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On strong convergence of an iterative algorithm for common fixed point and generalized equilibrium problems

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Abstract
In this article, an iterative algorithm for finding a common element in the solution set of generalized equilibrium problems and in the common fixed point set of a family of nonexpansive mappings. Strong convergence of the algorithm is established in the framework of Hilbert spaces.
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1 Introduction and preliminaries

Equilibrium problems which were introduced by Ky Fan [1] and further studied by Blum and Oettli [2] have intensively been investigated based on iterative methods. The equilibrium problems have emerged as an effective and powerful tool for studying a wide class of problems which arise in economics, ecology, transportation, network, elasticity, and optimization; see [3–6] and the references therein. It is well known that the equilibrium problems cover fixed point problems, variational inequality problems, saddle problems, inclusion problems, complementarity problems, and minimization problems; see [7–15] and the references therein.

In this paper, an iterative algorithm is proposed for treating common fixed point and generalized equilibrium problems. It is proved that the sequence generated in the algorithm converges strongly to a common element in the solution set of generalized equilibrium problems and in the common fixed point set of a family of nonexpansive mappings.

From now on, we always assume that H is a real Hilbert space, whose inner product and norm are denoted by $\langle\cdot,\cdot\rangle$ and $\|\cdot\|$, respectively. Let C be a nonempty closed convex subset of H and let P_C be the projection of H onto C.

Let $S: C \to C$ be a mapping. Throughout this paper, we use $F(S)$ to denote the fixed point set of the mapping S. Recall that $S: C \to C$ is said to be nonexpansive iff

$$\|Sx - Sy\| \le \|x - y\|, \quad \forall x, y \in C.$$

$S: C \to C$ is said to be firmly nonexpansive iff

$$\|Sx - Sy\|^2 \le \langle Sx - Sy, x - y\rangle, \quad \forall x, y \in C.$$

It is easy to see that every firmly nonexpansive mapping is nonexpansive.

Let $A : C \to H$ be a mapping. Recall that A is said to be monotone iff

$$\langle Ax - Ay, x - y\rangle \geq 0, \quad \forall x, y \in C.$$

A is said to be inverse-strongly monotone iff there exists a constant $\alpha > 0$ such that

$$\langle Ax - Ay, x - y\rangle \geq \delta \|Ax - Ay\|^2, \quad \forall x, y \in C.$$

For such a case, A is also said to be α-inverse-strongly monotone. It is known that if $S : C \to C$ is nonexpansive, then $A = I - S$ is $\frac{1}{2}$-inverse-strongly monotone. Recall that a set-valued mapping $T : H \to 2^H$ is called monotone if, for all $x, y \in H, f \in Tx$ and $g \in Ty$ imply $\langle x - y, f - g\rangle \geq 0$. A monotone mapping $T : H \to 2^H$ is maximal if the graph of $G(T)$ of T is not properly contained in the graph of any other monotone mapping. It is well known that a monotone mapping T is maximal if and only if for $(x, f) \in H \times H$, $\langle x - y, f - g\rangle \geq 0$ for every $(y, g) \in G(T)$ implies $f \in Tx$. Let B be a monotone map of C into H and let $N_C v$ be the normal cone to C at $v \in C$, *i.e.*, $N_C v = \{w \in H : \langle v - u, w\rangle \geq 0, \forall u \in C\}$ and define

$$Tv = \begin{cases} Bv + N_C v, & v \in C, \\ \emptyset, & v \notin C. \end{cases}$$

Then T is maximal monotone and $0 \in Tv$ if and only if $\langle Av, u - v\rangle \geq 0$, for $\forall u \in C$; see [16] and the references therein

Recall that the classical variational inequality is to find $u \in C$ such that

$$\langle Au, v - u\rangle \geq 0, \quad \forall v \in C. \tag{1.1}$$

In this paper, we use $VI(C, A)$ to denote the solution set of the variational inequality (1.1). One can see that the variational inequality (1.1) is equivalent to a fixed point problem. The element $u \in C$ is a solution of the variational inequality (1.1) if and only if $u \in C$ is a fixed point of the mapping $P_C(I - \lambda A)$, where $\lambda > 0$ is a constant and I denotes the identity mapping. If A is an α-inverse strongly monotone, we remark here that the mapping $P_C(I - \lambda A)$ is nonexpansive iff $0 < \lambda < 2\alpha$. Indeed,

$$\begin{aligned} \left\|P_C(I - \lambda A)x - P_C(I - \lambda A)y\right\|^2 &\leq \left\|(I - \lambda A)x - (I - \lambda A)y\right\|^2 \\ &= \|x - y\|^2 - 2\lambda\langle x - y, Ax - Ay\rangle + \lambda^2\|Ax - Ay\|^2 \\ &\leq \|x - y\|^2 - \lambda(2\alpha - \lambda)\|Ax - Ay\|^2. \end{aligned}$$

This alternative equivalent formulation has played a significant role in the studies of the variational inequalities and related optimization problems.

Let A be an inverse-strongly monotone mapping, F a bifunction of $C \times C$ into $\mathbb{R}$, where $\mathbb{R}$ is the set of real numbers. We consider the following equilibrium problem:

$$\text{Find } z \in C \text{ such that } F(z, y) + \langle Az, y - z\rangle \geq 0, \quad \forall y \in C. \tag{1.2}$$

In this paper, the set of such $z \in C$ is denoted by $EP(F,A)$, *i.e.*,

$$EP(F,A) = \{z \in C : F(z,y) + \langle Az, y - z\rangle \geq 0, \forall y \in C\}.$$

If the case of $A \equiv 0$, the zero mapping, the problem (1.2) is reduced to

$$\text{Find } z \in C \text{ such that } F(z,y) \geq 0, \quad \forall y \in C. \tag{1.3}$$

In this paper, we use $EP(F)$ to denote the solution set of the problem (1.3). The problem of (1.2) and (1.3) have been considered by many authors; see, for example, [17–29] and the references therein. In the case of $F \equiv 0$, the problem (1.2) is reduced to the classical variational inequality (1.1).

To study the equilibrium problems, we assume that the bifunction $F : C \times C \to \mathbb{R}$ satisfies the following conditions:

(A1) $F(x,x) = 0$ for all $x \in C$;
(A2) F is monotone, *i.e.*, $F(x,y) + F(y,x) \leq 0$ for all $x, y \in C$;
(A3) for each $x, y, z \in C$,

$$\limsup_{t \downarrow 0} F\big(tz + (1-t)x, y\big) \leq F(x,y);$$

(A4) for each $x \in C$, $y \mapsto F(x,y)$ is convex and lower semi-continuous.

The well-known convex feasibility problem which captures applications in various disciplines such as image restoration, and radiation therapy treatment planning is to find a point in the intersection of common fixed point sets of a family of nonlinear mappings. In this paper, we propose an iterative algorithm for finding a common element in the solution set of the generalized equilibrium problem (1.2) and in the common fixed point set of a family of nonexpansive mappings. Strong convergence of the algorithm is established in the framework of Hilbert spaces.

In order to prove our main results, we need the following definitions and lemmas.

A space X is said to satisfy Opial's condition [30] if for each sequence $\{x_n\}_{n=1}^{\infty}$ in X which converges weakly to point $x \in X$, we have

$$\liminf_{n\to\infty} \|x_n - x\| < \liminf_{n\to\infty} \|x_n - y\|, \quad \forall y \in X, y \neq x.$$

It is well known that the above inequality is equivalent to

$$\limsup_{n\to\infty} \|x_n - x\| < \limsup_{n\to\infty} \|x_n - y\|, \quad \forall y \in X, y \neq x.$$

The following lemma can be found in [2].

Lemma 1.1 *Let C be a nonempty closed convex subset of H ad let $F : C \times C \to \mathbb{R}$ be a bifunction satisfying* (A1)-(A4). *Then, for any $r > 0$ and $x \in H$, there exists $z \in C$ such that*

$$F(z,y) + \frac{1}{r}\langle y - z, z - x\rangle \geq 0, \quad \forall y \in C.$$

Further, if $T_r x = \{z \in C : F(z,y) + \frac{1}{r}\langle y - z, z - x\rangle \geq 0, \forall y \in C\}$, *then the following hold*:

$$T_r(x) = \left\{z \in C : F(z,y) + \frac{1}{r}\langle y - z, z - x\rangle \geq 0, \forall y \in C\right\}$$

for all $z \in H$. *Then the following hold*:

1. T_r *is single-valued*;
2. T_r *is firmly nonexpansive, i.e., for any* $x, y \in H$,

$$\|T_r x - T_r y\|^2 \leq \langle T_r x - T_r y, x - y\rangle;$$

3. $F(T_r) = EP(F)$;
4. $EP(F)$ *is closed and convex*.

Lemma 1.2 [20] *Let* C, H, F *and* T_r *be as in Lemma* 1.1. *Then the following holds*:

$$\|T_s x - T_t x\|^2 \leq \frac{s - t}{s}\langle T_s x - T_t x, T_s x - x\rangle$$

for all $s, t > 0$ *and* $x \in H$.

Lemma 1.3 [31] *Assume that* $\{\alpha_n\}$ *is a sequence of nonnegative real numbers such that*

$$\alpha_{n+1} \leq (1 - \gamma_n)\alpha_n + \delta_n,$$

where $\{\gamma_n\}$ *is a sequence in* $(0,1)$ *and* $\{\delta_n\}$ *is a sequence such that*

(a) $\sum_{n=1}^{\infty} \gamma_n = \infty$;
(b) $\limsup_{n\to\infty} \delta_n/\gamma_n \leq 0$ *or* $\sum_{n=1}^{\infty} |\delta_n| < \infty$.

Then $\lim_{n\to\infty} \alpha_n = 0$.

Definition 1.4 [32] Let $\{S_i : C \to C\}$ be a family of infinitely nonexpansive mappings and $\{\gamma_i\}$ be a nonnegative real sequence with $0 \leq \gamma_i < 1$, $\forall i \geq 1$. For $n \geq 1$ define a mapping $W_n : C \to C$ as follows:

$$\begin{aligned}
&U_{n,n+1} = I,\\
&U_{n,n} = \gamma_n S_n U_{n,n+1} + (1 - \gamma_n)I,\\
&U_{n,n-1} = \gamma_{n-1} S_{n-1} U_{n,n} + (1 - \gamma_{n-1})I,\\
&\vdots\\
&U_{n,k} = \gamma_k S_k U_{n,k+1} + (1 - \gamma_k)I,\\
&u_{n,k-1} = \gamma_{k-1} S_{k-1} U_{n,k} + (1 - \gamma_{k-1})I,\\
&\vdots\\
&U_{n,2} = \gamma_2 S_2 U_{n,3} + (1 - \gamma_2)I,\\
&W_n = U_{n,1} = \gamma_1 S_1 U_{n,2} + (1 - \gamma_1)I.
\end{aligned} \tag{1.4}$$

Such a mapping W_n is nonexpansive from C to C and it is called a W-mapping generated by $S_n, S_{n-1}, \dots, S_1$ and $\gamma_n, \gamma_{n-1}, \dots, \gamma_1$.

Lemma 1.5 [32] *Let C be a nonempty closed convex subset of a Hilbert space H, $\{S_i : C \to C\}$ be a family of infinitely nonexpansive mappings with $\bigcap_{i=1}^{\infty} F(S_i) \neq \emptyset$, $\{\gamma_i\}$ be a real sequence such that $0 < \gamma_i \leq l < 1$, $\forall i \geq 1$. Then*

(1) *W_n is nonexpansive and $F(W_n) = \bigcap_{i=1}^{\infty} F(S_i)$, for each $n \geq 1$;*
(2) *for each $x \in C$ and for each positive integer k, the limit $\lim_{n\to\infty} U_{n,k}$ exists;*
(3) *the mapping $W : C \to C$ defined by*

$$Wx := \lim_{n\to\infty} W_n x = \lim_{n\to\infty} U_{n,1} x, \quad x \in C, \tag{1.5}$$

is a nonexpansive mapping satisfying $F(W) = \bigcap_{i=1}^{\infty} F(S_i)$ and it is called the W-mapping generated by $S_1, S_2, \dots$ and $\gamma_1, \gamma_2, \dots$.

Lemma 1.6 [27] *Let C be a nonempty closed convex subset of a Hilbert space H, $\{S_i : C \to C\}$ be a family of infinitely nonexpansive mappings with $\bigcap_{i=1}^{\infty} F(S_i) \neq \emptyset$, $\{\gamma_i\}$ be a real sequence such that $0 < \gamma_i \leq l < 1$, $\forall i \geq 1$. If K is any bounded subset of C, then*

$$\lim_{n\to\infty} \sup_{x\in K} \|Wx - W_n x\| = 0.$$

Throughout this paper, we always assume that $0 < \gamma_i \leq l < 1$, $\forall i \geq 1$.

Lemma 1.7 [33] *Let $\{x_n\}$ and $\{y_n\}$ be bounded sequences in a Hilbert space H and let $\{\beta_n\}$ be a sequence in $[0,1]$ with $0 < \liminf_{n\to\infty} \beta_n \leq \limsup_{n\to\infty} \beta_n < 1$. Suppose that $x_{n+1} = (1-\beta_n) y_n + \beta_n x_n$ for all $n \geq 0$ and*

$$\limsup_{n\to\infty} \bigl(\|y_{n+1} - y_n\| - \|x_{n+1} - x_n\|\bigr) \leq 0.$$

Then $\lim_{n\to\infty} \|y_n - x_n\| = 0$.

2 Main results

Theorem 2.1 *Let C be a nonempty closed convex subset of a Hilbert space H and let F be a bifunction from $C \times C$ to $\mathbb{R}$ which satisfies (A1)-(A4). Let $A : C \to H$ be an α-inverse-strongly monotone mapping and let $\{S_i : C \to C\}$ be a family of infinitely nonexpansive mappings. Assume that $\Omega := \bigcap_{i=1}^{\infty} F(S_i) \cap EP(F, A) \neq \emptyset$. Let $f : C \to C$ be a κ-contractive mapping. Let $\{x_n\}$ be a sequence generated in the following process: let it be a sequence generated in*

$$\begin{cases} x_1 \in C, \quad \text{chosen arbitrarily}, \\ F(y_n, y) + \langle Ax_n, y - y_n \rangle + \frac{1}{r_n} \langle y - y_n, y_n - x_n \rangle \geq 0, \quad \forall y \in C, \\ x_{n+1} = \beta_n x_n + (1-\beta_n) W_n(\alpha_n f(W_n x_n) + (1-\alpha_n) y_n), \quad \forall n \geq 1, \end{cases}$$

where $\{W_n\}$ is the mapping sequence defined by (1.4), $\{\alpha_n\}$, and $\{\beta_n\}$ are sequences in $[0,1]$ and $\{r_n\}$ is a positive number sequence. Assume that the above control sequences satisfy the following restrictions:

(a) $0 < a \le \beta_n \le b < 1$, $0 < c \le r_n \le d < 2\alpha$;
(b) $\lim_{n\to\infty} \alpha_n = 0$, *and* $\sum_{n=1}^{\infty} \alpha_n = \infty$;
(c) $\lim_{n\to\infty}(r_n - r_{n+1}) = 0$.

Then $\{x_n\}$ *converges strongly to a point* $x \in \Omega$, *where* $x = P_\Omega f(x)$.

Proof First, we show that the sequence $\{x_n\}$ and $\{y_n\}$ are bounded. Fixing $x^* \in \Omega$, we find that

$$\begin{aligned}
\|y_n - x^*\|^2 &= \|T_{r_n}(x_n - r_nAx_n) - T_{r_n}(x^* - r_nAx^*)\|^2 \\
&\le \|(x_n - r_nAx_n) - (x^* - r_nAx^*)\|^2 \\
&= \|(x_n - x^*) - r_n(Ax_n - Ax^*)\|^2 \\
&= \|x_n - x^*\|^2 - 2r_n\langle x_n - x^*, Ax_n - Ax^*\rangle + r_n^2\|Ax_n - Ax^*\|^2 \\
&\le \|x_n - x^*\|^2 - 2r_n\alpha\|Ax_n - Ax^*\|^2 + r_n^2\|Ax_n - Ax^*\|^2 \\
&= \|x_n - x^*\|^2 + r_n(r_n - 2\alpha)\|Ax_n - Ax^*\|^2. \qquad (2.1)
\end{aligned}$$

Using the restriction (a), we find that

$$\|y_n - x^*\| \le \|x_n - x^*\|. \qquad (2.2)$$

From the above, we also find that the mappings $I - r_nA$ is nonexpansive. Putting $z_n = \alpha_n f(W_nx_n) + (1 - \alpha_n)y_n$, we find from (2.2) that

$$\begin{aligned}
\|z_n - x^*\| &= \|\alpha_n f(W_nx_n) + (1 - \alpha_n)y_n - x^*\| \\
&\le \alpha_n\|f(W_nx_n) - x^*\| + (1 - \alpha_n)\|y_n - x^*\| \\
&\le \alpha_n\kappa\|x_n - x^*\| + \alpha_n\|f(x^*) - x^*\| + (1 - \alpha_n)\|y_n - x^*\| \\
&\le (1 - \alpha_n(1 - \kappa))\|x_n - x^*\| + \alpha_n\|f(x^*) - x^*\|. \qquad (2.3)
\end{aligned}$$

It follows from (2.3) that

$$\begin{aligned}
\|x_{n+1} - x^*\| &\le \beta_n\|x_n - x^*\| + (1 - \beta_n)\|W_nz_n - x^*\| \\
&\le \beta_n\|x_n - x^*\| + (1 - \beta_n)\|z_n - x^*\| \\
&\le (1 - \alpha_n(1 - \beta_n)(1 - \kappa))\|x_n - x^*\| + \alpha_n(1 - \beta_n)\|f(x^*) - x^*\| \\
&\le \cdots \\
&\le \max\left\{\|x_1 - x^*\|, \frac{\|f(x^*) - x^*\|}{1 - \kappa}\right\}.
\end{aligned}$$

This shows that the sequence $\{x_n\}$ is bounded, and so are $\{y_n\}$ and $\{z_n\}$. Without loss of generality, we can assume that there exists a bounded set $K \subset C$ such that $x_n, y_n, z_n \in K$;

$$\begin{aligned}
\|y_{n+1} - y_n\| = \|&T_{r_{n+1}}(x_{n+1} - r_{n+1}Ax_{n+1}) - T_{r_{n+1}}(x_n - r_nAx_n) \\
&+ T_{r_{n+1}}(x_n - r_nAx_n) - T_{r_n}(x_n - r_nAx_n)\|
\end{aligned}$$

$$
\begin{aligned}
&\leq \big\|(x_{n+1}-r_{n+1}Ax_{n+1})-(x_n-r_nAx_n)\big\| \\
&\quad + \big\|T_{r_{n+1}}(x_n-r_nAx_n)-T_{r_n}(x_n-r_nAx_n)\big\| \\
&\leq \|x_{n+1}-x_n\|+|r_{n+1}-r_n|\|Ax_n\| \\
&\quad + \big\|T_{r_{n+1}}(x_n-r_nAx_n)-T_{r_n}(x_n-r_nAx_n)\big\|. \qquad (2.4)
\end{aligned}
$$

It follows that

$$
\begin{aligned}
&\|z_{n+1}-z_n\| \\
&\quad\leq \alpha_{n+1}\big\|f(W_{n+1}x_{n+1})-f(W_nx_n)\big\|+|\alpha_{n+1}-\alpha_n|\big(\big\|f(W_{n+1}x_{n+1})\big\|+\|y_n\|\big) \\
&\qquad + (1-\alpha_{n+1})\|y_{n+1}-y_n\| \\
&\quad\leq \alpha_{n+1}\kappa\|W_{n+1}x_{n+1}-W_nx_n\|+|\alpha_{n+1}-\alpha_n|\big(\big\|f(W_{n+1}x_{n+1})\big\|+\|y_n\|\big) \\
&\qquad + \|x_{n+1}-x_n\|+|r_{n+1}-r_n|\|Ax_n\|+\big\|T_{r_{n+1}}(x_n-r_nAx_n)-T_{r_n}(x_n-r_nAx_n)\big\|. \qquad (2.5)
\end{aligned}
$$

Note that

$$
\begin{aligned}
&\|W_{n+1}z_{n+1}-W_nz_n\| \\
&\quad = \|W_{n+1}z_{n+1}-Wz_{n+1}+Wz_{n+1}-Wz_n+Wz_n-W_nz_n\| \\
&\quad \leq \|W_{n+1}z_{n+1}-Wz_{n+1}\|+\|Wz_{n+1}-Wz_n\|+\|Wz_n-W_nz_n\| \\
&\quad \leq \sup_{x\in K}\big\{\|W_{n+1}x-Wx\|+\|Wx-W_nx\|\big\}+\|z_{n+1}-z_n\|. \qquad (2.6)
\end{aligned}
$$

Combing (2.5) with (2.6) yields

$$
\begin{aligned}
&\|W_{n+1}z_{n+1}-W_nz_n\|-\|x_{n+1}-x_n\| \\
&\quad \leq \sup_{x\in K}\big\{\|W_{n+1}x-Wx\|+\|Wx-W_nx\|\big\}+\alpha_{n+1}\kappa\|W_{n+1}x_{n+1}-W_nx_n\| \\
&\qquad + |\alpha_{n+1}-\alpha_n|\big(\big\|f(W_{n+1}x_{n+1})\big\|+\|y_n\|\big) \\
&\qquad + |r_{n+1}-r_n|\|Ax_n\|+\big\|T_{r_{n+1}}(x_n-r_nAx_n)-T_{r_n}(x_n-r_nAx_n)\big\|.
\end{aligned}
$$

From the restrictions (a), (b), and (c), we find from Lemma 1.6 that

$$
\limsup_{n\to\infty}\big\{\|W_{n+1}z_{n+1}-W_nz_n\|-\|x_{n+1}-x_n\|\big\}\leq 0.
$$

Using Lemma 1.7, we obtain

$$
\lim_{n\to\infty}\|W_nz_n-x_n\|=0. \qquad (2.7)
$$

It follows that

$$
\lim_{n\to\infty}\|x_{n+1}-x_n\|=0. \qquad (2.8)
$$

Using (2.1), we find that

$$\begin{aligned}\|x_{n+1}-x^*\|^2 &= \|\beta_n x_n+(1-\beta_n)W_n z_n-x^*\|^2\\ &\le \beta_n\|x_n-x^*\|^2+(1-\beta_n)\|z_n-x^*\|^2\\ &\le \beta_n\|x_n-x^*\|^2+(1-\beta_n)\big(\alpha_n\|f(W_nx_n)-x^*\|^2+(1-\alpha_n)\|y_n-x^*\|^2\big)\\ &\le \|x_n-x^*\|^2+\alpha_n\|f(W_nx_n)-x^*\|^2\\ &\quad+r_n(r_n-2\alpha)(1-\alpha_n)(1-\beta_n)\|Ax_n-Ax^*\|^2,\end{aligned}$$

which in turn yields

$$\begin{aligned}&r_n(2\alpha-r_n)(1-\alpha_n)(1-\beta_n)\|Ax_n-Ax^*\|^2\\ &\quad\le \|x_n-x^*\|^2-\|x_{n+1}-x^*\|^2+\alpha_n\|f(W_nx_n)-x^*\|^2\\ &\quad\le \big(\|x_n-x^*\|+\|x_{n+1}-x^*\|\big)\|x_n-x_{n+1}\|+\alpha_n\|f(W_nx_n)-x^*\|^2.\end{aligned}$$

Using (2.8), we find from the restrictions (a), (b), and (c) that

$$\lim_{n\to\infty}\|Ax_n-Ax^*\|=0. \tag{2.9}$$

On the other hand, we see that

$$\begin{aligned}\|y_n-x^*\|^2 &= \|T_{r_n}(I-r_nA)x_n-T_{r_n}(I-r_nA)x^*\|^2\\ &\le \langle (I-r_nA)x_n-(I-r_nA)x^*, y_n-x^*\rangle\\ &\le \frac{1}{2}\big(\|x_n-x^*\|^2+\|y_n-x^*\|^2-\|(x_n-y_n)-r_n(Ax_n-Ax^*)\|^2\big)\\ &= \frac{1}{2}\big(\|x_n-x^*\|^2+\|y_n-x^*\|^2-\|x_n-y_n\|^2\\ &\quad+2r_n\langle x_n-y_n, Ax_n-Ax^*\rangle-r_n^2\|Ax_n-Ax^*\|^2\big).\end{aligned}$$

Hence, we have

$$\|y_n-x^*\|^2\le\|x_n-x^*\|^2-\|x_n-y_n\|^2+2r_n\|x_n-y_n\|\|Ax_n-Ax^*\|.$$

It follows that

$$\begin{aligned}\|x_{n+1}-x^*\|^2 &\le \|x_n-x^*\|^2+\alpha_n\|f(W_nx_n)-x^*\|^2-(1-\alpha_n)(1-\beta_n)\|x_n-y_n\|^2\\ &\quad+2r_n(1-\alpha_n)(1-\beta_n)\|x_n-y_n\|\|Ax_n-Ax^*\|\\ &\le \|x_n-x^*\|^2+\alpha_n\|f(W_nx_n)-x^*\|^2-(1-\alpha_n)(1-\beta_n)\|x_n-y_n\|^2\\ &\quad+2r_n\|x_n-y_n\|\|Ax_n-Ax^*\|.\end{aligned}$$

This implies that

$$\begin{aligned}(1-\alpha_n)(1-\beta_n)\|x_n-y_n\|^2 &\le \big(\|x_n-x^*\|+\|x_{n+1}-x^*\|\big)\|x_n-x_{n+1}\|\\ &\quad+\alpha_n\|f(W_nx_n)-x^*\|^2+2r_n\|x_n-y_n\|\|Ax_n-Ax^*\|.\end{aligned}$$

Using (2.8) and (2.9), we find from the restrictions (a), (b), and (c) that

$$\lim_{n\to\infty} \|x_n - y_n\| = 0. \tag{2.10}$$

Since $z_n = \alpha_n f(W_n x_n) + (1-\alpha_n)y_n$, we find that

$$\lim_{n\to\infty} \|z_n - y_n\| = 0. \tag{2.11}$$

Notice that

$$\|x_{n+1} - x_n\| = (1-\beta_n)\|W_n z_n - x_n\|.$$

This implies from (2.8)

$$\lim_{n\to\infty} \|W_n z_n - x_n\| = 0. \tag{2.12}$$

Note that

$$\|W_n z_n - z_n\| \le \|z_n - y_n\| + \|y_n - x_n\| + \|x_n - W_n z_n\|.$$

From (2.10), (2.11), and (2.12), we obtain

$$\lim_{n\to\infty} \|W_n z_n - z_n\| = 0. \tag{2.13}$$

Since the mapping $P_\Omega f$ is contractive, we denote the unique fixed point by x. Next, we prove that $\limsup_{n\to\infty}\langle f(x) - x, z_n - x\rangle \le 0$. To see this, we choose a subsequence $\{z_{n_i}\}$ of $\{z_n\}$ such that

$$\limsup_{n\to\infty}\bigl\langle f(x) - x, z_n - x\bigr\rangle = \lim_{i\to\infty}\bigl\langle f(x) - x, z_{n_i} - x\bigr\rangle.$$

Since $\{z_{n_i}\}$ is bounded, there exists a subsequence $\{z_{n_{i_j}}\}$ of $\{z_{n_i}\}$ which converges weakly to z. Without loss of generality, we may assume that $z_{n_i} \rightharpoonup z$. Indeed, we also have $y_{n_i} \rightharpoonup f$.

First, we show that $z \in \bigcap_{i=1}^\infty F(S_i)$. Suppose the contrary, $Wz \ne z$. Note that

$$\begin{aligned}\|z_n - Wz_n\| &\le \|Wz_n - W_n z_n\| + \|W_n z_n - z_n\| \\ &\le \sup_{x\in K}\bigl\{\|Wx - W_n x\|\bigr\} + \|W_n z_n - z_n\|.\end{aligned}$$

Using Lemma 1.6, we obtain from (2.13) that $\lim_{n\to\infty}\|z_n - Wz_n\| = 0$. By Opial's condition, we see that

$$\begin{aligned}\liminf_{i\to\infty}\|z_{n_i} - z\| &< \liminf_{i\to\infty}\|z_{n_i} - Wz\| \\ &\le \liminf_{i\to\infty}\bigl\{\|z_{n_i} - Wz_{n_i}\| + \|Wz_{n_i} - Wz\|\bigr\} \\ &\le \liminf_{i\to\infty}\bigl\{\|z_{n_i} - Wz_{n_i}\| + \|z_{n_i} - z\|\bigr\}.\end{aligned}$$

This implies that $\liminf_{i\to\infty}\|z_{n_i}-z\| < \liminf_{i\to\infty}\|z_{n_i}-z\|$, which leads to a contradiction. Thus, we have $z \in \bigcap_{i=1}^{\infty} F(S_i)$.

Next, we show that $f \in EP(F,A)$. Note that $y_n \rightharpoonup z$. Since $y_n = T_{r_n}(x_n - r_n A x_n)$, we have

$$F(y_n,y) + \langle Ax_n, y-y_n\rangle + \frac{1}{r_n}\langle y-y_n, y_n-x_n\rangle \ge 0, \quad \forall y \in C.$$

From the condition (A2), we see that

$$\langle Ax_n, y-y_n\rangle + \frac{1}{r_n}\langle y-y_n, y_n-x_n\rangle \ge F(y,y_n), \quad \forall y \in C.$$

Replacing n by n_i, we arrive at

$$\langle Ax_{n_i}, y-y_{n_i}\rangle + \left\langle y-y_{n_i}, \frac{y_{n_i}-x_{n_i}}{r_{n_i}}\right\rangle \ge F(y,y_{n_i}), \quad \forall y \in C. \tag{2.14}$$

For t with $0 < t \le 1$ and $\rho \in C$, let $\rho_t = t\rho + (1-t)z$. Since $\rho \in C$ and $z \in C$, we have $\rho_t \in C$. It follows from (2.14) that

$$\begin{aligned}\langle \rho_t - y_{n_i}, A\rho_t\rangle &\ge \langle \rho_t - y_{n_i}, A\rho_t\rangle - \langle Ax_{n_i}, \rho_t - y_{n_i}\rangle - \left\langle \rho_t - y_{n_i}, \frac{y_{n_i}-x_{n_i}}{r_{n_i}}\right\rangle + F(\rho_t, y_{n_i}) \\ &= \langle \rho_t - y_{n_i}, A\rho_t - Ay_{n,i}\rangle + \langle \rho_t - y_{n_i}, Ay_{n,i} - Ax_{n_i}\rangle \\ &\quad - \left\langle \rho_t - y_{n_i}, \frac{y_{n_i}-x_{n_i}}{r_{n_i}}\right\rangle + F(\rho_t, y_{n_i}).\end{aligned} \tag{2.15}$$

Using (2.10), we have $Ay_{n,i} - Ax_{n_i} \to 0$ as $i \to \infty$. On the other hand, we get from the monotonicity of A that $\langle \rho_t - y_{n_i}, A\rho_t - Ay_{n,i}\rangle \ge 0$. It follows from (A4) and (2.15) that

$$\langle \rho_t - z, A\rho_t\rangle \ge F(\rho_t, z). \tag{2.16}$$

From (A1) and (A4), we see from (2.16) that

$$\begin{aligned}0 &= F(\rho_t,\rho_t) \le tF(\rho_t,\rho) + (1-t)F(\rho_t,z) \\ &\le tF(\rho_t,\rho) + (1-t)\langle \rho_t - z, A\rho_t\rangle \\ &= tF(\rho_t,\rho) + (1-t)t\langle \rho - z, A\rho_t\rangle,\end{aligned}$$

which yields $F(\rho_t,\rho) + (1-t)\langle \rho - f, A_3\rho_t\rangle \ge 0$. Letting $t \to 0$ in the above inequality, we arrive at $F(z,\rho) + \langle \rho - z, Az\rangle \ge 0$. This shows that $f \in EP(F,A)$. It follows that

$$\limsup_{n\to\infty}\langle f(x) - x, z_n - x\rangle \le 0. \tag{2.17}$$

Finally, we show that $x_n \to x$, as $n \to \infty$. Note that

$$\begin{aligned}\|z_n - x\|^2 &= \alpha_n\langle f(W_n x_n) - x, z_n - x\rangle + (1-\alpha_n)\langle y_n - x, z_n - x\rangle \\ &\le \bigl(1 - \alpha_n(1-\kappa)\bigr)\|x_n - x\|\|z_n - x\| + \alpha_n\langle f(x) - x, z_n - x\rangle \\ &\le \frac{1-\alpha_n(1-\kappa)}{2}\bigl(\|x_n - x\|^2 + \|z_n - x\|^2\bigr) + \alpha_n\langle f(x) - x, z_n - x\rangle.\end{aligned}$$

Hence, we have

$$\|z_n - x\|^2 \le \big(1-\alpha_n(1-\kappa)\big)\|x_n - x\|^2 + 2\alpha_n\big\langle f(x)-x, z_n - x\big\rangle.$$

This implies that

$$\begin{aligned}\|x_{n+1} - x\|^2 &= \big\|\beta_n x_n + (1-\beta_n)W_n z_n - x\big\|^2\\ &\le \beta_n\|x_n - x\|^2 + (1-\beta_n)\|z_n - x\|^2\\ &\le \big(1-\alpha_n(1-\beta_n)(1-\kappa)\big)\|x_n - x\|^2 + 2\alpha_n(1-\beta_n)\big\langle f(x)-x, z_n - x\big\rangle.\end{aligned}$$

Using Lemma 1.3 and (2.17), we find from the restrictions (a), (b), and (c) that $\lim_{n\to\infty}\|x_n - x\| = 0$. This completes the proof. □

3 Applications

For a single mapping, we find from Theorem 2.1 the following result.

Theorem 3.1 *Let C be a nonempty closed convex subset of a Hilbert space H and let F be a bifunction from $C\times C$ to $\mathbb{R}$ which satisfies* (A1)-(A4). *Let $A: C\to H$ be an α-inverse-strongly monotone mapping and let S be a nonexpansive mapping. Assume that $\Omega := F(S)\cap EP(F,A)\neq\emptyset$. Let $f: C\to C$ be a κ-contractive mapping. Let $\{x_n\}$ be a sequence generated in the following process*: *let it be a sequence generated in*

$$\begin{cases} x_1\in C, \quad \text{chosen arbitrarily,}\\ F(y_n,y) + \langle Ax_n, y-y_n\rangle + \frac{1}{r_n}\langle y-y_n, y_n - x_n\rangle \ge 0, \quad \forall y\in C,\\ x_{n+1} = \beta_n x_n + (1-\beta_n)S(\alpha_n f(Sx_n) + (1-\alpha_n)y_n), \quad \forall n\ge 1,\end{cases}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in $[0,1]$ and $\{r_n\}$ is a positive number sequence. Assume that the above control sequences satisfy the following restrictions:

(a) $0 < a \le \beta_n \le b < 1$, $0 < c \le r_n \le d < 2\alpha$;
(b) $\lim_{n\to\infty}\alpha_n = 0$, *and* $\sum_{n=1}^{\infty}\alpha_n = \infty$;
(c) $\lim_{n\to\infty}(r_n - r_{n+1}) = 0$.

Then $\{x_n\}$ converge strongly to a point $x\in\Omega$, where $x = P_\Omega f(x)$.

If S is the identity, we find the following result on the generalized equilibrium problem.

Corollary 3.2 *Let C be a nonempty closed convex subset of a Hilbert space H and let F be a bifunction from $C\times C$ to $\mathbb{R}$ which satisfies* (A1)-(A4). *Let $A: C\to H$ be an α-inverse-strongly monotone mapping. Assume that $EP(F,A)\neq\emptyset$. Let $f: C\to C$ be a κ-contractive mapping. Let $\{x_n\}$ be a sequence generated in the following process*: *let it be a sequence generated in*

$$\begin{cases} x_1\in C, \quad \text{chosen arbitrarily,}\\ F(y_n,y) + \langle Ax_n, y-y_n\rangle + \frac{1}{r_n}\langle y-y_n, y_n - x_n\rangle \ge 0, \quad \forall y\in C,\\ x_{n+1} = \beta_n x_n + (1-\beta_n)(\alpha_n f(x_n) + (1-\alpha_n)y_n), \quad \forall n\ge 1,\end{cases}$$

where $\{\alpha_n\}$ *and* $\{\beta_n\}$ *are sequences in* $[0,1]$ *and* $\{r_n\}$ *is a positive number sequence. Assume that the above control sequences satisfy the following restrictions*:

(a) $0 < a \le \beta_n \le b < 1$, $0 < c \le r_n \le d < 2\alpha$;
(b) $\lim_{n\to\infty} \alpha_n = 0$, *and* $\sum_{n=1}^{\infty} \alpha_n = \infty$;
(c) $\lim_{n\to\infty}(r_n - r_{n+1}) = 0$.

Then $\{x_n\}$ *converge strongly to a point* $x \in EP(F,A)$, *where* $x = P_{EP(F,A)}f(x)$.

Next, we give a result on the equilibrium problem (1.3).

Theorem 3.3 *Let* C *be a nonempty closed convex subset of a Hilbert space* H *and let* F *be a bifunction from* $C \times C$ *to* $\mathbb{R}$ *which satisfies* (A1)-(A4). *Let* $\{S_i : C \to C\}$ *be a family of infinitely nonexpansive mappings. Assume that* $\Omega := \bigcap_{i=1}^{\infty} F(S_i) \cap EP(F) \neq \emptyset$. *Let* $f : C \to C$ *be a* κ*-contractive mapping. Let* $\{x_n\}$ *be a sequence generated in the following process*: *let it be a sequence generated in*

$$\begin{cases} x_1 \in C, \quad \text{chosen arbitrarily}, \\ F(y_n, y) + \frac{1}{r_n}\langle y - y_n, y_n - x_n\rangle \ge 0, \quad \forall y \in C, \\ x_{n+1} = \beta_n x_n + (1-\beta_n)W_n(\alpha_n f(W_n x_n) + (1-\alpha_n)y_n), \quad \forall n \ge 1, \end{cases}$$

where $\{W_n\}$ *is the mapping sequence defined by* (1.4), $\{\alpha_n\}$ *and* $\{\beta_n\}$ *are sequences in* $[0,1]$ *and* $\{r_n\}$ *is a positive number sequence. Assume that the above control sequences satisfy the following restrictions*:

(a) $0 < a \le \beta_n \le b < 1$, $0 < c \le r_n \le d < +\infty$;
(b) $\lim_{n\to\infty} \alpha_n = 0$, *and* $\sum_{n=1}^{\infty} \alpha_n = \infty$;
(c) $\lim_{n\to\infty}(r_n - r_{n+1}) = 0$.

Then $\{x_n\}$ *converge strongly to a point* $x \in \Omega$, *where* $x = P_{\Omega}f(x)$.

Proof By putting $A_3 \equiv 0$, the zero operator, we can easily get the desired conclusion. This completes the proof. □

Next, we give a result on the classical variational inequality.

Theorem 3.4 *Let* C *be a nonempty closed convex subset of a Hilbert space* H. *Let* $A : C \to H$ *be an* α*-inverse-strongly monotone mapping and let* $\{S_i : C \to C\}$ *be a family of infinitely nonexpansive mappings. Assume that* $\Omega := \bigcap_{i=1}^{\infty} F(S_i) \cap VI(C,A) \neq \emptyset$. *Let* $f : C \to C$ *be a* κ*-contractive mapping. Let* $\{x_n\}$ *be a sequence generated in the following process*: *let it be a sequence generated in*

$$\begin{cases} x_1 \in C, \quad \text{chosen arbitrarily}, \\ y_n = P_C(x_n - r_n A x_n), \\ x_{n+1} = \beta_n x_n + (1-\beta_n)W_n(\alpha_n f(W_n x_n) + (1-\alpha_n)y_n), \quad \forall n \ge 1, \end{cases}$$

where $\{W_n\}$ *is the mapping sequence defined by* (1.4), $\{\alpha_n\}$ *and* $\{\beta_n\}$ *are sequences in* $[0,1]$ *and* $\{r_n\}$ *is a positive number sequence. Assume that the above control sequences satisfy the following restrictions*:

(a) $0 < a \le \beta_n \le b < 1$, $0 < c \le r_n \le d < 2\alpha$;

(b) $\lim_{n\to\infty} \alpha_n = 0$, *and* $\sum_{n=1}^{\infty} \alpha_n = \infty$;
(c) $\lim_{n\to\infty}(r_n - r_{n+1}) = 0$.

Then $\{x_n\}$ *converge strongly to a point* $x \in \Omega$, *where* $x = P_\Omega f(x)$.

Proof Putting $F \equiv 0$, we see from Theorem 2.1 that

$$\langle Ax_n, y - y_n\rangle + \frac{1}{r_n}\langle y - y_n, y_n - x_n\rangle \geq 0, \quad \forall y \in C, \forall y \in C, \forall n \geq 1.$$

This implies that

$$\langle y - y_n, x_n - r_n Ax_n - y_n\rangle \leq 0, \quad \forall y \in C.$$

It follows that

$$y_n = P_C(x_n - r_n Ax_n).$$

This completes the proof. □

Finally, we utilize the results presented in the paper to study the following optimization problem:

$$\min_{x\in C} h(x), \tag{3.1}$$

where C is a nonempty closed convex subset of a Hilbert space, and $h : C \to \mathbb{R}$ is a convex and lower semi-continuous functional. We use Ω to denote the solution set of the problem (3.1). Let $F : C \times C \to \mathbb{R}$ be a bifunction defined by $F(x, y) = h(y) - h(x)$. We consider the following equilibrium problem: to find $x \in C$ such that

$$F(x, y) \geq 0, \quad \forall y \in C.$$

It is easy to see that the bifunction F satisfies conditions (A1)-(A4) and $EP(F) = \Omega$.

Theorem 3.5 *Let* C *be a nonempty closed convex subset of a Hilbert space* H *and let* $h : C \to \mathbb{R}$ *be defined as above. Assume that* $\Omega \neq \emptyset$. *Let* $f : C \to C$ *be a* κ*-contractive mapping. Let* $\{x_n\}$ *be a sequence generated in the following process*: *let it be a sequence generated in*

$$\begin{cases} x_1 \in C, \quad \text{chosen arbitrarily}, \\ h(y) - h(u_n) + \frac{1}{r_n}\langle y - y_n, y_n - x_n\rangle \geq 0, \quad \forall y \in C, \\ x_{n+1} = \beta_n x_n + (1 - \beta_n)(\alpha_n f(x_n) + (1 - \alpha_n)y_n), \quad \forall n \geq 1, \end{cases}$$

where $\{\alpha_n\}$ *and* $\{\beta_n\}$ *are sequences in* $[0,1]$ *and* $\{r_n\}$ *is a positive number sequence. Assume that the above control sequences satisfy the following restrictions*:
(a) $0 < a \leq \beta_n \leq b < 1$, $0 < c \leq r_n \leq d < +\infty$;
(b) $\lim_{n\to\infty} \alpha_n = 0$, *and* $\sum_{n=1}^{\infty} \alpha_n = \infty$;
(c) $\lim_{n\to\infty}(r_n - r_{n+1}) = 0$.

Then $\{x_n\}$ *converges strongly to a point* $x \in \Omega$, *where* $x = P_\Omega f(x)$.

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The author declares to have no competing interests.

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