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Some $\mathbf{Z}_2$-invariant variables: constructed of the scalars dilaton and axion

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Abstract Using the dilaton scalar and axion pseudoscalar fields we construct a number of scalars and differential forms which are symmetric under the $\mathbf{Z}_2$-subgroup of the group $SL(2, \mathbf{R})$. These invariants enable us to establish various 10-dimensional invariant actions. Other invariants which are not independent from the previous ones will be detached.

1 Introduction

Deformation and generalization of the supergravity actions have been studied from various points of view. Among them the modified versions of the type IIB supergravity are more desirable. One reason is due to the fact that the type IIB theory contains the axion (the R–R pseudoscalar field) and dilaton (the NS–NS scalar field), which are corresponding to the D-instanton and the Hodge duals of their field strengths are associated with the D7-brane and NS7-brane. The role of the type IIB instanton has been extremely well studied in the subject of the AdS/CFT correspondence. In addition, the axion of the type IIB theory solves some cosmological problems [1–3].

On the other hand there is the $SL(2, \mathbf{R})$ group which is certainly a symmetry group of the type IIB supergravity. There is also an important special case, i.e. the S-duality, which relates strong and weak coupling phases of a given theory in some cases, whereas in some other situations strong and weak coupling regimes of two different theories are connected.

In this paper, from the dilaton and axion fields and their field strengths and also the Hodge duals of their field strengths we construct various differential r-forms with $r = 0, 1, 2, 10$ which are invariant under the $\mathbf{Z}_2$-subgroup of the group $SL(2, \mathbf{R})$. These differential forms enable us to obtain various independent 10-dimensional actions which separately have the above symmetry. In fact, many other invariant scalars, differential forms, and actions can be made which are not independent from the previous ones. We shall detach independent invariants.

The following facts motivated us to study the $\mathbf{Z}_2$-invariant variables specially the invariant actions. The first is that, since the Einstein–Hilbert action also is a $\mathbf{Z}_2$-invariant, it is relevant to add a subset of the invariant actions to this action. In this case, the full resulted theory possesses this symmetry. Thus, this subset of the actions becomes a source for the gravity. In addition, appearance of the 1-forms and 9-forms are appropriate for realizing the D-instantons [4], D7-branes, and NS7-branes. Furthermore, investigation on the dilaton and axion clarifies some properties of the D7-brane and NS7-brane, and hence may shed light on the F-theory [5]. Beside, we shall observe that the $\mathbf{Z}_2$-transformations can be viewed as gauge transformations. Finally, since the exponential of the dilaton defines the string coupling we should know all possible dynamics of this field, which are associated with various actions.

This paper is organized as follows. In Sect. 2, the $\mathbf{Z}_2$-invariant scalars will be constructed. In Sect. 3, the $\mathbf{Z}_2$-invariant differential forms will be made. In Sect. 4, by using the above scalars and differential forms, various $\mathbf{Z}_2$-invariant actions will be established. In Sect. 5, independent invariants will be detached from the dependent ones. Section 6 is devoted to the conclusions.

2 Invariant scalars

We remember that the bosonic massless excitations of the type IIB theory include the graviton $G_{\mu\nu}$, dilaton Φ, and antisymmetric tensor $B_{\mu\nu}$ in the NS–NS sector, and the R–R counterparts are the axion $C_0 \equiv C$, an antisymmetric tensor field $C_{(2)\mu\nu}$, and a four-index antisymmetric potential $C_{(4)\mu\nu\rho\lambda}$ which its field strength is self-dual. In addition, in both sectors the Hodge duality also procreates some other form fields.

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The $\mathbf{Z}_2$-duality is generated by the $\mathbf{Z}_2$-subgroup of the group $SL(2,\mathbf{R})$. This determines $\mathbf{Z}_2$-transformations of the scalar fields C and Φ as $\tau \to \tau' = -1/\tau$ where $\tau = C + ie^{-\Phi}$ is the complex axion–dilaton modulus. This transformation mixes the two scalars as in the following:

$$\mathrm{e}^{-\Phi'} = \frac{\mathrm{e}^{-\Phi}}{C^2 + \mathrm{e}^{-2\Phi}},$$
$$C' = -\frac{C}{C^2 + \mathrm{e}^{-2\Phi}}. \tag{1}$$

Since the string coupling is given by $g_s = \mathrm{e}^{\Phi}$, when the axion vanishes, the first equation relates the weak and strong coupling regimes of the type IIB superstring theory. The Einstein metric, which will be used, is an $\mathbf{Z}_2$-invariant variable

$$G'_{\mathrm{E}\mu\nu} = G_{\mathrm{E}\mu\nu}, \tag{2}$$

where the Einstein frame and string frame metrics are related by $G_{\mathrm{E}\mu\nu} = \mathrm{e}^{-\Phi/2} G_{\mu\nu}$. From now on we utilize the word "*invariant*" instead of "$\mathbf{Z}_2$*-invariant*", which means invariance under the transformations (1) and (2).

According to (1) we acquire the following equations:

$$\frac{\mathrm{e}^{-2\Phi'}}{C'^2 + \mathrm{e}^{-2\Phi'}} = \frac{\mathrm{e}^{-2\Phi}}{C^2 + \mathrm{e}^{-2\Phi}} \equiv \sigma_1(C,\Phi),$$
$$\frac{C'^2}{C'^2 + \mathrm{e}^{-2\Phi'}} = \frac{C^2}{C^2 + \mathrm{e}^{-2\Phi}} \equiv \sigma_2(C,\Phi). \tag{3}$$

These equations represent two invariant scalars σ_1 and σ_2, i.e. $\sigma_i(C',\Phi') = \sigma_i(C,\Phi)$ for $i = 1, 2$.

Besides, there are also the following invariant scalars which comprise derivatives of the fields Φ and C:

$$\mathrm{e}^{2\Phi'} F'_\mu F'^\mu + H'_\mu H'^\mu = \mathrm{e}^{2\Phi} F_\mu F^\mu + H_\mu H^\mu, \tag{4}$$
$$\mathrm{e}^{2\Phi'}[2C' F'_\mu H'^\mu + (C'^2 - \mathrm{e}^{-2\Phi'}) H'_\mu H'^\mu]$$
$$= \mathrm{e}^{2\Phi}[2C F_\mu H^\mu + (C^2 - \mathrm{e}^{-2\Phi}) H_\mu H^\mu], \tag{5}$$
$$\mathrm{e}^{4\Phi'}[(C'^2 - \mathrm{e}^{-2\Phi'}) F'_\mu F'^\mu - 2C' \mathrm{e}^{-2\Phi'} F'_\mu H'^\mu]$$
$$= \mathrm{e}^{4\Phi}[(C^2 - \mathrm{e}^{-2\Phi}) F_\mu F^\mu - 2C \mathrm{e}^{-2\Phi} F_\mu H^\mu], \tag{6}$$

where $H_\mu = \partial_\mu \Phi$, $F_\mu = \partial_\mu C$, $H'_\mu = \partial_\mu \Phi'$ and $F'_\mu = \partial_\mu C'$. The indices are raised by the Einstein metric, e.g. $F^\mu = G^{\mu\nu}_{\mathrm{E}} F_\nu$. Some other kinds of the invariant scalars can be seen in the integrand of the action (24) and in (27), where, in fact, the latter is anti-invariant.

Note that the 1-form field strength $F = F_\mu dx^\mu$ is corresponding to the D(-1)-brane (D-instanton). This brane is unique among the D-branes since it is localized in time as well as in space.

3 Invariant differential forms

Exterior derivatives of the scalar fields give 1-forms. Combination of these 1-forms and their Hodge duals determines some higher order invariant differential forms which will be used to obtain several invariant actions.

• **Invariant 2-form**

The exterior derivative of (1) defines the 1-forms

$$H' = \frac{1}{C^2 + \mathrm{e}^{-2\Phi}}(2CF + (C^2 - \mathrm{e}^{-2\Phi})H),$$
$$F' = \frac{1}{(C^2 + \mathrm{e}^{-2\Phi})^2}((C^2 - \mathrm{e}^{-2\Phi})F - 2C\mathrm{e}^{-2\Phi}H). \tag{7}$$

The wedge product of these forms specifies the following invariant 2-form:

$$\mathrm{e}^{\Phi'} F' \wedge H' = \mathrm{e}^{\Phi} F \wedge H \equiv \mathcal{F}. \tag{8}$$

This equation elaborates that $\mathcal{F}$ is the field strength of the 1-form $A = C\mathrm{e}^{\Phi} H$, and or $A' = C'\mathrm{e}^{\Phi'} H'$. These 1-forms are related to each other through the gauge transformation $A' = A + \Lambda$ where Λ is a closed 1-form. We shall investigate this.

• **Invariant 10-forms**

We know that Hodge duality definition of a differential form is characterized by the metric of the manifold. According to this, for the next purposes, we define this duality via the Einstein metric. For example, the Hodge dual of the 1-form F, which is a 9-form $\widetilde{F}$, has the components

$$\widetilde{F}_{\mu_1\mu_2\cdots\mu_9} = \sqrt{-G_{\mathrm{E}}}\, F_\mu G^{\mu\nu}_{\mathrm{E}} \varepsilon_{\nu\mu_1\mu_2\cdots\mu_9}, \tag{9}$$

where $G_{\mathrm{E}} = \det G_{\mathrm{E}\mu\nu}$. The Levi–Civita symbol $\varepsilon_{\mu_1\mu_2\cdots\mu_{10}}$ has the components ± 1 and 0. Therefore, the Hodge duality on (7) exhibits the following 9-forms:

$$\tilde{H}' = \frac{1}{C^2 + \mathrm{e}^{-2\Phi}}(2C\widetilde{F} + (C^2 - \mathrm{e}^{-2\Phi})\tilde{H}),$$
$$\widetilde{F}' = \frac{1}{(C^2 + \mathrm{e}^{-2\Phi})^2}((C^2 - \mathrm{e}^{-2\Phi})\widetilde{F} - 2C\mathrm{e}^{-2\Phi}\tilde{H}). \tag{10}$$

Let us give a brief description regarding the 9-forms $\widetilde{F}$ and $\tilde{H}$. Two different local combinations of these forms accompanied by other form fields have been associated with the D7-brane and NS7-brane. The NS7-brane is related to the D7-brane by S-duality. These transformations also imply that the D7-brane and NS7-brane do not form a doublet under S-duality. It has been commonly accepted that there are bound states of p D7-branes and q NS7-branes which transform as doublets. The D7-brane has a magnetic charge, whereas its dual is the D-instanton, which has an electric charge.

Combining (7) with (10) via the wedge product specifies the following invariant 10-forms:

$$\mathrm{e}^{2\Phi'} F' \wedge \widetilde{F}' + H' \wedge \tilde{H}' = \mathrm{e}^{2\Phi} F \wedge \widetilde{F} + H \wedge \tilde{H}. \tag{11}$$
$$\mathrm{e}^{2\Phi'}[2C' F' \wedge \tilde{H}' + (C'^2 - \mathrm{e}^{-2\Phi'}) H' \wedge \tilde{H}']$$
$$= \mathrm{e}^{2\Phi}[2CF \wedge \tilde{H} + (C^2 - \mathrm{e}^{-2\Phi}) H \wedge \tilde{H}], \tag{12}$$
$$\mathrm{e}^{4\Phi'}[(C'^2 - \mathrm{e}^{-2\Phi'}) F' \wedge \widetilde{F}' - 2C' \mathrm{e}^{-2\Phi'} H' \wedge \widetilde{F}']$$
$$= \mathrm{e}^{4\Phi}[(C^2 - \mathrm{e}^{-2\Phi}) F \wedge \widetilde{F} - 2C\mathrm{e}^{-2\Phi} H \wedge \widetilde{F}]. \tag{13}$$

These 10-forms have been precisely written on the basis of the Einstein frame. Note that there is also another invariant 10-form, $e^{\Phi}(H \wedge \widetilde{F} - F \wedge \tilde{H})$, which identically vanishes.

4 Invariant actions

Making use of the 10-forms (11)–(13) we establish the following actions, which have the $\mathbf{Z}_2$-symmetry:

$$I_1 = -\frac{1}{9!4\kappa^2}\int\limits_{\mathcal{M}} (F \wedge \widetilde{F}_s + e^{-2\Phi} H \wedge \tilde{H}_s), \quad (14)$$

$$I_2 = -\frac{1}{9!4\kappa^2}\int\limits_{\mathcal{M}} [2CF \wedge \tilde{H}_s + (C^2 - e^{-2\Phi}) H \wedge \tilde{H}_s], \quad (15)$$

$$I_3 = -\frac{1}{9!4\kappa^2}\int\limits_{\mathcal{M}} e^{2\Phi}[(C^2 - e^{-2\Phi}) F \wedge \widetilde{F}_s - 2Ce^{-2\Phi} \times H \wedge \widetilde{F}_s], \quad (16)$$

where κ is the 10-dimensional gravitational constant and $\mathcal{M}$ indicates the spacetime manifold. These actions have been written in the string frame, which is more usual. The 9-forms in the two frames are related by $\widetilde{F} = e^{-2\Phi}\widetilde{F}_s$ and $\tilde{H} = e^{-2\Phi}\tilde{H}_s$ where the subscript "s" refers to the string frame.

The other forms of the above actions can be obtained by the standard duality transformation

$$S_1 = -\frac{1}{4\kappa^2}\int\limits_{\mathcal{M}} d^{10}x\sqrt{-G}(F_\mu F_s^\mu + e^{-2\Phi} H_\mu H_s^\mu), \quad (17)$$

$$S_2 = -\frac{1}{4\kappa^2}\int\limits_{\mathcal{M}} d^{10}x\sqrt{-G}[2CF_\mu H_s^\mu + (C^2 - e^{-2\Phi}) \times H_\mu H_s^\mu], \quad (18)$$

$$S_3 = -\frac{1}{4\kappa^2}\int\limits_{\mathcal{M}} d^{10}x\sqrt{-G}e^{2\Phi}[(C^2 - e^{-2\Phi}) F_\mu F_s^\mu - 2Ce^{-2\Phi} F_\mu H_s^\mu], \quad (19)$$

where $G = \det G_{\mu\nu}$, and F_s^μ and H_s^μ refer to the string frame, i.e. $F_s^\mu = G^{\mu\nu}F_\nu$ and $H_s^\mu = G^{\mu\nu}H_\nu$. According to the invariant scalars (4)–(6) these actions also are symmetric under the $\mathbf{Z}_2$-transformations. The action S_1 in the Einstein frame can be written in the form

$$S_1 = -\frac{1}{4\kappa^2}\int\limits_{\mathcal{M}} d^{10}x\sqrt{-G_E}\,\frac{\partial_\mu\tau\partial^\mu\bar{\tau}}{(\mathrm{Im}\tau)^2}. \quad (20)$$

This clarifies that S_1 is a part of the action of the type IIB supergravity, which we found an alternative derivation and various features for it. Note that the actions S_1, S_2, and S_3 are not independent from the actions I_1, I_2, and I_3. In other words, the action S_i is an alternative version of the action I_i for $i \in \{1, 2, 3\}$.

The effective field theory actions describing the dynamics of the massless modes of the various string theories contain, in addition to the well-known supergravity terms, an infinite number of higher-derivative corrections. Symmetries and dualities of these effective field theories are useful tools to find such higher-derivative terms. Therefore, several explicit higher-derivative terms have been known or conjectured to exist by symmetry/duality arguments (e.g. see [6] and references therein). Specially, the effective action for bosonic massless modes of the type IIB supergravity at various levels of derivatives has been studied. Naturally, the dilaton and axion fields also contribute on higher order derivative corrections to the type IIB theory [7–9]. Though the proposed forms of the corrections to the effective action satisfy the $SL(2, R)$-invariance condition, but there is no proof for their validity [8]. On the other hand, string scattering diagrams describe a large number of Feynman diagrams in the Yang–Mills gauge theory [10–13]. It has been realized that only the bosonic degrees of freedom are relevant in the field theoretical limit of string amplitudes [14]. Thus, according to these facts, we proceed to construct a four-derivative action.

The 2-form (8) has the components

$$\mathcal{F}_{\mu\nu} = e^{\Phi}(F_\mu H_\nu - F_\nu H_\mu). \quad (21)$$

This indicates a field strength $\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ where the $U(1)$ vector field A_μ is specified by

$$A_\mu = C\partial_\mu e^{\Phi}. \quad (22)$$

We observe that under the transformations (1) the tensor $\mathcal{F}_{\mu\nu}$ remains invariant. In addition, the 1-form $A = A_\mu dx^\mu$ transforms to $A' = A + \Lambda$ where Λ is the following closed 1-form:

$$\Lambda = -\frac{2C^2e^{\Phi}}{C^2 + e^{-2\Phi}}(F + CH). \quad (23)$$

Thus, we are lead to introduce the invariant action

$$S_4 = -\frac{1}{4g_{10}^2}\int\limits_{\mathcal{M}} d^{10}x\sqrt{-G}e^{-3\Phi/2}\mathcal{F}_{\mu\nu}\mathcal{F}_s^{\mu\nu}, \quad (24)$$

where g_{10} is the 10-dimensional Yang–Mills coupling constant and $\mathcal{F}_s^{\mu\nu} = G^{\mu\rho}G^{\nu\lambda}\mathcal{F}_{\rho\lambda} = e^{-\Phi}\mathcal{F}_E^{\mu\nu}$. In the Einstein frame this action takes the form

$$S_4 = -\frac{1}{4g_{10}^2}\int\limits_{\mathcal{M}} d^{10}x\sqrt{-G_E}\mathcal{F}_{\mu\nu}\mathcal{F}_E^{\mu\nu}, \quad (25)$$

where $\mathcal{F}_E^{\mu\nu} = G_E^{\mu\rho}G_E^{\nu\lambda}\mathcal{F}_{\rho\lambda}$. From this point of view the transformations (1) can be interpreted as gauge transformations.

In fact, under the transformation $C \to C + 1$, which induces $\tau \to \tau + 1$, the actions S_1 and S_4 are symmetric. This invariance accompanied by the previous symmetry, i.e. invariance under $\tau \to -1/\tau$, demonstrate that these two actions possess the full $SL(2;\mathbf{Z})$ symmetry.

Now let us indicate the usefulness of the invariant actions. Since the spacetime is dynamical we should also include the kinetic term of the spacetime metric, i.e. the Einstein–Hilbert action

$$S_{\rm EH} = \frac{1}{2\kappa^2}\int\limits_{\mathcal{M}} {\rm d}^{10}x\sqrt{-G_{\rm E}}\; R_{\rm E}, \tag{26}$$

where the scalar curvature $R_{\rm E}$ is constructed from the Einstein metric $G_{{\rm E}\mu\nu}$. This action also has the $\mathbf{Z}_2$-symmetry. According to our requirement we can select a subset of the invariant actions $\{S_i|i = 1, 2, 3, 4\}$ as a source of gravity, i.e. the action $S_{\rm EH}$ should be added to that subset. Therefore, we obtain a gravity theory with the $\mathbf{Z}_2$-symmetry. In addition, due to the presence of the 1-forms and 9-forms field strengths, the actions (14)–(16) are appropriate for constructing various solitonic solutions of the D(−1), D7 and NS7-branes. More precisely, for realizing the D-instanton in its usual form as a solution of the field equations one requires the dual, i.e. the magnetic, formulation. The presence of the field strength $\widetilde{F}_{\rm s}$ in the actions (14) and (16) provide this requirement. From the instanton point of view the action S_1 reveals an electric picture, in which the instanton would be seen as an elementary source coupled to the R–R field C.

5 Dependent invariants

Making use of (1) we obtained the invariant scalars (3). It is possible to construct many other (anti-)invariant scalars from (1). They are not independent from the scalars σ_1 and σ_2. Consequently, the corresponding differential forms and actions usually are not independent from the previous ones. Let us illustrate this fact by the anti-invariant scalar

$$C'{\rm e}^{\Phi'} = -C{\rm e}^{\Phi}. \tag{27}$$

Extracting this equation via (3) ensures us that it is not independent. The effect of the exterior derivative on this equation, or on each of the equations in (3), leads to the following independent anti-invariant 1-form:

$${\rm e}^{\Phi'}(F' + C'H') = -{\rm e}^{\Phi}(F + CH). \tag{28}$$

The components of this 1-form inspire an invariant action, i.e.,

$$I = -\frac{1}{4\kappa^2}\int\limits_{\mathcal{M}} {\rm d}^{10}x\sqrt{-G}(F_\mu + CH_\mu)(F_{\rm s}^\mu + CH_{\rm s}^\mu). \tag{29}$$

We observe that this specific action is not independent, that is, $I = S_1 + S_2$.

The Hodge dual of (28) exhibits an independent anti-invariant 9-form

$${\rm e}^{\Phi'}(\widetilde{F}' + C'\tilde{H}') = -{\rm e}^{\Phi}(\widetilde{F} + C\tilde{H}). \tag{30}$$

The wedge product of (28) and (30) defines the invariant 10-form

$$\begin{aligned}&{\rm e}^{2\Phi'}(F' + C'H') \wedge (\widetilde{F}' + C'\tilde{H}')\\ &\quad = {\rm e}^{2\Phi}(F + CH) \wedge (\widetilde{F} + C\tilde{H}).\end{aligned} \tag{31}$$

By summing (11) and (12) and using the identity $H \wedge \widetilde{F} = F \wedge \tilde{H}$ this 10-form is reproduced and hence loses its independence. The action associated with this 10-form also is not independent, i.e. again it is given by $I = I_1 + I_2$.

In addition to the indicated invariant scalars, the 9-forms $\widetilde{F}$, $\widetilde{F}'$, $\tilde{H}$ and $\tilde{H}'$ also enable us to obtain the following invariant scalars:

$${\rm e}^{2\Phi'}\widetilde{F}'\cdot\widetilde{F}' + \tilde{H}'\cdot\tilde{H}' = {\rm e}^{2\Phi}\widetilde{F}\cdot\widetilde{F} + \tilde{H}\cdot\tilde{H}, \tag{32}$$

$$\begin{aligned}&{\rm e}^{2\Phi'}[2C'\widetilde{F}'\cdot\tilde{H}' + (C'^2 - {\rm e}^{-2\Phi'})\tilde{H}'\cdot\tilde{H}']\\ &\quad = {\rm e}^{2\Phi}[2C\widetilde{F}\cdot\tilde{H} + (C^2 - {\rm e}^{-2\Phi})\tilde{H}\cdot\tilde{H}],\end{aligned} \tag{33}$$

$$\begin{aligned}&{\rm e}^{4\Phi'}[(C'^2 - {\rm e}^{-2\Phi'})\widetilde{F}'\cdot\widetilde{F}' - 2C'{\rm e}^{-2\Phi'}\widetilde{F}'\cdot\tilde{H}']\\ &\quad = {\rm e}^{4\Phi}[(C^2 - {\rm e}^{-2\Phi})\widetilde{F}\cdot\widetilde{F} - 2C{\rm e}^{-2\Phi}\widetilde{F}\cdot\tilde{H}],\end{aligned} \tag{34}$$

where the dot product between any two 9-forms $\widetilde{A}$ and $\widetilde{B}$ is defined by

$$\widetilde{A}\cdot\widetilde{B} = \widetilde{A}_{\mu_1\cdots\mu_9}G_{\rm E}^{\mu_1\nu_1}\cdots G_{\rm E}^{\mu_9\nu_9}\widetilde{B}_{\nu_1\cdots\nu_9}. \tag{35}$$

For our case these 9-forms are Hodge duals of the 1-forms A and B, and hence we have $\widetilde{A}\cdot\widetilde{B} = -9!G_{\rm E}^{\mu\nu}A_\mu B_\nu$. Thus, the above invariant scalars are not independent from the previous ones, i.e. they are equivalent to (4)–(6), respectively.

According to the feature of S_4 it is obvious that it cannot be written in terms of two or three of the actions S_1, S_2, and S_3. In addition, vanishing of the linear combination $\sum_{i=1}^{3} a_i S_i$ also leads to $a_1 = a_2 = a_3 = 0$, which implies that the actions S_1, S_2, and S_3 also are linearly independent from each other. Therefore, we have four linearly independent actions.

6 Conclusions

The two cases of dilaton scalar and axion pseudoscalar enabled us to establish various adequate $\mathbf{Z}_2$-invariant and anti-invariant quantities such as scalars, differential forms, and actions. There are some other new invariants which are not independent from the previous ones.

Our method inspired the action S_1 which is a portion of the bosonic part of the type IIB supergravity. Since all the actions S_1–S_4 have the same derivation method they may possess the same pertinence. Therefore, from the point of view of the bosonic theories they may have the same desirability, but each action has its own properties. However, from the supersymmetry point of view they have a different desirability.

We observed that the invariant 2-form (i.e. (8)) can be interpreted as the field strength of an Abelian gauge field.

Therefore, we were guided to a $U(1)$ Yang–Mills theory. The $\mathbf{Z}_2$-transformations exhibit the corresponding gauge transformation of this theory.

The proliferation of the invariant variables demands its own subtlety. More independent invariants maybe exist. We illustrated that some of the invariant scalars and differential forms are not independent, and hence the invariant actions which are built from them can be extracted from the original actions. Note that by utilizing the self-$\mathbf{Z}_2$-dual forms C_4 and $F_5 = \mathrm{d}C_4 + \frac{1}{2}B \wedge \mathrm{d}C_2 - \frac{1}{2}C_2 \wedge \mathrm{d}B$ and combining them with the discussed invariant forms we can construct more invariant variables and actions.

Some notes

The goal of this paper is not an extension of the type IIB supergravity theory. In other words, the invariant actions (except S_1) are not parts of the type IIB supergravity. This implies that they do not need to be $SL(2, R)$ invariant. However, we observed that the actions S_1 and S_4 possess the $SL(2, Z)$ symmetry. We only borrowed the dilaton and axion fields of the type IIB theory but not more. In addition, since our setup is bosonic we neglected the supersymmetry. For constructing invariant supersymmetric actions one should add the Einstein–Hilbert action to each of the actions $\{S_i | i = 1, 2, 3, 4\}$, and then supersymmetrize the resulting theory.

References

1. A. Feinstein, M.A. Vazquez-Mozo, Phys. Lett. B **441**, 40 (1998)
2. E. Konishi, J. Maharana, Int. J. Mod. Phys. A **25**, 3797 (2010)
3. T.W. Grimm, Phys. Rev. D **77**, 126007 (2008)
4. J.Y. Kim, H.W. Lee, Y.S. Myung, Phys. Lett. B **400**, 32 (1997)
5. C. Vafa, Nucl. Phys. B **469**, 403 (1996)
6. K. Peeters, P. Vanhove, A. Westerberg, Class. Quant. Grav. **18**, 843 (2001)
7. G. Policastro, D. Tsimpis, Class. Quant. Grav. **26**, 125001 (2009)
8. A. Kehagias, H. Partouche, Phys. Lett. B **422**, 109 (1998)
9. A. Kehagias, H. Partouche, Int. J. Mod. Phys. A **13**, 5075 (1998)
10. Z. Bern, D.A. Kosower, Phys. Rev. D **38**, 1888 (1988)
11. Z. Bern, D.A. Kosower, Phys. Rev. Lett. **66**, 1669 (1991)
12. Z. Bern, D.A. Kosower, Nucl. Phys. B **379**, 451 (1992)
13. R.R. Metsaev, A.A. Tseytlin, Nucl. Phys. B **298**, 109 (1988)
14. A. Frizzo, L. Magnea, Nucl. Phys. B **604**, 92 (2001)