) + e(t)^{\mathrm{T}} e(t) - \gamma^2 \omega(t)^T \omega(t)\big]\,dt - V(\tilde{x}_t) \\ &\le \int_0^T \big[\dot{V}(\tilde{x}_t) + e(t)^{\mathrm{T}} e(t) - \gamma^2 \omega(t)^T \omega(t)\big]\,dt \\ &\le \int_0^T x_\alpha^T \big(\Gamma_1'' + \Gamma_2'\big) x_\alpha\,dt, \end{aligned}$$

where

$$x_\alpha^T = \begin{bmatrix} \tilde{x}^T(t) & \tilde{x}^T(t-d) & \omega^T(t) \end{bmatrix}, \qquad \Gamma_1'' = \begin{bmatrix} \Sigma_{11} + \tilde{L}^T \tilde{L} & \Sigma_{12} & P^T \tilde{B} \\ * & \Sigma_{22} & 0 \\ * & * & -\gamma^2 I \end{bmatrix}.$$

When $\sum < 0$, we have $J_T < 0$ for all $T > 0$, which implies that $\|e(t)\|_2 < \gamma \|\omega(t)\|_2$ for any nonzero $\omega(t) \in L_2[0,\infty)$. This completes the proof. □

Based on the sufficient conditions above, the design problem of robust H_∞ filter can be transformed into a problem of linear matrix inequality.

Theorem 2 *Given matrices Q, S, R, which Q, R are symmetric, and Q is negative definite for all uncertainties, the robust filtering issue is resolved for system* (6) *if there exist positive scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, symmetric positive definite matrices W, U, and invertible matrix $P = \operatorname{diag}(P_1, P_2)$, such that the following LMIs holds*:

$$\begin{bmatrix} G'_{ii} & \eta_1 & \eta_2^T \\ * & -\varepsilon_1 I & 0 \\ * & * & -\varepsilon_1^{-1} I \end{bmatrix} < 0, \tag{12}$$

$$\begin{bmatrix} G'_{ij} & \eta_3 & \eta_4^T \\ * & -\varepsilon_2 I & 0 \\ * & * & -\varepsilon_2^{-1} I \end{bmatrix} < 0, \tag{13}$$

where

$$G'_{ii} = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} & \theta_{14} & \theta_{15} & \theta_{16} & L_i^T \\ * & \theta_{22} & \theta_{23} & \theta_{24} & \theta_{25} & A_{fi}^T & -L_{fj}^T \\ * & * & \theta_{33} & \theta_{34} & 0 & \theta_{36} & 0 \\ * & * & * & \theta_{44} & 0 & C_{fi}^T & 0 \\ * & * & * & * & \theta_{55} & \theta_{56} & 0 \\ * & * & * & * & * & \theta_{66} & 0 \\ * & * & * & * & * & * & -I \end{bmatrix},$$

$$\eta_1 = \begin{bmatrix} P_1H_{1i} & 0 & 0 & 0 & 0 & 0 & 0 \\ M_{2i}H_{2i} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ H_{1i} + B_{fi}H_{2i} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & H_{3i} & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\eta_2 = \begin{bmatrix} E_{1i} & 0 & E_{2i} & 0 & E_{3i} & 0 & 0 \\ E_{4i} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$G'_{ij} = \begin{bmatrix} \bar{\theta}_{11} & \bar{\theta}_{12} & \bar{\theta}_{13} & \bar{\theta}_{14} & \bar{\theta}_{15} & \bar{\theta}_{16} & \bar{\theta}_{17} \\ * & \bar{\theta}_{22} & \bar{\theta}_{23} & \bar{\theta}_{24} & \bar{\theta}_{25} & \bar{\theta}_{26} & \bar{\theta}_{27} \\ * & * & \bar{\theta}_{33} & \bar{\theta}_{34} & 0 & \bar{\theta}_{36} & 0 \\ * & * & * & \bar{\theta}_{44} & 0 & \bar{\theta}_{46} & 0 \\ * & * & * & * & -2\gamma^2 I & \bar{\theta}_{56} & 0 \\ * & * & * & * & * & \bar{\theta}_{66} & 0 \\ * & * & * & * & * & * & -2I \end{bmatrix},$$

$$\eta_3 = \begin{bmatrix} P_1H_{1i} & P_1H_{1j} & 0 & 0 & \cdots & 0 \\ \bar{\theta}_{28} & \bar{\theta}_{29} & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ \bar{\theta}_{68} & \bar{\theta}_{69} & 0 & 0 & \cdots & 0 \\ 0 & 0 & H_{3i} & H_{3j} & \cdots & 0 \end{bmatrix}_{7\times 9},$$

$$\eta_4 = \begin{bmatrix} E_{1i} & 0 & E_{2i} & 0 & E_{3i} & 0 & 0 \\ E_{1j} & 0 & E_{2j} & 0 & E_{3j} & 0 & 0 \\ E_{4i} & 0 & 0 & 0 & 0 & 0 & 0 \\ E_{4j} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\theta_{11} = P_1A_{1i} + A_{1i}^TP_1 + W + M_1^T + M_1 + dZ_1, \qquad \theta_{12} = C_{1i}^TM_{2i}^T + W + M_1^T + M_1 + dZ_1,$$

$$\theta_{13} = P_1A_{2i} - M_1^T + M_2 + dZ_2, \qquad \theta_{14} = -M_1^T + M_2 + dZ_2, \qquad \theta_{15} = P_1B_i,$$

$$\theta_{16} = A_{1i}^T + C_{1i}^TB_{fi}^T, \qquad \theta_{22} = M_{1i} + M_{1i}^T + W + M_1^T + M_1 + dZ_1,$$

$$\theta_{23} = M_{2i}C_{2i} - M_1^T + M_2 + dZ_2, \qquad \theta_{24} = M_{3i} - M_1^T + M_2 + dZ_2,$$

$$\theta_{25} = M_{2i}D_i, \qquad \theta_{33} = \theta_{34} = -W - M_2^T - M_2 + dZ_3,$$
$$\theta_{36} = A_{2i}^T + C_{2i}^T B_{fi}^T, \qquad \theta_{44} = -W - M_2^T - M_2 + dZ_3,$$
$$\theta_{55} = -\gamma^2 I, \qquad \theta_{56} = B_i^T + D_i^T B_{fi}^T, \qquad \theta_{66} = -(dU)^{-1},$$
$$\bar{\theta}_{11} = P_1A_{1i} + A_{1i}^T P_1 + P_1 A_{1j} + A_{1j}^T P_1 + 2W + 2(M_1^T + M_1 + dZ_1),$$
$$\bar{\theta}_{12} = C_{1i}^T M_{2j}^T + C_{1j}^T M_{2i}^T + 2W + 2(M_1^T + M_1 + dZ_1),$$
$$\bar{\theta}_{13} = P_1A_{2i} + P_1A_{2j} + 2(-M_1^T + M_2 + dZ_2), \qquad \bar{\theta}_{14} = 2(-M_1^T + M_2 + dZ_2),$$
$$\bar{\theta}_{15} = P_1B_i + P_1B_j, \qquad \bar{\theta}_{16} = A_{1i}^T + A_{1j}^T + C_{1i}^T B_{fj}^T + C_{1j}^T B_{fi}^T,$$
$$\bar{\theta}_{17} = L_i^T + L_j^T, \qquad \bar{\theta}_{22} = M_{1i} + M_{1j} + M_{1i}^{\mathrm{T}} + M_{1j}^{\mathrm{T}} + 2W + 2(M_1^T + M_1 + dZ_1),$$
$$\bar{\theta}_{23} = M_{2j}C_{2i} + M_{2i}C_{2j} + 2(-M_1^{\mathrm{T}} + M_2 + dZ_2),$$
$$\bar{\theta}_{24} = M_{3i} + M_{3j} + 2(-M_1^T + M_2 + dZ_2),$$
$$\bar{\theta}_{25} = M_{2j}D_i + M_{2i}D_j, \qquad \bar{\theta}_{26} = A_{fi}^T + A_{fj}^T, \qquad \bar{\theta}_{27} = -L_{fi}^T - L_{fj}^T, \qquad \bar{\theta}_{28} = M_{2j}H_{2i},$$
$$\bar{\theta}_{29} = M_{2i}H_{2j}, \qquad \bar{\theta}_{33} = \bar{\theta}_{34} = -2W + 2(-M_2^T - M_2 + dZ_3),$$
$$\bar{\theta}_{36} = A_{2i}^T + A_{2j}^T + C_{2i}^T B_{fj}^T + C_{2j}^T B_{fi}^T,$$
$$\bar{\theta}_{44} = -2W + 2(-M_2^T - M_2 + dZ_3), \qquad \bar{\theta}_{46} = C_{fi}^T + C_{fj}^T,$$
$$\bar{\theta}_{56} = B_i^{\mathrm{T}} + B_j^T + D_i^T B_{fj}^T + D_j^T B_{fi}^T,$$
$$\bar{\theta}_{66} = -2(dU)^{-1}, \qquad \bar{\theta}_{68} = H_{1i} + B_{fj}H_{2i}, \qquad \bar{\theta}_{69} = H_{1j} + B_{fi}H_{2j}.$$

Proof From Theorem 1, the sufficient condition of solving robust H_∞ filtering problem is matrix inequality (9) holds. Then we have

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varepsilon(t))h_j(\varepsilon(t))G_{ij} = \sum_{i=1}^{r} h_i^2(\varepsilon(t))G_{ii} + \sum_{i<j}^{r} h_i(\varepsilon(t))h_j(\varepsilon(t))(G_{ji} + G_{ij}) < 0,$$

where

$$G_{ij} = \begin{bmatrix} \Xi_{11} & \Xi_{12} & P^T B_{ij} & A_{ij}^T H^T & L_{ij}^T \\ * & \Xi_{22} & 0 & A_{d_{ij}}^T H^T & 0 \\ * & * & -\gamma^2 I & B_{ij}^T H^T & 0 \\ * & * & * & -(dU)^{-1} & 0 \\ * & * & * & * & -I \end{bmatrix},$$
$$\Xi_{11} = P^T A_{ij} + A_{ij}^T P + H^T W H + H^T(M_1^T + M_1 + dZ_1)H,$$
$$\Xi_{12} = P^T A_{d_{ij}} + H^T(-M_1^T + M_2 + dZ_2)H, \qquad \Xi_{22} = H^T(-W - M_2^T - M_2 + dZ_3)H.$$

However, $h_i(\varepsilon(t)) \geq 0$ and $\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varepsilon(t))h_j(\varepsilon(t)) = 1$. So matrix inequality (9) holds as long as

$$\begin{aligned} &G_{ii} < 0, \quad i = 1, 2, \ldots, r, \\ &G_{ji} + G_{ij} < 0, \quad i < j \leq r. \end{aligned} \tag{14}$$

When $i = j$, from (14), adapt G_{ii}, we have $G_{ii} = G'_{ii} + G''_{ii}$, where

$$G'_{ii} = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} & \theta_{14} & \theta_{15} & \theta_{16} & L_i^T \\ * & \theta_{22} & \theta_{23} & \theta_{24} & \theta_{25} & A_{fi}^T & -L_{fi}^T \\ * & * & \theta_{33} & \theta_{34} & 0 & \theta_{36} & 0 \\ * & * & * & \theta_{44} & 0 & C_{fi}^T & 0 \\ * & * & * & * & \theta_{55} & \theta_{56} & 0 \\ * & * & * & * & * & \theta_{66} & 0 \\ * & * & * & * & * & * & -I \end{bmatrix},$$

$$G''_{ii} = \begin{bmatrix} \theta'_{11} & \theta'_{12} & \theta'_{13} & 0 & \theta'_{15} & \theta'_{16} & \Delta L_i^{\mathrm{T}} \\ * & 0 & \theta'_{23} & 0 & \theta'_{25} & 0 & 0 \\ * & * & 0 & 0 & 0 & \theta'_{36} & 0 \\ * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & \theta'_{56} & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix},$$

$$\theta_{11} = P_1A_{1i} + A_{1i}^TP_1 + W + M_1^T + M_1 + dZ_1, \qquad \theta_{12} = C_{1i}^TB_{fi}^TP + W + M_1^T + M_1 + dZ_1,$$
$$\theta_{13} = P_1A_{2i} - M_1^T + M_2 + dZ_2, \qquad \theta_{14} = -M_1^T + M_2 + dZ_2, \qquad \theta_{15} = P_1B_i,$$
$$\theta_{16} = A_{1i}^T + C_{1i}^TB_{fi}^T, \qquad \theta_{22} = P_2^TA_{fi} + A_{fi}^TP_2 + W + M_1^T + M_1 + dZ_1,$$
$$\theta_{23} = P_2^TB_{fi}C_{2i} - M_1^T + M_2 + dZ_2, \qquad \theta_{24} = P_2^TC_{fi} - M_1^T + M_2 + dZ_2,$$
$$\theta_{25} = P_2^TB_{fi}D_i, \qquad \theta_{33} = \theta_{34} = -W - M_2^T - M_2 + dZ_3, \qquad \theta_{36} = A_{2i}^T + C_{2i}^TB_{fi}^T,$$
$$\theta_{44} = -W - M_2^T - M_2 + dZ_3, \qquad \theta_{55} = -\gamma^2 I,$$
$$\theta_{56} = B_i^T + D_i^TB_{fi}^T, \qquad \theta_{66} = -(dU)^{-1},$$
$$\theta'_{11} = P_1\Delta A_{1i} + \Delta A_{1i}^TP_1, \qquad \theta'_{12} = \Delta C_{1i}^TB_{fi}^TP_2, \qquad \theta'_{13} = P_1\Delta A_{2i},$$
$$\theta'_{15} = P_1\Delta B_i, \qquad \theta'_{16} = \Delta A_{1i}^T + \Delta C_{1i}^TB_{fi}^T,$$
$$\theta'_{23} = P_2^TB_{fi}\Delta C_{2i}, \qquad \theta'_{25} = P_2^TB_{fi}\Delta D_i,$$
$$\theta'_{36} = \Delta A_{2i}^T + \Delta C_{2i}^TB_{fi}^T, \qquad \theta'_{56} = \Delta B_i^T + \Delta D_i^TB_{fi}^T.$$

Based on (2), we obtain that

$$G''_{ii} = \eta_1F_i\eta_2 + \eta_2^TF_i^T\eta_1^T.$$

Through Lemma 1, we can get that $G_{ii} < 0$ $(i = 1, 2, \dots, r)$ is equivalent to

$$G'_{ii} + \varepsilon_1^{-1}\eta_1\eta_1^T + \varepsilon_1\eta_2^T\eta_2 < 0.$$

Via the Schur complements, we obtain that

$$\begin{bmatrix} G'_{ii} & \eta_1 & \eta_2^T \\ * & -\varepsilon_1 I & 0 \\ * & * & -\varepsilon_1^{-1}I \end{bmatrix} < 0.$$

Let $M_{1i} = P_2^T A_{fi}$, $M_{2i} = P_2^T B_{fi}$, $M_{3i} = P_2^T C_{fi}$, (12) is completed.
Let $G_{ji} + G_{ij} = G'_{ij} + G''_{ij}$, where

$$G'_{ij} = \begin{bmatrix} \bar\theta_{11} & \bar\theta_{12} & \bar\theta_{13} & \bar\theta_{14} & \bar\theta_{15} & \bar\theta_{16} & \bar\theta_{17} \\ * & \bar\theta_{22} & \bar\theta_{23} & \bar\theta_{24} & \bar\theta_{25} & \bar\theta_{26} & \bar\theta_{27} \\ * & * & \bar\theta_{33} & \bar\theta_{34} & 0 & \bar\theta_{36} & 0 \\ * & * & * & \bar\theta_{44} & 0 & \bar\theta_{46} & 0 \\ * & * & * & * & \bar\theta_{55} & \bar\theta_{56} & 0 \\ * & * & * & * & * & \bar\theta_{66} & 0 \\ * & * & * & * & * & * & -2I \end{bmatrix},$$

$$G''_{ij} = \begin{bmatrix} \bar\theta'_{11} & \bar\theta'_{12} & \bar\theta'_{13} & 0 & \bar\theta'_{15} & \bar\theta'_{16} & \bar\theta'_{17} \\ * & 0 & \bar\theta'_{23} & 0 & \bar\theta'_{25} & 0 & 0 \\ * & * & 0 & 0 & 0 & \bar\theta'_{36} & 0 \\ * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & \bar\theta'_{56} & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix},$$

$$\bar\theta_{11} = P_1 A_{1i} + A_{1i}^T P_1 + P_1 A_{1j} + A_{1j}^T P_1 + 2W + 2\bigl(M_1^T + M_1 + dZ_1\bigr),$$

$$\bar\theta_{12} = C_{1i}^T B_{fj}^T P_2 + C_{1j}^T B_{fi}^T P_2 + 2W + 2\bigl(M_1^T + M_1 + dZ_1\bigr),$$

$$\bar\theta_{13} = P_1 A_{2i} + P_1 A_{2j} + 2\bigl(-M_1^T + M_2 + dZ_2\bigr), \qquad \bar\theta_{14} = 2\bigl(-M_1^T + M_2 + dZ_2\bigr),$$

$$\bar\theta_{15} = P_1 B_i + P_1 B_j, \qquad \bar\theta_{16} = A_{1i}^T + A_{1j}^T + C_{1i}^T B_{fj}^T + C_{1j}^T B_{fi}^T,$$

$$\bar\theta_{17} = L_i^T + L_j^T, \qquad \bar\theta_{22} = P_2^T A_{fi} + P_2^T A_{fj} + A_{fi}^T P_2 + A_{fj}^T P_2 + 2W + 2\bigl(M_1^T + M_1 + dZ_1\bigr),$$

$$\bar\theta_{23} = P_2^T B_{fj} C_{2i} + P_2^T B_{fi} C_{2j} + 2\bigl(-M_1^T + M_2 + dZ_2\bigr),$$

$$\bar\theta_{24} = P_2^T C_{fi} + P_2^T C_{fj} + 2\bigl(-M_1^T + M_2 + dZ_2\bigr), \qquad \bar\theta_{25} = P_2^T B_{fj} D_i + P_2^T B_{fi} D_j,$$

$$\bar\theta_{26} = A_{fi}^T + A_{fj}^T, \qquad \bar\theta_{27} = -L_{fi}^T - L_{fj}^T, \qquad \bar\theta_{33} = \bar\theta_{34} = -2W + 2\bigl(-M_2^T - M_2 + dZ_3\bigr),$$

$$\bar\theta_{36} = A_{2i}^T + A_{2j}^T + C_{2i}^T B_{fj}^T + C_{2j}^T B_{fi}^T, \qquad \bar\theta_{44} = -2W + 2\bigl(-M_2^T - M_2 + dZ_3\bigr),$$

$$\bar\theta_{46} = C_{fi}^T + C_{fj}^T, \qquad \bar\theta_{55} = -2\gamma^2 I, \qquad \bar\theta_{56} = B_i^T + B_j^T + D_i^T B_{fj}^T + D_j^T B_{fi}^T,$$

$$\bar\theta_{66} = -2(dU)^{-1}, \qquad \bar\theta'_{11} = P_1 \Delta A_{1i} + \Delta A_{1i}^T P_1 + P_1 \Delta A_{1j} + \Delta A_{1j}^T P_1,$$

$$\bar\theta'_{12} = \Delta C_{1i}^T B_{fj}^T P_2 + \Delta C_{1j}^T B_{fi}^T P_2, \qquad \bar\theta'_{13} = P_1 \Delta A_{2i} + P_1 \Delta A_{2j}, \qquad \bar\theta'_{15} = P_1 \Delta B_i + P_1 \Delta B_j,$$

$$\bar\theta'_{16} = \Delta A_{1i}^T + \Delta A_{1j}^T + \Delta C_{1i}^T B_{fj}^T + \Delta C_{1j}^T B_{fi}^T, \qquad \bar\theta'_{17} = \Delta L_i^T + \Delta L_j^T,$$

$$\bar\theta'_{23} = P_2^T B_{fj} \Delta C_{2i} + P_2^T B_{fi} \Delta C_{2j}, \qquad \bar\theta'_{25} = P_2^T B_{fj} \Delta D_i + P_2^T B_{fi} \Delta D_j,$$

$$\bar\theta'_{36} = \Delta A_{2i}^T + \Delta A_{2j}^T + \Delta C_{2i}^T B_{fj}^T + \Delta C_{2j}^T B_{fi}^T, \qquad \bar\theta'_{56} = \Delta B_i^T + \Delta B_j^T + \Delta D_i^T B_{fj}^T + \Delta D_j^T B_{fi}^T.$$

Based on (2), we obtain that

$$G''_{ij} = \eta_3 \bar F \eta_4 + \eta_4^T \bar F^T \eta_3^T,$$

where

$$\bar{F} = \begin{bmatrix} F_i & \\ & F_j \end{bmatrix}.$$

Through Lemma 1, we can get that $G_{ji} + G_{ij} < 0$ $(i < j \le r)$ is equivalent to

$$G'_{ij} + \varepsilon_2^{-1}\eta_3\eta_3^T + \varepsilon_2\eta_4^T\eta_4 < 0.$$

Via the Schur complement, we obtain that

$$\begin{bmatrix} G'_{ij} & \eta_3 & \eta_4^T \\ * & -\varepsilon_2 I & 0 \\ * & * & -\varepsilon_2^{-1} I \end{bmatrix} < 0.$$

Let $M_{1i} = P_2^T A_{fi}$, $M_{2i} = P_2^T B_{fi}$, $M_{3i} = P_2^T C_{fi}$, (13) is completed.

The parameters of the robust H_∞ filter are

$$A_{fi} = P_2^{-T} M_{1i}, \qquad B_{fi} = P_2^{-T} M_{2i}, \qquad C_{fi} = P_2^{-T} M_{3i}, L_{fi}. \qquad \square$$

4 Numerical example

Example 1 Consider system (6) described by

$$A_{11} = \begin{bmatrix} -1.2 & -0.8 \\ 1 & -2 \end{bmatrix}, \qquad A_{21} = \begin{bmatrix} -0.5 & 0.2 \\ 1 & 0 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0.4 & -0.1 \end{bmatrix},$$

$$C_{11} = \begin{bmatrix} 0.1 & 0.3 \\ 0.1 & 0.2 \end{bmatrix}, \qquad C_{21} = \begin{bmatrix} -0.2 & 0.4 \\ 0.1 & 0.2 \end{bmatrix}, \qquad D_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.1 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 1.5 & 0.1 \\ 0.1 & -0.2 \end{bmatrix}, \qquad A_{12} = \begin{bmatrix} -1.6 & 0.2 \\ 0.7 & -1 \end{bmatrix}, \qquad A_{22} = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & -0.3 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.7 & 0.1 \\ 0.2 & -0.1 \end{bmatrix}, \qquad C_{12} = \begin{bmatrix} 0.1 & 0.4 \\ 0.3 & 0.2 \end{bmatrix}, \qquad C_{22} = \begin{bmatrix} -0.3 & 0.4 \\ 0.1 & 0.2 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.4 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} -0.1 & 0.2 \\ 0.3 & 0.1 \end{bmatrix}, \qquad H_{11} = \begin{bmatrix} 0.1 \\ -0.2 \end{bmatrix}, \qquad H_{12} = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix},$$

$$H_{21} = \begin{bmatrix} -0.2 \\ 0.2 \end{bmatrix}, \qquad H_{22} = \begin{bmatrix} 0.2 \\ -0.1 \end{bmatrix}, \qquad H_{31} = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}, \qquad H_{32} = \begin{bmatrix} -0.2 \\ 0.3 \end{bmatrix},$$

$$E_{11} = [-0.2 \quad 0.1], \qquad E_{21} = [0.2 \quad -0.2], \qquad E_{31} = [-0.2 \quad 0.1],$$

$$E_{41} = [-0.2 \quad 0.2], \qquad E_{12} = [-0.2 \quad 0.1], \qquad E_{22} = [-0.1 \quad -0.2],$$

$$E_{32} = [-0.3 \quad 0.2], \qquad E_{42} = [-0.2 \quad 0.3].$$

The normalized membership functions of the first subsystem are

$$h_1\bigl(x(t)\bigr) = \frac{1 + \cos(x(t))}{2}, \qquad h_2\bigl(x(t)\bigr) = \frac{1 - \cos(x(t))}{2}.$$

Given $d = 8$, $\gamma = 0.25$. We can obtain the filter parameters as follows:

$$A_{f1} = \begin{bmatrix} 0.9585 & 0.3257 \\ 0.3600 & 0.3510 \end{bmatrix}, \qquad B_{f1} = \begin{bmatrix} 90.6474 & 170.4250 \\ -199.3139 & -298.7841 \end{bmatrix},$$
$$C_{f1} = \begin{bmatrix} -0.6731 & -0.5042 \\ -0.8152 & -0.5380 \end{bmatrix}, \qquad L_{f1} = \begin{bmatrix} 0.7716 & -0.0043 \\ 0.4243 & -0.0258 \end{bmatrix},$$
$$A_{f2} = \begin{bmatrix} -23.4914 & -18.7283 \\ -30.3710 & -25.2203 \end{bmatrix}, \qquad B_{f2} = \begin{bmatrix} -74.1374 & -86.4398 \\ 616.2515 & 655.3905 \end{bmatrix},$$
$$C_{f2} = \begin{bmatrix} 4.4241 & 4.4659 \\ 4.8130 & 6.1622 \end{bmatrix}, \qquad L_{f2} = \begin{bmatrix} -0.7979 & -0.0079 \\ -0.4663 & 0.0086 \end{bmatrix}.$$

Example 2 In order to show the advantage of the proposed method, we consider the T-S fuzzy time-delay system as the system show in simulation example in [14] with two rules.

Plant Rule 1: IF $x_1(t)$ is u_1 (*e.g.*, small) THEN

$$\begin{aligned} \dot{x}(t) &= \left[A_1 + \Delta A_1(t)\right]x(t) + \left[A_{d1} + \Delta A_{d1}(t)\right]x(t-1.5) + D_1\omega(t), \\ y(t) &= \left[C_1 + \Delta C_1(t)\right]x(t) + \left[C_{d1} + \Delta C_{d1}(t)\right]x(t-1.5) + E_1\omega(t), \\ z(t) &= L_1 x(t), \end{aligned}$$

and

Plant Rule 2: IF $x_1(t)$ is u_2 (*e.g.*, big) THEN

$$\begin{aligned} \dot{x}(t) &= \left[A_2 + \Delta A_2(t)\right]x(t) + \left[A_{d2} + \Delta A_{d2}(t)\right]x(t-1.5) + D_2\omega(t), \\ y(t) &= \left[C_2 + \Delta C_2(t)\right]x(t) + \left[C_{d2} + \Delta C_{d2}(t)\right]x(t-1.5) + E_2\omega(t), \\ z(t) &= L_2 x(t), \end{aligned}$$

where

$$\begin{aligned} &A_1 = \begin{bmatrix} -1.2 & -0.8 \\ 1 & -2 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -1.6 & 0.2 \\ 0.7 & -1 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} -0.5 & 0.2 \\ 1 & 0 \end{bmatrix}, \\ &A_{d2} = \begin{bmatrix} 0.6 & 0.2 \\ 0.5 & -0.8 \end{bmatrix}, \qquad D_1 = \begin{bmatrix} 1 \\ -0.2 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, \\ &C_1 = [1 \quad 0], \qquad C_2 = [0.5 \quad -0.6], \qquad C_{d1} = [-0.8 \quad 0.6], \\ &C_{d2} = [-0.2 \quad 1], \qquad E_1 = 0.3, \qquad E_2 = -0.6, \\ &L_1 = [1 \quad -0.5], \qquad L_2 = [-0.2 \quad 0.3], \\ &M_{11} = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix}, \qquad M_{21} = 0.8, \qquad N_{11} = [0 \quad 0.3], \qquad N_{21} = [0.2 \quad 0], \\ &M_{12} = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}, \qquad M_{22} = 0.6, \qquad N_{12} = [0.5 \quad 0], \qquad N_{22} = [0 \quad -0.2]. \end{aligned}$$

The normalized membership functions of the first subsystem are

$$h_1(x_1(t)) = \begin{cases} 1, & x_1 < -1, \\ \frac{1}{2} - \frac{1}{2}x_1, & |x_1| \le 1, \\ 0, & x_1 > 1, \end{cases} \quad \text{and} \quad h_2(x_1(t)) = \begin{cases} 0, & x_1 < -1, \\ \frac{1}{2} + \frac{1}{2}x_1, & |x_1| \le 1, \\ 1, & x_1 > 1. \end{cases}$$

Filter Rule 1: IF $x_1(t)$ is u_1 THEN

$$\begin{aligned} \dot{\hat{x}}(t) &= A_{f1}\hat{x}(t) + B_{f1}y(t), \\ \hat{z}(t) &= L_{f1}\hat{x}(t), \end{aligned}$$

and

Filter Rule 2: IF $x_1(t)$ is u_2 THEN

$$\begin{aligned} \dot{\hat{x}}(t) &= A_{f2}\hat{x}(t) + B_{f2}y(t), \\ \hat{z}(t) &= L_{f2}\hat{x}(t). \end{aligned}$$

Through Theorem 2 of this article, we can obtain $\gamma_{\min} = 0.0271$, it is less than the minimal level 0.4721 in simulation example in [14], this clearly shows the superiority of the results derived in this paper to those obtained from [14], and the filter parameters as follows:

$$\begin{aligned} &A_{f1} = \begin{bmatrix} -2.9870 & -0.7508 \\ 1.8256 & -1.5045 \end{bmatrix}, \qquad A_{f2} = \begin{bmatrix} -1.1822 & 0.3462 \\ 1.1459 & -1.8463 \end{bmatrix}, \\ &B_{f1} = \begin{bmatrix} -0.4195 \\ 1.3158 \end{bmatrix}, \qquad B_{f2} = \begin{bmatrix} -0.2797 \\ 1.1891 \end{bmatrix}, \\ &L_{f1} = [-0.6002 \quad 0.3459], \qquad L_{f2} = [-0.2737 \quad -0.3206]. \end{aligned}$$

Using the filter in (5), we can get $\gamma_{\min} = 0.0190$, it is less than the minimal level 0.0271 with $C_{fi} = 0$ in this paper, and the filter parameters as follows:

$$\begin{aligned} &A_{f1} = \begin{bmatrix} -1.0003 & -0.3517 \\ 1.8344 & -2.5099 \end{bmatrix}, \qquad A_{f2} = \begin{bmatrix} -1.1829 & 0.3544 \\ 1.6542 & -3.8510 \end{bmatrix}, \\ &B_{f1} = \begin{bmatrix} -0.4196 \\ 1.3157 \end{bmatrix}, \qquad B_{f2} = \begin{bmatrix} -0.2799 \\ 1.1890 \end{bmatrix}, \\ &C_{f1} = \begin{bmatrix} -1.6825 & -2.6491 \\ -0.0680 & -0.0098 \end{bmatrix}, \qquad C_{f2} = \begin{bmatrix} 5.0149 & 2.5331 \\ -0.0682 & -0.0121 \end{bmatrix}, \\ &L_{f1} = [-0.6030 \quad 0.3481], \qquad L_{f2} = [-0.2755 \quad -0.3215]. \end{aligned}$$

Therefore, we can see the advantage of the proposed method in this paper.

Example 3 In order to show the advantage of the proposed method, we consider the time-delay T-S fuzzy system as the system show in example 2 in [5] with two rules.

R_1: if $x_1(t)$ is about 0, then

$$\begin{aligned}
\dot{x}(t) &= (A_1 + \Delta A_1)x(t) + (A_{d1} + \Delta A_{d1})x(t-d) + (B_1 + \Delta B_1)w(t), \\
y(t) &= (C_1 + \Delta C_1)x(t) + (C_{d1} + \Delta C_{d1})x(t-d) + (D_1 + \Delta D_1)w(t), \\
z(t) &= (L_1 + \Delta L_1)x(t),
\end{aligned}$$

R_2: if $x_1(t)$ is about $\pm\frac{\pi}{2}$, then

$$\begin{aligned}
\dot{x}(t) &= (A_2 + \Delta A_2)x(t) + (A_{d2} + \Delta A_{d2})x(t-d) + (B_2 + \Delta B_2)w(t), \\
y(t) &= (C_2 + \Delta C_2)x(t) + (C_{d2} + \Delta C_{d2})x(t-d) + (D_2 + \Delta D_2)w(t), \\
z(t) &= (L_2 + \Delta L_2)x(t),
\end{aligned}$$

where

$$\begin{aligned}
&A_1 = \begin{bmatrix} 1 & 0.1 \\ -0.5 & 1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 1 & 0.5 \\ -0.1 & 1 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0.1 \end{bmatrix}, \\
&A_{d2} = \begin{bmatrix} 0 & 0.3 \\ 0 & 0.6 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad C_1 = [1 \quad 0], \\
&C_2 = [0.5 \quad -0.6], \qquad C_{d1} = [-0.2 \quad 0.6], \qquad C_{d2} = [-0.2 \quad 0.6], \\
&L_1 = [1 \quad -0.5], \qquad L_2 = [-0.2 \quad 0.3], \qquad D_1 = 0.3, \qquad D_2 = -0.6, \\
&H_{11} = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix}, \qquad H_{12} = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}, \qquad H_{21} = 0.8, \qquad H_{22} = 0.6, \\
&H_{L1} = 0.2, \qquad H_{L2} = 0.1, \qquad E_{11} = [0 \quad 0.3], \\
&E_{12} = [0.5 \quad 0], \qquad E_{21} = 0.1, \qquad E_{22} = 0.2, \qquad E_{31} = [0.1 \quad 0], \\
&E_{32} = [0.2 \quad 0], \qquad E_{d1} = [0.2 \quad 0], \qquad E_{d2} = [0 \quad -0.2].
\end{aligned}$$

The normalized membership functions of the first subsystem are

$$\begin{aligned}
h_1\bigl(x_1(t)\bigr) &= \left(1 - \frac{1}{1 + \exp(-7(x_1(t) - \pi/4))}\right) \times \left(\frac{1}{1 + \exp(-7(x_1(t) - \pi/4))}\right), \\
h_2\bigl(x_1(t)\bigr) &= h_1\bigl(x_1(t)\bigr).
\end{aligned}$$

Filter R_1: if $x_1(t)$ is μ_1 then

$$\begin{aligned}
\dot{\hat{x}}(t) &= A_{e1}\hat{x}(t) + B_{e1}y(t), \\
\hat{Z}(t) &= L_{e1}\hat{x}(t).
\end{aligned}$$

Filter R_2: if $x_1(t)$ is μ_2 then

$$\begin{aligned}
\dot{\hat{x}}(t) &= A_{e2}\hat{x}(t) + B_{e2}y(t), \\
\hat{Z}(t) &= L_{e2}\hat{x}(t).
\end{aligned}$$

Given $d = 0.5$, through Theorem 2 of this article, we can obtain minimum $\gamma = 0.0178$, and it is less than the minimal level 0.2453 in example 2 in [5], this clearly shows the superiority of the results derived in this paper to those obtained from [5], and the filter parameters as follows:

$$A_{e1} = \begin{bmatrix} -0.9836 & 0.3306 \\ -1.7660 & -3.0998 \end{bmatrix}, \qquad A_{e2} = \begin{bmatrix} -0.8186 & 0.4682 \\ -1.7634 & -4.4274 \end{bmatrix}, \qquad B_{e1} = \begin{bmatrix} 0.2093 \\ 0.1007 \end{bmatrix},$$
$$B_{e2} = \begin{bmatrix} -0.0132 \\ 0.4979 \end{bmatrix}, \qquad L_{e1} = [3.4626 \quad 4.3515], \qquad L_{e2} = [-0.1314 \quad -1.0629].$$

Using the filter in (5), we can get $\gamma_{\min} = 0.0144$, it is less than the minimal level 0.0178 with $C_{fi} = 0$ in this paper, therefore, using of the new filter, the obtained criterion is less conservative than those without the new filter, and the filter parameters are as follows:

$$A_{f1} = \begin{bmatrix} -0.9852 & 0.3396 \\ -1.7748 & -3.1052 \end{bmatrix}, \qquad A_{f2} = \begin{bmatrix} -0.8193 & 0.4764 \\ -1.7667 & -4.4321 \end{bmatrix},$$
$$B_{f1} = \begin{bmatrix} 0.2099 \\ 0.1027 \end{bmatrix}, \qquad B_{f2} = \begin{bmatrix} -0.0138 \\ 0.4980 \end{bmatrix},$$
$$C_{f1} = \begin{bmatrix} -1.5825 & -2.6501 \\ -0.0599 & -0.0089 \end{bmatrix}, \qquad C_{f2} = \begin{bmatrix} 4.9954 & 2.6001 \\ -0.0710 & -0.0098 \end{bmatrix},$$
$$L_{f1} = [3.4654 \quad 4.3537], \qquad L_{f2} = [-0.1332 \quad -1.0648].$$

Therefore, we can see the advantage of the proposed method in this paper.

5 Conclusion

In the paper, the problem of robust H_∞ filter with the exponential stability is investigated. Some criterion is proposed to ensure the considered system to be exponentially stable, and it satisfies a prescribed H_∞ performance in terms of LMIs. Thanks to the new filter, the obtained criterion is less conservative than the existing ones. Numerical examples show the effectiveness of the proposed methods.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
HY carried out the main part of this manuscript. YM participated in the discussion and corrected the main theorem. All authors read and approved the final manuscript.

Acknowledgements
This paper is supported by the National Natural Science Foundation of China (No. 61273004).

Received: 25 June 2013 Accepted: 25 September 2013 Published: 12 Dec 2013

References

1. Ma, YC, Yan, HJ: Delay-dependent non-fragile robust dissipative filtering for uncertain nonlinear stochastic singular time-delay systems with Markovian jump parameters. Adv. Differ. Equ. **2013**, 135 (2013). http://www.advancesindifferenceequations.com/content/2013/1/135
2. Wang, D, Shi, P, Wang, JL, Wang, W: Delay-dependent exponential H_∞ filtering for discrete-time switched delay systems. Int. J. Robust Nonlinear Control **22**(13), 1522-1536 (2012)
3. Kim, JH: Delay-dependent robust H_∞ filtering for uncertain discrete-time singular systems with interval time-varying delay. Automatica **46**(3), 591-597 (2010)

4. Gao, HJ, Wang, CH: Delay-dependent robust H_∞ and $L_2 - L_\infty$ filtering for a class of uncertain nonlinear time-delay systems. IEEE Trans. Autom. Control **48**(9), 1661-1666 (2003)
5. Chen, CL, Liu, HX, Guan, XP: H_∞ filtering of time-delay T-S fuzzy systems based on piecewise Lyapunov-Krasovskii functional. Signal Process. **89**(10), 1998-2005 (2009)
6. Su, YK, Chen, B, Lin, C, Zhang, HG: A new fuzzy H_∞ filter design for nonlinear continuous-time dynamic systems with time-varying delays. Fuzzy Sets Syst. **160**(24), 3539-3549 (2009)
7. Lu, RQ, Wang, JH, Xue, AK, Ye, SH, Chu, J: Robust H_∞ filtering for a class of uncertain Lurie time-delay singular systems. Acta Autom. Sin. **3**(33), 292-296 (2007)
8. Wu, ZG, Su, HY, Chu, J: Delay-dependent H_∞ filtering for singular Markovian jump time-delay systems. Signal Process. **90**(6), 1815-1824 (2010)
9. Kwon, OM, Park, MJ, Lee, SM, Park, JH: Augmented Lyapunov-Krasovskii functional approaches to robust stability criteria for uncertain Takagi-Sugeno fuzzy systems with time-varying delays. Fuzzy Sets Syst. **201**(2), 1-19 (2012)
10. Park, MJ, Kwon, OM, Park, JH, Lee, SM: Simplified stability criteria for fuzzy Markovian jumping Hopfield neural networks of neutral type with interval time-varying delays. Expert Syst. Appl. **39**(5), 5625-5633 (2012)
11. Park, MJ, Kwon, OM, Park, JH, Lee, SM, Cha, EJ: Synchronization criteria of fuzzy complex dynamical networks with interval time-varying delays. Appl. Math. Comput. **218**(23), 11634-11647 (2012)
12. Gong, CZ, Zeng, XW: Delay-dependent nonfragile robust H_∞ filtering of T-S fuzzy time-delay systems. Circuits Syst. Signal Process. **29**(3), 361-375 (2010)
13. Huang, SJ, Zhang, DQ, He, Q, Zhang, NN: An improved H_∞ filter design for nonlinear system with time-delay via T-S fuzzy models. Acta Autom. Sin. **36**(10), 1454-1459 (2010)
14. Xu, SY, Lam, J: Exponential H_∞ filter design for uncertain Takagi-Sugeno fuzzy systems with time delay. Eng. Appl. Artif. Intell. **17**(6), 645-659 (2004)
15. Zhao, Y, Gao, HJ, Lam, J: New results on H_∞ filtering for fuzzy systems with interval time-varying delays. Inf. Sci. **181**(11), 2356-2369 (2011)
16. Xia, ZL, Li, JM, Li, JR: Delay-dependent non-fragile H_∞ filtering for uncertain fuzzy systems based on switching fuzzy model and piecewise Lyapunov function. Int. J. Autom. Comput. **7**(4), 428-437 (2010)
17. Lu, RQ, Xu, Y, Xue, AK: H_∞ filtering for singular systems with communication delays. Signal Process. **90**(4), 1240-1248 (2010)
18. Xiao, SP, Wu, M, Zhang, XM: Non-fragile delay-dependent robust control for uncertain systems. J. Syst. Sci. Math. Sci. **27**(3), 401-411 (2007)

10.1186/1687-1847-2013-362
Cite this article as: Ma and Yan: Delay-dependent robust H_∞ filter for T-S fuzzy time-delay systems with exponential stability. *Advances in Difference Equations* 2013, 2013:362