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Energy consumption and lifetime analysis in clustered multi-hop wireless sensor networks using the probabilistic cluster-head selection method

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Abstract

Clustering sensor nodes into groups is an effective way of reducing the transmission of duplicated information in energy-constraint wireless sensor networks (WSNs). The performance of clustering is greatly influenced by the selection of cluster-heads, which are in charge of creating clusters and controlling member nodes. In selecting cluster-heads, a probabilistic method where each sensor node selects itself as a cluster-head with a given probability is often used in large-scale and dense WSNs because it enables all nodes to independently decide their roles while keeping the signaling overhead low. In this method, the probability of being a cluster-head should be optimally chosen to maximize the energy efficiency of the nodes. In this article, we propose a novel energy model to estimate the energy consumed in a multi-hop WSN clustered with probabilistic cluster-head selection. Then, based on our model, we determine optimal probability that maximizes the lifetime of a network. Simulation results achieved by the Monte Carlo method show that our model estimates well in energy consumption from a network and also predicts the optimal clustering probability accurately.

Keywords: clustered multi-hop wireless sensor networks, energy modeling, probabilistic cluster-head selection, optimal number of clusters

1 Introduction

Wireless sensor networks (WSNs) consist of spatially distributed autonomous sensor nodes with sensing, processing, and wireless communicating capabilities to cooperatively monitor physical or environmental conditions such as temperature, humidity, pressure, motion, and others in a specified sensing field. Since battery-powered sensor nodes are constrained by energy supply, it is important to investigate energy consumption optimization methods to prolong the lifetime of WSNs [1].

In most applications of WSNs, the sensed information is usually correlated both spatially and temporally, and it is transported only to a sink node. Thus, to reduce the energy waste, it is advantageous for several nodes to aggregate the information and send it to the sink node on behalf of other nodes [2,3]. In cluster-based networks, sensor nodes first send the sensed information to their cluster-heads. Then, after locally aggregating the received information, the cluster-heads transmit the aggregated information to a sink node on behalf of the cluster members.

In selecting cluster-heads, a probabilistic method where each node elects itself as a cluster-head with the same probability is often used in large-scale and homogenous WSNs because it enables all nodes to independently decide their roles while keeping the signaling overhead low. The method ensures rapid clustering while achieving favorable properties such as stable number of clusters and rotation of the cluster-heads. To evenly distribute the energy load among the nodes, the cluster-heads are re-selected at a regular interval [4,5].

In the probabilistic method, since the energy efficiency of the nodes is influenced by the number of clusters, it is important to optimally choose the probability to maximize the lifetime of the network [4-7]. To appropriately

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select the number of clusters, a number of studies have focused on derivation of energy models to estimate the energy consumed in the network with respect to the number of clusters [5-10]. However, accuracies of the existing models are not satisfactory because they make flawed assumptions. For example, some of them assume that all clusters have the same shapes (in particular, disc-shaped), and each cluster has the same number of member nodes [5,6]. However, the shape of clusters and the number of members in each cluster are arbitrary in practice. Furthermore, clustering of distributed nodes generally results in a large signaling overhead but most of the studies neglect the signaling overhead in modeling [5-10]. Finally, most studies simply derive the number of hops between the nodes by dividing the distance between them into a radio range, thus the accuracies of their models are not satisfactory [5,8-10].

In this article, we investigate major factors that influence the energy consumed in clustered multi-hop WSNs using the probabilistic cluster-head selection method and propose a novel energy model to correctly estimate the energy consumed in a network. Then, based on our model, we determine the optimal probability of a node to become a cluster-head that minimizes the energy consumption of the nodes, which in turn maximizes the lifetime of the network. Our model considers various factors such as different shapes (with varying cluster-members) of clusters, signaling overhead, and MAC inefficiency. Moreover, by properly deriving the number of hops from each node to its destination, our model gives a better approximation to the energy consumption than the previous models. Simulation results achieved by a Monte Carlo method show that our model estimates well in energy consumption from a network, and it also predicts the optimal probability of a node to become a cluster-head accurately.

The rest of the article is organized as follows. We introduce several important clustering schemes and energy models in Section 2. In Section 3, we introduce the overall procedures of the clustering scheme, assumptions for modeling, and formulate the problem. Then, we describe our energy model in detail in Section 4. Simulation results are shown in Section 5. Finally, we conclude our article in Section 6.

2 Related work

LEACH [4] is the first research to probabilistically select cluster-heads for WSNs. It assumes that all nodes are equipped with the capability of tuning the power, and they can send the collected data to a destination in one hop. For energy load balancing, LEACH cyclically switches the cluster-head role among the nodes and guarantees that each node equally becomes a cluster-head. The cluster-head selection is determined in a distributed autonomous fashion. An energy model to determine the suitable probability of a node to become a cluster-head is shown in [6]. The energy model of [6] only focuses on the energy consumed in transmitting data and derives the expected squared distance from a sensor node to its cluster-head using a simple stochastic method. Then, it considers that the energy consumption of the nodes is proportional to the derived value. This model is made on the assumption that the areas of all clusters are equal. However, the cluster areas are arbitrary in reality, and consequently, the model of [6] is not practical [11].

LEACH allows only single-hop clusters to be constructed. On the other hand, in EEHCA [5], it is assumed that all the nodes in the network transmit at a fixed power level; data between two communicating nodes which are out of each other's radio range are forwarded by other nodes. EEHCA also selects probabilistically the cluster-heads as in LEACH. Then, each non-cluster-head node becomes a member of a cluster with a cluster-head which is the closest in number of hops. Ref. [5] considers the energy consumed in transmitting data over the network is proportion to the number of hops between the communicating end-to-end nodes, i.e., each member (head) and its head (sink). To derive the number of hops between the end-to-end nodes, the energy model of [5] divides the average distance between the nodes by the radio range. However, this approach holds only when the relaying nodes are placed on a straight line between the end-to-end nodes. Thus, the model is inaccurate in estimating the number of hops between the nodes which are randomly placed. Furthermore, the model only considers the energy consumed in transmitting data without taking the data-receiving energy consumption into account. If the data-receiving energy is ignored, the important fact that the cluster-head spends more energy than a cluster-member, except for the part consumed for data aggregation, may mistakenly be neglected [8,10].

The weak points of EEHCA are improved by other studies. For example, to give an better approximation to the energy consumption, in CRS [8] and OCND [9], energy models which consider data-receiving energy are extended. On the other hand, the energy model of ECTC [10] considers the energy consumed by a radio during an idle state which refers to the state when the radio is on but not transmitting nor receiving any data. In CRS, the errors of EEHCA in deriving the number of hops between the end-to-end nodes are improved by compensating with the consideration of node density. This is because, when the node density is lower (higher), the possibilities of transmission detour become higher (lower), and thus the real number of hops between the nodes may be larger than (close to) the theoretical value derived from EEHCA. By additionally taking various factors which influence the energy consumption of nodes into consideration, the aforementioned models give better approximations to the energy consumption than the model of EEHCA. However, their approaches to the number of hops between the nodes are based on EEHCA's approach, thus significantly degrading the accuracies of the energy models.

In [11], the accuracy of deriving the number of hops is improved by individually deriving the number of hops from each node to its destination. However, this model only focuses on the energy consumed by the cluster-members, and lacks a complete energy model including the energy consumed by cluster-heads to predict the network lifetime. Ref. [12] takes into account that sensor nodes near the sink node suffer from heavy traffic load imposed on them, and therefore their energy is quickly depleted. So, [12] focuses on the energy consumed by the nodes in a bottleneck zone which is an area within the radio range from a sink node, and derives an upper bound for the lifetime of the network. However, the energy model of [12] holds on the assumptions that both the clusters and the bottleneck zone are disc-shaped, and the member nodes in each cluster are uniformly distributed. Due to such impractical assumptions, it may not properly determine the optimal probability of a node to become a cluster-head.

3 Preliminaries

In this section, we introduce the overall procedures of the clustering scheme and assumptions for modeling. Then, we formulate the problem.

3.1 Clustering algorithm

The clustering algorithm used in this article is referred to EEHCA's framework as a basis. The clustering algorithm is a distributed scheme that utilizes randomized selection of cluster-heads to distribute energy consumption among sensor nodes. The nodes share a single transmission channel and on the channel the nodes cannot transmit and receive simultaneously. Each sensor selects itself as a cluster-head with a predefined probability p without any information exchange with other nodes. Then, each cluster-head advertises itself as a cluster-head to other nodes within its radio range. Each node receives advertisements during a certain period from the arrival of the first received advertisement, and then chooses a cluster-head with the smallest number of hops from it and advertises its cluster-head to other nodes within its radio range. If cluster-heads with the smallest number of hops from a sensor node are more than two, then the node randomly selects one of them. This repeats until each node selects its cluster-head or become a cluster-head. All nodes communicate according to TDMA schedules organized by the cluster-heads or the sink node. Thus, data collision can be prevented.

Algorithm execution is divided into a number of rounds. Each round includes a set-up phase followed by a steady-state phase. In the set-up phase, the nodes are organized into clusters. After clusters are created, each cluster-head sets up a TDMA schedule for its members and the sink node sets up a TDMA schedule for the cluster-heads. Then, the TDMA schedules are distributed to the nodes. In the steady-state phase, according to the TDMA schedules, each member node forwards sensed data to its cluster-head and then each cluster-head aggregates data from its members and finally forwards to the sink node.

3.2 Assumptions for energy model

To determine the optimal parameters for our model, we make the following assumptions:

AS 1. n homogeneous sensor nodes in the network are distributed as per a homogeneous spatial Poisson process of intensity λ in a two-dimensional area A; hence, on average, the number of nodes is λA.

AS 2. All nodes transmit at a fixed power level and have the same radio range R.

AS 3. Data exchanged between two communicating sensor nodes not within each others's radio range are forwarded by other nodes.

AS 4. The sink node that ultimately processes the collected data is located in the center of the sensor field.

AS 5. The amount of data is fixed to l bits.

AS 6. The shortest path routing infrastructure is in place; hence, when a sensor node transmits data to another node, only the nodes on the shortest routing path forward the data.

AS 7. The data aggregation efficiency of cluster-heads is 100%; although a cluster-head receives a number of data, it aggregates them into one unit of data.

AS 8. The transmissions between nodes are over additive white Gaussian noise (AWGN) channels with path loss. The communication environment is contention-based and error-free; hence, sensor nodes do not have to retransmit any data.

AS 9. Each sensor node spends one (0.69) unit of energy to transmit (receive) one unit of data to (from) another node.

Assumptions (ASs) 1-8 are generally accepted in modeling of the energy consumption in clustered multi-hop WSNs [5,8,10]. The transmissions from the cluster-members to their cluster-head are usually of short-distance thus they are assumed over AWGN channels. In contrast, the transmissions from the cluster-heads to the sink node are of long distance and are assumed over fading channels [13]. In this article, the radio range of nodes is restricted to a short distance because of energy constraints. Thus, we assume that the transmissions among the nodes are over AWGN channels with path loss. When the cost for data transmission to the next hop is assumed to be one unit of energy, the cost for data reception is approximated to be 0.73 for the IEEE 802.11 2Mbps wireless network [14] and 0.69 for the MICA2 sensor mote [15].

3.3 Network model and problem formulation

In this article, we model the energy consumed in the network during a single round, because the energy consumption of each round is statistically identical. For the radio hardware energy consumption, we use a well-known model [16]. Let $C_{tx}(l, d)$ and $C_{rx}(l)$ denote, respectively, the energy required for a node to transmit and receive an l bits message over the distance (d). These are given as follows:

$$\begin{cases} C_{tx}(l,d) = (\alpha_0 + \beta d^t)l, \\ C_{rx}(l) = \alpha_1 l, \end{cases} \tag{1}$$

where α_0, α_1, and β denote, respectively, the energy required to run the transmitter circuitry, the receiver circuitry, and the transmitter amplifier, and t is the path attenuation exponent which depends on the distance between the transmitter and the receiver. Either the free space *(d^2* energy loss) or the multi-path fading *(d^4* energy loss) channel models can be used. According to AS 8, a free space model is considered in this article.

The cluster-head is in charge of data aggregation. Let $C_{\text{agg}}(s,l)$ be the energy spent in aggregating s streams of l bits raw information into a single stream of l bits of aggregated information. Then,

$$C_{\text{agg}}(s, l) = \gamma sl, \tag{2}$$

where γ is the energy required to aggregate one bit of data.

In this article, since the radio range of the nodes is restricted, several relay nodes may be required to successfully report the collected information from a node to its destination, i.e, from a member (head) to its head (sink). Thus, the energy consumed in a network increases in proportion to the number of hops between two end-to-end nodes. Let $C_{\text{req}}(i)$ denote the energy to be consumed in a network to report the data from node i to its destination. Since each node can transmit data to a node within the radio range *(R)*, from Equations 1 and 2, we have

$$C_{\text{req}}(i) = \begin{cases} h_1(i)(\alpha_0 + \alpha_1 + \beta R^t)l & \text{if } i \in G_1, \\ h_2(i)(\alpha_0 + \alpha_1 + \beta R^t)l + s(i)\gamma l & \text{if } i \in G_2, \\ 0 & \text{otherwise}, \end{cases} \tag{3}$$

where $h_1(i)$ and $h_2(i)$ represent the number of hops between node i and its destination (head or sink) when node i is a cluster-member or a cluster-head, respectively. According to AS 6, we only consider the minimum number of hops between the nodes. G_1 and G_2 denote the sets of cluster-members and cluster-heads, respectively. $s(i)$ denotes the number of data streams to be aggregated by node i when it is a cluster-head. In Equation 3, all variables except $h_1(i)$, $h_2(i)$, and $s(i)$ are constant. The total number of data streams to be aggregated in a network is identical to the number of the nodes. Thus, it is important to properly derive the number of hops to accurately estimate the total energy consumed in a network.

From our assumptions, the number of hops between the nodes is directly related to the probability of a node to become a cluster-head. Consequently, the probability of becoming a cluster-head is a unique factor in determining the average energy consumed in a network. Let $C(p)$ denote the average energy consumed in a network when the probability of becoming a cluster-head is p. Then, the optimal probability p^* to minimize the average energy consumption can be expressed as follows:

$$p* = \arg \min_{0 \le p \le 1} C(p). \tag{4}$$

In the next section, we introduce our model to derive $C(p)$ and find the optimal probability p^*.

4 A new energy model for clustered multi-hop WSNs

4.1 Energy consumption model

4.1.1 Total number of hops between the cluster-heads and the sink node

The average number of hops between the cluster-heads and the sink node depends on the sink node's location. We consider a disc-shaped sensing terrain with radius r. According to AS 4, the sink node is placed at the center of the sensing terrain. Any cluster-head having one hop to the sink node may be placed to an area which is disc-shaped with radius R. In the same manner, any cluster-head having two hops to the sink node may be placed to a ring-shaped area whose outer radius is $2R$ and inner radius is R. Consequently, cluster-heads with k hops to the sink node may be placed in a ring-shaped area whose outer radius is kR and whose inner radius is $(k - 1)R$. We depict this approach in Figure 1a.

Let λ_{CH} denote the density of the cluster-heads in the network. Define I_k to be the number of cluster-heads with k hops from the sink node. Then,

$$E[I_k] = \lambda_{\text{CH}} \int_{(k-1)R}^{kR} 2\pi r \, dr. \tag{5}$$

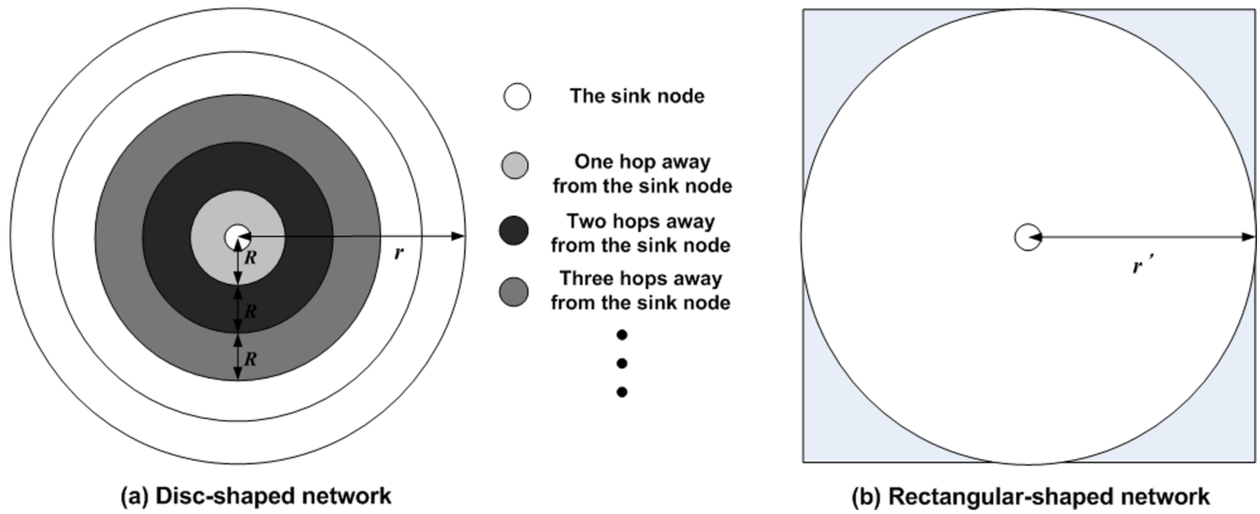

Figure 1 Deriving the number of hops from the cluster-heads to the sink node. **(a)** Disc-shaped network, **(b)** rectangular-shaped network.

Let X be the total number of hops from all the cluster-heads to the sink node in the network. Since the total number of hops of cluster-heads with k hops to the sink node is given as kI_k, we have

$$\begin{aligned} E[X] &= \lambda_{\mathrm{CH}} \sum_{k=1}^{u} k \int_{(k-1)R}^{kR} 2\pi r\,dr \\ &= \pi\lambda_{\mathrm{CH}} \sum_{k=1}^{u} k(2k-1)R^2 \\ &= \pi\lambda_{\mathrm{CH}} R^2 \frac{u(u+1)(4u-1)}{6}, \end{aligned} \quad (6)$$

where $u = r/R$.

Our modeling approach can be applied to an arbitrary-shaped network with a mathematical modification though the model becomes more complicated. For example, in the case of a rectangular-shaped sensing terrain with side $2r'$ as shown in Figure 1b, deriving the number of hops from the cluster-heads inside a circle with radius r' to the sink node is referred to as the modeling approach of a disc-shaped network. Then, the number of hops from the other cluster-heads located outside the circle to the sink node is derived in a mathematical modification which considers the area of the outside region and the distance from the cluster-heads in the outside to the sink node. In this article, we deal with a disc-shaped network for mathematical simplicity.

4.1.2 Total number of hops between the cluster-members and their respective cluster-heads

Generally, as the distance between a sensor node and a cluster-head increases, a possibility that the sensor node becomes a member of a cluster with the cluster-head decreases. This is because as two nodes become more distant, the number of hops between them is likely to become larger and in the clustering algorithm, any node that is not a cluster-head joins a cluster with a cluster-head that has the smallest number of hops from it.

Now, we will derive a probability a node to join a cluster with a cluster-head when the distance between the node and the cluster-head is given. Let x be the distance from a node to a cluster-head, and it can be ranged from a to b, i.e, $a \le x \le b$. If a is not zero, then a region where the node can be placed is ring-shaped as shown in Figure 2. Let $A^{[a,b]}$ be an area of the ring-shaped region. We can divide the interval $[a, b]$ into m subintervals of equal length $\Delta x = (b - a)/m$. Let $x_0(= a)$, x_1, x_2,..., $x_m(= b)$ be the end point of these subintervals. Then, $A^{[a,b]}$ is equivalent to $\lim_{m\to\infty} \sum_{i=0}^{m} \pi \left(x_{i+1}^2 - x_i^2\right)$. Since the density of the nodes is given as λ, the average number of nodes in the ring-shaped region can be approximated to $\lambda A^{[a,b]}$.

Though two sensor nodes are placed within the same distance x from a cluster-head, they can be members of different clusters as illustrated in Figure 3. This shows that the probability that a sensor node joins a cluster with a cluster-head is influenced by the existence of other cluster-heads as well. To deal with such problem, we employ a probability that a node becomes a member of a certain cluster with the consideration of cluster-head density. Let CH(1) be the cluster-head of the cluster(1). As shown in Figure 3, when the distance from a node to CH(1) is x, let $P\,\{(x, \mathrm{CH}(1)) \in \text{cluster}(1)\}$ be the probability that the node becomes a member of cluster (1). Then, let $M^{[a,b]}$ be the number of the member nodes which belong to cluster(1) and are located in a ring-shaped area whose the inner radius is a and the outer radius is b. Then, we have

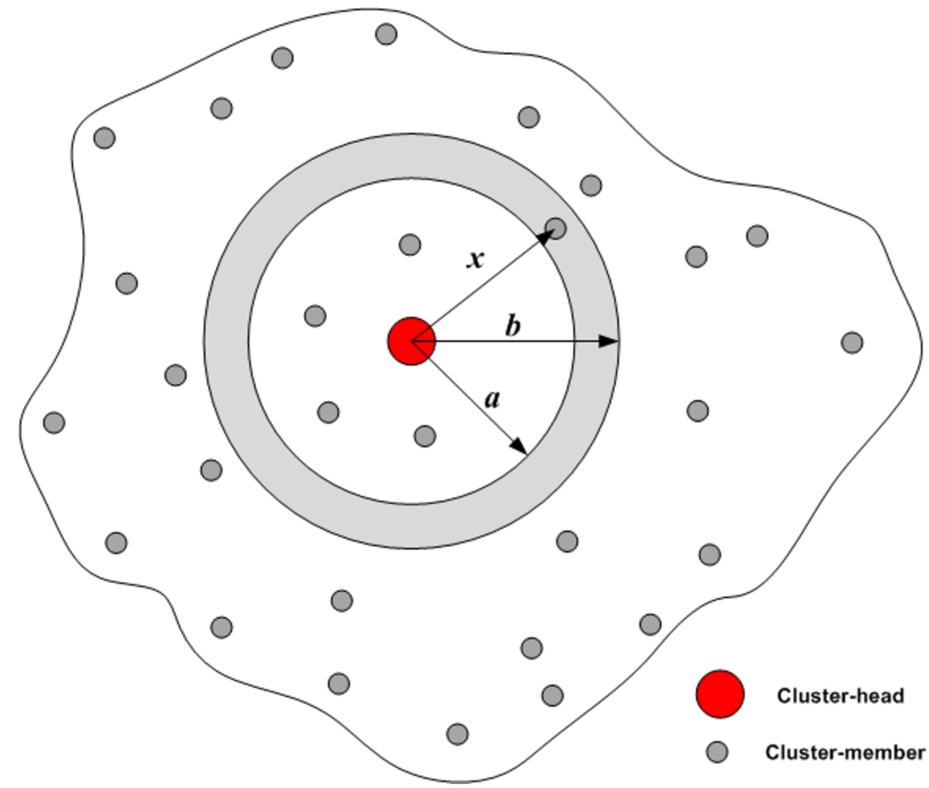

Figure 2 A probability a node to join a cluster with a cluster-head when the distance between the nodes is given.

$$\begin{aligned} E[M^{[a,b]}] &= \lambda_{\mathrm{CM}} \lim_{x\to\infty} \sum_{i=0}^{m} \pi\left(x_{i+1}^2 - x_i^2\right) \cdot P\left\{\left(x_i + \tfrac{\Delta x}{2}, \mathrm{CH}(1)\right) \in \mathrm{cluster}(1)\right\} \\ &= \lambda_{\mathrm{CM}} \lim_{x\to\infty} \sum_{i=0}^{m} \pi\left(2x_i\Delta x + \Delta x^2\right) \cdot P\left\{\left(x_i + \tfrac{\Delta x}{2}, \mathrm{CH}(1)\right) \in \mathrm{cluster}(1)\right\}, \end{aligned} \tag{7}$$

where λ_{CM} denotes the density of the cluster-members in the network. As m goes to infinity, Δx becomes extremely small, and we can ignore Δx^2. Similarly, we can regard $x_i + \Delta x/2$ as x_i. Then,

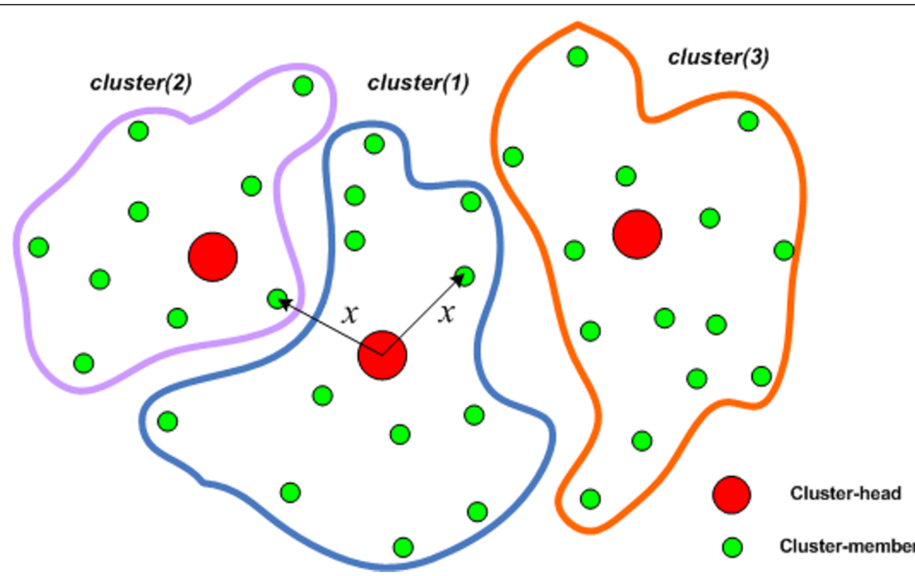

Figure 3 An example of arbitrary-shaped clusters.

$$E\left[M^{[a,b]}\right] = 2\pi\lambda_{\mathrm{CM}} \lim_{x\to\infty} \sum_{i=0}^{m} x_i \Delta x \cdot P\left\{(x_i, \mathrm{CH}(1)) \in \mathrm{cluster}(1)\right\}. \quad (8)$$

The probability $P\{(x,\mathrm{CH}(1)) \in \mathrm{cluster}(1)\}$ can be approximated to the probability that any cluster-head does not exist within distance x from a non-cluster-head node. Since the area of the sensing terrain is A, the number of cluster-heads can be approximated to $\lambda_{\mathrm{CH}}\ A$ According to Campbell's theorem and the results in [17], we get

$$P\{(x, \mathrm{CH}(1)) \in \mathrm{cluster}(1)\} = \left(1 - \frac{\pi x^2}{A}\right)^{\lambda_{\mathrm{CH}} A}. \quad (9)$$

When the sensor field is large, we approximately have

$$P\{(x, \mathrm{CH}(1)) \in \mathrm{cluster}(1)\} \approx \lim_{A\to\infty}\left(1 - \frac{\pi x^2}{A}\right)^{\lambda_{\mathrm{CH}} A} = e^{-\pi\lambda_{\mathrm{CH}} x^2}. \quad (10)$$

From Equations 8 and 10, we have

$$E\left[M^{[a,b]}\right] = 2\pi\lambda_{\mathrm{CM}} \int_a^b x \cdot e^{-\pi\lambda_{\mathrm{CH}} x^2} dx. \quad (11)$$

If we set $a = 0$ and $b = \infty$, then $M^{[0,\infty]}$ is the number of member nodes in an arbitrary-shaped cluster.

The total number of the member nodes having k hops from a cluster-head can be expressed as $M^{[(k-1)R,kR]}$, and thus, the total number of hops between the member nodes and the cluster-head is approximated to $kM^{[(k-1)R,kR]}$. Since p is the probability of being a cluster-head, the density of cluster-heads and cluster-members can be expressed as $p\lambda$ and $(1-p)\lambda$, respectively. Let Y_0 be the total number of hops between all member nodes and the cluster-head in a cluster. Then, we have

$$\begin{aligned} E[Y_0] &= 2\pi(1-p)\lambda \sum_{k=1}^{\infty} k \int_{(k-1)R}^{kR} x \cdot e^{-\pi p\lambda x^2} dx \\ &= \frac{(1-p)}{p} \sum_{k=0}^{\infty} e^{-\pi\lambda(kR)^2 p}. \end{aligned} \quad (12)$$

Let Y be the total number of hops between all the cluster-members and their respective cluster-heads in a network. Since there are λAp clusters on average, the expected value of Y is as follows:

$$\begin{aligned} E[Y] &= \lambda Ap \cdot E[Y_0] \\ &= \lambda A(1-p) \sum_{k=0}^{\infty} e^{-\pi\lambda(kR)^2 p}. \end{aligned} \quad (13)$$

4.1.3 MAC inefficiency and signaling overhead

The energy loss due to inefficient operations in MAC, such as idle listening or overhearing, and clustering overhead may depend on the MAC protocol, the routing protocol and the clustering algorithm that are used [12]. We define e_{wt} and e_{wr} as the energy wasted by a transmitter due to MAC inefficiency for transmitting a bit and the energy wasted by a receiver due to MAC inefficiency for receiving a bit in one-hop communication, respectively. Then, we replace α_0 and α_1 with α_0' and α_1', where $\alpha_0' = \alpha_0 + e_{\mathrm{wt}}$ and $\alpha_1' = \alpha_1 + e_{\mathrm{rt}}$.

The signaling overhead associated with clustering consists of two major factors: one for the cluster-head selection and another for the distribution of TDMA schedules. To select the cluster-head, each sensor node receives advertisement messages from its neighboring nodes and the node forwards a message which advertises its cluster-head to the other nodes. Let the length of an advertisement message be l_1 bits. Then, the energy consumed for the cluster-head selection in a network, S_1, is defined as follows:

$$E[S_1] = \varphi_1 \lambda A, \quad (14)$$

where $\varphi_1 = \left(\alpha'_0 + \lambda\pi R^2 \alpha'_1 + \beta R^t\right) l_1$. ϕ_1 represents the energy consumed for data processing, receiving an l_1 bits message from the neighboring nodes, and transmitting l_1 bits message over the radio range *(R)*.

To avoid data collision, the cluster-heads and the sink node set up TDMA schedules for each node in their respective clusters and for the cluster-heads, respectively. Then, the cluster-heads and the sink node distribute the schedules to their clustered nodes and the cluster-heads in the network, respectively. In the case of the cluster-heads, the schedules are distributed twice; one for data collection and another for aggregated data report. Let the length of a TDMA schedule message be l_2 bits. Then, the energy consumed for the distribution of the TDMA schedules in a network, S_2, can be expressed using the total energy consumed to collect the sensed information. Then, we have

$$E[S_2] = \varphi_2 (E[X] + 2E[Y]), \quad (15)$$

where $\varphi_2 = (\alpha_0' + \alpha_1' + \beta R^t) l_2$. ϕ_2 represents the energy consumed for data processing, transmitting, and receiving an l_2 bits message over the radio range *(R)*.

4.1.4 Total energy consumption in the network

In Equation 3, we showed that the energy required for data transmission and reception depends on the number of hops between the end-to-end nodes. In addition, the cluster-heads consume additional energy due to data aggregation. Since we are interested in the total energy consumption in a network, we need to derive the total number of data streams to be aggregated by all cluster-heads, which equals to the number of nodes, i.e., λA. Let Z be the total energy consumed by the cluster-heads for aggregating l bits messages in a network. Then, from

Equation 2, we have

$$E[Z] = \gamma \lambda A l. \tag{16}$$

Then, we can derive the total energy consumed by all nodes in a single round as the sum of the energy consumed for processing, transmitting, receiving, aggregating, and signaling. From Equations 3, 6, 13-16, we can derive *C(p)* as follows:

$$\begin{aligned} C(p) &= \varphi_0 E[X] + \varphi_0 E[Y] + E[Z] + E[S_1] + E[S_2] \\ &= \pi \lambda R^2 (\varphi_0 + \varphi_2) \tfrac{u(u+1)(4u-1)}{6} p + \lambda A (1-p)(\varphi_0 + 2\varphi_2) \sum_{k=0}^{\infty} e^{-\lambda \pi (kR)^2 p} \\ &+ \varphi_1 \lambda A + \mu, \end{aligned} \tag{17}$$

where $\varphi_0 = \left(\alpha'_0 + \alpha'_1 + \beta d^t\right) l$ and $\mu = \gamma \lambda A l$. ϕ_0 and μ represent the energy consumed for data processing, transmitting, and receiving an l bits message, and the total energy consumption by the cluster-heads for aggregating information in a network, respectively.

4.2 Optimal clustering

From Equations 4 and 17, we can determine the optimal probability p^* to minimize the total energy consumption. According to the Galois Theory [18], p^* cannot be obtained by elementary algebra. However, we can use numerical methods to solve a general polynomial equation [9]. Since *C(p)* is a convex function, we use Newton's method to find a minimum of *C(p)*. The proof of the convexity is shown in the Appendix.

Though we assume a disc-shaped sensing terrain for mathematical simplicity, our model enables to simply determine the optimal number of clusters because it only requires information on the node density, the area of sensing terrain, and the radio range to find a solution.

5 Evaluation of the energy model

5.1 Simulation environment

To evaluate the accuracy of our energy model, we compare it with the energy models of EEHCA [5], CRS [8], and the results from a Monte Carlo simulation [19]. Since the signaling overhead for clustering is not considered in the existing energy models, to compare the accuracy of the energy models under the same conditions, we evaluate the energy models ignoring the energy spent for signaling. Other multi-hop clustering algorithms such as OCND [9] and ECTC [10] adopt the same modeling approach as in EEHCA. Thus, their accuracies are almost identical to that of EEHCA. Hence, we compare our model with the model of EEHCA on their behalf.

In the simulation, nodes are randomly distributed in a disc-shaped area with a radius of 50 m. The nodes are assumed to be homogeneous, omnidirectional, and stationary. The radio range of all the nodes is set to 10 m. The sink node is placed at the center of the disc-shaped sensing terrain. The nodes share a single transmission channel on which they cannot transmit and receive simultaneously. Data collision is prevented by TDMA schedules organized by the cluster-heads or the sink node. Thus, energy consumption caused by packet retransmission is disregarded. Network parameters used for the evaluation are shown in Table 1.

The cost for data transmission to the next hop is set to one unit of energy. On the other hand, the costs for data reception and data aggregation are set to 0.69 for the MICA2 sensor mote [15] and 0.1 for each stream [6], respectively. The energy models of EEHCA and CRS are transformed to be adequate for the disc-shaped sensing terrain. In simulation where the Monte Carlo method is used, the nodes are randomly distributed, and the average of 100 repeated simulations is taken as the total energy consumption of the nodes.

5.2 Evaluation of the energy model

Figure 4 shows the total number of hops between the cluster-heads and the sink node in the network, where the number of hops increases linearly with the probability of being a cluster-head. This is because the average number of cluster-heads increases in proportion to the probability. Figure 4 shows that our model provides the most precise estimation among all the models. Furthermore, the results of our model are very close to those of Monte Carlo simulation when the number of nodes is large. However, modeling errors of EEHCA and CRS may increase.

Figure 5 shows the total number of hops between all cluster-members and their respective cluster-heads in the network, where the number of hops decreases with the increase of the probability of a node to become a cluster-head. This is because, as the number of clusters increases, the average number of hops between cluster-members and the cluster-head decreases. According to Figure 5, the results of our model nearly match with those of Monte Carlo simulation, except when the probability is very small. On the other hand, the models of CRS and EEHCA considerably underestimate the number of hops. Although CRS compensates the underestimation errors with consideration of node density, it is not sufficient to redeem the errors. Figure 5 also shows that our model becomes more accurate as the number of nodes increases. As the number of nodes increases, it is more

Table 1 The network parameters for the evaluation

Parameter	Values
Number of nodes (*n*)	500, 1500
Radius of the covered disc-shaped field (*r*)	50m
Radio range (*R*)	10m

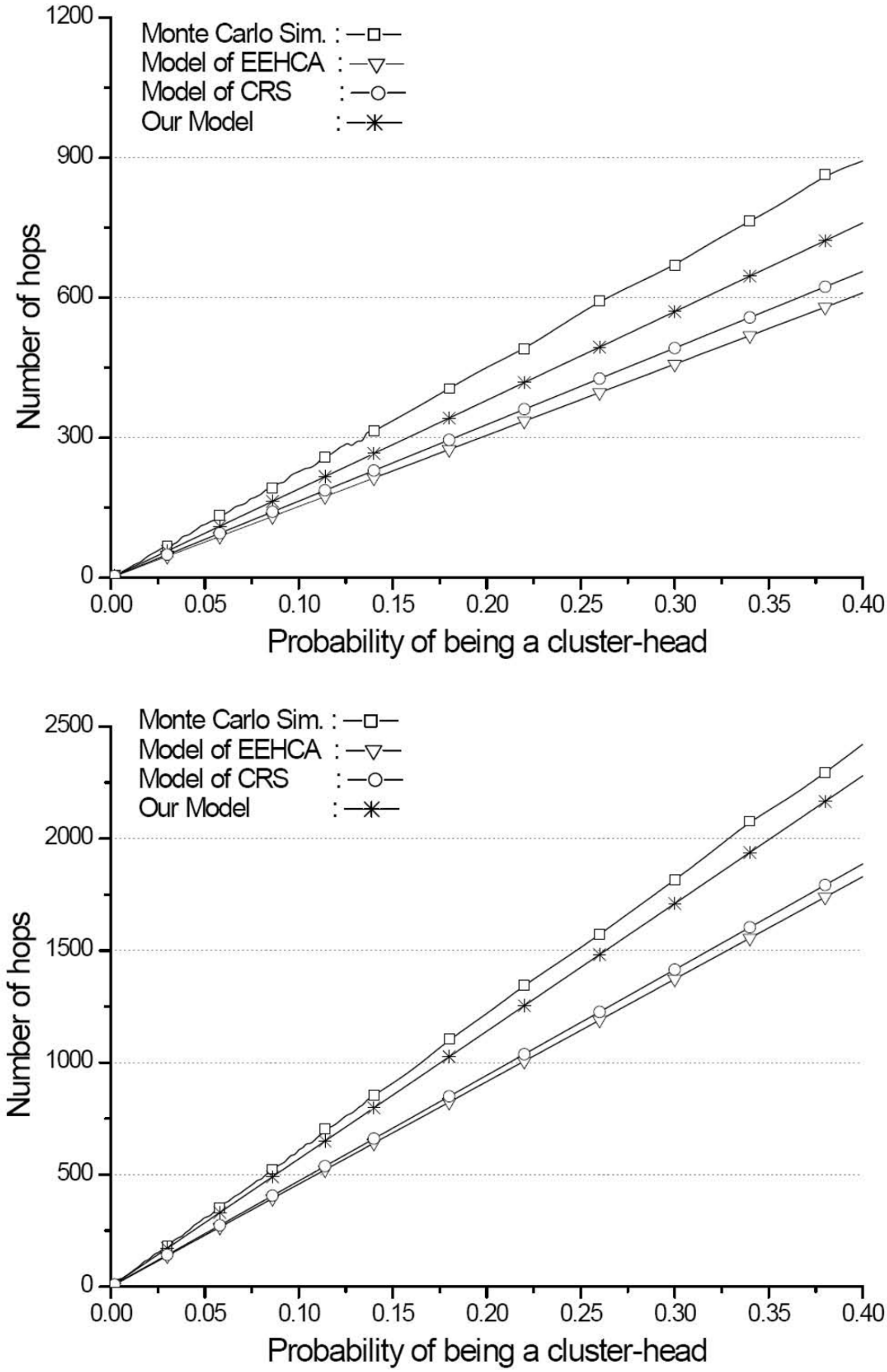

Figure 4 Total number of hops from the cluster-heads to the sink node in the network. **(a)** n = 500, (b) n= 1500.

likely to find relay nodes in a shortest path to the destination, consequently, the modeling errors decrease. However, we cannot observe the same behavior from the models of EEHCA and CRS because they simply obtain the number of hops by dividing the radio range into the average distance between the end-to-end nodes.

The total energy consumption in the network is shown in Figure 6. We can observe that our model gives

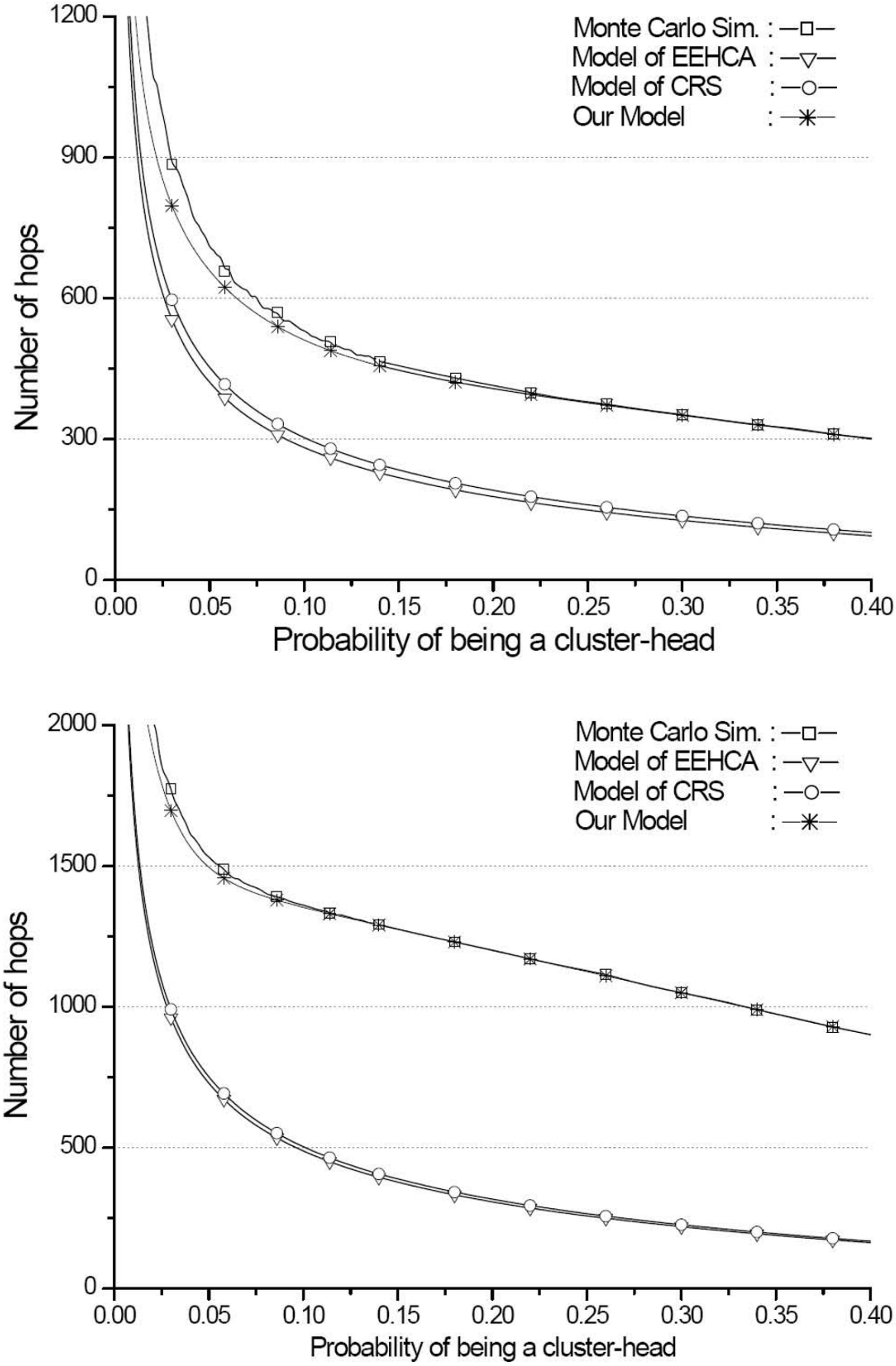

Figure 5 Total number of hops from all cluster-members to their respective cluster-heads in the network. **(a)** n= 500, **(b)** n= 1500.

a better approximation of the energy consumption than the existing models. Furthermore, our model provides a better prediction than the other models in determining the optimal probability of being a cluster-head, thus minimizing the energy consumed in the network. The optimal probability p^* obtained from the Newton method is provided in Table 2. To compare the accuracy of the energy models in detail, we analyze how

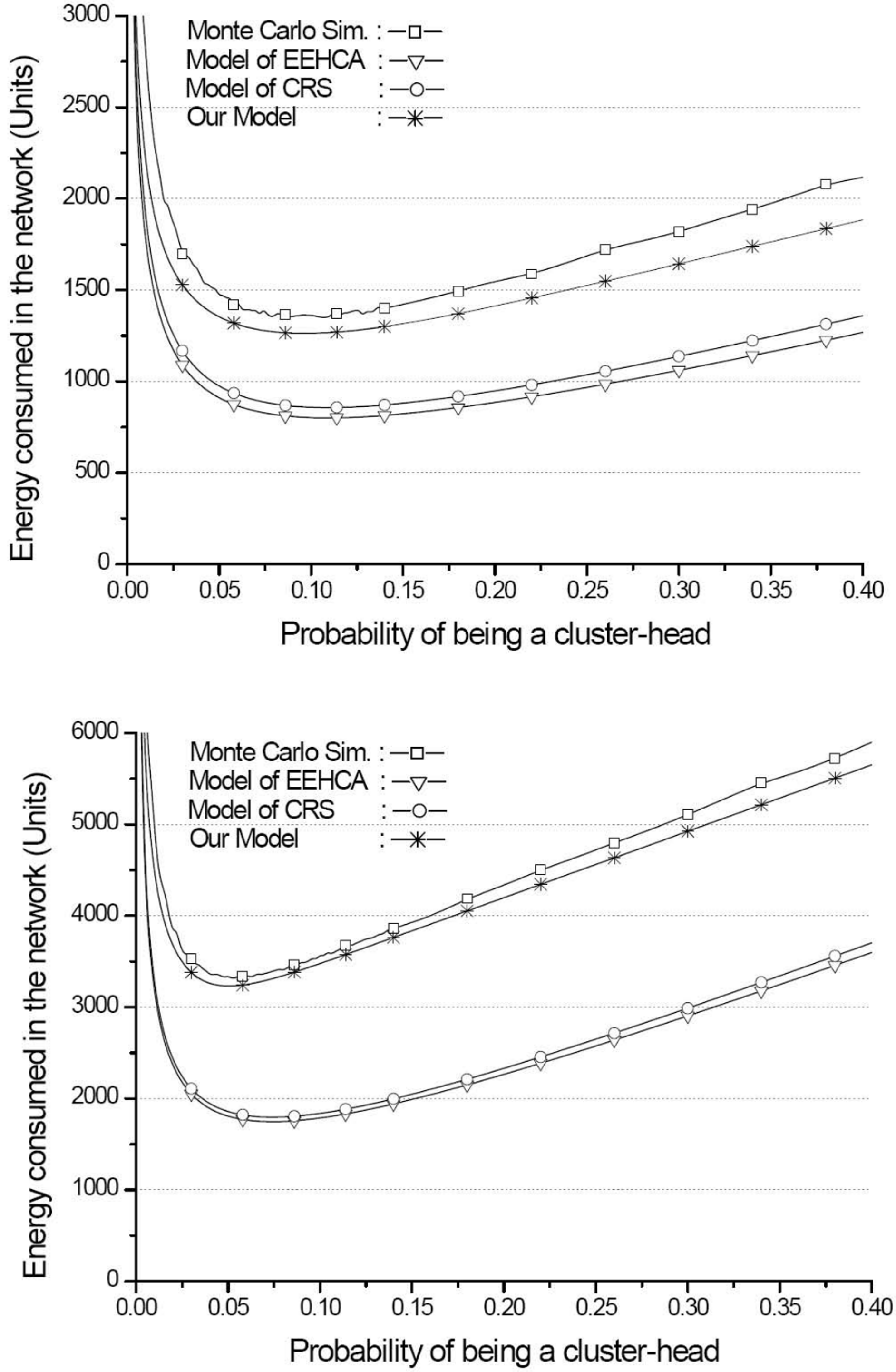

Figure 6 Total energy consumed by nodes in the network. **(a)** n = 500, **(b)** n = 1500.

close the models are to the Monte Carlo simulation. To do that we divided the simulation result by the models' predictions, and called it the Monte Carlo similarity. In Figure 7, we compare the Monte Carlo similarities of the models. Figure 7 shows that the Monte Carlo similarity of our model reaches about 90-95%, except when the probability is very small. On the contrary, the Monte Carlo similarities of the other two models are at

Table 2 Optimal values of probability p^* for multi-hop with clustering

Sensing terrain	Number of nodes (n)	Density (λ)	Probability (p^*)
A disc-shaped field	500	$0.2/\pi$ (nodes/m^2)	0.096
50m	1500	$0.6/\pi$ (nodes/m^2)	0.050

most about 70%. This result demonstrates that our model may give more precise prediction for optimal probability than other models.

6 Conclusions

In this article, by properly deriving the number of hops between end-to-end nodes, we have derived an accurate

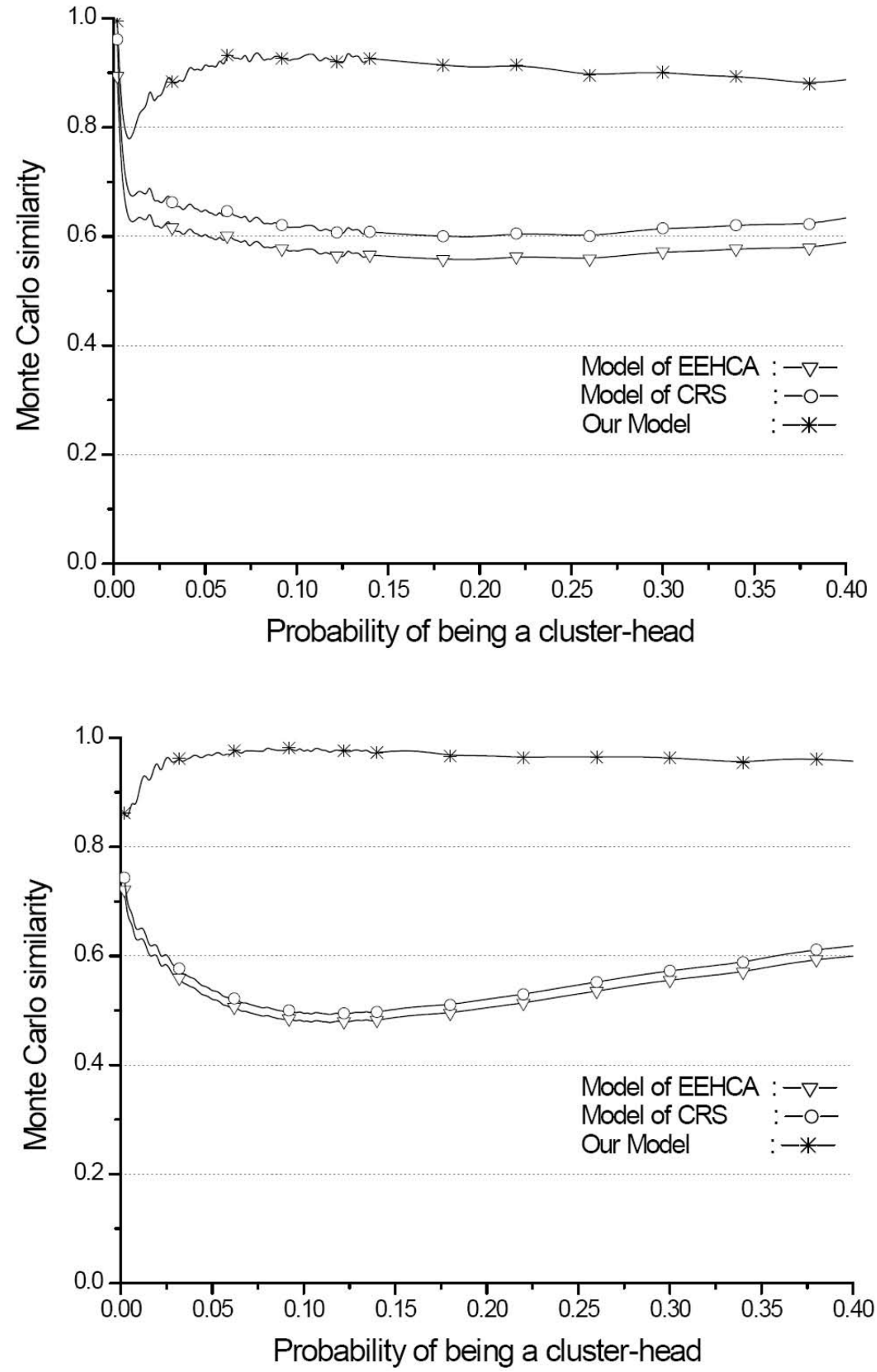

Figure 7 Monte Carlo similarities of the energy models. **(a)** n= 500, **(b)** n= 1500.

energy model for clustered multi-hop WSNs using a probabilistic cluster-head selection method. Using this model, we have determined the optimal number of clusters in a network, thus minimizing the total energy consumption to maximize the lifetime of the network. In our model, we have assumed that the sensing terrain is disc-shaped for mathematical simplicity. For other shapes of sensing terrain, our modeling approach can be applied with a mathematical modification though the model becomes more complicated.

On the nature of modeling, our model may be accompanied by errors in derivation. Nevertheless, the simulation results have showed that our energy model gives a better approximation of the energy consumed in a network than the other models. In this article, though we have assumed that a node can reach its destination by exactly n hops if the destination is away from it between n - 1 and n times the radio range, we have found our model more accurate. For more precise modeling, other modeling approaches for deriving the number of hops may be necessary.

Appendix

Proof of the convexity of the function *C(p)*

To prove that the function $C(p)$ is convex, we show that $C''(p)$ is non-negative for $0 \le p \le 1$. From Equation 15, $C'(p)$ and $C''(p)$ are given by

$$C(p) = \pi\lambda R^2(\kappa + \varphi_2)\frac{u(u+1)(4u-1)}{6}p + \lambda A(1-p)(\kappa + 2\varphi_2)\sum_{k=0}^{\infty} e^{-\lambda\pi(kR)^2 p} + \varphi_1\lambda A + \mu,$$

$$C'(p) = \lambda A(\kappa + 2\varphi_2)\sum_{k=0}^{\infty}\{(1-p)(-\pi\lambda(kR)^2) - 1\}e^{-\pi\lambda(kR)^2 p} + \pi\lambda R^2(\kappa + \varphi_2)\frac{u(u+1)(4u-1)}{6},$$

$$C''(p) = \lambda A(\kappa + 2\varphi_2)\sum_{k=0}^{\infty}\left\{(1-p)\left(\pi\lambda(kR)^2\right)^2 + 2\left(\pi\lambda(kR)^2\right)\right\}e^{-\pi\lambda(kR)^2 p}.$$

From the above equations, it is evident that $C''(p)$ is great than zero for $0 \le p \le 1$.

Since C"(0) <0, if $C'(1)$ >0, then the global minimum exists at $0 < p < 1$. If $C'(1) \le 0$, then the minimum exists at $p = 1$.

Competing interests

The authors declare that they have no competing interests.

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