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## Onset of cavitation in the quark-gluon plasma

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**ABSTRACT:** We study the onset of bubble formation (cavitation) in the quark-gluon plasma as a result of the reduction of the effective pressure from bulk-viscous corrections. By calculating velocity gradients in typical models for quark-gluon plasma evolution in heavy-ion collisions, we obtain results for the critical bulk viscosity above which cavitation occurs. Since present experimental data for heavy-ion collision seems inconsistent with the presence of bubbles above the phase transition temperature of QCD, our results may be interpreted as an upper limit of the bulk viscosity in nature. Our results indicate that bubble formation is consistent with the expectation of hadronisation in low-temperature QCD.

**KEYWORDS:** QCD Phenomenology, Heavy Ion Phenomenology

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## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Bulk-viscous bubble formation in relativistic hydrodynamics</b>	<b>2</b>
2.1	Thermodynamics of cavitation in QCD	3
2.2	Critical bulk viscosity in first and second order hydrodynamics	3
<b>3</b>	<b>Cavitation in heavy-ion collisions</b>	<b>5</b>
3.1	Bjorken flow	5
3.2	Gubser flow	5
3.3	Numerical results	6
3.4	QCD equation of state	7
<b>4</b>	<b>Conclusion</b>	<b>7</b>

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## 1 Introduction

The experimental heavy-ion collisions programme conducted at the Relativistic Heavy Ion Collider and the Large Hadron Collider strongly suggests that the quark-gluon plasma (QGP) formed in these collisions behaves like an almost ideal fluid [1–4]. This fluid is very well-described by relativistic hydrodynamics [5]. For an ordinary fluid such as water, the effective pressure can be different than the equilibrium pressure and, in particular, in some situations it can drop below the vapour pressure. In this case, the thermodynamically preferred phase becomes the gas phase, and a vapour bubble forms inside the fluid, a phenomenon known as ‘cavitation’.<sup>1</sup> Mainly, high fluid velocities trigger cavitation in liquids. In the case of relativistic fluids such as the QGP studied in heavy-ion collisions, cavitation would imply a phase transition from a deconfined plasma phase of quarks and gluons to a confined hadron-gas phase. The resulting medium would be highly inhomogeneous with (possibly short-lived) hadron gas bubbles expanding and collapsing in an otherwise laminar fluid. Maybe more importantly, hadron gas dynamics would take over at temperatures above the QCD phase transition, which would have immediate consequences on the measured particle spectra. The apparent success of describing experimental data by relatively simple, laminar fluid flows and subsequent freeze-out at the QCD phase transition temperature seems inconsistent with the presence of hadron gas bubbles, or even the onset of fluid instabilities in the high-temperature QGP. In the present work, we will thus make the assumption that cavitation does *not* occur in the experimentally observed QGP. We

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<sup>1</sup>We note that if the bubble is unstable, then it quickly collapses. Nevertheless, the onset of cavitation signals an instability in the fluid evolution.

will study cavitation in relativistic fluids with a given bulk viscosity coefficient and then proceed to rule out bulk viscosity values under the above assumption.

In this article, the effective pressure is defined as one third of the trace of the pressure tensor [6]. For this definition, the shear viscous contribution to the pressure cancels, whereas the bulk-viscous contribution remains present. This differs from other approaches taken in the literature, which focused on single components of the pressure tensor [7–11] where the shear viscous contribution is present and important. Because shear-viscous effects will add on to the effects considered in this work, cavitation could occur in regions which — in our analysis — are found to be stable, but not the other way around. One caveat of our approach is that in the calculations that follow, the bulk-viscous contributions to the fluid flow profiles themselves have been neglected for simplicity. In principle, these contributions should be taken into account, but in practice, one expects the corrections to be small as long as the bulk viscosity coefficient itself is small [12]. Thus, our approach essentially amounts to a linear-response treatment of bulk-viscous effects in fully non-linear, shear-viscous fluid dynamics.

This work is organised as follows: in section 2, cavitation for relativistic hydrodynamics is defined. In section 2.2, the main idea of constraining bulk viscosity is elucidated for 1st and 2nd order hydrodynamics. Section 3 applies this framework to heavy-ion collisions, i.e., the critical bulk viscosity for cavitation is calculated for analytical and numerical flow profiles, as well as different equations of state. We present our conclusions and an upper limit for the QCD bulk viscosity in section 4.

## 2 Bulk-viscous bubble formation in relativistic hydrodynamics

Cavitation can be defined as the drop of pressure below the saturated vapour pressure of the particular liquid (see p. 6 in ref. [13]). This definition needs to be revisited for relativistic fluids which can have a pressure tensor that differs strongly from equilibrium. Formally, starting from a standard decomposition of the energy-momentum tensor

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + (p - \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu}, \tag{2.1}$$

with  $\epsilon, p, u^\mu$  the energy density, pressure and fluid four velocity, and the projector  $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$  with a mostly minus signature metric tensor  $g^{\mu\nu}$ , we identify  $\Pi, \pi^{\mu\nu}$  as the bulk- and shear-viscous stress tensor components. In this work, we concentrate on the effective, local pressure defined in three dimensions as

$$p_{\text{eff}} \equiv -\frac{1}{3} \Delta_{\mu\nu} T^{\mu\nu} = p - \Pi. \tag{2.2}$$

This preserves the normal, intuitive definition of pressure in the rest frame and, being a scalar, is easy to interpret in non-equilibrium situation.

Akin to ref. [13], the *occurrence of cavitation* can be mathematically defined as

$$p_{\text{eff}} < p_v, \tag{2.3}$$

with  $p_v$  the ‘vapour’ pressure of a different thermodynamic phase having the same temperature. In words, this means that if the effective pressure  $p_{\text{eff}}$  of a QGP falls below  $p_v$ , then

the liquid will undergo a phase transition to the hadron-gas phase (often referred to as ‘freeze-out’ in the language of heavy-ion collisions) and a (small) gas bubble will form. The definition (2.3) is intuitively much easier to interpret than the case considered by most other authors [7–11], where a single component of the pressure tensor drops below the vapour pressure, whereas other component(s) will generally be larger than the vapour pressure.

## 2.1 Thermodynamics of cavitation in QCD

In ordinary fluids such as water, the transition from vapour to liquid is first order, and cavitation leads to the formation of vapour bubbles having the same temperature but different density than the surrounding liquid. The confinement-deconfinement transition in QCD at vanishing baryon density has been calculated to be an analytic crossover from hadron gas to QGP using lattice gauge theory [14]. As a consequence, one may worry if cavitation in QCD may occur at all, given that the thermodynamic properties such as energy density are continuous across the phase boundary.

In the following, we will assume that cavitation in QCD is possible in principle in the sense of forming hadron gas domains (‘cavities’) inside the QGP liquid. Our assumption is based on two arguments: first, note that while the confinement-deconfinement transition in QCD is an analytic crossover in equilibrium situations at vanishing baryon density, our present work focusses on *non-equilibrium transitions*<sup>2</sup> induced by strong fluid flow gradients. In particular, these strong fluid gradients may lead to confinement-deconfinement transitions induced at temperatures well above the critical temperature of equilibrium QCD  $T_c$ . The nature of the non-equilibrium confinement-deconfinement transition induced at  $T > T_c$  is not known, but it is possible to be a true phase transition, given that one may expect the energy density between QGP and hadron gas phase to be discontinuous at constant temperatures  $T > T_c$ .

Second, if the non-equilibrium confinement-deconfinement transition at  $T > T_c$  *continued to be* an analytic crossover, one would still expect the drop in effective liquid pressure to be associated with phase-separated domains, possibly with extended domain walls (thick-walled bubbles), or more in line with elements found in spinodal decomposition if there is no energy barrier separating the two phases. This would be in-line with the expectation for  $Z(3)$  domains studied in [15].

Once created, we expect the inhomogeneous liquid-gas domains to evolve in a non-trivial fashion; we reserve a study of this fascinating subject for future work.

## 2.2 Critical bulk viscosity in first and second order hydrodynamics

After establishing a criterion for cavitation, the critical bulk viscosity for the onset of cavitation can be calculated by assuming the validity of hydrodynamics.

**For 1st order hydrodynamics.** The effective pressure (2.2) up to 1st order gradients [16] for a non-conformal fluid is

$$p_{\text{eff}} = p - \Pi \approx p - \zeta \nabla_\mu u^\mu. \quad (2.4)$$

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<sup>2</sup>Note that by *non-equilibrium*, we mean slightly off equilibrium for which hydrodynamics is still valid, in contrast to far-from-equilibrium transitions.

The critical bulk viscosity is defined as the maximum value of  $\zeta$  for which the fluid flow is still *non-cavitating*. Assuming the sign of the gradient  $\nabla_\mu u^\mu$  to be positive (this is the case for all the scenarios we consider below), one finds

$$\left. \frac{\zeta}{s} \right|_{\text{crit}} \equiv \frac{(p - p_v) T}{(\epsilon + p) \nabla_\mu u^\mu}, \quad (2.5)$$

where the bulk viscosity was divided by the entropy density  $s = (\epsilon + p)/T$ , yielding a dimensionless ratio.

**For 2nd order hydrodynamics.** In order to assess the accuracy of 1st order calculation, one can consider the effect of 2nd order gradients. Expanding the viscosity scalar  $\Pi$  of eq. (2.2) up to 2nd order in gradients yields for a flat space-time [16]

$$p_{\text{eff}} = p - \zeta \nabla_\mu u^\mu + \zeta \tau_\Pi D(\nabla_\mu u^\mu) + \xi_1 \sigma^{\mu\nu} \sigma_{\mu\nu} + \xi_2 (\nabla_\mu u^\mu)^2. \quad (2.6)$$

Most of the 2nd order transport coefficients  $\tau_\Pi, \xi_1, \xi_2$  are poorly known for most quantum field theories. However, in a particular strong coupling<sup>3</sup> (see ref. [17]), these have been calculated [16]:

$$\eta = \frac{3\zeta}{2(1 - 3c_s^2)}, \quad (2.7)$$

$$\zeta \tau_\Pi = \zeta \tau_\pi = \frac{\zeta}{\epsilon + p} \eta (4 - \ln 4), \quad (2.8)$$

$$\xi_1 = \frac{\lambda_1}{3} (1 - 3c_s^2) = \frac{2\eta^2}{3(\epsilon + p)} (1 - 3c_s^2) = \frac{\zeta}{\epsilon + p} \eta, \quad (2.9)$$

$$\xi_2 = \frac{2\eta \tau_\Pi c_s^2}{3} (1 - 3c_s^2) = \frac{\zeta \tau_\Pi c_s^2}{1 - 3c_s^2} = \frac{\zeta}{\epsilon + p} \eta c_s^2 (4 - \ln 4). \quad (2.10)$$

By expressing these transport coefficients in terms of the speed of sound squared  $c_s^2$ ,  $\zeta$ , and shear viscosity  $\eta$ , one finds the critical bulk viscosity in 2nd order hydrodynamics:

$$\left. \frac{\zeta}{s} \right|_{\text{crit}} \equiv \frac{(p_v - p) T}{(\epsilon + p)} \left[ \nabla_\mu u^\mu - \frac{\eta (4 - \ln 4) (D \nabla_\mu u^\mu + c_s^2 (\nabla_\mu u^\mu)^2) + \sigma^{\mu\nu} \sigma_{\mu\nu}}{T} \right]^{-1}. \quad (2.11)$$

Note that in the 2nd order result, there is a pole in the critical bulk viscosity once the 2nd order gradient terms become as large as the 1st order terms. We find that for QCD this typically happens at very low temperatures (far below  $T_c$ ), where one does not expect a hydrodynamic description to be applicable in the first place. In weak coupling, the transport coefficients typically lead to a quadratic dependence of  $\zeta/s$  [18, 19] which is beyond the scope of our linear-response treatment.

To recapitulate, this method assumes a conformal, hydrodynamic description and uses the resulting flow profile to constrain the maximum value of  $\zeta/s$  by requiring that the *effective pressure does not drop below zero*.

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<sup>3</sup>Note that by construction, this particular theory only includes terms of linear order in bulk viscosity, e.g., terms of the form  $\zeta^2$  are absent in the transport coefficients.

### 3 Cavitation in heavy-ion collisions

In this section, the critical bulk viscosity coefficient for the onset of cavitation is calculated for specific flow profiles used in the modelling of heavy-ion collisions. Specifically, the flow gradients are calculated for Bjorken flow [20]; Gubser flow [21, 22]; and a numerical solver for relativistic, viscous hydrodynamics in 2+1 dimensions [23, 24]. All these flow profiles are for *conformal fluids*, e.g., they ignore effects of bulk viscosity in the flow itself (see the discussion in section 1). In this entire section, the vapour pressure  $p_v$  is chosen to be zero:

$$p_v \equiv 0, \tag{3.1}$$

which will result in the most conservative estimates of cavitation since higher values of  $p_v$  would decrease  $\zeta/s|_{\text{crit}}$  (e.g., see eq. (2.5)).

#### 3.1 Bjorken flow

The set-up that was suggested by Bjorken [20] in 1982 can be utilised to extract a benchmark value on the bulk viscosity. It is particularly simple to use Milne coordinates  $x^\mu = (\tau, x, y, \eta)$  because the fluid velocity becomes  $u^\mu = (1, 0, 0, 0)^T$ , i.e., the fluid is locally at rest. The velocity gradient simply depends on the Christoffel symbols for Milne coordinates

$$\nabla_\mu u^\mu = \frac{1}{\tau}, \tag{3.2}$$

whereas the temperature evolution is governed by

$$T = T_0 (\tau/\tau_0)^{-c_s^2}. \tag{3.3}$$

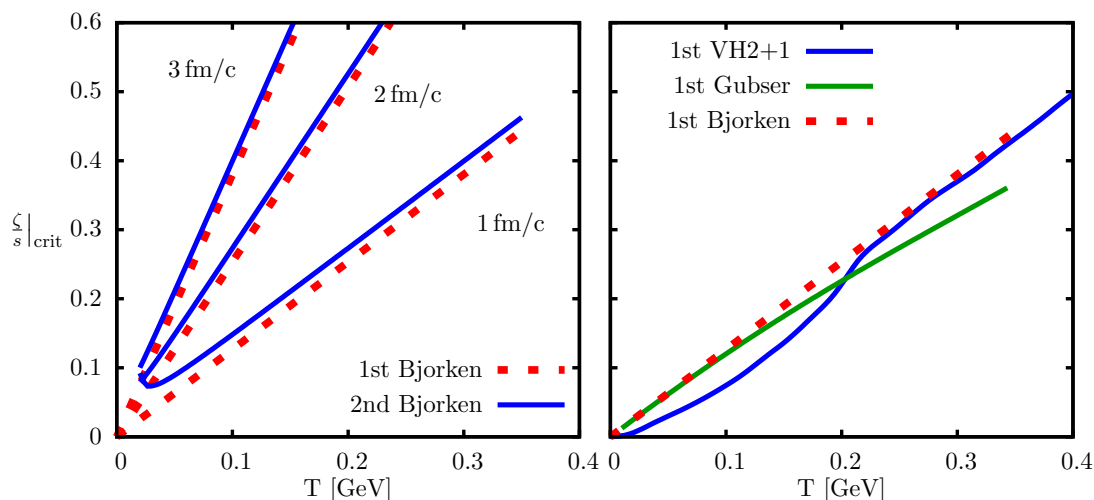
This framework is comparatively simple because it entirely neglects transverse, spatial dynamics of the QGP. For an ideal equation of state the dynamics is completely governed by the gradients: evidently, the gradient is always positive for  $\tau > 0$  for 1st order hydrodynamics; however, for 2nd order gradients, the gradients become negative after diverging for early times and/or low temperatures ( $\tau T \ll 1$ ).

The critical bulk viscosity for Bjorken flow is monotonously increasing with temperature. By taking 2nd order gradients into account, only small changes are present, as can be seen in figure 1. The effect of these higher-order terms is small due to the small numerical values of the 2nd order transport coefficients.

#### 3.2 Gubser flow

By expressing the flow profile that was proposed by Gubser [21] in Milne coordinates, one finds a fluid flow profile

$$u^\mu = \left( \frac{1 + q^2 r^2 + q^2 \tau^2}{2q\tau\sqrt{1+g^2}}, \frac{qr}{\sqrt{1+g^2}}, 0, 0 \right)^T \tag{3.4}$$



**Figure 1.** Critical bulk viscosity as a function of temperature for ideal equation of state. Areas above respective lines of  $\zeta/s|_{\text{crit}}$  are regions where cavitation occurs. Left: comparison of 1st and 2nd order results for Bjorken flow. Right: comparison between Bjorken flow, Gubser flow, and numerical computations.

and a temperature profile

$$T = \frac{1}{\tau f_*^{1/4}} \left\{ \frac{\hat{T}_0}{(1+g^2)^{1/3}} + \frac{H_0}{\sqrt{1+g^2}} \left[ 1 - (1+g^2)^{1/6} {}_2F_1 \left( \frac{1}{2}, \frac{1}{6}, \frac{3}{2}, -g^2 \right) \right] \right\}, \quad (3.5)$$

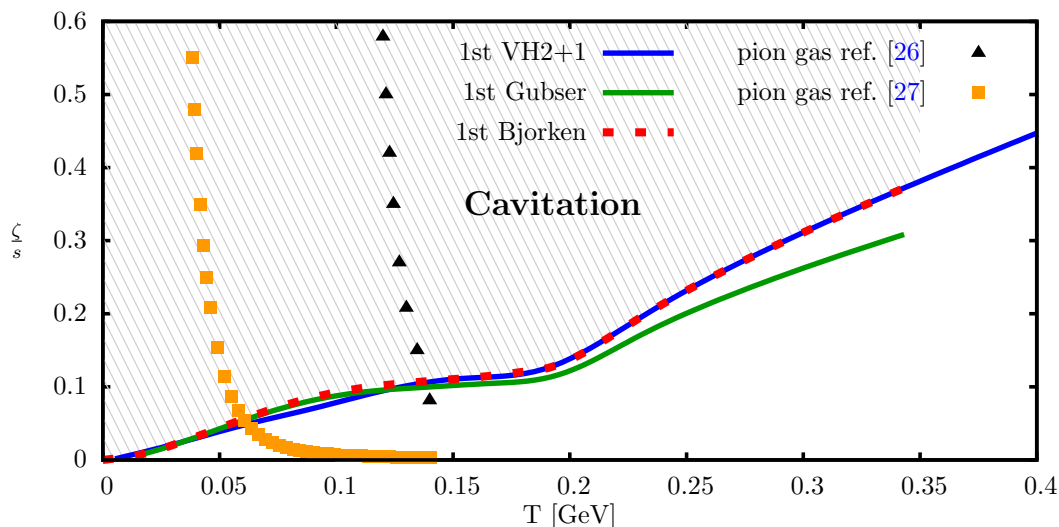
where

$$g = \frac{1 + q^2 r^2 - q^2 \tau^2}{2q\tau}, \quad f_*^{1/4}, \quad \hat{T} = 5.55, \quad H_0 = 0.33, \quad 1/q = 4.3\text{fm}.$$

The variables  $\tau$  and  $r$  denote the proper time and radial distance, respectively. The specific form of fluid velocity and temperature arises from symmetry considerations [21, 22]. In comparison to Bjorken flow, this flow profile is more realistic because radial velocities are non-vanishing; hence, the transverse dynamics is not neglected but fixed to have a unique, analytical form. By comparing the different orders of Gubser flow, one sees a similar behaviour to Bjorken flow, i.e., higher-order gradients decrease the denominator; thus, 2nd order terms increase the value of  $\zeta/s|_{\text{crit}}$  (see figure 1).

### 3.3 Numerical results

The last flow profile stems from a numerical simulation that fully includes transverse dynamics (“VH2+1”, see refs. [23, 24] for details) for an initial condition of a central  $Au + Au$  collision at  $\sqrt{s} = 200$  GeV per nucleon pair. It was initialised with vanishing flow at early times, such that the high-temperature  $\zeta/s|_{\text{crit}}$  behaviour matches the Bjorken flow result, as it should. For late times, the significant fluid velocity gradients differ from both the Bjorken and Gubser flow results, resulting in a different  $\zeta/s|_{\text{crit}}$  behaviour at low temperatures (see figure 1).



**Figure 2.** Bulk viscosity over entropy density ratio as a function of temperature. Shown are results for the lowest critical bulk viscosity coefficient  $\zeta/s|_{\text{crit}}$  using different flow profiles and a QCD equation of state. For higher viscosity values, we predict bubbles to form in the liquid (‘cavitation’). For comparison we also show the result of a calculation of  $\zeta/s$  for two pion gases from refs. [26, 27]. The pion gas is calculated up to  $T = 140$  MeV.

### 3.4 QCD equation of state

For a realistic model of the QGP, we have repeated the above calculations for  $\zeta/s|_{\text{crit}}$  with a QCD equation of state (see ref. [25]). In figure 2, the *lowest* value of  $\zeta/s|_{\text{crit}}$  for Bjorken, Gubser and numerical flow profiles, respectively, is shown. For comparison, we also show the result of calculations of  $\zeta/s$  for a pion gas from refs. [26, 27]. The former is performed at chemical equilibrium; whereas, the latter being an out-of-chemical equilibrium calculation for which elastic scattering is the dominant process [27, 28]. In dynamical heavy-ion collisions  $\zeta/s$  is most likely to lie between these curves. At very high temperature, one could also compare to perturbative QCD calculations from ref. [29] where  $\zeta/s \sim 0.01\alpha_s^2$ , which tends to fall as a function of temperature, whereas  $\zeta/s|_{\text{crit}}$  rises with temperature. Thus, we presume that cavitation is a phenomenon of low temperatures — not of high temperatures.

## 4 Conclusion

In this work, we have studied the onset of bubble formation (cavitation) in the QGP resulting from the presence of bulk-viscous terms in relativistic hydrodynamics. We found that at temperatures  $T < 140$  MeV, a bulk viscosity coefficient smaller than that expected from a pion gas leads to the formation of hadron gas bubbles in the QGP liquid (see also refs. [6–10, 26, 27, 30, 31]). This may be interpreted as the known freeze-out phenomenon in heavy-ion collisions where the plasma undergoes a phase transition to a hadron gas. At around the QCD phase transition temperature, we predict that for values of  $\zeta/s \gtrsim 0.1$ , cavitation in the QGP will occur. Under the assumption that experimental data on the



QGP from heavy-ion collisions is inconsistent with the presence of hadron gas bubbles, our results for  $\zeta/s|_{\text{crit}}$  may be interpreted as an upper bound on the bulk viscosity in high-temperature QCD. At very high temperatures, this interpretation seems consistent with known perturbative values of  $\zeta/s$ . Several aspects of our work can and should be improved in subsequent studies: first, it is possible to implement the corrections from bulk viscosity in the flow profiles used in the calculations of  $\zeta/s|_{\text{crit}}$ , eliminating the approximation we have used in this work. Second, one can repeat our study with more realistic approximations for the hadron gas pressure than our choice:  $p_v \equiv 0$ . Ultimately, and maybe most importantly, it would be interesting to calculate the particle spectra from a numerical simulation including the presence of hadron gas bubbles. This could potentially be done using state-of-the-art numerical hydrodynamical solvers [5, 32, 33] and could verify the assumption that cavitating fluids are inconsistent with experimental data on heavy-ion collisions.

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