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Letter

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Transverse-mass effective temperature in heavy-ion collisions from AGS to SPS

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Abstract. Transverse-mass spectra in Au + Au and Pb + Pb collisions in the incident energy range from $2A$ to $160A$ GeV are analyzed within the model of 3-fluid dynamics (3FD). It is shown that the dynamical description of freeze-out, accepted in this model, naturally explains the incident-energy behavior of the inverse-slope parameters of these spectra observed in experiment. The simultaneous reproduction of the inverse slopes of all considered particles (p, π and K) suggests that these particles belong to the same hydrodynamic flow at the instant of their freeze-out.

PACS. 24.10.Nz Hydrodynamic models – 25.75.-q Relativistic heavy-ion collisions

Experimental data on transverse-mass spectra of kaons produced in central Au + Au [1] or Pb + Pb [2] collisions reveal peculiar dependence on the incident energy. The inverse-slope parameter (the so-called effective temperature T) of these spectra at mid rapidity increases with incident energy in the energy domain of BNL Alternating Gradient Synchrotron (AGS) and then saturates at the energies of CERN Super Proton Synchrotron (SPS). In refs. [3, 4] it was assumed that this saturation is associated with the deconfinement phase transition. This assumption was indirectly confirmed by the fact that microscopic transport models, based on hadronic degrees of freedom, failed to reproduce the observed behavior of the kaon inverse slope [5,6]. Hydrodynamic simulations of ref. [7] succeeded to describe this behavior. However, in order to reproduce it these hydrodynamic simulations required incident-energy dependence of the freeze-out temperature which almost repeated the shape of the corresponding kaon effective temperature. This happened even in spite of using the equation of state (EoS) involving the phase transition into quark-gluon plasma (QGP). This way, the puzzle of kaon effective temperatures was just translated into a puzzle of freeze-out temperatures. Moreover, the results of ref. [7] imply that the peculiar incident-energy dependence of the kaon effective temperature may be associated with the dynamics of freeze-out.

In this paper we would like to present calculations of effective temperatures within the 3FD model [8–11] which is suitable for simulating heavy-ion collisions in the range from AGS to SPS energies. We perform our simulations [8, 10,11] with a simple, hadronic EoS [12], which involves only a density-dependent mean field providing saturation of cold nuclear matter at normal nuclear density and with the proper binding energy. The 3FD model with the intermediate EoS turned out to be able to reasonably reproduce a great body of experimental data [8] in a wide energy range from AGS to SPS. In particular, transverse-mass spectra of protons were reproduced. This was done with the unique set of model parameters summarized in ref. [8]. Problems were met only in the description of the transverse flow [10]. The directed flow requires a softer EoS at top AGS and SPS energies (in particular, this desired softening may signal the occurrence of the phase transition into QGP). A similar softening is needed for the reproduction of recent data on the rapidity distributions of the net-baryon number in central Pb + Pb collisions at energies $20A$–$80A$ GeV [11].

The transverse-mass spectra are most sensitive to the freeze-out parameters of the model. In fact, the inverse slopes of these spectra represent a combined effect of the temperature and collective transverse flow of expansion. Figure 1 demonstrates this important interplay. Had it been only the effect of thermal excitation, inverse slopes for different hadronic species would approximately equal.

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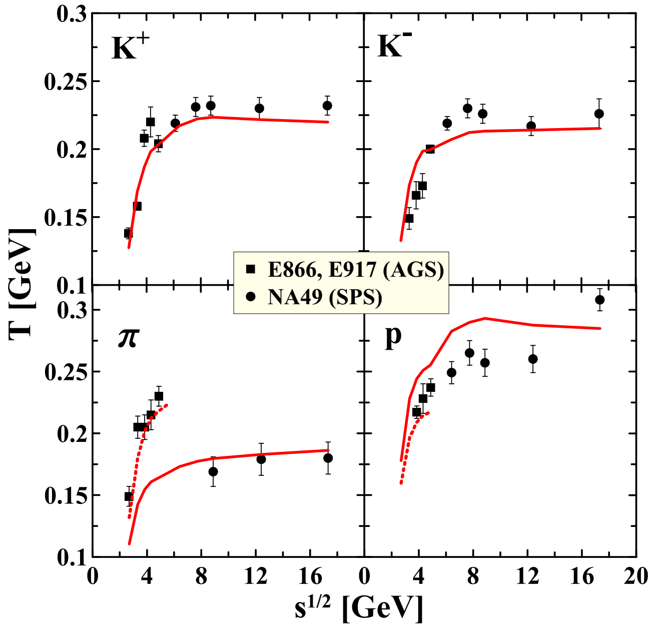

Fig. 1. Inverse-slope parameters of transverse-mass spectra of kaons, pions and protons at mid rapidity produced in central Au + Au and Pb + Pb collisions as a function of the invariant incident energy. The solid lines correspond to the purely exponential fit ($\lambda = 0$, see eq. (1)), while the dashed lines present results with $\lambda = -1$ for pions and with $\lambda = 1$ for protons. Experimental data are from refs. [1,2,13,14].

The collective transverse flow makes them different. These two effects partially compensate each other: the later the freeze-out occurs, the lower the temperature and the stronger the collective flow are. Nevertheless, transverse-mass spectra turn out to be sensitive to the instant of the freeze-out.

3FD results for inverse-slope parameters of transverse-mass spectra of kaons, pions and protons produced in central Au + Au and Pb + Pb collisions are presented in fig. 1. The inverse slopes T were deduced by fitting the calculated spectra by the formula

$$\frac{\mathrm{d}^2 N}{m_T\, \mathrm{d}m_T\, \mathrm{d}y} \propto (m_T)^\lambda \exp\left(-\frac{m_T}{T}\right), \tag{1}$$

where m_T and y are the transverse mass and rapidity, respectively. Though the purely exponential fit with $\lambda = 0$ does not always provide the best fit of the spectra, it allows a systematic way of comparing spectra at different incident energies[1]. In order to comply with experimental fits at AGS energies (and hence with displayed experimental points), we also present results with $\lambda = -1$ for pions[2] [1] and with $\lambda = 1$ for protons [14]. These results are obtained with precisely the same EoS and set of parameters (friction, freeze-out and formation time) as those used in ref. [8], which was found to be the best for other observables. No special tuning was done to reproduce these effective temperatures.

Numerical problems, discussed in ref. [8], prevented us from simulations at RHIC energies. Already for the central Pb + Pb collision at the top SPS energy the code requires 7.5 GB of (RAM) memory. At the top RHIC energy, the required memory is three order of magnitude higher, which is unavailable in modern computers.

As seen from fig. 1, the reproduction of effective temperatures is quite reasonable. Moreover, the pion and proton effective temperatures also reveal saturation at SPS energies, if they are deduced from the purely exponential fit with $\lambda = 0$. It is important that it is achieved with a single freeze-out parameter $\varepsilon_{\mathrm{frz}} = 0.4\ \mathrm{GeV/fm^3}$, the critical freeze-out energy density, which is the same for all considered incident energies above $2A$ GeV, both for chemical and thermal freeze-out. Only for smaller energies we used smaller values: $\varepsilon_{\mathrm{frz}}(2A\ \mathrm{GeV}) = 0.3\ \mathrm{GeV/fm^3}$ and $\varepsilon_{\mathrm{frz}}(1A\ \mathrm{GeV}) = 0.2\ \mathrm{GeV/fm^3}$. In order to clarify why this happens, let us turn to the 3FD freeze-out procedure, which is analyzed in ref. [15] in more detail.

The freeze-out criterion we use is

$$\varepsilon < \varepsilon_{\mathrm{frz}}, \tag{2}$$

where $\varepsilon = u_\mu T^{\mu\nu} u_\nu$ is the total energy density of all three fluids in the proper reference frame, where the composed matter is at rest. This total energy density is defined in terms of the total energy momentum tensor $T^{\mu\nu} \equiv T^{\mu\nu}_{\mathrm{p}} + T^{\mu\nu}_{\mathrm{t}} + T^{\mu\nu}_{\mathrm{f}}$ being the sum of energy momentum tensors $T^{\mu\nu}_\alpha$ of separate fluids (projectile-like, target-like and fireball ones) and the total collective 4-velocity of the matter $u^\mu = u_\nu T^{\mu\nu} / u_\lambda T^{\lambda\kappa} u_\kappa$. Note that the latter definition is, in fact, an equation determining u^μ. A very important feature of our freeze-out procedure is an anti-bubble prescription. The matter is allowed to be frozen out only if a) either the matter is located near the boarder with vacuum (this piece of matter gets locally frozen out), b) or the maximal value of the total energy density in the system is less than $\varepsilon_{\mathrm{frz}}$:

$$\max \varepsilon < \varepsilon_{\mathrm{frz}} \tag{3}$$

(the whole system gets instantly frozen out).

In the 3FD model this freeze-out simultaneously terminates both chemical and kinetic processes.

Before the instant of the global freeze-out, cf. (3), the freeze-out removes matter from the surface of the hydrodynamically expanding system. This removed matter gives rise to observable spectra of hadrons. This kind of freeze-out is similar to the model of "continuous emission" proposed in ref. [16]. There the particle emission occurs from a surface layer of the mean-free-path width. In our case the physical pattern is the same, only the mean free path is shrunk to zero.

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[1] Accordingly to the experimental fit of proton spectra at SPS energies [13], we fitted the calculated spectra by function (1) with $\lambda = 0$ only at $m_T - m_N > 0.2\ \mathrm{GeV}/c^2$, where m_N is the nucleon mass, and did not care of low-m_T parts of these spectra.

[2] In fact, λ was treated as a fit parameter in ref. [1]. However, since the resulting values of the λ parameter turned out to be compatible with -1 within the error bars, we performed our fit with $\lambda = -1$.

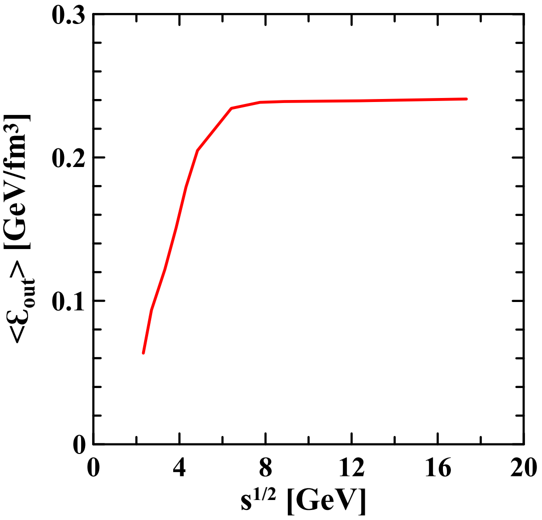

Fig. 2. Actual average freeze-out energy density in central (zero impact parameter) Pb + Pb collisions as a function of invariant incident energy.

Condition (2) ensures only that the actual freeze-out energy density (let us call it $\varepsilon_{\rm out}$), at which the freeze-out actually occurs, is less than $\varepsilon_{\rm frz}$. Therefore, $\varepsilon_{\rm frz}$ can be called a "trigger" value of the freeze-out energy density. As explained in ref. [15], a natural value of this actual freeze-out energy density is $\varepsilon_{\rm out} \approx \varepsilon_s/2$, *i.e.* at that the middle of the fall from the near-surface value of the energy density, ε_s, to zero. To find out the actual value of $\varepsilon_{\rm out}$, we have to analyze the results of a particular simulation. In our previous paper [8] we have performed only a rough analysis of this kind. This is why in the main text of ref. [8] we mentioned the value of approximately 0.2 GeV/fm^3 for $\varepsilon_{\rm out}$ and in the appendix explained how the freeze-out actually proceeded. (In terms of ref. [8] ($\varepsilon_{\rm frz[1]}$ and $\varepsilon^{\rm code}_{\rm frz[1]}$) our present quantities are $\varepsilon_{\rm frz} = \varepsilon^{\rm code}_{\rm frz[1]}$ and $\varepsilon_{\rm out} = \varepsilon_{\rm frz[1]}$.) The results of a more comprehensive analysis for central ($b = 0$) Pb + Pb collisions are presented in fig. 2, which shows the $\varepsilon_{\rm out}$ value averaged over the space-time evolution of the collision: $\langle\varepsilon_{\rm out}\rangle$. As seen, $\langle\varepsilon_{\rm out}\rangle$ reveals saturation at the SPS energies, very similar to that in effective temperatures in fig. 1. This happens in spite of the fact that our freeze-out condition involves only a single constant parameter $\varepsilon_{\rm frz}$.

The "step-like" behavior of $\langle\varepsilon_{\rm out}\rangle$ is a consequence of the freeze-out dynamics, as was demonstrated in ref. [15]. At low (AGS) incident energies, the energy density achieved at the border with vacuum, ε_s, is lower than $\varepsilon_{\rm frz}$. Therefore, the surface freeze-out starts at lower energy densities. It further proceeds at lower densities up to the global freeze-out because the freeze-out front moves not faster than with the speed of sound, like any perturbation in the hydrodynamics. Hence it cannot overcome the supersonic barrier and reach dense regions inside the expanding system. With the incident-energy rise the energy density achieved at the border with vacuum gradually reaches the value of $\varepsilon_{\rm frz}$ and then even overshoots it. If the overshoot happens, the system first expands without

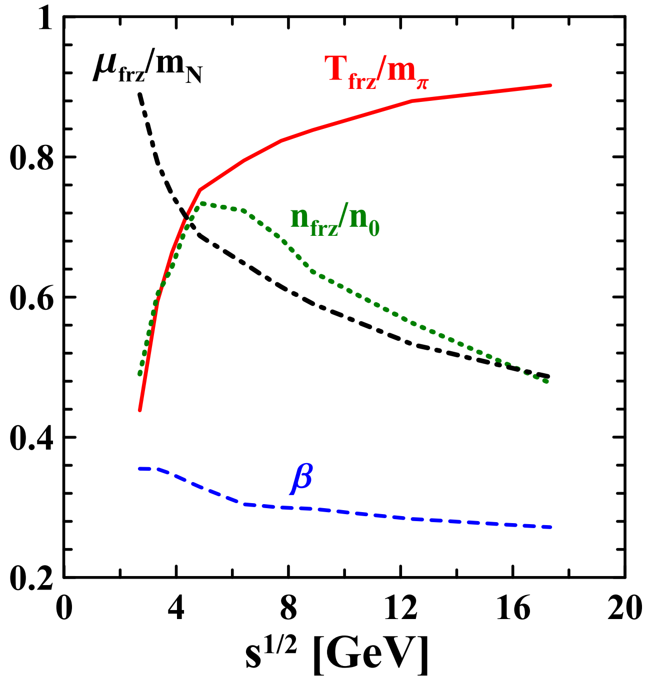

Fig. 3. Average temperature $T_{\rm frz}/m_\pi$ (over the pion mass), transverse velocity $\beta = v_T/c$, baryon density $n_{\rm frz}/n_0$ (over the normal nuclear density) and baryon chemical potential $\mu_{\rm frz}/m_N$ (over the nucleon mass) at the freeze-out in central Au + Au (at AGS energies, $b = 2$ fm) and Pb + Pb (at SPS energies, $b = 2.5$ fm) collisions as a function of the invariant incident energy.

freeze-out. The freeze-out starts only when ε_s drops to the value of $\varepsilon_{\rm frz}$. Then the surface freeze-out occurs really at the value $\varepsilon_s \approx \varepsilon_{\rm frz}$ and thus the actual freeze-out energy density saturates at the value $\langle\varepsilon_{\rm out}\rangle \approx \varepsilon_{\rm frz}/2$. This freeze-out dynamics is quite stable with respect to numerics [15].

Figure 3 presents average temperatures, transverse velocities, baryon densities and chemical potentials achieved at the freeze-out in central collisions. Hadronic gas EoS was used to determine these quantities. The freeze-out temperature $T_{\rm frz}$ has a similar "step-like" behavior. However, its absolute values are essentially lower than the effective temperatures in fig. 1. This fact once again illustrates that inverse slopes represent a combined effect of the temperature and collective transverse flow of expansion associated with the collective transverse velocity β. Note that β even slightly decreases with incident energy. At SPS energies the freeze-out temperatures in fig. 3 are noticeably lower than those deduced from hadron multiplicities in the statistical model [17,18]. The reason for this is as follows. Whereas the statistical model assumes a single uniform fireball, in the 3FD simulations at the late stage of the evolution the system effectively consists of several "fireballs": two (one baryon-rich and one baryon-free) fireballs at lower SPS energies and three (two baryon-rich and one baryon-free) fireballs at top SPS energies [15]. Therefore, whereas high multiplicities of mesons and antibaryons are achieved by means of high temperatures in the statistical model, the 3FD model explains them by an additional contribution of the baryon-free fireball at a lower temperature. In particular, this is the reason why two different freeze-out points (chemical and kinetic ones) are not needed in the 3FD model. The freeze-out baryon density n_{frz} exhibits a maximum at incident energies of $E_{\mathrm{lab}} = 10A$–$30A$ GeV which are well within range of the planned FAIR in GSI. This observation agrees with that deduced from the statistical model [19], even baryon density values in the maximum are similar to those presented in ref. [19].

Returning to the question if the considered "step-like" behavior of effective temperatures is a signal of phase transition into QGP, we should admit that this is not quite clear as yet. It depends on the nature of the freeze-out parameter $\varepsilon_{\mathrm{frz}} = 0.4$ GeV/fm^3 which should be further clarified. EoS is not of prime importance for this behavior. The only constrain on the EoS is that it should be in some way reasonable. Moreover, our preliminary results indicate that a completely different EoS [20] with 1st-order phase transition to QGP still reasonably reproduces this "step-like" behavior even in spite of the fact that it fails to describe a large body of other data. This happens because the same freeze-out pattern is accepted there.

In fact, EoS is just the pressure as a function of baryon and energy densities: $P(n_B, \varepsilon)$. In this calculation we used hadronic EoS [8,12]. However, this is just *an interpretation* of the function $P(n_B, \varepsilon)$, which we use, in hadronic terms. Moreover, our EoS is too soft at high densities to be matched with even heavy-quark bag-model EoS [20] or quasiparticle fits to lattice QCD data [21] in order to construct the 1st-order phase transition. (The hadronic and quark pressures as functions of the baryon chemical potential should have a crossing point in order to construct the 1st-order phase transition.) Therefore, it would not be surprising if the same EoS can be reinterpreted also in terms of a very smooth cross-over phase transition to quark-gluon matter.

This hydrodynamic explanation of the considered "step-like" behavior of effective temperatures together with the failure of kinetic approaches implies that a heavy nuclear system really reveals a hydrodynamic motion during its expansion. The simultaneous reproduction of the inverse slopes of all considered particles (p, π and K) implies that these particles belong to the same hydrodynamic flow at the instant of their freeze-out. An indirect support of this conjecture is the recent success of the GiBUU model [22] in the reproduction of kaon inverse slopes. That was achieved by taking into account three-body interactions, which essentially increased the equilibration rate.

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