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Alternative relaying for cooperative multiple-access channels in wireless vehicular networks

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Abstract

In this paper, a novel spectrally efficient half-duplex cooperative transmission protocol is proposed for cooperative multiple-access channels in wireless vehicular networks, where multiple sources (vehicles) deliver messages to a common destination (roadside base station or roadside infrastructure) with the help of multiple decode-and-forward relays (roadside stations). The basic idea is to apply superposition coding at each transmitter in order to achieve the full diversity gain, where the linear zero-forcing detection is used at each relay to combat inter-relay interference. Compared to existing uplink cooperative protocols, the proposed scheme can exploit the cooperation involving both the relays and the sources. An achievable diversity-multiplexing trade-off is developed for the proposed transmission protocol. Even with strong inter-relay interference which has been ignored by many existing works, the proposed scheme can still approach the optimal multiple-input single-output upper bound. Numerical results have also been provided to demonstrate the performance of the proposed protocol.

Keywords: Cooperative multiple access; Wireless vehicular network; Alternative relaying; Diversity-multiplexing trade-off

1 Introduction

1.1 Background

Recent researches have paid much attention to wireless vehicular *ad hoc* networks (VANETs) since they are important to intelligent transportation, environment monitoring, etc [1,2]. Unlike the cellular networks, there does not exist a powerful base station in a vehicular *ad hoc* network, so peer-to-peer transmission is relied on. Correspondingly, a key problem is how to design the routing protocols to efficiently forward the source messages to the destination. One potential method is to design cross-layer protocols for mobile vehicular *ad hoc* networks. For example, the work in [3] has shown that cooperative transmission, a low-cost and effective alternative to multiple-input multiple-output (MIMO) techniques to enhance the physical layer performance, can bring its physical layer benefits to the upper layer, such as the design of routing protocols.

Motivated by these benefits of cooperative transmission, this paper aims to design a novel cooperative transmission scheme for the multiple access channel (MAC) in the wireless vehicular networks.

Exploring cooperation among the source nodes has been studied in [4,5] for the MACs. The work in [4] proposed a cooperative multiple-access (CMA) transmission protocol using amplify-and-forward (AF) strategy, and its decode-and-forward (DF) version has been proposed in [5] by using superposition (SP) coding at each source, where the idea of superposition coding was first proposed in [6] to increase the system capacity. Specifically, the key idea of such superimposed cooperative schemes is that each user will share parts of its power with other users and transmit a mixture of its own and others' information. The diversity-multiplexing trade-off (DMT) [7] achieved by these schemes can approach the optimal multiple-input single-output (MISO) upper bound. However, such schemes only exploit the cooperation among the sources and are therefore constrained by the number of sources.

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1.2 Related work

To further improve the reception robustness, exploiting relays as an extra dimension has been recognized as a cost-effective way, since the number of idle users is always larger than the active ones in a typical wireless network. However, extra time slots may be consumed to repeat source messages in relay transmission. For example, due to the half-duplex constraint, there exists a loss of spectral efficiency for the classical orthogonal transmission schemes [8,9]. The standard non-orthogonal transmission strategies in [4,10] can make some improvement and achieve the full multiplexing gain but still suffer a significant diversity loss for large multiplexing gains, if compared with the optimal MISO upper bound. To overcome limitation of the available diversity, a *successive relaying* concept which aims to physically reuse the relay-destination channels was studied in [11-14]. The basic idea is to arrange two or more relays to alternatively retransmit the source messages. To deal with the inter-relay interference, the inter-relay link was usually assumed to be either sufficiently strong or weak in these DF-relaying works. A cooperative transmission protocol named as shifted successive DF relaying (SSDFR) was proposed in [14], where maximum likelihood (ML) decoding is utilized at two multiple-antenna relays. In [15], another cooperative transmission protocol, called relay-reuse DF (RRDF), has been proposed to the scenario with a single source node, which yields furthermore improvement in comparison with [14] due to the linear zero-forcing (ZF) detection which can reduce the computational complexity.

For a general uplink scenario with multiple users and relays, it is still a challenging problem to integrate the cooperation among the sources and the cooperation from the relays. In [16,17], each relay utilized superposition coding to assist each source. However, the developed upper bound of DMT in [16,17] can only be achieved with some assumptions on the conditions of inter-relay channels. The work in [18] considered a more general scenario and proposed a cooperative protocol, called relay-assisted CMA (R-CMA), which carefully schedules the multiple sources and relays' transmission, but the optimal MISO bound can only be approached for a large number of relays. More works that considered such multi-user multi-relay networks can be found in [19-21], where the assistance of relays has been well exploited. Generally, these works only use the assistance of relays without the cooperation between sources, which cannot fully exploit the capacity of the uplink system.

1.3 Contribution

This paper aims to design a novel cooperative transmission protocol for a CMA channel to integrate the

cooperation from dedicated relays and the mutual cooperation between users.

Specially, we consider a cooperative multiple-access communication scenario in which multiple source nodes communicate with the common destination with multiple half-duplex relays. Such a communication pattern is an important building block in wireless VANETs. For example, some proposals of intelligent vehicle highway systems (IVHS) have introduced a variety of vehicle-to-vehicle and vehicle-to-roadside-infrastructure communications. In this case, cooperation can take place between vehicles or between vehicles and roadside stations. When some vehicles wish to send messages to a common roadside base-station, they can cooperate with each other and also obtain external assistances from more powerful roadside stations with multiple antennas.

The proposed cooperative transmission protocol is termed as alternative-relaying decode-and-forward (ARDF) throughout this paper. The main novelty is that a new cooperative multiple-access transmission protocol has been designed by carefully applying these low-cost tools (zero-forcing detection, antenna selection, superposition coding, repetition-coded decode-and-forward, etc.) and fully exploiting the potentials of relay and source cooperation. To the best of the authors' knowledge, no existing works can effectively achieve this *full* cooperation. The basic idea of the proposed protocol is to combine the superposition coding [6] with RRDF scheme [15]. By carefully arranging the transmission process and applying superposition coding at transmitters, each source can be assisted by all the other nodes, including relays and other sources; by utilizing the linear ZF decoding in terms of RRDF transmission scheme, the reuse of the relay-destination links can be achievable in the scenario with a general inter-relay interference. Hence, the potential of such a system can be fully exploited, and the full diversity gain can be achievable. Antenna selection is also applied at each relay to select the best antenna for transmitting, which can preserve the full diversity gain and save transmit power.

DMT is used to evaluate the spectral efficiency of the proposed CMA protocol, since it has been recognized as an effective information-theoretic criterion. The achievable DMT for the proposed ARDF protocol for the CMA channel is developed by characterizing the associated outage probabilities of outage events. From the derived result, we can show that the DMT of the proposed scheme can approach the optimal MISO upper bound when the length of one cooperative data frame is large. Compared to the previous CMA schemes in [4,5] and R-CMA in [18], the proposed cooperative scheme achieves a better DMT curve at most multiplexing gain ranges, especially when the frame length is large enough. Monte Carlo simulation results have also been presented to demonstrate the

outage probability performance of the proposed transmission protocol.

Compared to the point-to-point communication in [15], we consider a multi-user case for the addressed MAC scenario in this paper. As a consequence, DMT for the MAC channel in [22] should be adopted, the information-theoretic criterion, which is developed based on the capacity region of MAC [23]. Specifically, define $X_{\mathcal{S}} = \{X_i : i \in \mathcal{S}\}$, and X_i (Y) is the channel input (output) variable. For a instantaneous channel state information (CSI) which is known at the receiver, the capacity region of the M-user MAC can be written as following [23]:

$$\sum_{i \in \mathcal{S}} R_i < I(X_{\mathcal{S}}; Y | X_{\mathcal{S}^c}, \mathbf{H} = H) \text{ for all}$$

$$\mathcal{S} \subseteq \{1, 2, \cdots, M\} \tag{1}$$

for some product distribution $p_1(x_1) \cdots p_M(x_M)$, where R_i the data rate of source i ($i = 1, \cdots, M$) and $\mathbf{H}$ is the corresponding source-destination channel matrix. As can be observed from the above equation, the capacity region of MAC is a set of many inequalities with different conditions, which makes the calculation of DMT much more difficult than the single-user scenario in [15]. These challenges also make the corresponding analysis more valuable.

This paper is organized as follows. Section 2 describes the proposed CMA-ARDF transmission protocol. Section 3 presents the details of the ARDF transmission process of the proposed scheme. Section 4 outlines the achievable DMT and the Monte Carlo simulation results for the proposed protocol. Section 5 offers concluding remarks. The proofs of the final DMT result are provided in the 'Appendix' section.

2 Protocol description

A cooperative multiple-access scenario is considered in this paper, where multiple sources wish to communicate a common destination under the help of relays. In particular, the addressed scenario consists of two single-antenna sources (i.e., vehicles, denoted as S_1 and S_2), two M_r-antenna ($M_r \geq 2$) relays (i.e., fixed roadside stations, denoted as R_1 and R_2), and one common single-antenna destination (i.e., base station, denoted as D). The abstract system model and its application to the wireless vehicular networks are shown in Figures 1 and 2, respectively. All channels are assumed to be frequency-flat quasi-static Rayleigh fading. Such a Rayleigh fading assumption is valid in a practical vehicular network since it is very likely that there does not exist a line-of-sight (LOS) component between a pair of VANET transceivers due to their relatively low antenna height [3,24,25]. All nodes are constrained by the half-duplex assumption and use the same

transmission power. Time division multiplexing access (TDMA) is utilized in this scenario for its simplicity.

Every antenna at each relay is marked with a distinct index number from 1 to M_r. As shown in Figure 1, the column vector $\mathbf{h}_{i,Rj}$ denotes the channel vector between the i-th source and R_j, where $i, j = 1, 2$, and h_{Si} represent the source-destination channel. The channel vector between R_j and D is denoted as a column vector $\mathbf{g}_j = \{g_{j,1}, \cdots, g_{j,M_r}\}$. Moreover, the column vector $\mathbf{h}_{j,m}$ denotes the inter-relay channel vector between the m-th antenna of R_j and the other relay. Each relay uses all the M_r antennas to receive messages but only chose the *best* antenna to transmit signals for simplicity. The index of such a transmit antenna is defined as r_j at R_j. Note that this antenna selection method can preserve the diversity gain achieved by all the antennas at each relay in the high SNR regions.

The proposed protocol consists of two stages, initialization and data transmission.

2.1 Initialization

During the initialization stage, each relay channel is assumed to have the knowledge of its incoming and outgoing channel state information (CSI). Based on the source-relay CSI, using the relay selection method in [8], we known that R_j can decode the messages from S_i when the source-relay channel satisfies $\log(1 + \rho|\mathbf{h}_{i,Rj}|^2) > \tilde{R}$. Here, $\log(\cdot)$ is taken to base 2, ρ denotes the transmit signal-to-noise ratio (SNR), where we have assumed that all transmitters are under the same and fixed power constraint, $|\cdot|$ denotes the Euclidean norm of a vector, and $\tilde{R}$ denotes the number of bits in each codeword transmitted by sources[a]. If R_j can correctly decode the messages from both sources, i.e., $|\mathbf{h}_{i,Rj}|^2 \geq \frac{2^{\tilde{R}}-1}{\rho}$ for $\forall i \in \{1, 2\}$, it will broadcast a one-bit indicator, which is denoted as ACK1. Otherwise, R_j broadcasts a one-bit indicator NACK1, assumed that all the other nodes can correctly receive each feedback signal. Denote A_k as the situation in which k relays broadcast ACK1 signals. On the other hand, based on the relay-destination CSI, each relay can determine the index of the best antenna as $r_j \triangleq \arg\max_{r=1\ldots M_r} |g_{j,r}|^2$.

For situation A_2 in which both relays broadcast ACK1 signals, two sub-events will be further considered. According to the source-relay/inter-relay CSI, R_j will determine whether it can use ZF detection to separate the mixture of two unknown streams which are from S_j and the other relay, respectively. For the ZF receiver, the qualification criterion [26] is $\left[(\mathbf{H}_{Rj}^H \mathbf{H}_{Rj})^{-1}\right]_{k,k} < \rho/2^{\tilde{R}-1}$, where $[\mathbf{X}]_{k,k}$ represents the k-th diagonal element of matrix $\mathbf{X}$, $\mathbf{H}_{Rj} = \begin{bmatrix} \beta_1 \mathbf{h}_{2,r_2} & \alpha_1 \mathbf{h}_{1,R1} \end{bmatrix}$ for $j = 1$, and $\begin{bmatrix} \beta_1 \mathbf{h}_{1,r_1} & \alpha_1 \mathbf{h}_{2,R2} \end{bmatrix}$ for $j = 2$. Note that α_i and β_i are the power weighting factors at the sources and relays, respectively, which are constrained by $0 < \alpha_i, \beta_i < 1$ and $\sum_{i=1}^2 \alpha_i^2 = \sum_{i=1}^2 \beta_i^2 = 1$. If R_j satisfies this ZF condition, it will send back another one-bit

(a) When S_2 and R_1 transmit signals

(b) When S_1 and R_2 transmit signals
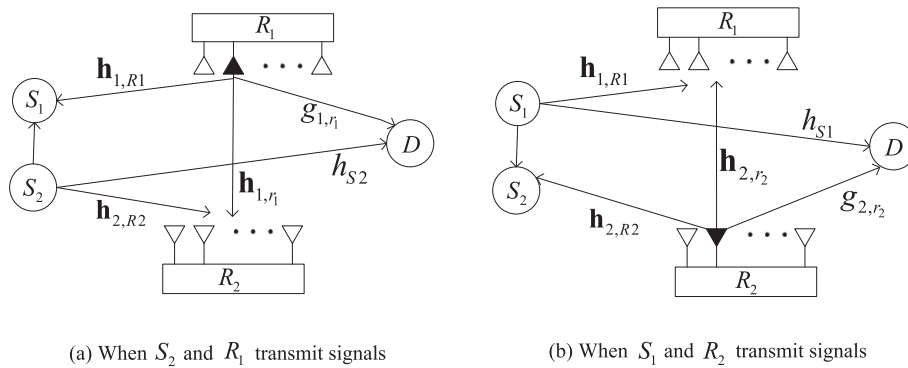
Figure 1 The system model and ARDF process at two successive time slots (a-b). Each relay is equipped with multiple antennas. The antenna marked with black color for each relay is selected as the transmit antenna.

indicator ACK2. Otherwise, it sends back NACK2. Let E denote the event that both relays send back ACK2 signals, and such an event can be represented as [26]

$$E \triangleq \left\{ \log \left(1 + \frac{\rho}{\left[(\mathbf{H}_{Rj}^H \mathbf{H}_{Rj})^{-1} \right]_{k,k}} \right) > \tilde{R}, \forall k, j \in \{1, 2\} \right\},$$
(2)

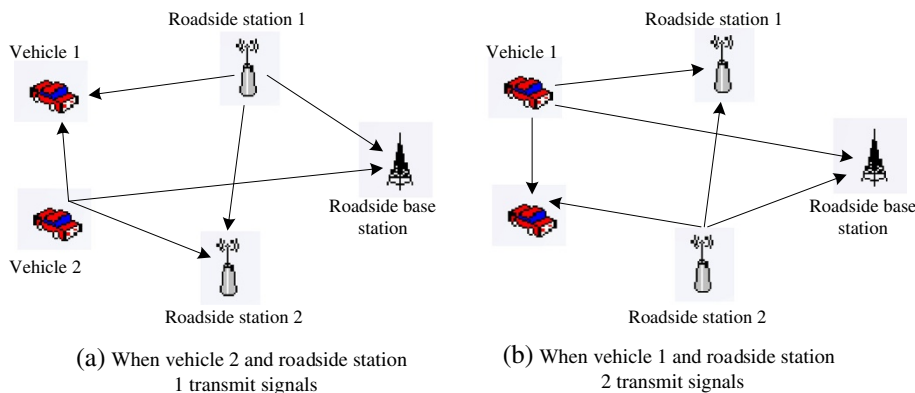Otherwise, the system lies in event $\overline{E}$ if any relay sends back a NACK2 signal.

2.2 Cooperative transmission

Based on the initialization stage, the transmission is divided into four different modes according to the indicators sent back from the two relays. The flow chart of the proposed protocol is shown in Figure 3, where each of the sources is assumed to correctly decode the messages of the other source, which is valid for high SNR regions as shown in [5]. In the next paragraphs, we will describe
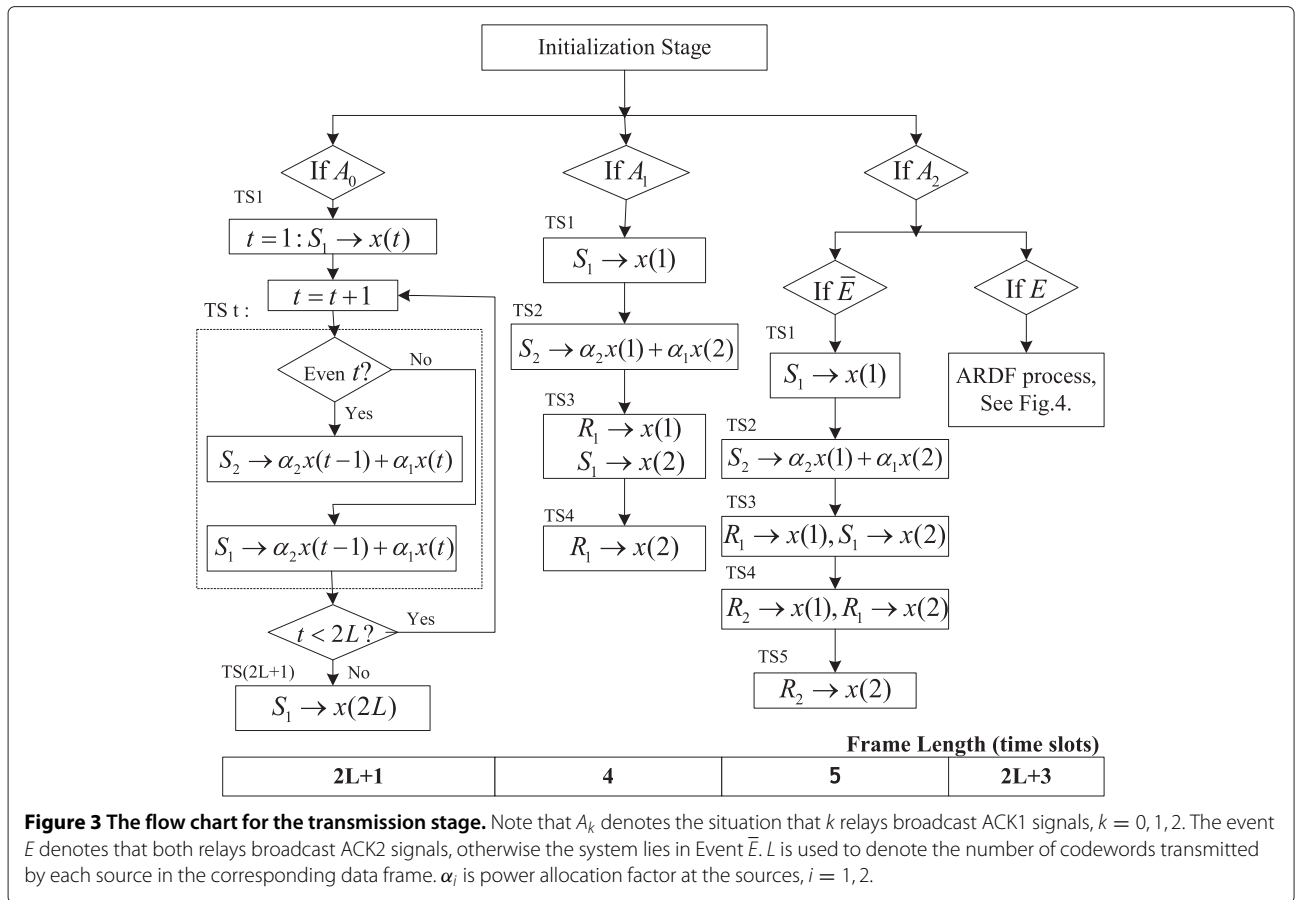
the cooperative transmission mode for each situation in details.

For situation A_0 in which both relays send back NACK1 signals, we can only utilize the mutual cooperation between the two sources. In one cooperative data frame, each source intends to send L codewords, and the system works in mode 1. In time slot n ($2 \leq n \leq 2L$), S_i will transmit a mixed message[b] $\alpha_2 x(n-1) + \alpha_1 x(n)$ and the other source listens, where $i = 2$ if n is even and $i = 1$ if n is odd. The data rate per channel use here is $\frac{2L}{2L+1}\tilde{R}$.

For situation A_1 in which only one relay sends back ACK1 signal, we assume that R_1 is this relay without loss of generality. Each source intends to transmit only one codeword, and the system works in mode 2. For the first two time slots, the transmission process is the same as that in mode 1, and R_1 can correctly decode $x(1)$ and $x(2)$ since it sends back ACK1. In the third time slot, S_1 and R_1 will retransmit $x(2)$ and $x(1)$, respectively. In the fourth time slot, R_1 will retransmit $x(2)$. The transmission data rate in this situation is $\frac{2}{4}\tilde{R}$.

(a) When vehicle 2 and roadside station 1 transmit signals

(b) When vehicle 1 and roadside station 2 transmit signals

Figure 2 The application of the system model in Figure 1 to the wireless vehicular networks (a-b). Two vehicles communicate with a common roadside base station with the help two roadside stations.

Figure 3 The flow chart for the transmission stage. Note that A_k denotes the situation that k relays broadcast ACK1 signals, $k = 0, 1, 2$. The event E denotes that both relays broadcast ACK2 signals, otherwise the system lies in Event $\bar{E}$. L is used to denote the number of codewords transmitted by each source in the corresponding data frame. α_i is power allocation factor at the sources, $i = 1, 2$.

For situation A_2, if the event $\bar{E}$ occurs, the system works in mode 3. The transmission scheduling during the first three time slots is the same as that in mode 2. Then, during the fourth time slot, R_1 and R_2 will retransmit $x(2)$ and $x(1)$, respectively. During the fifth time slot, R_2 retransmits $x(2)$. The data rate for the event $\bar{E}$ is $\frac{2}{5}\tilde{R}$. For the event E, both relays broadcast ACK2 signals. As shown in Equation (2), each relay can correctly decode two unknown codewords transmitted by a source and the other relay, so the ARDF transmission process can be performed. The two sources will transmit $2L$ codewords to the destination during $2L + 3$ time slots, so the data rate is $\frac{2L}{2L+3}\tilde{R}$. The details of such an ARDF process are put into in to the next section, i.e., Section 3, and the system works in mode 4.

From the protocol description, the transmission mode and equivalent channel matrix for each situation can be easily shown in Table 1.

3 Alternative-relaying DF process
3.1 Transmission process
The proposed ARDF transmission process follows the concept of successive relaying [13], where the two half-

duplex relays alternatively retransmit the source messages to mimic a full-duplex node. As shown in Figure 1, the *sending source* sends a superposition codeword $\alpha_2 x(n-3) + \alpha_1 x(n)$ in any one time slot n ($n > 3$), where $x(n)$ is its own codeword and $x(n-3)$ is the codeword it received from the other source in time slot $n-3$. At the same time, the *sending relay* will transmit a superposition codeword $\beta_2 x(n-1) + \beta_1 x(n)$, where $x(n-1)$ and $x(n-2)$ are codewords it received from the source during the previous two time slots. The other source (i.e., the *listening source*) and the other relay (i.e., the *listening relay*) listen to the transmissions in this time slot. In the next time slot, i.e., time slot $n + 1$, both the two sources and relays will exchange their working roles, where the listening ones become the sending ones and the sending ones become the listening ones. At the sources, different to [5,6], the sending source retransmit the codeword $x(n - 3)$ instead of $x(n - 1)$ in time slot n. This is because we have arranged the sending relay to transmit $x(n - 1)$, so such a codeword should not be transmitted by the sending source again in this time slot to avoid the potential mutual cancellation. In the proposed ARDF process, each codeword can be transmitted by both the sources and relays during four successive time slots, which ensures that the 'full' cooperation can be achieved.

Table 1 The transmission modes for each situation, where $y_i, n_i \in \mathcal{C}^i, i = 4, 5, 2L + 1, 2L + 3; x_j \in \mathcal{C}^j, j = 2, 2L$

Situation	Transmission mode	Equivalent channel matrix
Situation A_0	$\underbrace{\mathbf{y}_{2L+1} = \mathbf{H}_{A_0}\mathbf{x}_{2L} + \mathbf{n}_{2L+1}}_{\text{Mode } 1}$	$\mathbf{H}_{A_0} = \underbrace{\begin{bmatrix} h_{S1} & 0 & 0 & \cdots & 0 \\ \alpha_2 h_{S2} & \alpha_1 h_{S2} & 0 & \cdots & 0 \\ 0 & \alpha_2 h_{S1} & \alpha_1 h_{S1} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \alpha_2 h_{S2} & \alpha_1 h_{S2} \\ 0 & \cdots & 0 & 0 & h_{S1} \end{bmatrix}}_{(2L+1) \times 2L \text{ Matrix}}$
Situation A_1	$\underbrace{\mathbf{y}_4 = \mathbf{H}_{A_1}\mathbf{x}_2 + \mathbf{n}_4}_{\text{Mode } 2}$	$\mathbf{H}_{A_1} = \begin{bmatrix} h_{S1} & 0 \\ \alpha_2 h_{S2} & \alpha_1 h_{S2} \\ g_{1,r_1} & h_{S1} \\ 0 & g_{1,r_1} \end{bmatrix}$
Situation A_2 event $\bar{E}$	$\underbrace{\mathbf{y}_5 = \mathbf{H}_{A_2,\bar{E}}\mathbf{x}_2 + \mathbf{n}_5}_{\text{Mode } 3}$	$\mathbf{H}_{A_2,\bar{E}} = \begin{bmatrix} h_{S1} & 0 \\ \alpha_2 h_{S2} & \alpha_1 h_{S2} \\ g_{1,r_1} & h_{S1} \\ g_{2,r_2} & g_{1,r_1} \\ 0 & g_{2,r_2} \end{bmatrix}$
Situation A_2 event E	$\underbrace{\mathbf{y}_{2L+3} = \mathbf{H}_{A_2,E}\mathbf{x}_{2L} + \mathbf{n}_{2L+3}}_{\text{Mode } 4}$	$\mathbf{H}_{A_2,E} = \mathbf{H}$, refer to Eqution (3).

In the following paragraphs, we will describe the transmission process during the four successive time slots in details.

Assume n to be odd without loss of generality. In time slot n ($1 \leq n \leq 2L$), the sending source S_1 first transmits $x(n)$, while R_1 and S_2 decode $x(n)$ from S_1. In time slot $n + 1$, R_1 is the sending relay and uses its best antenna to retransmit $x(n)$; R_2 receives $x(n)$ from R_1. In time slot $n+2$, R_2 becomes the sending relay and uses its best antenna to repeat $x(n)$. In time slot $n + 3$, the sending source S_2 retransmits $x(n)$ again. Therefore, during such four successive time slots, $x(n)$ can be transmitted four times, by both two sources and relays. Similar to the transmission of $x(n)$, the codeword of S_2, $x(n + 1)$, will be transmitted by S_2, R_2, R_1, and S_1 in time slot $n + 1$, $n + 2$, $n + 3$, and $n + 4$, respectively. When a transmitting node wishes to transmit two codewords simultaneously, it will transmit the superposition of these two codewords and use an amplitude factor to constrain the power of each codeword. Figure 4 illustrates the whole ARDF transmission process, where $2L + 3$ time slots are required for a cooperative data frame.

From the above ARDF transmission process, one can observe that the listening relay may need to decode at most two unknown codewords in a time slot. For instance, during time slot n ($2 \leq n \leq 2L$), in addition to the new source codeword $x(n)$, the listening relay also does not know $x(n - 1)$ transmitted by the sending

relay. This is because it is transmitting during the previous time slot and cannot simultaneously receive the source codeword $x(n - 1)$ due to the half-duplex constraint. After canceling the known codewords, this relay will decode two independent unknown codewords, i.e., $x(n)$ and $x(n - 1)$, from one source and the other relay, respectively. Such a transmission mode mimics a two-user MAC with two single-antenna transmitters and one M_r-antenna receiver. At each relay, simple linear zero-forcing detection approaches can be used to separate the source message and the interference from the other relay, where each relay is equipped with multiple antennas and therefore has the capability to separate the messages under the ZF condition in Equation (2).

3.2 Data model

Note that we have used a fixed power allocation strategy in each time slot, and the use of more advanced adaptive power allocation shall yield better outage performance at finite SNRs. However, for the infinite SNR region, the precise power allocation does not affect the final DMT result [27].

For the ARDF process with respect to situation A_2 event E, the data model can be expressed as $\mathbf{y} = \mathbf{Hx} + \mathbf{n}$, where $\mathbf{x} = [x(1), \cdots, x(2L)]^T$, which is the codeword vector of the two sources transmitted during a data frame; $\mathbf{y} = [y(1), \cdots, y(2L+3)]^T$, which is the observation vector at the destination during a data frame that lasts $2L+3$ time
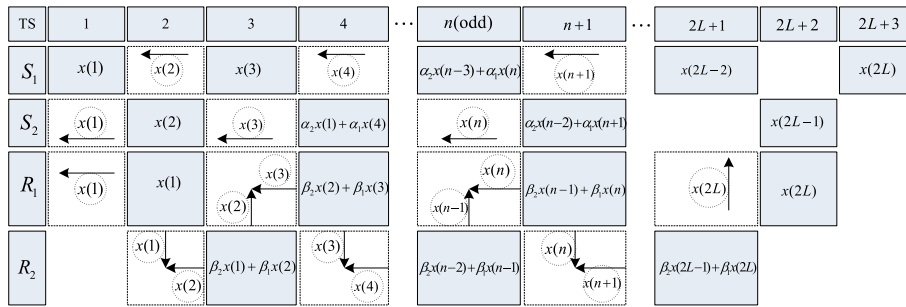
Figure 4 The ARDF process during each time slot (TS). Here n is an odd number and $5 \leq n \leq 2L - 1, L \geq 3$. The solid blocks denote that the sending relays transmit the codewords using the best antenna. Dashed blocks denote that the listening relays receive signals using all the antennas, and each codeword within the dashed circle is an unknown codeword to be decoded. The horizontal and vertical arrows imply that a node receives the corresponding codewords from sending source and the sending relay, respectively.

slots; $\mathbf{n} = [n(1), \cdots, n(2L + 3)]^T$, which is the Gaussian additive noise vector; and the $(2L + 3) \times (2L)$ equivalent channel matrix is as follows:

$$\mathbf{H} = \begin{bmatrix} h_{S1} & 0 & 0 & 0 & 0 & \cdots & 0 \\ g_{1,r_1} & h_{S2} & 0 & 0 & 0 & \cdots & 0 \\ \beta_2 g_{2,r_2} & \beta_1 g_{2,r_2} & h_{S1} & 0 & 0 & \cdots & 0 \\ \alpha_2 h_{S2} & \beta_2 g_{1,r_1} & \beta_1 g_{1,r_1} & \alpha_1 h_{S2} & 0 & \cdots & 0 \\ 0 & \alpha_2 h_{S1} & \beta_2 g_{2,r_2} & \beta_1 g_{2,r_2} & \alpha_1 h_{S1} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \alpha_2 h_{S2} & \beta_2 g_{1,r_1} & \beta_1 g_{1,r_1} & \alpha_1 h_{S2} \\ 0 & \cdots & 0 & 0 & h_{S1} & \beta_2 g_{2,r_2} & \beta_1 g_{2,r_2} \\ 0 & \cdots & 0 & 0 & 0 & h_{S2} & g_{1,r_1} \\ 0 & \cdots & 0 & 0 & 0 & 0 & h_{S1} \end{bmatrix}. \quad (3)$$

4 Diversity-multiplexing trade-off and numerical results

In this section, the analytical performance evaluation of the proposed protocol will be first shown by using DMT. Then, the Monte Carlo simulations will be carried out to demonstrate the outage performance of the proposed cooperative ARDF protocol.

4.1 Achievable diversity-multiplexing trade-off

Firstly, recall that the diversity and multiplexing gain are defined as [7]

$$d \triangleq -\lim_{\rho \to \infty} \frac{\log[P_e(\rho)]}{\log \rho} \quad \text{and} \quad r \triangleq \lim_{\rho \to \infty} \frac{R(\rho)}{\log \rho}, \quad (4)$$

where P_e is the ML detection error probability and R is the target data rate per channel use. Following similar steps in [22], it can be proved that the ML error probability in the MAC scenario can be tightly bounded by the outage probability at high SNR. So the outage probability will be focused in this paper. When $\lim_{\rho \to \infty} \frac{\log f(\rho)}{\log \rho} = d, f(\rho)$ is said to be exponentially equal to ρ^d, denoted as $f(\rho) \doteq \rho^d$ ($\stackrel{.}{\leq}$

and $\stackrel{.}{\geq}$ are similarly defined). The following theorem gives the final DMT performance of the proposed cooperative protocol.

Theorem 1. *When each relay is equipped with at least two antennas, i.e., $M_r \geq 2$, the achievable DMT of the proposed ARDF protocol for the two-user and two-relay CMA scenario can be expressed as*

$$d^*(r) = (2M_r + 2)\left(1 - \frac{2L + 3}{2L}r\right)^+, \quad (5)$$

where $(x)^+$ denotes $\max\{x, 0\}$.

Proof 1. *We have briefly provided the proof of this theorem in [28] without formal derivations. In this journal paper, the details of the proof for this theorem will be blue presented in Appendix 2.*

- *Remark 1:* The optimal MISO DMT for the addressed CMA scenario can be expressed as

$$d_{MISO}(r) = (2M_r + 2)(1 - r), \quad 0 \leq r \leq 1.$$

According to Equation (5), when the frame length L is sufficiently large, the achievable DMT of the proposed scheme yields $\frac{2L+3}{2L} \approx 1$, which demonstrates that the optimal MISO trade-off $d_{MISO}(r)$ can be asymptotically approached by the proposed scheme.

- *Remark 2:* The achievable DMT of the CMA schemes in [4,5] can be written as

$$d_{CMA}(r) = M(1 - r), \quad 0 \leq r \leq 1,$$

where M denotes the number of the sources. When there only exist two active users in a network, $d_{CMA}(r) = 2(1 - r)$. Hence, the schemes in [4,5] cannot perform well for a small number of sources, whereas the proposed ARDF-CMA scheme can

further enhance the reception robustness by exploiting the relay nodes.

- *Remark 3:* On the other hand, the R-CMA protocol in [18] can be straightforwardly extended to the two-user two-relay scenario considered in this paper. It means that each relay apply a simple antenna selection, and the DMT result presented in Equation (7) in [18] can be accordingly updated as
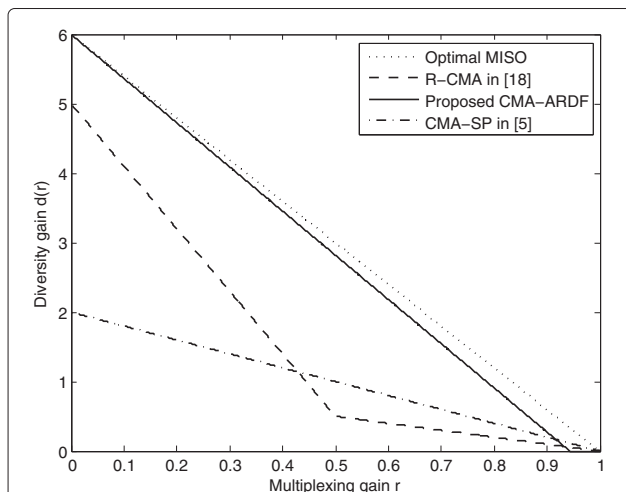
$$d_{R\_CMA}(r) = (1-r) + M_r (2 - 4r)^+, \quad 0 \le r \le 1.$$

Compared to the R-CMA scheme, the proposed scheme achieves one more diversity gain, because it exploits the help of two M_r-antenna relays while preserves the cooperation between two sources. Although the proposed protocol can only achieve a maximal multiplexing gain $\frac{2L}{2L+3}$ while the schemes in [18] can achieve 1, the difference is negligible for a large L.

Figure 5 illustrates the achievable DMTs of various cooperative schemes. As can be seen from this figure, the proposed scheme can approach the optimal MISO upper bound for a sufficiently large L, and hence, outperforms the comparable ones in most multiplexing range.
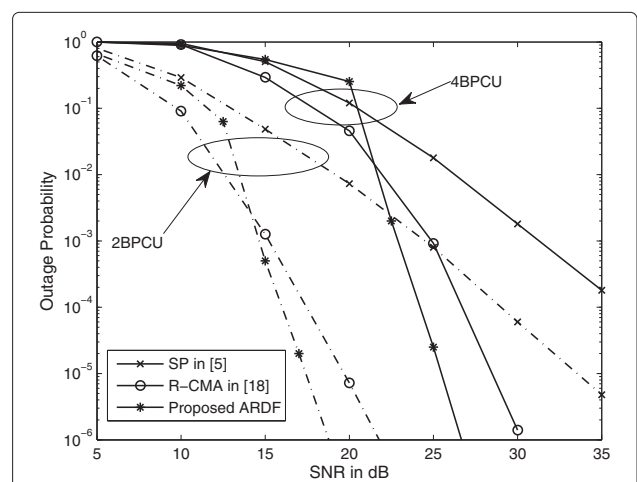
4.2 Numerical results
In this section, some numerical results based on the Monte Carlo simulations are provided for the addressed CMA scenario. The targeted data rate is set as 2 or 4 bits per channel use (BPCU). Each relay is equipped with two antennas, i.e., $M_r = 2$. For the proposed CMA-ARDF protocol, set $L = 25, \alpha_1 = \beta_1 = 0.87$ for simplicity. Note that all the channels are assumed to be Rayleigh fading with unit variance and all the nodes have the same transmit power.

In addition to the superposition-coding-based CMA in [5], the R-CMA scheme in [18] is also used to compare with the proposed ARDF protocol. As shown in Figure 6, ARDF has limited performance at low SNR. This is because the performance of the proposed scheme is much more sensitive to the source-relay/inter-relay channel condition, and the outage event is prone to occur at each relay at low SNR, which results in some degradation to the outage performance. However, the proposed scheme achieves the smallest outage probability among the comparative ones when we increase SNR. This performance gain is due to the fact that the proposed ARDF scheme can exploit the available relays to provide an extra dimension to improve the outage performance, whereas the scheme in [5] only considers the cooperation between the two sources and the R-CMA scheme in [18] does not consider the mutual source cooperation. Therefore, larger diversity gains can be achieved by the proposed transmission protocol, which guarantee the superior performance particularly at high SNR. In Figure 7, different antenna numbers, i.e., M_r, is consider at each relay to demonstrate the outage performance of the proposed scheme. One can observe that increasing the number of relay antennas can further improve the robustness of the transmission scheme, since more diversity gains can be provided.

Figure 5 The DMT performance of different protocols for addressed CMA channel. Here each relay is equipped with two antennas. For the proposed protocol, the number of codewords transmitted by each source in one cooperative data frame is fixed as $L = 25$. The two sources communicate with the destination using TDMA for all the protocols.

Figure 6 The outage probability for the proposed protocol, the superposition cooperative scheme in [5] and non-cooperative transmission. Note that each relay is equipped with two antennas. For the proposed protocol, we set $L = 25$. The power allocation at the sources for both the proposed ARDF scheme and the SP scheme in [5] is fixed as $\alpha_1^2 = 0.87$. Moreover, we set $\beta_1^2 = 0.87$ at the relays.
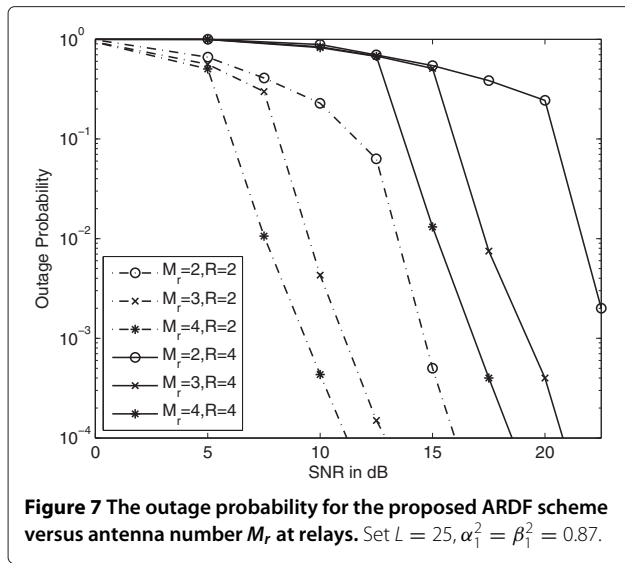
Figure 7 The outage probability for the proposed ARDF scheme versus antenna number M_r at relays. Set $L = 25$, $\alpha_1^2 = \beta_1^2 = 0.87$.

5 Conclusions

In this paper, a new cooperative transmission protocol has been proposed for a CMA scenario in the wireless vehicular networks. Without the use of assumptions of strong inter-relay channel in [13], the relay reuse has still been realized using the linear ZF detection at the half-duplex relays to combat a general inter-relay interference. In addition, to fully exploit the cooperation between all the transmitters, the superposition coding strategy has also been carefully applied at both sources and relays . To evaluate the spectral efficiency, an achievable DMT of the proposed scheme was developed, which demonstrated that the proposed ARDF protocol can outperform the existing related schemes and approximately achieve the optimal MISO upper bound. The derived analytical result and the numerical results have demonstrated that the proposed scheme has a better performance in comparison with the existing related schemes in most conditions. The impact of adaptive power allocation has not been analyzed due to the high SNR considered in this paper, where a promising future direction is to carry out the study for the optimal design of power allocation to improve the overall system throughput at intermediate SNR.

Endnotes

[a]For a particular situation, the data rate should be $\frac{\tilde{R} \times N_x}{N_t}$, where N_x and N_t are the number of transmitted codewords and the number of required time slots, respectively. So the *targeted* data rate R (i.e., the average data rate) can be calculated from $\tilde{R}$. We will provide more discussions about the relationship between the two rates later in this paper.

[b]In this paper, $x(n)$ denotes the new source codeword sent by S_i ($i = 1, 2$) in time slot n ($1 \leq n \leq 2L$). Obviously, the mapping criterion is $x(n) = \begin{cases} s_1(\frac{n+1}{2}), & \text{if } n \text{ is odd} \\ s_2(\frac{n}{2}), & \text{if } n \text{ is even} \end{cases}$, where $s_i(l)$ denotes the l-th ($1 \leq l \leq L$) codeword of S_i in one cooperative frame.

Appendices

Appendix 1

Preliminary results

We first present four lemmas and their proof steps in this section. These results will be useful for the proofs of Theorem 1 in the next section.

Lemma 2. *By assuming quasi-static and flat Rayleigh-fading channels, the probability of each situation A_k with k-qualified relays broadcasting ACK1 signals can be expressed as $P(A_k) \doteq \rho^{-(2-k)M_r(1-\tilde{r})}$, where $\tilde{r} = \lim_{\rho \to \infty} \frac{\tilde{R}(\rho)}{\log \rho}$.*

Lemma 3. *By assuming quasi-static and flat Rayleigh-fading channels and $\alpha_1 = \beta_1 = \sqrt{\frac{1}{2}}$ for simplicity, the probability of the event E that both relays can successfully perform ZF detection and broadcast ACK2 signals, as expressed in Equation (2), can be bounded as*

$$1 - \frac{4\gamma^{M_r - 1}}{(M_r - 1)!} \leq P(E) \leq 1 - \frac{2\gamma^{M_r - 1}}{(M_r - 1)!}, \quad (A.1)$$

where $\gamma = \frac{2^{\tilde{R}} - 1}{\rho \alpha_1^2 \delta^2}$, δ^2 is the variance of each channel coefficient. Furthermore, the probability of the event $\overline{E}$ can be obviously obtained as $P(\overline{E}) \doteq \rho^{-(M_r - 1)(1 - \tilde{r})}$.

Lemma 4. *When X_1 and X_2 are subjected to exponential distribution, the probability can be approximated as $P(X_1 X_2 < a) \approx -a \ln a$, where $a > 0$ and is sufficiently small.*

Lemma 5. *Assuming that $z = \max\{z_1, \cdots, z_{M_r}\}$, where z_i is exponentially distributed with unit variance, the expectation of e^{-cz} ($c > 0$) can be revealed as $\epsilon[e^{-cz}] = \frac{M_r!}{\Pi_{i=1}^{M_r}(c+i)}$, $\epsilon[\cdot]$ denotes the expectation of a random variable.*

The proofs steps of the above lemmas are provided in the following section.

Proof of Lemma 2

Denote B as the event that a relay R_j is not qualified, i.e., it can be expressed as the event: $B \triangleq \bigcup_{i=1}^{2} \{\log(1 + \rho \alpha_i^2 |h_{i,Rj}|^2) < \tilde{R}\}$. The probability of the event B can be easily calculated as $P(B) \doteq \rho^{-M_r(1-\tilde{r})}$.

Thus, the probability of each situation A_k can be expressed as follows:

$$P(A_k) \doteq \binom{2}{k} [1 - P(B)]^k [P(B)]^{2-k} \doteq \rho^{-(2-k)M_r(1-\tilde{r})}. \tag{A.2}$$

Proof of Lemma 3

Consider a signal model in which the r_2-th antenna of R_2 transmits a message $\tilde{s}_1$ with a power lever β_1^2 while the first source S_1 broadcasts a message $\tilde{s}_2$ with a power lever α_1^2. We set $\alpha_1 = \beta_1 = \sqrt{\frac{1}{2}}$ for ease of explanation. Such a signal model can be found in most time slots with odd numbers in situation A_2 event E. At this time, the signal vector received by R_1 can be written as $\mathbf{r}_{R1} = \mathbf{H}_{R1}\tilde{\mathbf{s}} + \mathbf{w}_1$, where $\mathbf{r}_{R1}, \mathbf{w}_1 \in \mathcal{C}^{M_r}$, $\mathbf{H}_{R1} = [\beta_1 \mathbf{h}_{2,r_2} \ \alpha_1 \mathbf{h}_{R1}]$, $\tilde{\mathbf{s}} = [\tilde{s}_1, \tilde{s}_2]^T$, and $\mathbf{w}_1$ is the noise vector. This transmission model can be recognized as a special MIMO system with two transmit, and M_r receive antennas when ZF detection is applied. Denote the event as $E_{R1} \triangleq \left\{ \log\left(1 + \frac{\rho}{[(\mathbf{H}_{R1}^H \mathbf{H}_{R1})^{-1}]_{k,k}}\right) > \tilde{R}, \forall k \in \{1, 2\}\right\}$. Applying ZF detection at the receiver and according to [26], the probability of E_{R1} can be expressed as

$$P(E_{R1}) = 1 - \frac{2\gamma^{M_r - 1}}{(M_r - 1)!}, \tag{A.3}$$

when ρ is sufficiently large, where $\gamma = \frac{2^{\tilde{R}} - 1}{\rho \alpha_1^2 \delta^2}$. Note that the similar ZF decoding can be applied at R_2 when R_1 and S_2 are transmitting messages at the same time, and E_{R2} is similarly defined as $E_{R2} \triangleq \left\{ \log\left(1 + \frac{\rho}{[(\mathbf{H}_{R2}^H \mathbf{H}_{R2})^{-1}]_{k,k}}\right) > \tilde{R}, \forall k \in \{1, 2\}\right\}$, whose probability is the same as $P(E_{R1})$, i.e.,

$$P(E_{R2}) = 1 - \frac{2\gamma^{M_r - 1}}{(M_r - 1)!}. \tag{A.4}$$

Then, the probability of the event E defined in Equation (2) can be revealed as $P(E) = P\left(E_{R1} \bigcap E_{R2}\right)$, where the events E_{R1} and E_{R2} are not strictly independent. Since reciprocal channel is assumed, the channel vectors $\mathbf{h}_{1,r_1}$ and $\mathbf{h}_{2,r_2}$ have one common element which is the channel coefficient between the r_1-th antenna of R_1 and the r_2-th antenna of R_2, so that it is difficult to obtain the accurate value of $P(E)$. However, when we notice that $P(E)$ can be bounded as $P(E_{R1}) + P(E_{R2}) - 1 \leq P(E) \leq P(E_{R1})$, we have

$$P(E_{R1}) - \frac{2\gamma^{M_r - 1}}{(M_r - 1)!} \leq P(E) \leq P(E_{R1}). \tag{A.5}$$

According to Equation (A.3), Lemma 3 can be proved.

Proof of Lemma 4

Assume X_1 and X_2 to be independently exponentially distributed with unit variance without loss of generality. Let $D = \{(x_1, x_2) \in \mathbb{R}^{2+} | x_1 x_2 < a\}$ and $f_{x_1 x_2}(x_1, x_2) = f_{x_1}(x_1) f_{x_2}(x_2)$ which is the joint density of $\{x_1, x_2\}$, where $f_{x_i}(x_i)$ denotes X_i's probability density function (PDF) $f_{x_i} = e^{-x_i}$, $i = 1, 2$. Then, the probability $P(X_1 X_2 < a)$ can be expressed as

$$\begin{aligned} \Phi_a &= P(X_1 X_2 < a) = \iint_D f_{x_1 x_2}(x_1, x_2) \mathrm{d}x_1 \mathrm{d}x_2 \\ &= \int_0^{+\infty} e^{-x}(1 - e^{-a/x}) \mathrm{d}x \\ &= \sum_{i=1}^{\infty} \frac{(-1)^{i+1}}{i!} a^i \int_0^{+\infty} \frac{e^{-x}}{x^i} \mathrm{d}x. \end{aligned} \tag{A.6}$$

Now, the function $\Phi_a(u)$ is first defined as

$$\Phi_a(u) = \sum_{i=1}^{\infty} \frac{(-1)^{i+1}}{i!} a^i B_i(u), \tag{A.7}$$

where $B_i(u) = \int_u^{+\infty} \frac{e^{-x}}{x^i} \mathrm{d}x$. Then, the *improper integral* Φ_a can be calculated as $\Phi_a = \lim_{u \to 0^+} \Phi_a(u)$.

Moreover, from [29] (Equation 3.351.4), $B_i(u)$ can be calculated by the exponential integral function as

$$\begin{aligned} B_i(u) = {}&(-1)^i \frac{\mathrm{Ei}(-u)}{(i-1)!} \\ &+ e^{-u} \sum_{n=0}^{i-2} \frac{(-1)^n u^{-(i-1-n)}}{(i-1)(i-2)\cdots(i-1-n)}, \end{aligned} \tag{A.8}$$

where $i \geq 2$, and exponential integral function can be shown as $\mathrm{Ei}(-u) = \ln(u) + \sum_{k=1}^{\infty} \frac{(-u)^k}{k \cdot k!}$. Now that, $B_i(u)$ can be obtained as

$$\frac{B_i(u)}{e^{-u}} = \begin{cases} -\ln(u) + o(1) & \text{if } i = 1 \\ \frac{(-1)^i \ln(u)}{(i-1)!} + \sum_{n=0}^{i-2} \frac{(-1)^n u^{-(i-1-n)}}{(i-1)(i-2)\cdots(i-1-n)} + o(1) & \text{if } i \geq 2 \end{cases}, \tag{A.9}$$

where $o(1) \to 0$ when $u \to 0^+$. By substituting $B_i(u)$ into Equation (A.7) and rearranging the infinite series, $\Phi_a(u)$ can be rewritten as

$$\Phi_a(u) = e^{-u} \sum_{n=0}^{\infty} \phi_n(u) + o(1), \tag{A.10}$$

where

$$\begin{aligned} \phi_n(u) = {}&\frac{-a^{n+1}}{n!(n+1)!} \ln(u) \\ &+ \sum_{i=n+2}^{\infty} \frac{(-1)^{i+n+1} a^i u^{-(i-1-n)}}{i!(i-1)(i-2)\cdots(i-1-n)}. \end{aligned} \tag{A.11}$$

Let $v = au^{-1}$, $\phi_n(v)$ can be expressed as

$$\phi_n(v) = \frac{-a^{n+1}\ln a}{n!\,(n+1)!} + a^{n+1}F_n(v), \tag{A.12}$$

where

$$F_n(v) = \frac{\ln v}{n!\,(n+1)!} + \sum_{i=n+2}^{\infty} \frac{(-1)^{i+n+1}v^{i-1-n}}{i!\,(i-1)(i-2)\cdots(i-1-n)}. \tag{A.13}$$

Following similar proof steps of Corollary 1 in [15], the limit $\lim_{v\to+\infty} F_n(v)$ can be proved to exist. Hence, by recalling Equation (A.10), Φ_a can be expressed as

$$\Phi_a = \lim_{u\to 0^+} \Phi_a(u) = \sum_{n=0}^{\infty} \left[\frac{-a^{n+1}\ln a}{n!\,(n+1)!} + C_n a^{n+1} \right]. \tag{A.14}$$

For a sufficiently large ρ, Φ_a can be approximately calculated as $\Phi_a \approx -a\ln a$.

Proof of Lemma 5

It is not difficult to obtain the PDF of z as $f_z(z) = M_r e^{-z}(1 - e^{-z})^{M_r-1}$, $z > 0$, since $z = \max\{z_1, \cdots, z_{M_r}\}$ and $f_{z_r}(z_r) = e^{-z_r}$, $r \in \{1, \cdots, M_r\}$, so that

$$\epsilon[e^{-cz}] = \int_0^{\infty} e^{-cz} f_z(z)\mathrm{d}z = M_r \int_0^1 t^c(1-t)^{M_r-1}\mathrm{d}t. \tag{A.15}$$

Using binomial theorem, $\epsilon[e^{-cz}]$ can be written as

$$\epsilon[e^{-cz}] = M_r \sum_{r=0}^{M_r-1} \int_0^1 C_{M_r-1}^r (-1)^r t^{c+r}\mathrm{d}t$$
$$= M_r \sum_{r=0}^{M_r-1} C_{M_r-1}^r \frac{(-1)^r}{c+r+1} \tag{A.16}$$

where $C_p^q = \frac{p!}{(p-q)!\,q!}$, p and q are positive integers, and $p \geq q$. Splitting each term on the right side, Equation (A.16) can be calculated as

$$\frac{\epsilon[e^{-cz}]}{M_r} = \left(\frac{1}{c+1} - \frac{1}{c+2} \right) + (-1)\cdot\underbrace{(M_r-2)}_{C_{M_r-2}^1}$$
$$\times\left(\frac{1}{c+2} - \frac{1}{c+3} \right) + \cdots + (-1)^r\underbrace{\left(C_{M_r-1}^r - C_{M_r-2}^{r-1}\right)}_{C_{M_r-2}^r}$$
$$\times\left(\frac{1}{c+r+1} - \frac{1}{c+r+2} \right) + \cdots + (-1)^{M_r-2}$$
$$\times\left(\frac{1}{c+M_r-1} - \frac{1}{c+M_r} \right)$$
$$= \sum_{r=0}^{M_r-2} C_{M_r-2}^r \frac{(-1)^r}{(c+r+1)(c+r+2)} \tag{A.17}$$

Furthermore, by iteratively repeating the similar process in Equation (A.17), the expectation of e^{-cz} can be finally obtained as $\epsilon[e^{-cz}] = \frac{M_r!}{\prod_{i=1}^{M_r}(c+i)}$.

Appendix 2
Proof of theorem 1

As shown in [7,22], the ML error probability can be tightly bounded by the outage probability at high SNR, so the outage probability will be analyzed in this section. According to [22] and the protocol description in Section 2, we can define the outage event of the proposed ARDF protocol as $\mathcal{O} \triangleq \bigcup_{k=0}^{2} \mathcal{O}_{A_k}$, where $\mathcal{O}_{A_2} \triangleq \mathcal{O}_{A_2,\overline{E}}\bigcup\mathcal{O}_{A_2,E}$. Here, $\mathcal{O}_{A_k}$ denotes the outage event in the situation A_k at the destination, $\mathcal{O}_{A_2,\overline{E}}$ and $\mathcal{O}_{A_2,E}$ are similarly defined. Thus, the overall outage probability of the proposed protocol can be expressed as

$$P(\mathcal{O}) = \sum_{k=0}^{1} P(\mathcal{O}_{A_k})P(A_k) + P(A_2,\overline{E})P(\mathcal{O}_{A_2,\overline{E}})$$
$$+ P(A_2, E)P(\mathcal{O}_{A_2,E})$$
$$\leq \sum_{k=0}^{1} P(\mathcal{O}_{A_k})P(A_k) + P(\overline{E})P(\mathcal{O}_{A_2,\overline{E}}) + P(E)P(\mathcal{O}_{A_2,E}). \tag{B.1}$$

The probabilities of each situation A_k and the event E have been presented in Lemma 2 and Lemma 3, respectively. Moveover, the outage probability in each situation will be analyzed in the terms of the MAC capacity region. Based on the definition in Equation (1), the source data rate constrains for the proposed-ARDF protocol can be further calculated as follows:

$$|\mathcal{S}|Q\tilde{R} < \log\left(\det\left(\mathbf{I} + \rho \sum_{l\in\mathcal{L}} \mathbf{h}_l\mathbf{h}_l^H \right) \right), \forall \mathcal{S} \subseteq \{1,2\} \tag{B.2}$$

where $|\mathcal{S}|$ denotes number of users in $\mathcal{S}$, Q denotes the number of codewords transmitted by each source in one cooperative frame, and $\mathbf{h}_l$ is a channel vector, both the structure of $\mathbf{h}_l$ and the set $\mathcal{L}$ are a function of $\mathcal{S}$ and the details of their relationship to $\mathcal{S}$ will be discussed in the next few subsections. The outage events occur when any constraint in Equation (B.2) is not met, and the highest outage probability achieved by each constraint is the dominant factor [22]. In the following subsections, different values of each parameter in Equation (B.2) will be considered for different situations.

Situation A_0

$\mathcal{S} \subseteq \{1,2\}$, $Q = L$, and $\mathbf{h}_l$ denotes the l-th column vector of $\mathbf{H}_{A_0}$ in Table 1. When $|\mathcal{S}| = 1$, $\mathcal{S}$ is assumed to be $\{1\}$ without loss of generality, so $\mathcal{L} = \{1, 3, \cdots, 2L-1\}$, and

the outage probability at the destination in such a case can be calculated as

$$P_1(\mathcal{O}_{A_0}) = P\left\{\left[1+\rho(X_1+\alpha_2^2 X_2)\right]\left[1+\rho(\alpha_1^2 X_1+\alpha_2^2 X_2)\right]^{L-1}\right.$$

$$\left. < 2^{L\tilde{R}}\right\} \overset{(a)}{\leq} P\left\{\left[1+\rho(\alpha_1^2 X_1+\alpha_2^2 X_2)\right]^L < 2^{L\tilde{R}}\right\}$$

$$\leq P\left\{\alpha_1^2 X_1 < \frac{2^{\tilde{R}}-1}{\rho}, \alpha_2^2 X_2 < \frac{2^{\tilde{R}}-1}{\rho}\right\}$$

$$\overset{(b)}{\leq} P\left\{X_1 < \frac{2^{\tilde{R}}-1}{\alpha_1^2 \rho}\right\} P\left\{X_2 < \frac{2^{\tilde{R}}-1}{\alpha_2^2 \rho}\right\}$$

$$\doteq \rho^{-2(1-\tilde{r})}, \tag{B.3}$$

where $X_i = |h_{Si}|^2$, $i = 1, 2$; (a) holds since $0 < \alpha_1^2 < 1$, and (b) holds since X_1 is independent of X_2. Otherwise, $\mathcal{S} = \{1, 2\}$ and $\mathcal{L} = \{1, 2, \cdots, 2L\}$,

$$P_2(\mathcal{O}_{A_0}) = P\left\{\log\left(\det(\mathbf{I}_{2L+1} + \rho \mathbf{H}_{A_0}\mathbf{H}_{A_0}^H)\right) < 2L\tilde{R}\right\}. \tag{B.4}$$

In order to make the analysis more tractable, a $(2L + 1) \times (2L + 1)$ square matrix $\widetilde{\mathbf{H}}_{A_0}$ is first defined as $\widetilde{\mathbf{H}}_{A_0} \triangleq [\mathbf{H}_{A_0}, \mathbf{0}]$, where the zero column vector is $(2L+1)$-dimensional. It is easy to show that $\mathbf{H}_{A_0}\mathbf{H}_{A_0}^H = \widetilde{\mathbf{H}}_{A_0}\widetilde{\mathbf{H}}_{A_0}^H$ and $[\mathbf{I}_{2L+1} + \rho \widetilde{\mathbf{H}}_{A_0}\widetilde{\mathbf{H}}_{A_0}^H]$ is a tridiagonal matrix. According to [30], the determinant of the tridiagonal matrix can be shown iteratively as

$$D_n = [1 + \rho(x_n + y_{n-1})] D_{n-1} - \rho^2 x_n y_{n-1} D_{n-2},$$

where $D_n = \det[\mathbf{I}_{2L+1} + \rho \widetilde{\mathbf{H}}_n \widetilde{\mathbf{H}}_n^H]$, and $\widetilde{\mathbf{H}}_n$ denotes the $n \times n$ top-left submatrix from $\widetilde{\mathbf{H}}_{A_0}$, x_n and y_n are the n-th element on the principle diagonal and subdiagonal of $\widetilde{\mathbf{H}}_{A_0}$, respectively. By using such a property and note that $x_{2L+1} = 0$, the following inequality can be obtained

$$D_{2L+1} \geq (1 + \rho X_1)(1 + \rho\alpha_1 X_1)^{L-1}(1 + \rho\alpha_1 X_2)^L$$

$$+ \alpha_2^{2L-1}\left(\rho^2 X_1 X_2\right)^L. \tag{B.5}$$

From Equation (B.5) and Lemma 4, $P_2(\mathcal{O}_{A_0})$ can be upper bounded as

$$P_2(\mathcal{O}_{A_0}) \leq P\left\{(\rho^2 \alpha_2^2 X_1 X_2)^L < 2^{2L\tilde{R}}\right\}$$

$$\approx -\frac{2^{2\tilde{R}}}{\alpha_2^2 \rho^2} \ln\left(\frac{2^{2\tilde{R}}}{\alpha_2^2 \rho^2}\right)$$

$$\doteq \rho^{-2(1-\tilde{r})}. \tag{B.6}$$

Thus, $P(\mathcal{O}_{A_0}) \overset{.}{\leq} \rho^{-2(1-\tilde{r})}$ can be easily obtained by combining $P_1(\mathcal{O}_{A_0})$ and $P_2(\mathcal{O}_{A_0})$.

Situation A_1

$\mathcal{L} = \mathcal{S} \subseteq \{1, 2\}$, $Q = 1$, $\mathbf{h}_l$ denotes the l-th column vector of $\mathbf{H}_{A_1}$ in Table 1. When $|\mathcal{S}| = 1$, assume that $\mathcal{S} = \{1\}$, $P_1(\mathcal{O}_{A_1})$ can be easily obtained as

$$P_1(\mathcal{O}_{A_1}) = P\left\{X_1 + \alpha_2^2 X_2 + G_1 < \frac{2^{\tilde{R}}-1}{\rho}\right\}$$

$$\overset{.}{\leq} \rho^{-2(1-\tilde{r})} \prod_{m=1}^{M_r} P\left\{|g_{1,m}|^2 < \frac{2^{\tilde{R}}-1}{\rho}\right\}$$

$$\doteq \rho^{-(M_r+2)(1-\tilde{r})}, \tag{B.7}$$

where $G_j = |g_{j,r_j}|^2 \triangleq \max\{|g_{j,1}|^2, \cdots, |g_{j,M_r}|^2\}, j = 1, 2$. Otherwise, $\mathcal{S} = \{1, 2\}$, $P_2(\mathcal{O}_{A_1})$ is

$$P_2(\mathcal{O}_{A_1}) = P\left\{\log\left(\det(\mathbf{I}_2 + \rho \mathbf{H}_{A_1}^H \mathbf{H}_{A_1})\right) < 2\tilde{R}\right\}. \tag{B.8}$$

The determinant of $[\mathbf{I}_2 + \rho \mathbf{H}_{A_1}^H \mathbf{H}_{A_1}]$ can be first calculated as

$$\det\left(\mathbf{I}_2 + \rho \mathbf{H}_{A_1}^H \mathbf{H}_{A_1}\right) = \prod_{i=1}^{2}\left[1 + \rho(X_1 + G_1 + \alpha_i^2 X_2)\right]$$

$$- \rho^2 \left|\alpha_1\alpha_2 X_2 + g_{M1}^* h_{S1}\right|^2, \tag{B.9}$$

where

$$|\alpha_1\alpha_2 X_2 + g_{M1}^* h_{S1}|^2 \leq (\alpha_1\alpha_2 X_2)^2 + G_1 X_1 + 2\alpha_1\alpha_2 X_2\sqrt{G_1 X_1}$$

$$\leq (\alpha_1\alpha_2 X_2)^2 + G_1 X_1 + \alpha_1\alpha_2 X_2(X_1 + G_1). \tag{B.10}$$

So that $\det\left(\mathbf{I}_2 + \rho \mathbf{H}_{A_1}^H \mathbf{H}_{A_1}\right) > \rho^2\left[G_1^2 + (1-\alpha_1\alpha_2)X_1 X_2\right]$ can be obtained, where $0 < \alpha_1\alpha_2 \leq \frac{1}{2}$, and $P_2(\mathcal{O}_{A_1})$ can be upper bounded as

$$P_2(\mathcal{O}_{A_1}) \leq P\left\{G_1^2 + (1-\alpha_1\alpha_2)X_1 X_2 < \frac{2^{2\tilde{R}}}{\rho^2}\right\}$$

$$\leq P\left\{X_1 X_2 < \frac{2^{2\tilde{R}}}{(1-\alpha_1\alpha_2)\rho^2}\right\} P\left\{G_1 < \frac{2^{\tilde{R}}}{\rho}\right\}$$

$$\doteq \rho^{-(M_r+2)(1-\tilde{r})}, \tag{B.11}$$

where the last relationship is based on Lemma 4. Hence, $P(\mathcal{O}_{A_1}) \overset{.}{\leq} \rho^{-(M_r+2)(1-\tilde{r})}$ can be obtained.

Situation A_2 event $\bar{E}$

$\mathcal{L} = \mathcal{S} \subseteq \{1, 2\}$, $Q = 1$, $\mathbf{h}_l$ denotes the l-th column vector of $\mathbf{H}_{A_2,\bar{E}}$ in Table 1. When $|\mathcal{S}| = 1$, $P_1(\mathcal{O}_{A_2,\bar{E}}) \doteq \rho^{-(2M_r+2)(1-\tilde{r})}$ can be easily obtained by following the

similar steps in Equation (B.7). Otherwise, $\mathcal{S} = \{1, 2\}$, $P_2(\mathcal{O}_{A_2,\overline{E}})$ can be written as

$$P_2(\mathcal{O}_{A_2,\overline{E}}) = P\left\{\log\left(\det(\mathbf{I}_2 + \rho\mathbf{H}_{A_2,\overline{E}}^H\mathbf{H}_{A_2,\overline{E}})\right) < 2\tilde{R}\right\}.$$
(B.12)

In the above equation, $\det\left(\mathbf{I}_2 + \rho\mathbf{H}_{A_2,\overline{E}}^H\mathbf{H}_{A_2,\overline{E}}\right) = K_1 - K_2$, where

$$K_1 = \prod_{i=1}^{2}[1 + \rho(X_1 + G_1 + G_2 + \alpha_i^2 X_2)]$$
(B.13)

$$\geq \rho^2\left[(X_1+G_2+G_1)^2 + X_2(X_1+G_2+G_1)+\alpha_1\alpha_2 X_2^2\right],$$

$$K_2 = \rho^2\left|\alpha_1\alpha_2 X_2 + g_{M1}^* h_{S1} + g_{M2}^* g_{M1}\right|^2$$
(B.14)

$$\leq \rho^2\left[(\alpha_1\alpha_2 X_2) + \sqrt{G_1}(\sqrt{X_1} + \sqrt{G_2})\right]^2$$

$$\leq \rho^2\left[(\alpha_1\alpha_2 X_2)^2 + \alpha_1\alpha_2 X_2(X_1 + G_2 + 2G_1)\right.$$

$$\left. + 2G_1(X_1 + G_2)\right],$$

so that $\det\left(\mathbf{I}_2 + \rho\mathbf{H}_{A_2,\overline{E}}^H\mathbf{H}_{A_2,\overline{E}}\right) > \rho^2\left[G_1^2 + G_2^2 + (1 - \alpha_1\alpha_2)X_1 X_2\right]$ can be obtained. By using Lemma 4 and following the similar steps in Equation (B.11), $P_2(\mathcal{O}_{A_2,\overline{E}})$ can be upper bounded as

$$P_2(\mathcal{O}_{A_2,\overline{E}}) \leq P\left\{G_1^2 + G_2^2 + (1 - \alpha_1\alpha_2)X_1 X_2 < \frac{2^{2\tilde{R}}}{\rho^2}\right\}$$

$$\doteq \rho^{-(2M_r+2)(1-\tilde{r})}.$$
(B.15)

Therefore, it can be shown that $P(\mathcal{O}_{A_2,\overline{E}})\dot{\leq}$ $\rho^{-(2M_r+2)(1-\tilde{r})}$.

Situation A_2 event E

The outage probability in situation A_2 event E is difficult to be obtained using the above method, but the upper bound can be calculated by following the similar analysis in [17]. Firstly, model 4 in Table 1 is assumed to be a symmetric $2L$-user multiple-access system where the codeword $x(n)$ in $\mathbf{x}_{2L}$ is transmitted by S_n, and such an assumption will make the analysis tractable. For the two-user case considered in this paper, the performance would not be worse than the performance of the former one. So that $\mathcal{L} = \mathcal{S} \subseteq \{1, 2, \cdots, 2L\}$, $Q = 1$, $\mathbf{h}_l$ denotes the l-th column vector of $\mathbf{H}$ in Equation (3), and there are $(2^{2L}-1)$ source data constrains in Equation (B.2). For each constraint, there exists a probability that the channel condition cannot satisfy it, and the highest outage probability is the dominant factor and achieve the system's DMT.

In order to calculate the outage probability of each constraint, a $(m + 3) \times m$ MIMO channel is first considered as $\mathbf{y}_m = \mathbf{F}_m\mathbf{s}_m + \mathbf{w}_m$, where $\mathbf{s}_m \in \mathcal{C}^m, \mathbf{y}_m, \mathbf{w}_m \in \mathcal{C}^{m+3}$ and $\mathbf{F}_m$ is the $(m + 3) \times m$ top-left submatrix from $\mathbf{H}$

in Equation (3). According to [17], the outage probability achieved by this system for every $1 \leq m \leq 2L$ is the same as the highest outage probability for each constraint in Equation (B.2). When $m = 1$, $P_1(\mathcal{O}_{A_2,E}) \doteq \rho^{-(2M_r+2)(1-\tilde{r})}$ can be easily obtained, following the similar steps in Equation (B.7).

When $m > 1$, following the similar DMT analysis for the inter-symbol interference (ISI) channel in [31] and the proof steps in [17], the average error probability can be upper bounded by

$$P_e \leq 2(m + 3) \cdot \epsilon\left[\exp(-c_1\overline{\lambda}\rho^{1-\tilde{r}}|\mathbf{f}|^2)\right],$$
(B.16)

where c_1 is a constant, $\mathbf{f} = [h_{S1}, h_{S2}, g_{1,r_1}, g_{2,r_2}]^T$, $\epsilon[\cdot]$ denotes the expectation of a random variable, and $\overline{\lambda} = \inf_{\mathbf{f}\in\mathcal{C}^4}\lambda_{\min}(\frac{\mathbf{F}_m}{|\mathbf{f}|}) > 0$, $\lambda_{\min}(\cdot)$ denotes the minimum singular value of a matrix. By using Lemma 5, P_e can be upper bounded as

$$P_e \leq 2(m + 3)\left(\frac{1}{c_1\overline{\lambda}\rho^{1-\tilde{r}} + 1}\right)^2\left[\frac{M_r!}{\Pi_{i=1}^{M_r}(c_1\overline{\lambda}\rho^{1-\tilde{r}} + i)}\right]^2$$

$$\doteq \rho^{-(2M_r+2)(1-\tilde{r})}.$$
(B.17)

By observing the fact that $P(\mathcal{O}_{A_2,E})\dot{\leq}P_e$ [7], the outage probability of model 4 in situation A_2 event E can be upper bounded as $P(\mathcal{O}_{A_2,E})\dot{\leq}\rho^{-(2M_r+2)(1-\tilde{r})}$.

Now that recalling Equation (B.1), Lemma 2, and Lemma 3 and considering all the situations, the overall outage probability can be written as

$$P(\mathcal{O})\dot{\leq}\rho^{-\min\{(2M_r+2)(1-\tilde{r}),(3M_r+1)(1-\tilde{r})\}} \doteq \rho^{-(2M_r+2)(1-\tilde{r})}.$$
(B.18)

Depending on the variable-rate strategy in Equation (34) of [8] and integrating the four transmission modes, the target transmission data rate R BPCU can be expressed as

$$R = P(A_0)\frac{(2L)\tilde{R}}{2L+1} + P(A_1)\frac{\tilde{R}}{2} + P(A_2, \overline{E})\frac{2\tilde{R}}{5} + P(A_2, E)\frac{(2L)\tilde{R}}{2L+3}.$$
(B.19)

One can also refer to [8] to get the mapping criterion from R to $\tilde{R}$. It is not difficult to prove the inequality $P(\overline{E}) + P(A_2) - 1 \leq P(A_2, \overline{E}) \leq P(\overline{E})$, so $P(A_2, \overline{E}) \doteq P(\overline{E})$ can be obtained in the large-SNR region. From Equation (A.2) and Lemma 3 and substituting $R = r\log\rho$, $\tilde{R} = \tilde{r}\log\rho$, $P(A_2, E) = P(A_2) - P(A_2, \overline{E})$ into Equation (B.19), r can be revealed as

$$r \doteq \tilde{r}\left\{\frac{2L}{2L+1}P_1^2 + P_1(1 - P_1) + \frac{2}{5}P_2\right.$$

$$\left. + \frac{2L}{2L+3}[(1 - P_1)^2 - P_2]\right\},$$
(B.20)

where $P_1 = \rho^{-M_r(1-\tilde{r})}, P_2 = \rho^{-(M_r-1)(1-\tilde{r})}$. Following the analysis in Claim 3 of [8] and the similar steps in

[14], $d(r) = (2M_r + 2)\left(1 - \frac{2L+3}{2L}r\right)^+$ can be proved by substituting $\tilde{r} = \frac{2L+3}{2L}r$ into Equation (B.18).

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