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The type IIB string axiverse and its low-energy phenomenology

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ABSTRACT: We study closed string axions in type IIB orientifold compactifications. We show that for natural values of the background fluxes the moduli stabilisation mechanism of the LARGE Volume Scenario (LVS) gives rise to an axiverse characterised by the presence of a QCD axion plus many light axion-like particles whose masses are logarithmically hierarchical. We study the phenomenological features of the LVS axiverse, deriving the masses of the axions and their couplings to matter and gauge fields. We also determine when closed string axions can solve the strong CP problem, and analyse the first explicit examples of semi-realistic models with stable moduli and a QCD axion candidate which is not eaten by an anomalous Abelian gauge boson. We discuss the impact of the choice of inflationary scenario on the LVS axiverse, and summarise the astrophysical, cosmological and experimental constraints upon it. Moreover, we show how models can be constructed with additional light axion-like particles that could explain some intriguing astrophysical anomalies, and could be searched for in the next generation of axion helioscopes and light-shining-through-a-wall experiments.

KEYWORDS: Strings and branes phenomenology

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1 Introduction

The QCD axion ‘ a ’ is predicted [1, 2] in the most plausible explanation of the strong CP problem, that is the non-observation of a θ -angle term. To accomplish this, it must act as a field-dependent θ -angle term, i.e. it must have a coupling to the topological charge density in QCD:

$$\mathcal{L} \supset \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{g_3^2}{32\pi^2} \frac{a C_{a3}}{f_a} F_{3,\mu\nu}^b \tilde{F}_3^{b,\mu\nu}, \quad (1.1)$$

where $F_{3,\mu\nu}^b$ is the b th component of the gluon field strength tensor, $\tilde{F}_3^{b,\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{3,\rho\sigma}^b$ its dual, g_3 is the strong coupling, and f_a is the axion decay constant. We take conventions such that the axion is periodic under $a \rightarrow a + 2\pi f_a$; C_{a3} is then a constant describing the coupling of the axion to QCD, which for a field theory model is equal to the anomaly of the Peccei-Quinn field under QCD. It is usually absorbed into the definition of f_a but it shall be more convenient for string models to retain it.

Non-perturbative QCD effects at low energies then generate a potential for the QCD axion plus the θ angle, stabilising the physical value at zero, thereby solving the strong CP problem [3], and giving the axion a parametrically small mass $m_a \sim m_\pi f_\pi / f_a$, where m_π and f_π are the pion mass and decay constant, respectively.

There are beam dump, astrophysical and experimental constraints on the decay constant of the QCD axion and axion-like particles (ALPs) a_i via their couplings to photons and electrons:

$$\mathcal{L} \supset -\frac{e^2}{32\pi^2} \frac{C_{i\gamma}}{f_{a_i}} a_i F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{C_{ie}}{2f_{a_i}} \bar{e} \gamma^\mu \gamma_5 e \partial_\mu a_i, \quad (1.2)$$

that are summarised in appendix B. There are also searches underway in light-shining-through-a-wall (LSW) experiments [4], in searches for solar axions [5] and in direct halo-scope searches for axion cold dark matter (CDM) [6]. Direct searches for the QCD axion in beam dumps and the non-observation of its effect on the cooling of stars constrain the QCD axion decay constant to be $f_a/C_{a3} \gtrsim 10^9$ GeV. Furthermore, there are also several

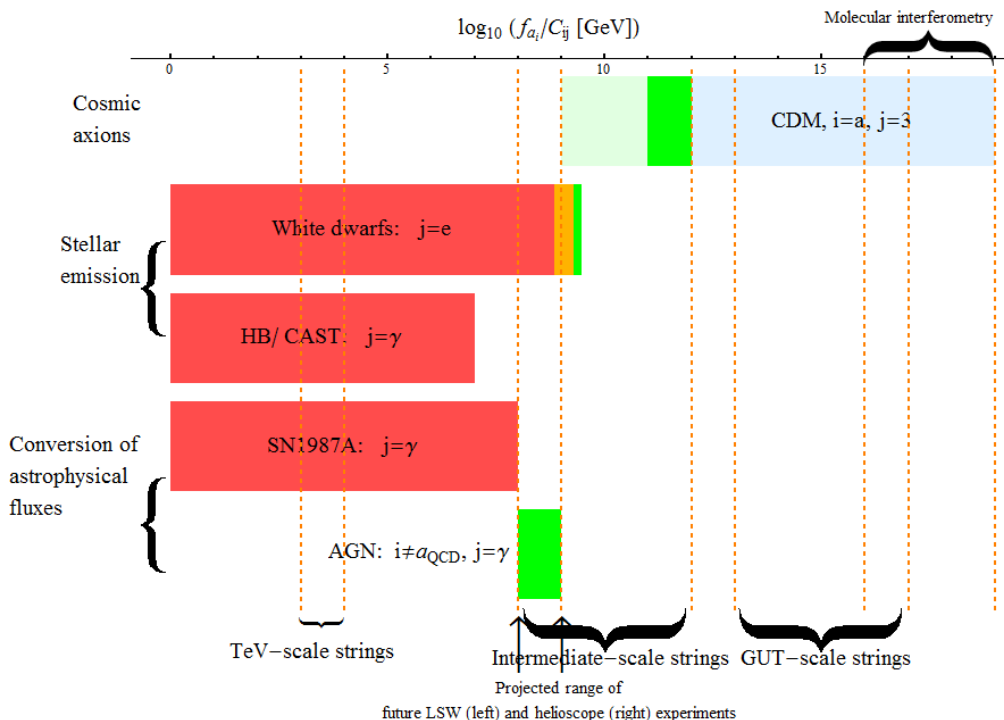


Figure 1. A summary of constraints on and hints for the couplings f_{a_i}/C_{ij} of axion-like particles a_i to fields j . The green regions from top to bottom correspond respectively to the classic ‘axion dark matter window’, hints of an axion from white dwarf cooling and transparency of the Universe to very high energy gamma rays. Red regions are excluded, and the orange region would be excluded by red giants but is compatible with the hints from white dwarfs. The blue region would be excluded by dark matter overproduction in the absence of a dilution mechanism or tuning of the misalignment angle. Legend: ‘Molecular interferometry’ refers to reach of future experiments discussed in [13], see appendix B.2; ‘CDM’ is constraints on the QCD axion coupling from dark matter overproduction in the standard cosmology, see section B.2; constraints from ‘White dwarfs’ are discussed in section B.1.1; ‘HB/CAST’ are constraints from horizontal branch stars and the CAST experiment respectively, see section B.1.1; ‘SN1987A’ are the bounds from observation of high energy gamma rays from that supernova, see section B.1.2; AGN are the hints from observation of gamma rays from active galactic nuclei, also in section B.1.2.

tantalising astrophysical hints, which together point to a light ALP with couplings and mass [7–12]:

$$\frac{f_{a_i}}{C_{ie}} \simeq (0.7 \div 2.6) \times 10^9 \text{ GeV}, \quad \frac{f_{a_i}}{C_{i\gamma}} \sim 10^8 \text{ GeV}, \quad m_{a_i} \lesssim 10^{-9} \div 10^{-10} \text{ eV},$$

hinting at $C_{i\gamma}/C_{ie} \sim 10$, which we shall review in appendix B. A summary of constraints and hints is given in figure 1.

As a result of these constraints and hints, the QCD axion is necessarily associated with a very high energy scale, and so it is natural to search for it in ultra-violet completions of the Standard Model (SM) such as string theory. Indeed, it has long been known that the low-energy effective field theory (EFT) of string compactifications predicts natural candidates

for the QCD axion [14–18], often even an ‘axiverse’ [19] containing many additional light ALPs whose masses are logarithmically hierarchical.

Despite all of these promising considerations, it has been very hard to construct explicit string theoretic examples with a successful QCD axion candidate, let alone additional light ALPs. In order to understand the origins of these difficulties, we focus on type IIB flux compactifications since this is the theory where moduli stabilisation is best understood. Let us summarise the main features and problems encountered in the study of closed string axions from type IIB compactifications:

1. The low-energy spectrum below the compactification scale generically contains many axion-like particles which are the Kaluza-Klein zero modes of antisymmetric form fields belonging to the massless spectrum of the bosonic type IIB string. Their number is related to the topology of the internal manifold, namely to the number of harmonic forms which can be defined on a Calabi-Yau. Given that this number is generically of the order of hundreds, we expect the low-energy theory to be populated by many axion-like particles. These are all closed string axions which live in the bulk. On top of them there can also be open string axions living on a brane but their presence is more model-dependent, and so we shall focus just on closed string axions.
2. Type IIB string theory compactified on a Calabi-Yau three-fold gives rise to an $\mathcal{N} = 2$ 4D EFT which is not chiral. An $\mathcal{N} = 1$ EFT can be obtained via an orientifold projection which projects out many degrees of freedom of the low-energy theory. The kind of axions which are projected out depends on the particular orientifold projection considered, but certainly several axions are removed from the low-energy spectrum.
3. Each axion comes with a real scalar field, the corresponding ‘saxion’, forming a complex field which is the lowest component of an $\mathcal{N} = 1$ chiral superfield. The real part of this field is the saxion whereas the axion is the imaginary part. The saxions are moduli which parameterise the size of the extra dimensions and their VEV determines crucial features of the EFT like the gauge couplings and the mass spectrum. It is then of primary importance to develop a potential for these moduli also because, if massless, they would mediate long-range unobserved fifth-forces. Moreover the moduli may suffer from the famous cosmological moduli problem (CMP) either decaying after Big-Bang Nucleosynthesis (BBN) has begun or, if stable, overclosing the Universe. Typically this is solved if they have masses larger than $\mathcal{O}(10)$ TeV.
4. The mechanism which stabilises the moduli, giving them a mass of the order $m_{\text{mod}} \gtrsim 10$ TeV, might also develop a potential for the axions rendering them too heavy. In fact, the axions enjoy a shift symmetry which forbids the presence of axion-dependent operators but only at the perturbative level since this symmetry is broken by non-perturbative effects. Therefore if the saxions are fixed perturbatively, the axions do not develop any potential, whereas if the moduli are fixed by non-perturbative effects, the axions might obtain a mass of the same order of magnitude. This is indeed the case for KKL'T [20] scenarios where the Kähler moduli are fixed by assuming that the

superpotential has non-perturbative contributions for each Kähler modulus. All the corresponding axions would then have the same mass as the Kähler moduli, which is generically of the order the gravitino mass, rendering all of them too heavy.

5. In type IIB string compactifications, the visible sector (the MSSM or GUT-like generalisations thereof) is built via space-time filling D7-branes wrapping internal four-cycles whose volumes are measured by the saxions (the so-called Kähler moduli). Each stack of N D7-branes supports a $U(N) = SU(N) \times U(1)$ field theory where the $U(1)$ factor is generically anomalous. This anomaly is cancelled via the Green-Schwarz mechanism which induces a mass term for the $U(1)$ gauge bosons. The gauge boson masses generated in this way may be as small as an meV [21] for a low string scale, but will often be much heavier than observable physics. This is a standard Stückelberg mass term since the Abelian gauge boson is made massive by eating the axion which is the imaginary part of the Kähler modulus parameterising the size of the wrapped four-cycle. Thus all the axions which are eaten disappear from the low-energy EFT.
6. Even if some axions are not projected out by the orientifold projection, do not become too heavy due to non-perturbative moduli stabilisation and are not eaten by anomalous $U(1)$ s, it is very hard to obtain axion decay constants $f_a \lesssim 10^{12}$ GeV, where this bound comes from the requirement in standard thermal cosmology of avoiding the overclosure of the Universe by axion CDM produced by the misalignment mechanism (see appendix B.2 for a brief review). In fact, in many string compactifications the QCD axion and other ALPs generically have a larger decay constant of the order the GUT scale $M_{\text{GUT}} \sim 10^{16}$ GeV.¹ This is not necessarily a phenomenological disaster if the axions are diluted by the late out-of-equilibrium decay of some heavy moduli, or have their initial misalignment angles tuned small. Nonetheless only ALPs with an intermediate scale decay constant allow a possible observation of axion related effects in astrophysics, cosmology and even in the laboratory.

At present, the best realisation of the axiverse from type IIB string compactifications is based on the generation of a single non-perturbative correction to the superpotential which fixes all the $h^{1,1}$ Kähler moduli plus one axion combination, leaving all the other $h^{1,1} - 1$ axions massless at leading order [22]. These axions then develop a potential at subleading order via higher order instanton effects. In order to fix the moduli within the regime of validity of the EFT, the tree-level superpotential W_0 has to be fine-tuned very small, $W_0 \ll 1$, of the order the leading non-perturbative correction. This effect can be generated by a Euclidean D3-brane instanton or gaugino condensation on D7-branes wrapping an ‘ample divisor’ which is given by a combination of all the Kähler moduli with positive coefficients. A microscopic realisation of this interesting scenario based on a Calabi-Yau example with explicit brane set-up and fluxes has not been found yet due to the difficulty

¹This is true not just for type IIB strings but also for heterotic orbifolds [18] or M-theory compactifications [22].

to find a rigid ample divisor and to avoid chiral intersections between the instanton and the visible sector.

In this paper we shall present a different realisation of the axiverse from type IIB strings which is valid in the natural regime $W_0 \sim \mathcal{O}(1)$. It is based on a moduli stabilisation mechanism, the so-called LARGE Volume Scenario (LVS), which exploits both perturbative and non-perturbative effects and is characterised by an exponentially large volume of the extra dimensions [23, 24].

Explicit LVS examples of globally consistent Calabi-Yau constructions with magnetised D-branes and chirality have been recently derived in [25]. The same paper presents a general discussion of various tensions between brane fluxes and moduli stabilisation (chirality *vs* non-vanishing non-perturbative effects, cancellation of Freed-Witten anomalies *vs* generation of more than one non-perturbative contribution, D-term induced shrinking of the cycles supporting the visible sector) showing how the LARGE Volume Scenario provides a very attractive solution to these problems. Then ref. [25] outlined a general strategy to fix the moduli within the LVS. We shall now show how this moduli fixing mechanism gives rise to an axiverse:

- In the LVS, usually all the $h_{1,1}^-$ moduli are projected out by the orientifold involution so that $h_{1,1}^+ = h_{1,1}$.
- The only condition to realise the LVS is the presence of a del Pezzo four-cycle τ_{dP} supporting a single non-perturbative effect whose interplay with the leading order α' correction to the EFT naturally yields an exponentially large internal volume $\mathcal{V} \sim W_0 e^{2\pi\tau_{dP}/n}$. Notice that the rigidity of a del Pezzo divisor guarantees the generation of this non-perturbative effect, and the fact that the LVS does not require more than one non-perturbative contribution implies that there is no problem with the cancellation of Freed-Witten anomalies. Moreover, a del Pezzo divisor is intrinsically ‘local’, in the sense that a basis can always be found where this four-cycle enters the volume in a completely diagonal way. Therefore it is easy to decouple this modulus from the one supporting the visible sector, avoiding problems arising from chiral intersections. The final upshot is that this blow-up mode does not appear in the D-terms and the corresponding axion is very heavy, of the order of the gravitino mass.
- The D-terms are the leading order effects in an expansion in inverse powers of the huge internal volume and they might in principle generate a potential for $h^{1,1} - 1$ moduli via the flux induced generation of moduli-dependent Fayet-Iliopoulos (FI) terms. However, if this is the case, the only solution to $D = 0$ in the absence of SM singlets which can have non-zero VEVs, is the one where all the cycles shrink to zero size. This can be avoided if the FI-terms depend only on d moduli with $d < h^{1,1} - 1$, which can be obtained by an appropriate choice of brane set-up and fluxes. Thus exactly d axions are eaten up, corresponding to the d combinations of Kähler moduli fixed by the D-terms.
- All the remaining $n_{ax} = h^{1,1} - 1 - d$ flat directions have to be fixed perturbatively via α' or g_s effects. As we have already pointed out, the volume is fixed by the leading order α' effects whereas all the other cycles are stabilised by string loops.

- The number of massless axions is n_{ax} which for an arbitrary Calabi-Yau with $h^{1,1} \sim \mathcal{O}(100)$ is likely to be very large. Subleading higher-order non-perturbative effects then slightly lift the axion directions giving rise to the ‘LVS axiverse’.

As explained in [25], the LARGE Volume Scenario requires $n_{\text{ax}} \geq 2$, implying that the simplest model involves at least two light axions. If $n_{\text{ax}} = 1$, this remaining flat direction would parameterise the overall volume and be constrained to be fixed at small size by the requirement of obtaining a visible sector gauge coupling g_{vs} of the correct order of magnitude, given that the D-term relations force the volume to scale as the visible sector cycle: $\mathcal{V}^{2/3} \sim \tau_{\text{vs}} \sim g_{\text{vs}}^{-2}$. Hence the volume would be rather small in string units and the tree-level superpotential W_0 fine-tuned small otherwise the relation $\mathcal{V} \sim W_0 e^{2\pi\tau_{\text{dP}}/n}$ would yield an exponentially small gauge coupling. However, if $n_{\text{ax}} \geq 2$, only one of these flat directions is forced to be small to obtain the correct value of g_{vs} while the others can be exponentially large allowing a standard realisation of the LVS for $W_0 \sim \mathcal{O}(1)$.

Clearly, providing a string realisation of the axiverse is not enough if none of these light axions can behave as the QCD axion. This depends mainly on the strength of the corresponding axion coupling to QCD which, in turn, depends on the topology of the Calabi-Yau three-fold and the choice of brane set-up and fluxes. In fact, if the visible sector wraps a small rigid divisor which does not intersect the cycles controlling the size of the large overall volume, the decay constant of the corresponding axion is of the order the string scale $f_a \sim M_s/\sqrt{4\pi}$ due to the local nature of this interaction. In all the other cases, this interaction is non-local resulting in axion couplings suppressed by the Planck scale. We shall discuss the consequences of this in section 3.2 where we shall find that, in the cases when non-local axions can play the rôle of the QCD axion, their decay constants are of the order $f_a \sim M_{\text{GUT}} \sim 10^{16}$ GeV.

Given that by dimensional reduction the string scale can be expressed as $M_s = M_P/\sqrt{4\pi\mathcal{V}}$, exponentially large values of the volume can naturally yield a very low string scale. The gravitino mass is instead $m_{3/2} = \sqrt{g_s/2}W_0M_P/\mathcal{V}$, and so TeV-scale soft-terms $M_{\text{soft}} \sim m_{3/2} \sim 1$ TeV can be obtained for $\mathcal{V} \sim 10^{14}$ if $W_0 \sim \mathcal{O}(1)$ and $g_s \simeq 0.1$. Consequently the string scale turns out to be intermediate, $M_s \sim 5 \cdot 10^{10}$ GeV, or a few orders of magnitude larger as we moderately tune W_0 smaller, providing an axion decay constant in the classic phenomenological window for the case of a local interaction [15]. In this paper we shall describe how to obtain a QCD axion candidate plus additional ALPs which are not eaten by any anomalous U(1)s and exhibit these interesting phenomenological features.

We stress that LVS models with an intermediate string scale involve light moduli which may suffer from the CMP. For example, the mass of the volume mode \mathcal{V} is $m_{\mathcal{V}} = m_{3/2}/\mathcal{V}^{1/2} \sim 0.1$ MeV. In order to solve the CMP, these light moduli could be diluted either by the entropy released by the decay of some heavy moduli [26] or by a late time inflationary period caused by thermal effects [27]. If a non-local axion with a GUT-scale decay constant plays the rôle of the QCD axion, this dilution mechanism would also dilute this axion, potentially allowing it to evade the present bounds from dark matter overproduction. Consequently, non-local axions might form dark matter and be detectable via future molecular interferometry experiments (see appendix B.2). On the other hand, if the QCD

axion is realised as a local axion with an intermediate scale decay constant at the lower end discussed above, $f_a \sim M_s/\sqrt{4\pi} \sim 10^{10}$ GeV, this opens up the intriguing possibility to detect it directly in the laboratory in the next generation of LSW experiments. Furthermore, these local axions can provide a nice explanation to several astrophysical puzzles, such as the anomalous transparency of the Universe for TeV photons and the anomalous cooling of white dwarfs; all of these hints and constraints are summarised in appendix B. Moreover, it is worth stressing that in this case axions do not form a sizeable contribution to dark matter, since the latter requires a decay constant in the upper end discussed above, $f_a \sim M_s/\sqrt{4\pi} \sim 10^{12}$ GeV, cf. appendix B.2.

We shall illustrate our general claims with four Calabi-Yau examples with stabilised moduli and an explicit choice of brane set-up and fluxes. The main features of each example are:

1. GUT-like model with $n_{\text{ax}} = 1$ non-local axion with $f_a \sim M_{\text{KK}}^{10\text{D}}/(4\pi) \simeq 5 \cdot 10^{14}$ GeV that behaves as the QCD axion in the presence of a dilution mechanism.
2. MSSM-like model with $n_{\text{ax}} = 2$ non-local axions, a_f with $f_{a_f} \simeq 10^{16}$ GeV, and a_b with $f_{a_b} \sim M_{\text{KK}}^{6\text{D}}/(4\pi) \simeq 5$ TeV. The axion a_f can be the QCD axion in the presence of a dilution mechanism, whereas a_b is a very light and almost decoupled ALP ($C_{bj} \ll 1$).
3. Chiral model with $n_{\text{ax}} = 3$ light axions: one local axion a_s with $f_{a_s} \sim M_s/\sqrt{4\pi} \simeq 10^{10}$ GeV, and two non-local axions, a_f and a_b , with $f_{a_f} \sim f_{a_b} \sim M_{\text{KK}}^{10\text{D}}/(4\pi) \simeq 10^8$ GeV. The local axion a_s provides the first explicit example of a closed string QCD axion which is not eaten by any anomalous U(1) and has an intermediate scale decay constant. The two non-local axions are very light and almost decoupled ALPs ($C_{fj}, C_{bj} \ll 1$).
4. Chiral model with more than one light local axion with $f_a \sim M_s/\sqrt{4\pi} \simeq 10^{10}$ GeV.

We stress again that in this analysis we focused only on axions which come from closed strings since only this case is model-independent. In fact, axion-like particles can also be realised as open strings which live on D-branes but this realisation is more model-dependent, since it depends crucially on the particular D-brane construction considered.

This paper is organised as follows. Section 2 presents a simple review of the low-energy 4D EFT of type IIB string theory compactified on Calabi-Yau orientifolds paying particular attention to axions and their interplay with massive anomalous U(1)s. In section 3 we describe in detail how the LVS moduli stabilisation gives rise to an axiverse giving formulae for the sizes of decay constants, couplings to gauge and matter fields for Swiss-cheese and fibred Calabi-Yau compactifications. The explicit LVS examples mentioned above are analysed in section 4, whereas we present our conclusions in section 5. Due to this success to derive the axiverse from type IIB strings, in appendix A, we work out the low-energy effective action of many ALPs computing in particular their couplings to photons and electrons which are particularly relevant for phenomenology. In appendix B we summarise the results of axion searches in astrophysics, cosmology, and laboratory arguing that various astrophysical hints point to an axion decay constant at the intermediate scale. Finally, in appendix C we compute the axionic couplings to matter at one-loop, and in appendix D we discuss the axion decay constants for general fibred Calabi-Yau manifolds.

2 Closed string axions from type IIB strings

In this section we will describe how axion-like fields arise in type IIB string theory models and how to determine which of the axion-like fields are eaten by massive U(1) gauge bosons. The low-energy couplings, decay constants, and masses can be obtained by comparing the EFT obtained in this section from dimensional reduction of the IIB string from 10 to 4 dimensions with the general low-energy effective Lagrangian of axion-like particles (A.1) which we shall derive in appendix A.

2.1 Effective axionic action

In type IIB flux compactifications on Calabi-Yau orientifolds, the Standard Model or a suitable extension like a Grand Unified Theory (GUT) is localised on a stack of space-time filling D3 or D7 branes. Pseudo-scalar axion-like fields c^a and c_α arise as Kaluza-Klein zero modes of the Ramond-Ramond (RR) antisymmetric tensor fields C_2 and C_4 [28, 29] (for a short review, see the appendices in [21, 30]), that is as coefficients of the two- and four-form term, respectively, in their expansion in harmonics of the Calabi-Yau orientifold:

$$C_2 = c^a(x)\omega_a, \quad a = 1, \dots, h_-^{1,1} \quad \text{and} \quad C_4 = c_\alpha(x)\tilde{\omega}^\alpha + \dots, \quad \alpha = 1, \dots, h_+^{1,1}.$$

The axionic couplings to gauge field strengths $\propto F \wedge F$ arise then from the Kaluza-Klein reduction of the Chern-Simons term in the action of the D-branes.

The effective Lagrangian including U(1) gauge bosons A_i and couplings to axions c^a and c_α can be written as:

$$\begin{aligned} \mathcal{L} \supset & -\frac{e^\Phi}{4\mathcal{V}^2} \left(dc^a + \frac{e^{-\Phi}}{2\pi} M_P A_i r^{ia} \right) \mathcal{K}^{ab} \wedge \star \left(dc^b + \frac{e^{-\Phi}}{2\pi} M_P A_j r^{jb} \right) \\ & + \frac{2\pi M_P^2}{\mathcal{V}^2 (2\pi)^3} e^{-\Phi} A_i A_j r^{ia} r^{jb} \mathcal{K}^{ab} - \left(dc_\alpha + \frac{M_P}{\pi} A_i q_{i\alpha} \right) \frac{\mathcal{K}_{\alpha\beta}}{8} \wedge \star \left(dc_\beta + \frac{M_P}{\pi} A_j q_{j\beta} \right) \\ & + \frac{M_P^2}{2(2\pi)^2} A_i A_j q_{i\alpha} \mathcal{K}_{\alpha\beta} q_{j\beta} + \frac{1}{4\pi M_P} (r^{i\alpha} c_\alpha + q_{i\alpha} c^a) \text{tr}(F \wedge F) - \frac{r^{i\alpha} \tau_\alpha}{4\pi M_P} \text{tr}(F_i \wedge \star F_i). \end{aligned} \quad (2.1)$$

The quantities in the above expression are defined as follows. M_P is the reduced Planck mass $(8\pi G_N)^{-1/2} \simeq 2.4 \cdot 10^{18}$ GeV, Φ is the axio-dilaton, \mathcal{V} is the Einstein-frame volume of the internal space in string units, while q_{iA} and r^{iA} are the couplings of the two-forms ω_A and four-forms $\tilde{\omega}^A$ to branes, ignoring any globally-trivial forms, and are given by:

$$\begin{aligned} q_{iA} &\equiv \ell_s^{-2} \int_{D_i} \omega_A \wedge \frac{\mathcal{F}}{2\pi} = \ell_s^{-4} \int_{D_i} \omega_A \wedge \ell_s^2 \mathcal{F}, \\ r^{iA} &\equiv \ell_s^{-4} \int_{D_i} \tilde{\omega}^A = \ell_s^{-4} \int \hat{D}_i \wedge \tilde{\omega}^A, \quad A = 1, \dots, h^{1,1} = h_-^{1,1} + h_+^{1,1}, \end{aligned} \quad (2.2)$$

where \mathcal{F} is the gauge flux on D_i which is a generic divisor whose Poincaré dual two-form can be expressed in terms of the basis forms as $\hat{D}_i = r^{iA} \omega_A$ with $\ell_s^{-4} \int \omega_B \wedge \tilde{\omega}^A = \delta_B^A$. Writing the Kähler form as $J = t^A \omega_A$, we have:

$$\begin{aligned} K &= -2 \ln \mathcal{V}, & \tau_A &\equiv \frac{1}{2} k_{ABC} t^A t^C, & \mathcal{T}_{AB} &\equiv k_{ABC} t^C, \\ \mathcal{K}_{AB} &\equiv \frac{\partial^2 K}{\partial \tau_A \partial \tau_B} = \frac{t_A t_B}{2\mathcal{V}^2} - \frac{(\mathcal{T}^{-1})_{AB}}{\mathcal{V}}, & \mathcal{K}^{AB} &= \tau_A \tau_B - \mathcal{T}_{AB} \mathcal{V}, \end{aligned} \quad (2.3)$$

where $k_{ABC} = \ell_s^{-6} \int \omega_A \wedge \omega_B \wedge \omega_C$ are the triple intersection numbers.

The above expression makes clear that some axions are eaten by the U(1) gauge symmetries in the model, while the axionic couplings to other gauge groups are inversely proportional to the Planck mass. However, the physical axion decay constants will be determined by the kinetic terms, which - particularly for large volumes - can differ from unity by many orders of magnitude. In this work, we are interested in ALPs whose decay constants may be equal to or less than the string scale, so that their couplings may be equal to or enhanced relative to the QCD axion; an examination of the above action and the expressions for the metrics (2.3) will make clear that the orientifold-odd fields c^a will have decay constants equal to or parametrically greater than the string scale (similar to the case for U(1) masses [30]) and so we will neglect them, or focus on models with $h_-^{1,1} = 0$. In this case the Kähler moduli are defined as:

$$T_\alpha = \tau_\alpha + i c_\alpha \quad \text{with} \quad \alpha = 1, \dots, h_+^{1,1} = h^{1,1}. \quad (2.4)$$

However, it should be stated that the analysis of the odd two-form fields c^a would proceed almost identically to that of the even fields.

2.2 Canonical normalisation

From (2.2) it can be seen via topological charge quantisation that the axions have (unconventionally for axions, but conventionally for Kähler moduli) periods equal to *integer* multiples of M_P ; this derives from the quantisation of the Ramond-Ramond forms from which they descend. We diagonalise the axions in two steps: first we make an orthogonal transformation, after which the new fields are still periodic under integer shifts by M_P . This leads to kinetic terms:

$$\mathcal{L} \supset -\frac{1}{8} \lambda_\alpha \partial_\mu c'_\alpha \partial^\mu c'_\alpha,$$

where λ_α are the eigenvalues of $\mathcal{K}_{\alpha\beta}$. We then canonically normalise by defining $c'_\alpha \equiv 2\lambda_\alpha^{-1/2} a_\alpha$; the new fields are then periodic under shifts by $\frac{1}{2}\lambda_\alpha^{1/2} M_P$ - this allows us to define the decay constants:

$$f_\alpha \equiv \frac{\sqrt{\lambda_\alpha} M_P}{4\pi} \quad (2.5)$$

so that the fields have the conventional periodicity $a_\alpha = a_\alpha + 2\pi f_\alpha$.

Some of these fields will be eaten by a U(1) gauge boson and thus cannot enter into a term in the superpotential of the form $W \supset A e^{-aT}$ where A is a constant (not a function of matter fields) due to gauge invariance - the shift symmetry is gauged. Hence these axions are in some sense orthogonal to their light ALP brothers, and we must first remove them from our consideration. To find the massless axions coupling to a given gauge group, we must find the null vectors to eliminate the $dc \wedge \star A$ couplings. Hence the full transformation, leaving only the ungauged axions, is:

$$\begin{aligned} c_\alpha &= 2 a_\gamma V_{\gamma\beta} \mathcal{C}_{\beta\alpha}, & \mathcal{C}_{\gamma'\alpha} \mathcal{K}_{\alpha\beta} \mathcal{C}_{\beta\delta'}^T &= \delta_{\gamma'\delta'}, & \mathcal{C}_{\gamma'\alpha} \mathcal{C}_{\alpha\delta'}^T &= \lambda_{\gamma'}^{-1} \delta_{\gamma'\delta'}, \\ V_{\gamma\gamma'} \mathcal{C}_{\gamma'\beta} \mathcal{K}_{\beta\alpha} a_{\alpha i}^T &= 0, & (VV^T)_{\alpha\beta} &= \begin{cases} \delta_{\alpha\beta} & a \text{ ungauged} \\ 0 & a \text{ gauged} \end{cases}, \end{aligned} \quad (2.6)$$

i.e. the matrix V is the set of null vectors of the mass couplings. When both axions and gauge fields are canonically normalised we have a term $\mathcal{L} \supset \frac{g_i^2}{2\pi M_P} \text{tr}(F_i \wedge F_i) r^{i\alpha} \mathcal{C}_{\alpha\beta}^T V_{\beta\gamma}^T a_\gamma$, so in the form of couplings (1.2) this gives:

$$\frac{f_{a_j}}{C_{ji}} = \frac{1}{8\pi} \frac{M_P}{r^{j\alpha} \mathcal{C}_{\alpha\beta}^T V_{\beta i}^T} \times \begin{cases} 1/2 & \text{U}(1) \\ 1 & \text{SU}(N) \end{cases} . \quad (2.7)$$

Note that for $\text{U}(1) \supset \text{U}(N)$ we normalise the generators so that $\text{tr}(T_{\text{U}(1)} T_{\text{U}(1)}) = 1$, which means that $T_{\text{U}(1)} = \mathbb{1}/\sqrt{N}$ and $\text{tr}(T_{\text{U}(1)}) = \sqrt{N}$. There is an alternative notation much used in the literature, involving physical couplings g_{ji} :

$$\begin{aligned} \mathcal{L} &\supset -\frac{1}{4} a_j g_{ji} F_{i,\mu\nu}^b \tilde{F}_i^{b,\mu\nu} \\ \Rightarrow g_{ji} &= -\frac{2g_i^2}{\pi M_P} r^{i\alpha} \mathcal{C}_{\alpha\beta}^T V_{\beta j}^T \times \begin{cases} 1 & \text{U}(1) \\ \frac{1}{2} & \text{SU}(N) \end{cases} = -\frac{4}{M_P} \frac{1}{\tau_i} r^{i\alpha} \mathcal{C}_{\alpha\beta}^T V_{\beta j}^T, \end{aligned} \quad (2.8)$$

where τ_i is the volume of the four-cycle wrapped by the brane in string units.

It is often convenient to make a redefinition of the axions choosing a basis where $r^{i\alpha} a_\alpha \equiv \{\hat{a}_i, 0\}$ depending on whether the cycle is wrapped by a brane or not. Under this transformation, we must not forget to transform the charges appropriately.

With these couplings, we can work out which axions will obtain a mass through any non-perturbative effects - such as coupling to a gaugino condensate or D-brane instanton - since these axions cannot be gauged (for masses that arise through superpotential terms of the form $W \supset A e^{-bT}$ where A is independent of matter fields). In determining the light axion and ALPs we must therefore make a further redefinition of the fields to eliminate the heavy massive ones.

2.3 Volume scaling of axion decay constants and couplings to gauge bosons

Being the imaginary component of the moduli, whose interaction with matter is gravitational, we expect the physical couplings of axions to matter fields to be suppressed by the Planck scale. This applies for moduli which are able to propagate throughout the Calabi-Yau volume. However, for axion fields that only propagate over so-called ‘local’ cycles (such as small rigid divisors), i.e. four-cycles which do not intersect any divisor which controls the overall volume, the natural coupling strength is the string scale. Using conventional definitions, the physical coupling is inversely proportional to the axion decay constant f_a and proportional to the square of the gauge coupling $g^2 \simeq \tau_a^{-1}$. However, this argument only holds for the scaling with respect to the overall volume, rather than the volumes of small cycles - we expect:

$$f_a \simeq \begin{cases} M_P/\tau_a & \text{non-local axion} \\ M_P/\sqrt{\mathcal{V}} \simeq M_s & \text{local axion} \end{cases} \quad (2.9)$$

As pointed out in section 1, local interactions open up the possibility to obtain axion decay constants which are much lower than the Planck or GUT scale, since their size is

controlled by M_s instead of M_P [15]. Hence if $M_s \sim 10^{10}$ GeV, f_a turns out to be intermediate in a regime which is particularly interesting for phenomenology. Notice that this value of M_s can be naturally obtained for exponentially large values of the overall volume which characterise LVS scenarios. A range of values of f_a are then possible depending upon the exact numerical coefficients, the value of the gravitino mass and the tuning of W_0 ; $M_s \simeq \sqrt{\frac{M_P m_{3/2}}{W_0}}$ so values of decay constants spanning the classic axion window ($10^9 \div 10^{12}$ GeV) are possible without fine tuning.

On the other hand, the decay constants of non-local axions may be even smaller - since τ_a in this case can scale as $\mathcal{V}^{2/3}$ or even \mathcal{V} for anisotropic compactifications.² This has important consequences for the amount of dark matter that they can produce, which we shall discuss in section 3.3.

We shall now illustrate these general claims and derive the physical couplings to gauge bosons, C_{ij} , for explicit Swiss-cheese compactifications and in section 2.3.2 for fibred Calabi-Yau examples.

2.3.1 Swiss-cheese Calabi-Yau manifolds

In the simplest Swiss-cheese case, the Calabi-Yau volume takes the form [31]:

$$\mathcal{V} = \frac{1}{9\sqrt{2}} \left(\tau_b^{3/2} - \tau_s^{3/2} \right), \tag{2.10}$$

where τ_b is a large cycle controlling the overall size of the internal manifold whereas the small divisor τ_s is a local blow-up mode. Writing $\epsilon^2 \equiv \tau_s/\tau_b \ll 1$, and working at leading order in a large volume expansion, we find:

$$\mathcal{K}_0 = \frac{3}{2\tau_b^{3/2}\sqrt{\tau_s}} \begin{pmatrix} 2\epsilon & -3\epsilon^2 \\ -3\epsilon^2 & 1 \end{pmatrix}. \tag{2.11}$$

The eigenvalues of this are $3\tau_b^{-2}, \frac{3}{2}\tau_b^{-3/2}\tau_s^{-1/2}$, and so the decay constants of the relevant axions will be:

$$f_{a_b} = \frac{\sqrt{3} M_P}{4\pi \tau_b} \simeq \frac{M_P}{4\pi \mathcal{V}^{2/3}} \simeq \frac{M_{\text{KK}}^{10\text{D}}}{4\pi}, \quad f_{a_s} = \frac{1}{\sqrt{6} (2\tau_s)^{1/4}} \frac{M_P}{4\pi\sqrt{\mathcal{V}}} \simeq \frac{M_s}{\sqrt{4\pi\tau_s}^{1/4}}, \tag{2.12}$$

in agreement with (2.9). In (2.12), $M_{\text{KK}}^{10\text{D}}$ denotes the 10D Kaluza-Klein scale:

$$M_{\text{KK}}^{10\text{D}} \simeq \frac{M_s}{\mathcal{V}^{1/6}} \simeq \frac{M_P}{\mathcal{V}^{2/3}}. \tag{2.13}$$

The diagonalising matrix is given by:

$$\mathcal{C}^T = \tau_b \begin{pmatrix} -\frac{1}{\sqrt{6}} \left(\frac{1-9\epsilon^3}{\sqrt{1-\frac{9}{2}\epsilon^3}} \right) & \sqrt{3} \epsilon^{3/2} \\ -\epsilon^2 \sqrt{\frac{3}{2}} \left(1 - \frac{9}{2}\epsilon^3 \right) & \frac{1}{\sqrt{3}} \epsilon^{1/2} \end{pmatrix}, \tag{2.14}$$

²For anisotropic compactifications we show in appendix D that there is only one axion that has a Planck-sized decay constant - that corresponding to the fibre - with the others being much smaller.

and the matrix $r^{i\alpha}$ is the identity. Hence the original axion fields c_b and c_s can be written in terms of the canonically normalised fields a_b and a_s as:

$$\frac{c_b}{\tau_b} \simeq -\frac{1}{\sqrt{6}} a_b + \sqrt{3} \epsilon^{3/2} a_s \simeq \mathcal{O}(1) a_b + \mathcal{O}\left(\tau_s^{3/4} \mathcal{V}^{-1/2}\right) a_s, \quad (2.15)$$

$$\frac{c_s}{\tau_s} \simeq -\sqrt{\frac{3}{2}} a_b + \frac{1}{\sqrt{3} \epsilon^{3/2}} a_s \simeq \mathcal{O}(1) a_b + \mathcal{O}\left(\tau_s^{-3/4} \mathcal{V}^{1/2}\right) a_s, \quad (2.16)$$

where we have used $\mathcal{V} \simeq \tau_b^{2/3}$ and we have neglected subleading corrections proportional to ϵ . Thus the axionic couplings to gauge bosons scale as:

$$\begin{aligned} \mathcal{L} \supset & \frac{c_b}{M_P} g_b^2 \text{tr}(F_b \wedge F_b) + \frac{c_s}{M_P} g_s^2 \text{tr}(F_s \wedge F_s) \\ & \simeq \left[\mathcal{O}\left(\frac{1}{M_P}\right) a_b + \mathcal{O}\left(\frac{\tau_s^{3/4}}{\mathcal{V}^{1/2} M_P}\right) a_s \right] \text{tr}(F_b \wedge F_b) \\ & \quad + \left[\mathcal{O}\left(\frac{1}{M_P}\right) a_b + \mathcal{O}\left(\frac{1}{\tau_s^{3/4} M_s}\right) a_s \right] \text{tr}(F_s \wedge F_s). \end{aligned} \quad (2.17)$$

The volume scaling of the leading terms of these couplings are exactly in accordance with (2.9). The constants describing the coupling of the axions to gauge bosons instead become:

$$\begin{aligned} C_{bb} & \simeq g_b^{-2} \frac{f_{a_b}}{M_P} \simeq \mathcal{O}(1), & C_{sb} & \simeq g_b^{-2} \frac{f_{a_s} \tau_s^{3/4}}{\mathcal{V}^{1/2} M_P} \simeq \mathcal{O}(\epsilon) \simeq \mathcal{O}(\mathcal{V}^{-1/3}), \\ C_{bs} & \simeq g_s^{-2} \frac{f_{a_b}}{M_P} \simeq \mathcal{O}(\epsilon^2) \simeq \mathcal{O}(\mathcal{V}^{-2/3}), & C_{ss} & \simeq g_s^{-2} \frac{f_{a_s}}{\tau_s^{3/4} M_s} \simeq \mathcal{O}(1). \end{aligned}$$

2.3.2 Fibred Calabi-Yau manifolds

Now let us consider fibred Calabi-Yau manifolds with a del Pezzo divisor. The volume is given by [32]:

$$\mathcal{V} = t_b t_f^2 + t_s^3 = t_b \tau_f + t_s^3, \quad (2.18)$$

where t_b is the size of the \mathbb{P}^1 base, $\tau_f = t_f^2$ the volume of the K3 or T^4 fibre and $\tau_s = 3t_s^2$ parameterises the volume of a small del Pezzo four-cycle. The volume of the divisor including the base of the fibration is instead given by $\tau_b = 2t_b t_f = 2t_b \sqrt{\tau_f}$. We treat more general fibred examples in appendix D.

In the limit $t_s \tau_f \gg \tau_s^{3/2}$, the eigenvalues of the mixing matrix are to leading order τ_f^{-2} , $2\tau_b^{-2}$, $\frac{1}{2\sqrt{3}} \mathcal{V} \sqrt{\tau_s}$, and so the decay constants are:

$$f_{a_f} = \frac{1}{4\pi} \frac{M_P}{\tau_f}, \quad f_{a_b} = \frac{\sqrt{2}}{4\pi} \frac{M_P}{\tau_b}, \quad f_{a_s} = \frac{1}{\sqrt{2} (3\tau_s)^{1/4}} \frac{M_P}{4\pi \sqrt{\mathcal{V}}} \simeq \frac{M_s}{\sqrt{4\pi} \tau_s^{1/4}},$$

again in agreement with (2.9): the first two axions are non-local. Note that the fibre axion has a potentially large decay constant in the anisotropic limit when $\tau_f \sim \tau_s \ll \tau_b$; we show in appendix D that this is the only axion that can have a decay constant parametrically above the string scale.

We should, however, distinguish the decay constants from the physical couplings. To determine these we can use the results of [27], by which the original axion fields c_b , c_f and c_s can be written in terms of the canonically normalised fields a_b , a_f and a_s as:

$$\frac{c_f}{\tau_f} \simeq \mathcal{O}(1) a_f + \mathcal{O}\left(\tau_s^{3/2} \mathcal{V}^{-1}\right) a_b + \mathcal{O}\left(\tau_s^{3/4} \mathcal{V}^{-1/2}\right) a_s, \quad (2.19)$$

$$\frac{c_b}{\tau_b} \simeq \mathcal{O}\left(\tau_s^{3/2} \mathcal{V}^{-1}\right) a_f + \mathcal{O}(1) a_b + \mathcal{O}\left(\tau_s^{3/4} \mathcal{V}^{-1/2}\right) a_s, \quad (2.20)$$

$$\frac{c_s}{\tau_s} \simeq \mathcal{O}(1) a_f + \mathcal{O}(1) a_b + \mathcal{O}\left(\tau_s^{-3/4} \mathcal{V}^{1/2}\right) a_s. \quad (2.21)$$

The axionic couplings to gauge bosons following from (2.19)–(2.21) scale as:

$$\begin{aligned} \mathcal{L} \supset & \frac{c_f}{M_P} g_f^2 \text{tr}(F_f \wedge F_f) + \frac{c_b}{M_P} g_b^2 \text{tr}(F_b \wedge F_b) + \frac{c_s}{M_P} g_s^2 \text{tr}(F_s \wedge F_s) \\ & \simeq \left[\mathcal{O}\left(\frac{1}{M_P}\right) a_f + \mathcal{O}\left(\frac{\tau_s^{3/2}}{\mathcal{V} M_P}\right) a_b + \mathcal{O}\left(\frac{\tau_s^{3/4}}{\mathcal{V}^{1/2} M_P}\right) a_s \right] \text{tr}(F_f \wedge F_f) \\ & + \left[\mathcal{O}\left(\frac{\tau_s^{3/2}}{\mathcal{V} M_P}\right) a_f + \mathcal{O}\left(\frac{1}{M_P}\right) a_b + \mathcal{O}\left(\frac{\tau_s^{3/4}}{\mathcal{V}^{1/2} M_P}\right) a_s \right] \text{tr}(F_b \wedge F_b) \\ & + \left[\mathcal{O}\left(\frac{1}{M_P}\right) a_f + \mathcal{O}\left(\frac{1}{M_P}\right) a_b + \mathcal{O}\left(\frac{1}{\tau_s^{3/4} M_s}\right) a_s \right] \text{tr}(F_s \wedge F_s). \end{aligned} \quad (2.22)$$

Notice that volume scaling of the leading terms of these couplings are exactly in accordance with (2.9). The constants describing the coupling of the axions to gauge bosons instead become:

$$\begin{aligned} C_{ff} & \simeq \mathcal{O}(1), & C_{bf} & \simeq \mathcal{O}\left(\frac{(\tau_s \tau_f)^{3/2}}{\mathcal{V}^2}\right), & C_{sf} & \simeq \mathcal{O}\left(\frac{\tau_f \sqrt{\tau_s}}{\mathcal{V}}\right) \\ C_{fb} & \simeq \mathcal{O}\left(\left(\frac{\tau_s}{\tau_f}\right)^{3/2}\right), & C_{bb} & \simeq \mathcal{O}(1), & C_{sb} & \simeq \mathcal{O}\left(\sqrt{\frac{\tau_s}{\tau_f}}\right), \\ C_{fs} & \simeq \mathcal{O}\left(\frac{\tau_s}{\tau_f}\right), & C_{bs} & \simeq \mathcal{O}\left(\frac{\tau_s \sqrt{\tau_f}}{\mathcal{V}}\right), & C_{ss} & \simeq \mathcal{O}(1). \end{aligned} \quad (2.23)$$

Let us now see how these quantities behave in the two different cases when the Calabi-Yau has an isotropic or anisotropic shape.

- *Isotropic case:* in the isotropic limit $t_b \sim \sqrt{\tau_f} \sim \mathcal{V}^{1/3}$ the decay constants have the same volume scaling as the Swiss-cheese example:

$$f_{a_f} \simeq f_{a_b} \simeq \frac{M_P}{4\pi \mathcal{V}^{2/3}} \simeq \frac{M_{\text{KK}}^{10\text{D}}}{4\pi}, \quad f_{a_s} \simeq \frac{M_s}{\sqrt{4\pi} \tau_s^{1/4}}, \quad (2.24)$$

and the anomaly coefficients (2.23) reduce to:

$$\begin{aligned} C_{ff} & \simeq C_{bb} \simeq \mathcal{O}(1), & C_{bf} & \simeq C_{fb} \simeq \mathcal{O}(\mathcal{V}^{-1}), & C_{sf} & \simeq C_{sb} \simeq \mathcal{O}(\mathcal{V}^{-1/3}) \\ C_{fs} & \simeq C_{bs} \simeq \mathcal{O}(\mathcal{V}^{-2/3}), & C_{ss} & \simeq \mathcal{O}(1). \end{aligned} \quad (2.25)$$

- *Anisotropic case:* in the anisotropic limit $t_b \sim \mathcal{V} \gg \sqrt{\tau_f} \sim \sqrt{\tau_s}$ considered in [21, 33], the decay constants become:

$$f_{a_f} \simeq \frac{M_P}{4\pi\tau_s}, \quad f_{a_b} \simeq \frac{M_P}{4\pi\tau_b} \simeq \frac{M_{\text{KK}}^{6\text{D}}}{4\pi}, \quad f_{a_s} \simeq \frac{M_s}{\sqrt{4\pi\tau_s^{1/4}}}, \quad (2.26)$$

where f_{a_b} is now scaling as the Kaluza-Klein scale of the 6D EFT given by:

$$M_{\text{KK}}^{6\text{D}} \simeq \frac{M_s}{\sqrt{t_b}} \simeq \frac{M_P}{\sqrt{\mathcal{V}t_b}} \simeq \frac{M_P}{t_b t_f} \simeq \frac{M_P}{\tau_b}. \quad (2.27)$$

Moreover, the anomaly coefficients (2.23) take the simplified form:

$$\begin{aligned} C_{ff} &\simeq C_{fb} \simeq C_{bb} \simeq C_{sb} \simeq C_{fs} \simeq C_{ss} \simeq \mathcal{O}(1), \\ C_{sf} &\simeq C_{bs} \simeq \mathcal{O}(\mathcal{V}^{-1}), \quad C_{bf} \simeq \mathcal{O}(\mathcal{V}^{-2}). \end{aligned} \quad (2.28)$$

2.4 Axionic couplings to matter fields

After having computed the axionic couplings to gauge fields, let us now move to the computation of the axionic couplings to matter fields. Written in terms of two-component Weyl spinors, these couplings take the form

$$\mathcal{L} \supset \frac{\hat{X}_\psi^i}{f_{a_i}} (\psi \sigma^\mu \bar{\psi}) \partial_\mu a_i. \quad (2.29)$$

We discuss the low energy phenomenology of these couplings in appendix A, where we also explain that we shall neglect ‘Yukawa’ couplings of the form $a_i \psi \tilde{\psi}$ - since they arise from non-generic and model dependent non-perturbative terms.

In string theory, closed string axions couple generically to matter fields from higher-order Kähler potential terms, and can thus be derived from the effective supergravity action. In a globally supersymmetric theory, the axion couplings arise from the $i\theta\sigma^\mu\bar{\theta}\partial_\mu T_\alpha$ component of the superfield $T_\alpha = \tau_\alpha + i c_\alpha + \dots$:

$$\int d^4\theta \Phi\bar{\Phi} (T_\alpha + \bar{T}_\alpha) \supset (\psi \sigma^\mu \bar{\psi}) \partial_\mu c_\alpha. \quad (2.30)$$

If we write for simplicity a diagonal matter Kähler metric as $\partial_{\Psi\bar{\Psi}} K \equiv \hat{K}_\Psi$ (the generalisation to non-diagonal cases being obvious) then the moduli dependence of this gives us the couplings:

$$\mathcal{L} \supset \int d^4\theta \hat{K}_i \Phi\bar{\Phi} \supset \hat{K}_\Phi|_{\theta=\bar{\theta}=0} i\psi\sigma^\mu\partial_\mu\bar{\psi} + \left. \frac{\partial\hat{K}_\Phi}{\partial T_\alpha} \right|_{\theta=\bar{\theta}=0} (\psi\sigma^\mu\bar{\psi})\partial_\mu c_\alpha. \quad (2.31)$$

So in the physical basis (when we canonically normalise the matter fields) the coupling of axions to fermions is $\partial_{T_\alpha} (\ln \hat{K}_\Phi)$. Given that the above expression has to be gauge invariant, the axions can only appear in the combination $\partial_\mu c_\alpha + \frac{M_P}{\pi} q_\alpha A_\mu$, and so we have additional anomalous gauge couplings. However, in a *local* supergravity theory, there is an additional contribution:

$$\mathcal{L} \supset \hat{K}_\Phi i\psi\sigma^\mu\partial_\mu\bar{\psi} + \psi\sigma^\mu\bar{\psi}\partial_\mu c_\alpha \left(\hat{K}_{\Phi T} - \frac{1}{2} K_{0,T} \hat{K}_\Phi \right), \quad (2.32)$$

and so when we canonically normalise the matter fields we obtain physical couplings:

$$\mathcal{L} \supset \partial_{T_\alpha} \left[\ln \left(e^{-\frac{K_0}{2}} \hat{K}_\Phi \right) \right] (\psi \sigma^\mu \bar{\psi}) \partial_\mu c_\alpha. \quad (2.33)$$

Note that these are different to the coupling to the real parts of the moduli, which couple to the fields via the kinetic term only.

The best conjecture for the Kähler metric of chiral matter supported on curves at the intersection of cycles a and b is [34, 35]:

$$\hat{K}_\Phi = e^{K_0/3} t_{ab}^{1/3} \quad \text{where} \quad t_{ab} = \int [D_a] \wedge [D_b] \wedge J = r^{ai} r^{bj} k_{ijk} t^k. \quad (2.34)$$

Thus:

$$\frac{\partial \left[\ln \left(e^{-\frac{K_0}{2}} \hat{K}_\Phi \right) \right]}{\partial T_\alpha} = \frac{1}{6} \left[-\frac{1}{2} \frac{\partial K_0}{\partial \tau_\alpha} + \frac{1}{t_{ab}} \frac{\partial t_{ab}}{\partial \tau_\alpha} \right] = \frac{1}{6} \left[\frac{1}{2} \frac{t_\alpha}{\mathcal{V}} + \frac{r^{ai} r^{bj}}{t_{ab}} k_{ijk} \mathcal{T}^{k\alpha} \right],$$

For matter supported all on one cycle we have $\hat{K}_\Phi = e^{K_0/3} \tau_a^{1/3}$ where $\tau_a \equiv r^{ai} \tau_i$, so:

$$\frac{\partial \left[\ln \left(e^{-\frac{K_0}{2}} \hat{K}_\Phi \right) \right]}{\partial T_\alpha} = \frac{1}{6} \left[-\frac{1}{2} \frac{\partial K_0}{\partial \tau_\alpha} + \frac{r^{a\alpha}}{\tau_a} \right] = \frac{1}{6} \left[\frac{1}{2} \frac{t_\alpha}{\mathcal{V}} + \frac{g_a^2}{4\pi} \right]. \quad (2.35)$$

We then translate these into ALP-matter couplings (to axions a_i):

$$\frac{\hat{X}_\psi^i}{f_{a_i}} = \frac{1}{3M_P} V_{i\beta} \mathcal{C}_{\beta\alpha} \begin{cases} \frac{t_\alpha}{2\mathcal{V}} + \frac{r^{ai} r^{bj}}{t_{ab}} k_{ijk} \mathcal{T}^{k\alpha} & \text{matter on curve } t_{ab} \\ \frac{t_\alpha}{2\mathcal{V}} + \frac{r^{a\alpha}}{\tau_a} & \text{matter on cycle } a \\ \frac{t_\alpha}{2\mathcal{V}} & \text{matter at a singularity} \end{cases} \quad (2.36)$$

It is thus necessary to consider the canonical normalisation of the axions. A local axion will thus couple typically as $\frac{\hat{X}_\psi^i}{f_{a_i}} \sim \frac{\sqrt{\mathcal{V}}}{3M_P} \mathcal{O}(g^2)$ in the first two cases, and non-local ones will couple much weaker than Planck strength (or not at all). Note that we expect these couplings to *violate parity* — the parity violating couplings are proportional to $\hat{X}_{\psi_L}^i - \hat{X}_{\psi_R}^i$, which are generically non-zero since the left- and right-handed fields will be localised on different curves or branes - but as discussed earlier these should not have severe consequences. We can parameterise the coupling as:

$$\mathcal{L} \supset \frac{C_{i\psi}^A}{2f_{a_i}} \bar{\psi} \gamma^\mu \gamma_5 \psi \partial_\mu a_i + \frac{C_{i\psi}^V}{2f_{a_i}} \bar{\psi} \gamma^\mu \psi \partial_\mu a_i, \quad (2.37)$$

where $C_{i\psi}^{A,V} = \frac{1}{2} (\hat{X}_{\psi_R}^j \pm \hat{X}_{\psi_L}^j)$.

An important special case is the couplings of the ‘large’ cycle axion. When the SM is realised on a local cycle, $\mathcal{C}_{bi} \simeq \mathcal{O}(1)$ and only the universal term contributes; the couplings are then highly suppressed, with $\frac{\hat{X}_\psi^i}{f_{a_i}} \sim \frac{1}{3M_P \mathcal{V}^{2/3}}$ for a Swiss-cheese compactification. The coupling to matter fields will then be dominated by the effect generated at one loop via

the photon coupling; most important among these for phenomenology is the coupling to electrons. We shall give a full discussion of the one-loop effects in appendix C, but here present a useful approximation to the one-loop corrections to the above couplings:

$$\begin{aligned}\Delta_{i\gamma\gamma}[C_{ie}^A] &\approx \frac{3\alpha^2}{4\pi^2}C_{i\gamma}\ln\left(\frac{M_{\text{soft}}}{m_e}\right) + \frac{2\alpha^2}{4\pi^2}C_{i\gamma}\ln\left(\frac{\Lambda}{M_{\text{soft}}}\right), \\ \Delta_{i\gamma\gamma}[C_{ie}^V] &\approx \frac{2\alpha^2}{4\pi^2}C_{i\gamma}\ln\left(\frac{\Lambda}{M_{\text{soft}}}\right),\end{aligned}\tag{2.38}$$

while there is an additional correction from pion loops to the QCD axion to electron coupling, unchanged in SUSY theories, equal to $\Delta_{\text{QCD}}[C_{ae}^A] = \frac{3\alpha^2}{4\pi}\Delta C_{a\gamma}\ln(\Lambda_{\text{QCD}}/m_a)$.

3 The axiverse in the LARGE Volume Scenario

In this section we will focus on the type IIB LARGE Volume Scenario and show how this moduli stabilisation mechanism gives rise to an axiverse with many light axions. We shall then discuss the phenomenology and cosmology of local and non-local axions in different scenarios where the Calabi-Yau has a isotropic or anisotropic shape and the SM is supported on a local or non-local four-cycle.

3.1 The axiverse and moduli stabilisation

As we have already pointed out in section 1, there are two regimes for the type IIB axiverse depending on the value of the VEV of the tree-level superpotential W_0 .

Case $W_0 \ll 1$: the axiverse from an ample divisor. In the case where $W_0 \ll 1$, the axiverse can be realised by an E3-instanton or gaugino condensation on D7-branes wrapping an ample divisor D_{am} whose volume is given by τ_{am} [22]. If D_i is a basis of $H_4(X, \mathbb{Z})$ where X is a generic Calabi-Yau three-fold, then:

$$D_{\text{am}} = \sum_{i=1}^{h^{1,1}} \lambda_i D_i,\tag{3.1}$$

is ‘ample’ if and only if $\lambda_i > 0 \forall i = 1, \dots, h^{1,1}$. The non-perturbative effects supported by this divisor yield a superpotential which looks like [36]:

$$W = W_0 + A e^{-a T_{\text{am}}} = W_0 + A e^{-a \sum_{i=1}^{h^{1,1}} \lambda_i T_i},\tag{3.2}$$

where $a = 2\pi/n$ with $n = 1$ for an E3-instanton and $n = N$ for gaugino condensation on N D7-branes. By fine-tuning $W_0 \sim A e^{-a T_{\text{am}}} \ll 1$, this single non-perturbative effect can generate a minimum for all the Kähler moduli within the regime of validity of the EFT. However it can lift only one axion corresponding to the imaginary part of the ample divisor modulus: $c_{\text{am}} = \text{Im}(T_{\text{am}})$. All the remaining $h^{1,1} - 1$ axions are massless at leading order and develop a potential only via tiny higher order instanton effects of the form [22]:

$$W = W_0 + A e^{-a T_{\text{am}}} + \sum_{i=1}^{h^{1,1}-1} A_i e^{-n_i a_i T_i},\tag{3.3}$$

where T_i is a combination of moduli orthogonal to $T_{\text{am}} \forall i = 1, \dots, h^{1,1} - 1$. Two main concerns regarding the microscopic realisation of this scenario are related to the difficulty to find an ample divisor which is rigid (and so definitely receives non-perturbative effects), and the possibility to choose gauge fluxes that avoid chiral intersections between the instanton and the visible sector.

Case $W_0 \sim \mathcal{O}(1)$: the LVS axiverse. In the natural regime where $W_0 \sim \mathcal{O}(1)$, the LVS gives rise to an axiverse since only one Kähler modulus, a del Pezzo divisor T_{dP} , is fixed by non-perturbative effects whereas all the other moduli (if not fixed by D-terms) are stabilised perturbatively by α' or g_s effects. Hence only one axion, $c_{\text{dP}} = \text{Im}(T_{\text{dP}})$, becomes heavy whereas all the other ones (except those eaten up by anomalous U(1)s) remain light and develop a potential via subleading higher order instanton effects.

Before providing the details of this mechanism, let us summarise three issues that have to be addressed when combining moduli stabilisation with the presence of brane fluxes within the same compactification [25, 32]:

1. *Tension between chirality and non-vanishing non-perturbative effects*

Chiral modes at the intersection of D-branes are induced by a non-zero world-volume flux $\mathcal{F} \neq 0$. In the presence of chiral intersections between the visible sector and a cycle T_{np} supporting non-perturbative effects of the form:

$$W_{\text{np}} = A e^{-a T_{\text{np}}}, \tag{3.4}$$

the prefactor A depends on visible sector modes ϕ_i : $A = A(\phi_i)$. However, in order to prevent the breaking of the visible sector gauge group at a high scale, $\langle \phi_i \rangle = 0$, and in turn $A = 0$, implying the vanishing of W_{np} .

This observation sets the following constraint on the flux choice: there has to be no chiral intersection between the visible sector and the cycle supporting non-perturbative effects. A straightforward solution to this problem is obtained by placing the non-perturbative effects on a del Pezzo divisor D_{dP} since, due to the local nature of this blow-up mode, it is always possible to find a basis of $H_4(X, \mathbb{Z})$ such that D_{dP} is an element of the basis that intersects no other element. Moreover, due to the rigidity of this four-cycle, the generation of the non-perturbative effect is definitely under control.

2. *Tension between the cancellation of Freed-Witten (FW) anomalies and the generation of more than one non-perturbative effect*

The cancellation of worldsheet FW anomalies requires to turn on a half-integer flux on any divisor D_{ns} which is ‘non-spin’, i.e. its first Chern class $c_1(D_{\text{ns}}) = -\hat{D}_{\text{ns}}$ is odd:

$$F_{\text{FW}} = -\frac{1}{2} c_1(D_{\text{ns}}) = \frac{1}{2} \hat{D}_{\text{ns}}. \tag{3.5}$$

In order to obtain a non-zero contribution to the superpotential, the total flux $\mathcal{F} = F - B$ on an E3-instanton or gaugino condensation stack wrapping an invariant

four-cycle D_a , has to vanish: $\mathcal{F}_a = 0$. However, in the case of a non-spin divisor, $F_a = \frac{1}{2} \hat{D}_a \neq 0$, forcing to choose $B = F_a$ in order to obtain $\mathcal{F}_a = 0$. Notice, however, that once B is fixed in this way, generically it cannot be used anymore to cancel the FW flux on a second non-spin divisor D_b . Thus the total flux on D_b is non-zero, $\mathcal{F}_a = \frac{1}{2} (\hat{D}_b - \hat{D}_a) \neq 0$, preventing the generation of a second non-perturbative contribution to W .

This tension is clearly absent if the Kähler moduli are frozen by using only one non-perturbative effect. If this is supported by a del Pezzo divisor, then there is also no problem with chirality and the rigidity of this divisor guarantees the generation of W_{np} .

3. *D-term induced shrinking of the cycles supporting the visible sector*

The world-volume flux turned on to produce chirality generates also moduli-dependent Fayet-Iliopoulos (FI) terms:

$$\xi_a = \frac{1}{4\pi\mathcal{V}} \int_{D_a} J \wedge \mathcal{F}_a = \frac{1}{4\pi\mathcal{V}} q_{aj} t^j \quad \text{where} \quad q_{aj} \equiv k_{ajk} \mathcal{F}_a^k, \quad (3.6)$$

resulting in a total D-term potential which looks like:

$$V_D = \sum_a \frac{g_a^2}{2} \left(\sum_j \tilde{q}_{aj} |\phi_j|^2 - \xi_a \right)^2. \quad (3.7)$$

For vanishing VEVs of charged open string fields, $\langle \phi_j \rangle = 0$, the supersymmetric D-term conditions give rise to a system of a homogeneous linear equations $\xi_a = 0 \Leftrightarrow q_{aj} t^j = 0$ which might in general depend on all the $h^{1,1}$ Kähler moduli.

Notice that in order to avoid unwanted matter in the adjoint representation, the visible sector is generically built by wrapping D7-branes around rigid divisors. If this divisor is a del Pezzo T_{dP} , then one equation reduces just to $t_{\text{dP}} = 0$, forcing the shrinking of the four-cycle supporting the visible sector. Hence the EFT is driven to a regime where the supergravity approximation is not under good control. For this reason, the visible sector should be built on rigid but not del Pezzo divisors. On the other hand, as we have stressed above, a del Pezzo divisor should support the non-perturbative effects. Thus this four-cycle does not enter in the D-term equations which turn out to depend in general on $h^{1,1} - 1$ unknowns. In order to prevent the shrinking of some four-cycles, the brane configuration and the fluxes have to be chosen such that the rank d of the matrix of the system is $d < h^{1,1} - 1$, implying that the D-term conditions stabilise d combinations of Kähler moduli leaving $n_{\text{ax}} = h^{1,1} - 1 - d$ flat directions.

Notice that the combinations of two forms \tilde{c}_j , dual to the axions c_j , which couple to the anomalous U(1)s are also given by $q_{aj} \tilde{c}^j$. Therefore the d combinations of axions which are eaten up are the same as the d combinations of Kähler moduli which are fixed by the FI-terms.

As explained in [25], the type IIB LARGE Volume Scenario provides a promising way to address these issues, allowing the construction of explicit Calabi-Yau examples with magnetised D-branes and stabilised moduli which are chiral and globally consistent.

The scalar potential receives several contributions which have a different scaling with the exponentially large overall volume \mathcal{V} :

$$V = V_D + V_F^{\text{tree}} + V_F^{\text{np}} + V_F^{\text{p}}.$$

Let us briefly describe each of these contributions:

- V_D is D-term potential (3.7) which scales as $V_D \sim \mathcal{O}(\mathcal{V}^{-2})$. Given that it is the leading order effect in an expansion in inverse powers of \mathcal{V} , it is minimised supersymmetrically at first approximation by imposing $V_D = 0$. This condition fixes d combinations of Kähler moduli with $d < h^{1,1} - 1$. Therefore d axions are eaten up. Notice that if $n_{\text{ax}} = h^{1,1} - 1 - d = 1$, the D-term equations force all the moduli to be of the same size: $\tau_i = \tau_* \forall i = 1, \dots, h^{1,1} - 1$. In turn, τ_* is required to be small in order to obtain a visible sector gauge coupling $g_{\text{vs}}^{-2} \sim \tau_*$ of the observed order of magnitude. This observation then prevents the volume to be exponentially large. This is because, requiring that there is a standard model which does not wrap at least one “small” diagonal del-Pezzo divisor, if $d = h^{1,1} - 2$ then the large cycle will be fixed relative to the small cycles. This is not the case if $n_{\text{ax}} \geq 2$, and so we conclude that the simplest realisation of a standard LVS requires at least $n_{\text{ax}} = 2$, and so $h^{1,1} = 3 + d$, giving $h^{1,1} = 4$ for $d = 1$.³ Notice that this bound applies only for models with the visible sector in the geometric regime. For example, in the case of a canonical strong Swiss-cheese geometry, D-terms force the visible sector to the quiver locus, leaving just one light axion (the volume axion). The visible sector could be kept in the geometric regime by giving a VEV to a open string singlet, resulting again in at least two light axions: the volume axion and the phase of the open string mode.
- V_F^{tree} is the tree-level F-term scalar potential which also scales as $V_F^{\text{tree}} \sim \mathcal{O}(\mathcal{V}^{-2})$ but it is zero due to the ‘no-scale structure’.
- V_F^{np} scales as $V_F^{\text{np}} \sim \mathcal{O}(\mathcal{V}^{-3})$ and it is the non-perturbative scalar potential arising from corrections to the superpotential W coming from an E3-instanton or gaugino condensation on a D7-stack wrapping a single del Pezzo divisor T_{dP} :

$$W = W_0 + A e^{-a T_{\text{dP}}}. \tag{3.8}$$

This effect fixes T_{dP} at a small size giving it a mass of the order the gravitino mass. Hence the corresponding axion becomes very heavy [37]:

$$m_{a_{\text{dP}}} \sim m_{\tau_{\text{dP}}} \sim \frac{W_0 \sqrt{\ln \mathcal{V}}}{\mathcal{V}} M_P \sim m_{3/2} \ln \left(\frac{M_P}{m_{3/2}} \right). \tag{3.9}$$

³Note that the classic realisations of the LVS on a two-modulus strong swiss cheese have $n_{\text{ax}} = 1$, which violate our condition because there is no standard model cycle.

- V_F^p is the perturbative potential which is derived from the α' and g_s corrections to the Kähler potential K . This potential lifts all the n_{ax} flat directions left over by the D-terms and the non-perturbative effects, leaving all the corresponding axions massless. This is the origin of the ‘LVS axiverse’.

The α' correction to K is of the form [38]:

$$K = -2 \ln \left(\mathcal{V} + \frac{\zeta}{2g_s^{3/2}} \right) \simeq -2 \ln \mathcal{V} - \frac{\zeta}{g_s^{3/2} \mathcal{V}}, \quad (3.10)$$

with $\zeta \propto (h^{1,2} - h^{1,1})$. It gives rise to a scalar potential which scales as $V_F^{\alpha'} \sim \mathcal{O}(\mathcal{V}^{-3})$ and its interplay with W_{np} , for natural values $W_0 \sim \mathcal{O}(1)$, yields a supersymmetry-breaking AdS minimum at exponentially large volume [23]:

$$\mathcal{V} \sim W_0 e^{a \tau_{\text{dP}}}. \quad (3.11)$$

Consequently, the volume mode obtains a mass of the order [37]:

$$m_{\mathcal{V}} \sim \frac{W_0 M_P}{\mathcal{V}^{3/2}} \sim m_{3/2} \sqrt{\frac{m_{3/2}}{M_P}}, \quad (3.12)$$

and so it is much lighter than the gravitino. This can cause a CMP if $M_{\text{soft}} \sim m_{3/2}$ [37] whereas there is no problem in sequestered models where $M_{\text{soft}} \sim m_{3/2}/\mathcal{V} \ll m_{3/2}$ [35]. Notice that this vacuum can be uplifted to a dS solution within a manifestly supersymmetric EFT by using the positive contribution coming from E(-1)/D3 non-perturbative effects at a singularity [39].

On the other hand, g_s corrections to K are subleading with respect to the α' -corrections due to the ‘extended no-scale structure’ which forces their contribution to V to scale as $V_F^{g_s} \sim \mathcal{O}(\mathcal{V}^{-(3+p)})$ with $p > 0$ [40]. Hence the remaining $n_{\text{ax}} - 1$ flat direction can be lifted at subleading order either by open string loops (if the corresponding cycle intersects a cycle wrapped by a brane or the cycle itself is wrapped by a brane with vanishing flux so that the axion is not eaten up) [24] or by closed string loops [41].⁴

Tiny subdominant higher-order non-perturbative effects similar to the ones in (3.3) then slightly lift the n_{ax} axion directions via the generation of the following potential:

$$V_{W_{\text{np}}}(c_i) \simeq - \sum_{i=1}^{n_{\text{ax}}} \frac{n_i a_i \tau_i W_0}{\mathcal{V}^2} e^{-n_i a_i \tau_i} \cos(n_i a_i c_i). \quad (3.13)$$

Notice, however, that the axions can become massive also by non-perturbative corrections to the Kähler potential of the form $K_{\text{np}} \simeq \mathcal{V}^{-1} \sum_i e^{-n_i a_i T_i}$ which are not required to be

⁴If there are no branes wrapping the cycles which do not intersect any cycle wrapped by branes, then there is no open string localised on these cycles, and so the open string loop corrections should not depend on these moduli. However the closed string loop corrections can still depend on these moduli. See for example the short discussion in the paragraph around eq. (2.48) in [33] where there is an example of a cycle which does not intersect the SM cycle and is not wrapped by any D7-brane but still obtains closed string loop corrections.

invariant under the orientifold involution [15]. These effects generate a potential for the axions which looks like:

$$V_{K_{np}}(c_i) \simeq \frac{W_0^2}{\mathcal{V}^3} \sum_{i=1}^{n_{ax}} e^{-n_i a_i \tau_i} \cos(n_i a_i c_i) . \quad (3.14)$$

In [15] it was determined that the size of the potential generated by the above effects could only dominate over the QCD axion potential for a single instanton superpotential contribution but the presence of such an instanton is incompatible with chirality. On the other hand, the mass generated by a single Kähler potential contribution would be:

$$m_a \lesssim \frac{W_0}{\mathcal{V}} e^{-\pi \alpha_{QCD}^{-1}} M_P \sim m_{3/2} e^{-\pi \alpha_{QCD}^{-1}} , \quad (3.15)$$

where we used the fact that the relevant four-cycle volume for the QCD axion is $\tau_a = \alpha_{QCD}^{-1}$. This gives $m_a \lesssim \mathcal{O}(10^{-11})$ eV for $m_{3/2} \sim 100$ TeV and the suppression would be even greater for higher order effects. Hence we expect ALP masses to be typically smaller than those of the QCD axion.

For non-local axions, given that they correspond in general to non-rigid cycles, single non-perturbative effects are likely to be absent due to their zero-mode structure. However, even if they are present, they are suppressed with respect to perturbative corrections by a huge exponential factor. Two typical examples of non-local cycles whose non-perturbative effects would be minuscule are the volume mode of Swiss-cheese manifolds $\tau_b \sim \mathcal{V}^{2/3}$ where $W_{np} = A_b e^{-a T_b} \propto e^{-a \mathcal{V}^{2/3}} \ll 1$ — in fact even for $\mathcal{V} \simeq 10^4$ this factor is less than 10^{-600} — and the fibre modulus T_f of fibred Calabi-Yau constructions which might receive only poly-instanton contributions of the form $W_{np} = A_s e^{-a_s (T_s + A_f e^{-a_f T_f})} \Rightarrow W_{np}(T_f) \sim -a_s A_s A_f e^{-a_s T_s} e^{-a_f T_f} \ll 1$ [33, 42]. On the other hand, non-local axions may obtain small masses via a QCD-like effect in the hidden sector, as we shall discuss in section 3.2; these are of the order $m_{non-local} \sim \Lambda^2/M_P$, where Λ is the relevant strong coupling scale. These last two mass contributions are very model dependent.

In summary, generic non-local axions are effectively massless (although in certain cases some may obtain non-negligible masses, still typically smaller than local axion masses) while local axions obtain masses of the form:

$$m_{local} \sim e^{-n\pi\tau_{local}} \times \begin{cases} M_P & \text{Superpotential terms or QCD-like masses} \\ m_{3/2} & \text{Kähler potential terms} \end{cases} \quad (3.16)$$

Therefore the axions acquire a mass spectrum which is logarithmically hierarchical giving rise to the ‘LVS axiverse’. As pointed out above, the LVS requires $n_{ax} \geq 2$, implying that the simplest realisation of the LVS axiverse involves at least two light axions with a QCD axion candidate plus one ALP. More generic models for $h^{1,1}$ very large will include an arbitrarily large number of light axions.

3.2 The QCD axion in the LVS axiverse

In order to have a phenomenologically viable axiverse, it is crucial to identify potential QCD axion candidates among all the light axions. A very important observation related

to this task is the following. If we restore the QCD vacuum angle θ , we have:

$$\mathcal{L} \supset \frac{g_3^2}{32\pi^2} \left(\theta - \frac{a C_{a3}}{f_a} \right) \text{tr}(F_3 \wedge F_3), \quad (3.17)$$

where the coefficient C_{a3} gives the effective QCD anomaly. Since QCD is periodic under shifts of θ by 2π , we have the standard axion situation only if $C_{a3} \simeq \mathcal{O}(1)$. However, given that a only varies through $[0, 2\pi f_a)$, if $C_{a3} \ll 1$, like in some cases described in sections 2.3.1 and 2.3.2 where $C_{a3} \simeq \mathcal{O}(\mathcal{V}^{-\alpha}) \ll 1$ for $\alpha > 0$, a vacuum angle $\theta > 2\pi C_{a3} \simeq 2\pi \mathcal{V}^{-\alpha}$ cannot be absorbed into the expectation value of a .

This implies that *axions with a volume suppressed anomaly coefficient C_{a3} cannot solve the strong CP problem*, and so they cannot be the QCD axion. Let us investigate which axions can play the rôle of the QCD axion in the case when the SM is on a local or a non-local cycle.

3.2.1 SM on a local cycle

In this case the visible sector is localised on a small rigid divisor which in sections 2.3.1 and 2.3.2 we have denoted as τ_s . The local axions a_s are natural QCD axion candidates since they have $C_{ss} \simeq \mathcal{O}(1)$ and an intermediate scale decay constant of the order the string scale $f_{a_s} \simeq M_s/\sqrt{4\pi} \sim 10^{10}$ GeV for $\mathcal{V} \sim 10^{14}$ which gives rise to TeV-scale SUSY for $W_0 = 1$, or f_{a_s} somewhat larger for smaller W_0 . This has already been pointed out in [15] but without providing an explicit chiral example where the local axion is not eaten up by an anomalous U(1). In sections 4.3 and 4.4 we shall present two explicit examples where the SM is on a local cycle and the QCD axion is given by a local axion which is not eaten up by any anomalous U(1) and does not receive large mass contributions from single instanton effects.

This is a crucial issue since non-local axions, like the volume axion a_b of section 2.3.1, cannot be the QCD axion when the standard model is built on a local cycle. In fact, consider the case where the local axion c_s coupling to QCD is eaten by an anomalous U(1), and the only other axion that can couple is the non-local one c_b . The theory can be written as:

$$\begin{aligned} \mathcal{L} \simeq & \frac{1}{8\mathcal{V}} \left(\partial_\mu c_s + \frac{M_P}{\pi} q A_\mu \right)^2 + \left(\partial_\mu c_s + \frac{M_P}{\pi} q A_\mu \right) \frac{\partial^\mu c_b}{4\mathcal{V}^{5/3}} + \frac{(\partial_\mu c_b)^2}{8\mathcal{V}^{4/3}} \\ & + \frac{g^2}{4\pi M_P} c_s \text{tr}(F \wedge F), \end{aligned} \quad (3.18)$$

which can be diagonalised via:

$$c_b \simeq 2\mathcal{V}^{2/3} a_b, \quad c_s \simeq 2\mathcal{V}^{1/2} a_s - 2a_b, \quad (3.19)$$

giving:

$$\mathcal{L} \simeq \frac{1}{2} \left(\partial_\mu a_s + \frac{M_P}{2\sqrt{\mathcal{V}}\pi} q A_\mu \right)^2 + \frac{1}{2} (\partial_\mu a_b)^2 + \frac{2g^2}{4\pi M_P} (\sqrt{\mathcal{V}} a_s - a_b) \text{tr}(F \wedge F).$$

We can then make a chiral rotation of the fermions charged under the massive U(1) to eliminate the axion a_s at low energies, leaving only $\frac{1}{2}(\partial_\mu a_b)^2 - \frac{2g^2}{4\pi M_P} a_b \text{tr}(F \wedge F)$.

Notice that the axion a_s , which is eaten up, corresponds to the axion c_s , which is gauged, only at leading order since the kinetic terms induce also a subleading mixing with c_b . In fact, the canonical normalisation (3.19) implies:

$$2\mathcal{V}^{1/2}a_s \simeq c_s + \mathcal{V}^{-2/3}c_b. \quad (3.20)$$

It is this subleading mixing that induces the coupling of a_b to QCD. However, as can be seen in (2.25), the non-local axion a_b has $C_{b_s} \simeq \mathcal{O}(\mathcal{V}^{-3/2}) \ll 1$, and so it cannot play the rôle of the QCD axion. Moreover, there is necessarily an anomalous *global* U(1) symmetry left over. When this remaining global symmetry is spontaneously broken, an additional axion will be introduced, and only one linear combination of this new axion and a_b is made massive by QCD.

If, on the other hand, the strongly coupled gauge group was not QCD, and the global U(1) was not spontaneously broken (but merely anomalous - and hence broken to a discrete subgroup by the strong coupling effect) then a_b could obtain a mass, of order

$$m_{a_b} \sim \Lambda^2/M_P \quad (3.21)$$

where Λ is the strong coupling scale. The dark matter density produced through misalignment would be set by sending $f \rightarrow M_P$, $\Theta_i \rightarrow \Theta_i/\mathcal{V}^{2/3}$ (where Θ_i is the initial misalignment angle) in the usual axion calculation, and would therefore be comfortably tiny. This is at first rather puzzling, since the initial Lagrangian is independent of the expectation value of c_b , and $a_b \propto c_b$. However, we note that a shift of a_b induces a shift in c_s - and this is where the mass generation arises. That both axions become massive appears because we have two different mass terms - and a *mass mixing* term (the combination eaten by the gauge field is a_s , which depends on both c_s and c_b).

On the other hand, in the presence of a non-perturbative superpotential of the form:

$$W_{np} = A_s e^{-a_s T_s} + A_b e^{-a_b T_b}, \quad (3.22)$$

the local axion c_s is made more massive than the QCD axion, while the non-local axion c_b develops a non-perturbative potential only at subleading order and its coupling to QCD is extremely small. In fact, the Lagrangian:

$$\mathcal{L} \simeq \frac{(\partial_\mu c_s)^2}{8\mathcal{V}} + \frac{\partial_\mu c_s \partial^\mu c_b}{4\mathcal{V}^{5/3}} + \frac{(\partial_\mu c_b)^2}{8\mathcal{V}^{4/3}} + \frac{g^2}{4\pi M_P} c_s \text{tr}(F \wedge F) - V(c_s, c_b) \quad (3.23)$$

would be diagonalised by a transformation which looks like:

$$\frac{c_b}{\tau_b} \simeq \mathcal{O}(1) a_b + \mathcal{O}(\mathcal{V}^{-1/2}) a_s, \quad (3.24)$$

$$\frac{c_s}{\tau_s} \simeq \mathcal{O}(\mathcal{V}^{1/2}) a_s + \mathcal{O}(e^{-a_b \tau_b}) a_b, \quad (3.25)$$

giving:

$$\mathcal{L} \sim \frac{1}{2} \left[(\partial_\mu a_s)^2 + (\partial_\mu a_b)^2 \right] + \frac{g^2}{2\pi M_P} \left(\sqrt{\mathcal{V}} a_s + \mathcal{O}(e^{-a_b \tau_b}) a_b \right) \text{tr}(F \wedge F) - V(a_s, a_b),$$

and so the coupling of a_b to $F \wedge F$ is suppressed by $e^{-a_b \tau_b} \ll 1$ (less than 10^{-600} for $\mathcal{V} \simeq 10^4$) for $\tau_b \simeq \mathcal{V}^{2/3} \gg 1$. This implies that the anomaly coefficient C_{bs} in this case would scale as $C_{bs} \sim e^{-a_b \tau_b} \mathcal{V}^{-2/3} \ll 1$, and so it is in practice zero for all intents and purposes. Such a light non-local axion would effectively produce no dark matter.

Thus we have shown that non-local axions cannot behave as the QCD axion if the SM is on a local cycle. There is however an exception to this conclusion. If the extra dimensions are anisotropic, as shown in (2.28), the fibre axion a_f has $C_{fs} \simeq \mathcal{O}(1)$, and so it can be the QCD axion in the presence of a dilution mechanism as its decay constant is of order $f_{a_f} \simeq M_P/\tau_s \sim M_{\text{GUT}} \sim 10^{16}$ GeV.

3.2.2 SM on a non-local cycle

In this case the SM is localised on a divisor which controls the overall size of the Calabi-Yau, and so non-local axions can play the rôle of the QCD axion. In fact, considering the Swiss-cheese example of section 2.3.1 or isotropic fibred manifolds of section 2.3.2, the visible sector is now supported on τ_b . As can be seen from (2.25) and (2.28), now $C_{bb} \simeq \mathcal{O}(1)$, and so the non-local axion a_b can now be the QCD axion. However, given that the visible sector gauge coupling scales as $g_{\text{vs}}^{-2} \simeq \tau_b$, the volume $\mathcal{V} \simeq \tau_b^{3/2}$ has to be set of the order $\mathcal{V} \sim 10^4$, while W_0 has to be fine-tuned small to obtain TeV-scale soft terms. In turn, the axion decay constant $f_{a_b} \sim M_P/\mathcal{V}^{2/3} \sim M_{\text{GUT}} \sim 5 \cdot 10^{15}$ GeV, and so a dilution mechanism is needed. We shall present an explicit example of this situation in section 4.1.

Alternatively, if the Calabi-Yau has an anisotropic shape, the SM can be localised on the fibre divisor τ_f and the volume can be set exponentially large $\mathcal{V} \sim 10^{14}$ without any need to tune W_0 since the visible sector gauge coupling does not depend on the overall volume $g_{\text{vs}}^{-2} \simeq \tau_f \sim \tau_s$. Moreover, as shown in (2.28), the anomaly coefficient of the fibre axion a_f is of order unity, $C_{ff} \simeq \mathcal{O}(1)$, allowing a_f to play the rôle of the QCD axion provided that it can be diluted by some mechanism as $f_{a_f} \sim M_P/\tau_s \sim M_{\text{GUT}} \sim 10^{16}$ GeV. We shall present an explicit example of this situation in section 4.2.

3.3 Cosmology of the LVS axiverse

The cosmology of the LVS axiverse takes a very different form depending on whether the value of the Hubble scale during inflation H_{inf} is larger or smaller than the gravitino mass $m_{3/2}$. The standard case in string cosmology is $H_{\text{inf}} \leq m_{3/2}$, creating a well-known tension between inflation and supersymmetry breaking since the requirement of generating enough density perturbations sets, in general, $H_{\text{inf}} \sim M_{\text{GUT}}$, forcing $m_{3/2}$ to be well above the TeV scale [43]. Given that the moduli tend to develop a mass of the order $m_{\text{mod}} \sim m_{3/2}$, the cosmological moduli problem and its solution are closely related to the above mentioned tension between inflation and TeV-scale supersymmetry. The best available solutions to this problem are:

1. The value of $m_{3/2}$ after inflation is very different from the value of $m_{3/2}$ during inflation, so that the relation $H_{\text{inf}} \leq m_{3/2}$ does not hold today where we can instead have $m_{3/2}^{\text{today}} < H_{\text{inf}}$ allowing TeV-scale SUSY for $\mathcal{V} \sim 10^{14}$. This is achieved if the

volume mode is the inflaton [44]. However this model is very fine-tuned and reheating is a big problem since the inflaton at the end of inflation becomes extremely light.

2. The value of $m_{3/2}$ after inflation is the same as the value of $m_{3/2}$ during inflation but one has a sequestered scenario where the soft terms are suppressed with respect to $m_{3/2}$ by some power of the volume [35]. Hence M_{soft} can be of order TeV even if $m_{3/2}$ is very large. The most promising sequestered models have $M_{\text{soft}} \sim m_{3/2}^2/M_P \sim M_P/\mathcal{V}^2$, and so TeV-scale SUSY requires $\mathcal{V} \sim 10^7$ instead of the ‘traditional’ $\mathcal{V} \sim 10^{14}$.

Let us now analyse the two different cases separately.

3.3.1 Case $H_{\text{inf}} > m_{3/2}$

In this case, as we have pointed out above, we can set $\mathcal{V} \sim 10^{14}$, so to obtain $M_{\text{soft}} \sim m_{3/2} \sim \mathcal{O}(\text{TeV})$,⁵ and the string scale turns out to be intermediate $M_s = M_P/\sqrt{4\pi\mathcal{V}} \sim 5 \cdot 10^{10}$ GeV. This is the case considered in subsection 3.2.1, and found in sections 4.3 and 4.4 where the SM is on a local cycle and the QCD axion is given by a local axion. We stress again that in the literature there is no explicit inflationary model with a successful reheating process that realises this scenario [45, 46]. We shall however assume that this is possible and proceed to analyse the cosmological implications.

1. The complex structure moduli and the dilaton do not play any significant cosmological rôle since they are trapped at their minima, i.e. their initial displacement is tiny [47].
2. The non-local Kähler moduli are all very light and suffer from the CMP. In particular, the mass of the volume mode is of the order $m_{\tau_b} = m_{3/2}/\sqrt{\mathcal{V}} \simeq 0.1$ MeV, while that of the fibre modulus is $m_{\tau_f} = m_{3/2}/\mathcal{V}^{2/3} \simeq 0.5$ keV for $m_{3/2} = \kappa M_P/\mathcal{V} \simeq 1$ TeV, where $\mathcal{V} = 2 \cdot 10^{14}$ and $\kappa \equiv \sqrt{g_s/(4\pi)}W_0$ with $g_s = 0.1$ and $W_0 = 1$.
3. The non-local axions associated with the base and the fibre divisors behave as very light and almost decoupled ALPs with a decay constant of the order the 10D KK scale, $f_{a_b} \sim f_{a_f} \sim M_{\text{KK}}^{10\text{D}} \sim M_P/(4\pi\mathcal{V}^{2/3}) \sim 10^8$ GeV. Notice that these non-local ALPs do not create problems with dark matter overproduction since they are very light and their decay constants are very low (see appendix B.2).
4. The del Pezzo modulus τ_{dP} and the corresponding axion a_{dP} fixed by non-perturbative effects, obtain a large mass of the order $m_{\tau_{\text{dP}}} \sim m_{a_{\text{dP}}} = m_{3/2} \ln \mathcal{V} \simeq 35$ TeV. However this modulus suffers from the CMP since it has a very small coupling with the SM, of the order $1/(M_P\sqrt{\mathcal{V}})$ [45, 46], reflecting the fact that this cycle blows up a point which is geometrically separated from the point blown up by the SM cycle. Thus τ_{dP} and a_{dP} would decay after BBN at a temperature of the order:

$$T_{\tau_{\text{dP}}} \sim T_{a_{\text{dP}}} \simeq \sqrt{\Gamma_{\tau_{\text{dP}}} M_P} \simeq \left(\frac{m_{\tau_{\text{dP}}}}{M_P \mathcal{V}} \right)^{1/2} m_{\tau_{\text{dP}}} \simeq 0.1 \text{ eV}. \quad (3.26)$$

⁵Due to the fact that the SM cycle is stabilised perturbatively, we expect $M_{\text{soft}} \sim m_{3/2}$ instead of the suppressed case $M_{\text{soft}} \sim m_{3/2}/\ln(M_P/m_{3/2})$ which is typical of scenarios with the SM cycle fixed by non-perturbative effects.

Notice that τ_{dP} and a_{dP} would have a much stronger coupling, of the order $1/M_s$, to hidden sector degrees of freedom living on this del Pezzo divisor, and so the CMP would be solved if they could decay to hidden particles. However, if the hidden sector consists of an E3-instanton or a pure $\mathcal{N} = 1$ SYM theory that develops a mass gap, no light hidden degree of freedom is present.

5. Some of the local Kähler moduli supporting the SM are fixed by the D-terms which induce masses of the order M_s , and so these moduli disappear from the EFT. The directions left massless by the D-terms (the ones corresponding to the uneaten axions) are instead fixed by string loops and obtains a mass of the order $m_{\tau_{\text{SM}}} \simeq \alpha_{\text{SM}} m_{3/2} \simeq 40 \text{ GeV}$ for $\alpha_{\text{SM}} \simeq 1/25$. This scaling can be understood from the fact that the g_s potential is of the order $V_{g_s}(\tau_{\text{SM}}) \simeq 1/(\mathcal{V}^3 \sqrt{\tau_{\text{SM}}})$ [25, 40], while the mass can be estimated as $m_{\tau_{\text{SM}}}^2 \sim K_{\tau_{\text{SM}}, \tau_{\text{SM}}}^{-1} V_{\tau_{\text{SM}}, \tau_{\text{SM}}} \sim 1/(\mathcal{V} \tau_{\text{SM}})^2 \simeq (m_{3/2} \alpha_{\text{SM}})^2$. These moduli couple locally as $1/M_s$ but, as shown in [27], they decay before dominating the energy density. Hence their decay does not reheat the Universe and cannot dilute any unwanted relic. Therefore the best way to solve the CMP seems to be the presence of a late time period of thermal inflation [48].
6. Not all the local closed string axions are eaten up, and so one of them can be the QCD axion with an intermediate scale decay constant $f_{a_s} \sim M_s/\sqrt{4\pi} \sim 10^{10} \text{ GeV}$ and an order unity anomaly coefficient $C_{ss} \sim \mathcal{O}(1)$. Notice, however, that this axion does not constitute a relevant component of dark matter, unless one moderately fine-tunes the parameters such that the string scale and correspondingly the decay constant is increased to $f_{a_s} \sim M_s/\sqrt{4\pi} \sim 10^{12} \text{ GeV}$ (see appendix B.2). Thus, the constraints from isocurvature fluctuations (see appendix B.2.2) do not apply to this model.

Different considerations apply to the case considered in subsection 3.2.2 and found in section 4.1 where the SM is supported on the non-local cycle τ_b controlling the overall volume of the Calabi-Yau. As we have already stressed, the volume cannot be set very large, otherwise the visible sector gauge coupling would be too small, whereas W_0 has to be fine tuned small in order to obtain TeV-scale soft terms. For $\mathcal{V} \simeq 10^4$, $g_s \simeq 0.1$ and $W_0 \simeq 1.5 \cdot 10^{-10}$, we would obtain $M_{\text{soft}} \sim m_{3/2} \simeq 3 \text{ TeV}$. The string scale turns out to be quite high: $M_s \sim 5 \cdot 10^{15} \text{ GeV}$. Let us present a brief analysis of the cosmology of this scenario:

1. The non-local Kähler modulus τ_b is light and suffers from the CMP given that its mass is of the order $m_{\tau_b} = m_{3/2}/\sqrt{\mathcal{V}} \simeq 30 \text{ GeV}$.
2. The non-local axion a_b associated with the large cycle can play the rôle of the QCD axion since $C_{bb} \simeq \mathcal{O}(1)$. Its decay constant is however rather high $f_{a_b} \sim M_P/(4\pi\mathcal{V}^{2/3}) \sim 5 \cdot 10^{14} \text{ GeV}$, and so it would naively lead to dark matter overproduction (without tuning the initial misalignment angle, or, as is in fact the case, entropy dilution).
3. The del Pezzo modulus τ_{dP} and the corresponding axion a_{dP} fixed by non-perturbative effects obtain a larger mass of the order $m_{\tau_{\text{dP}}} \sim m_{a_{\text{dP}}} = m_{3/2} \ln \mathcal{V} \simeq$

30 TeV. Hence, given that this modulus has an ordinary Planck-strength coupling to the SM, it does not suffer from the CMP since it would decay at a temperature of the order:

$$T_{\tau_{\text{dP}}} \sim T_{a_{\text{dP}}} \simeq \sqrt{\Gamma_{\tau_{\text{dP}}} M_P} \simeq \left(\frac{m_{\tau_{\text{dP}}}}{M_P} \right)^{1/2} m_{\tau_{\text{dP}}} \simeq 5 \text{ MeV}, \quad (3.27)$$

which is above the BBN temperature but below the freeze-out temperature of thermal dark matter. Thus, dark matter will be produced again non-thermally by the decay of τ_{dP} which will also dilute the modulus τ_b suffering from the CMP and the QCD axion a_b . Contrary to the previous case, a_b would constitute a relevant component of dark matter, and so the constraints from isocurvature fluctuations would apply to this model. Conventional haloscope searches exploiting microwave cavities, however, would not be sensitive to a_b dark matter due to its tiny mass and extremely small coupling. Molecular interferometry may be a future option to search for it if one allows for a bit of fine-tuning in W_0 such that $f_{a_b} \simeq 10^{16}$ GeV.

We finally briefly analyse the model of section 4.2, where the Calabi-Yau has an anisotropic shape and the SM is localised on the fibre divisor τ_f . In this case the visible sector gauge coupling does not depend on the overall volume $g_{\text{vs}}^{-2} \simeq \tau_f \sim \tau_s$, and so we can set $\mathcal{V} \simeq 10^{14}$ and $W_0 \simeq \mathcal{O}(1)$. The two non-local Kähler moduli, τ_f and τ_b are very light, with masses again of the order $m_{\tau_b} \simeq 1 \text{ MeV}$ and $m_{\tau_f} \simeq 1 \text{ keV}$. The axion of the fibre divisor, a_f , behaves as the QCD axion since $C_{ff} \simeq \mathcal{O}(1)$, but its decay constant is very high, $f_{a_f} \simeq M_P/(4\pi\tau_f) \simeq 10^{16} \text{ GeV}$ for $\tau_f \simeq \mathcal{O}(10)$, even if the string scale is intermediate. The axion of the base divisor, a_b , is a very light and decoupled ALP ($C_{bf} \sim \mathcal{V}^{-2}$) with $f_{a_b} \simeq M_P/(4\pi\tau_b) \simeq 5 \text{ TeV}$ for $\tau_b = \mathcal{V}/\sqrt{\tau_f} \simeq 5 \cdot 10^{13}$. The del Pezzo modulus and its axion have a mass of the order $m_{\tau_{\text{dP}}} \sim m_{a_{\text{dP}}} \simeq 35 \text{ TeV}$ and couple to the SM as $1/M_P$. Therefore they would decay at $T_{\tau_{\text{dP}}} \sim T_{a_{\text{dP}}} \simeq 5 \text{ MeV}$, producing non-thermal dark matter and diluting the abundance of both the light moduli and the QCD axion.

It is worth pointing out that the cosmology of an axiverse with a high string scale was also discussed in [22, 26]. As we have seen, our scenarios have some similarities and crucial differences to that case.

3.3.2 Case $H_{\text{inf}} \leq m_{3/2}$

In this case, inflation can be reconciled with TeV-scale SUSY if the SM is sequestered from the SUSY-breaking sector. This can be achieved if the SM is built via fractional D3-branes at a singularity obtained by shrinking a blow-up divisor. The most promising scenario involves the following hierarchy $M_{\text{soft}} \simeq m_{3/2}/\mathcal{V}$ which leads to $M_{\text{soft}} \simeq 2 \text{ TeV}$ for $m_{3/2} \simeq 10^{10} \text{ GeV}$ with $g_s = 0.1$, $W_0 = 1$ and $\mathcal{V} \simeq 10^7$. This is also the value of the volume which gives the right amount of density perturbations in the inflationary model of [51] where the inflaton is a blow-up mode fixed by non-perturbative effects. Therefore, contrary to the case $H_{\text{inf}} > m_{3/2}$, we now have a more robust construction with an explicit realisation of inflation and reheating [45, 46]. Notice that the string scale turns out to be relatively high: $M_s = M_P/\sqrt{4\pi\mathcal{V}} \simeq 10^{14} \text{ GeV}$.

Let us now present an outline analysis of the cosmological implications of this scenario.

1. The del Pezzo modulus τ_{dP} , and the corresponding axion a_{dP} fixed by non-perturbative effects, play the rôle of the inflaton and are very light during inflation. After the end of inflation, when the classical condensate oscillates coherently around the minimum, they obtain a large mass of the order $m_{\tau_{\text{dP}}} \sim m_{a_{\text{dP}}} = m_{3/2} \ln \mathcal{V} \simeq 5 \cdot 10^{11}$ GeV. Given that these moduli couple to the SM as $1/(M_P \sqrt{\mathcal{V}})$, their perturbative decay gives rise to reheating at a temperature:

$$T_{\tau_{\text{dP}}} \sim T_{a_{\text{dP}}} \simeq \sqrt{\Gamma_{\tau_{\text{dP}}} M_P} \simeq \left(\frac{m_{\tau_{\text{dP}}}}{M_P \mathcal{V}} \right)^{1/2} m_{\tau_{\text{dP}}} \simeq 40 \text{ TeV}. \quad (3.28)$$

Notice that τ_{dP} and a_{dP} have a much stronger coupling, of the order $1/M_s$, to hidden sector degrees of freedom living on this del Pezzo divisor, and so they would dump all their energy to hidden, instead of visible, particles. However, this problem is absent if the hidden sector consists of an E3-instanton or a pure $\mathcal{N} = 1$ SYM theory that develops a mass gap since no light hidden degree of freedom would be present [45, 46].

2. The non-local Kähler moduli are all very heavy in this case and do not suffer from the CMP. In fact, the mass of the volume and fibre moduli are $m_{\tau_b} = m_{3/2}/\sqrt{\mathcal{V}} \simeq 5 \cdot 10^6$ GeV and $m_{\tau_f} = m_{3/2}/\mathcal{V}^{2/3} \simeq 500$ TeV. Given that they both couple to the SM with Planck-strength they would decay respectively at:

$$T_{\tau_b} \simeq \sqrt{\Gamma_{\tau_b} M_P} \simeq \left(\frac{m_{\tau_b}}{M_P} \right)^{1/2} m_{\tau_b} \simeq 10 \text{ GeV}, \quad (3.29)$$

and:

$$T_{\tau_f} \simeq \sqrt{\Gamma_{\tau_f} M_P} \simeq \left(\frac{m_{\tau_f}}{M_P} \right)^{1/2} m_{\tau_b} \simeq 100 \text{ MeV}. \quad (3.30)$$

Since T_{τ_f} is above $T_{\text{BBN}} \simeq \mathcal{O}(1)$ MeV but below $T_{\text{freeze-out}} \sim \mathcal{O}(10)$ GeV, non-thermal dark matter will be produced by the decay of τ_f which will also dilute the abundance of any axion-like particle.⁶

3. If the Calabi-Yau is isotropic, the non-local axions associated with the base and the fibre divisors cannot be the QCD axion and behave as very light and almost decoupled ALPs with a decay constant of the order the 10D KK scale, $f_{a_b} \sim f_{a_f} \sim M_{\text{KK}}^{10\text{D}} \sim M_P/(4\pi \mathcal{V}^{2/3}) \sim 5 \cdot 10^{12}$ GeV. Notice that these non-local ALPs do not create problems with dark matter overproduction since they are very light and their decay constants are very low. On the other hand, if the Calabi-Yau is anisotropic, the fibre axion a_f can be the QCD axion since $C_{ff} \simeq \mathcal{O}(1)$, but its decay constant is very high: $f_{a_f} \simeq 10^{16}$ GeV. However this is not a problem since a_f will be diluted by the decay of the non-local Kähler moduli. The axion of the base divisor, a_b , is instead again a very light and decoupled ALP.
4. In the case of fractional D3-branes at del Pezzo singularities, all the local closed string axions are eaten up, and so cannot be the QCD axion. Correspondingly, all

⁶Ref. [49, 50] showed that the decay of τ_b gives rise to dark radiation made of light volume axions.

the local Kähler moduli are fixed by D-terms, and so obtain masses of the order M_s and disappear from the EFT. In fact, each dP_n has n two-cycles which can be trivial or non-trivial within the Calabi-Yau plus one two-cycle corresponding to the canonical class which is always also a two-cycle of the Calabi-Yau dual to the del Pezzo divisor. Moreover each dP_n has exactly two anomalous U(1)s which become massive by eating the axions given by the reduction of C_2 on the canonical two-cycle and C_4 on the del Pezzo divisor [52]. Given that these are the only two local cycles, all the local closed string axions are eaten up. In fact, if some of the n two-cycles are non-trivial within the Calabi-Yau, they would correspond to non-local cycles, and so to non-local axions. Two possible way-outs could be the following:

- Consider more complicated singularities whose resolution involves more than two local cycles. In this case, some local closed string axions might be left over, and they could play the rôle of the QCD axion. The corresponding decay constant would be rather high, $f_{a_s} \simeq M_s/\sqrt{4\pi} \simeq 5 \cdot 10^{13}$ GeV, but the abundance of the QCD axion would be diluted by the decay of the non-local Kähler moduli.
- Focus on the phase of an open string axion ϕ to realise the QCD axion in a more model-dependent way. In this case, one should check that the D-terms, which give a VEV to the radial part of ϕ that sets the axion decay constant, do not resolve the singularity obtained by setting the Fayet-Iliopoulos term ξ to zero. In fact, the D-term potential used to fix the blow-up mode τ_{blow} at the singularity looks like:

$$V_D \simeq g^2 (|\phi|^2 - \xi)^2, \quad \text{with} \quad \xi = \frac{\tau_{\text{blow}}}{\mathcal{V}}.$$

Hence, if $\langle |\phi| \rangle = 0$, $V_D = 0$ for $\xi = 0$ implying $\langle \tau_{\text{blow}} \rangle = 0$. However, if ϕ is a SM singlet which can acquire a non-zero VEV, $V_D = 0$ would set $\langle |\phi| \rangle = \sqrt{\xi} \simeq \langle \sqrt{\tau_{\text{blow}}} \rangle M_s$, creating a tension between $\langle \tau_{\text{blow}} \rangle = 0$ and $\langle |\phi| \rangle \neq 0$. However τ_{blow} could still have a non-zero VEV, as long as it is below the string scale, since we would still be at the quiver locus if $\langle \tau_{\text{blow}} \rangle = \mathcal{V}^{-2\alpha}$ with $\alpha > 0$. If this kind of VEV were induced by the F-terms, the VEV of $|\phi|$ would become $\langle |\phi| \rangle \simeq M_s/\mathcal{V}^\alpha$. Given that this VEV also sets the axion decay constant f_a , the volume suppression factor might bring f_a at the intermediate scale. At that point, the QCD axion would not form a relevant component of dark matter due to the dilution induced by the decay of the non-local Kähler moduli, and the constraints from isocurvature fluctuations would not apply to this case.

4 Explicit LVS models

Not all the axions of the examples studied in section 2.3 will survive in the EFT since some of them will become heavy due to non-perturbative effects whereas others will be eaten up by anomalous U(1)s. According to the general discussion of the axiverse and moduli stabilisation performed in section 3.1, only $n_{\text{ax}} = h^{1,1} - d - 1$ axions remain light. Moreover, the LVS requires $n_{\text{ax}} \geq 2$ guaranteeing the presence of a QCD axion candidate

and at least one ALP. However, the couplings of the n_{ax} light axions depends crucially on the Calabi-Yau topology and the particular choice of brane set-up and fluxes. In particular, it is important to determine under what circumstances one of these n_{ax} light axions can have an intermediate scale decay constant.

This axion should be associated to a local blow-up mode but if we consider the simplest case of a GUT model with $SU(5)$ that lies on a diagonal del Pezzo four-cycle τ_{dP} that does not intersect any other divisor, this axion is definitely eaten up. In fact, even if the D-term induced shrinking of τ_{dP} is avoided by giving a non-zero VEV to a SM singlet, since $SU(5)$ descends from a $U(5)$ gauge group, we must make the $U(1) \subset U(5)$ massive via a diagonal flux upon the stack, which gauges the axion corresponding to τ_{dP} .

The way to circumvent the above issue is to introduce additional local cycles that intersect the GUT/MSSM stack. This, then, introduces the additional problem of how to stabilise the corresponding moduli. In [15], this was performed by an instanton superpotential that included two blow-up moduli, and then their relative sizes were fixed by loop corrections, leaving a massless axion. However, there may be difficulties with that approach due to chirality, since it required an instanton coupling to the SM cycle. Moreover, the massless axion left over is likely to be eaten up as in the example above with a single del Pezzo divisor.

We shall instead advocate a different strategy, where D-term constraints are used to fix the relative sizes of local cycles, and then the overall size of these is fixed by loop corrections following the general strategy outlined in [25]. In this way, we are sure that the massless axion left over is definitely not eaten up by any anomalous $U(1)$, and so it can play the rôle of QCD axion with an intermediate scale decay constant.

On top of these local axions, as we discussed in previous sections, the LVS ensures the presence of at least one additional non-local axion associated to the overall volume. This axion is also a QCD axion candidate, albeit with a GUT-scale decay constant. Due to the large size of this decay constant (and also of the decay constants of other possible non-local ALPs), severe constraints from overproduction of dark matter have to be considered with care (for a complete set of constraints upon such ALPs see [6]). However, as explained in section 3.3, these constraints can be avoided in the presence of a late out-of-equilibrium decay of heavy moduli which dilute the non-local axions.

In the following sections, we shall discuss several explicit examples with different topologies and choices of brane set-up and fluxes which lead to different phenomenological features:

- We shall first focus on a fibred example with $h^{1,1} = 4$ taken from [25].
 - In section 4.1 we shall analyse an isotropic GUT-like model with $d = 2$ D-term constraints leading to $n_{\text{ax}} = 1$ light axion with $f_a \simeq M_{\text{KK}}^{10\text{D}}/(4\pi) \simeq 5 \cdot 10^{14}$ GeV which is associated to the overall volume. As we have seen in section 3.3, this axion can be the QCD axion in the presence of a dilution mechanism (or tuning of the initial misalignment angle).
 - On the other hand, in section 4.2, we shall study an anisotropic $SU(3) \times SU(2)$ model with $d = 1$ D-term equation leading to $n_{\text{ax}} = 2$ light axions: the fibre

axion a_f with $f_{a_f} \simeq 10^{16}$ GeV, and the base axion a_b with $f_{a_b} \simeq M_{\text{KK}}^{6\text{D}}/(4\pi) \simeq 5$ TeV. As explained in section 3.3, a_f can be the QCD axion in the presence of a dilution mechanism, whereas a_b is a very light and almost decoupled ALP.

- In section 4.3 we shall then consider another fibred Calabi-Yau example with $h^{1,1} = 5$ taken from [53], which allows a chiral model with $n_{\text{ax}} = 3$ light axions. One of them is a local axion a_s which plays the rôle of the QCD axion with an intermediate scale decay constant: $f_{a_s} \simeq M_s/\sqrt{4\pi} \sim 10^{10}$ GeV, whereas the other two light axions, a_f and a_b , are non-local and have $f_{a_f} \sim f_{a_b} \simeq M_{\text{KK}}^{10\text{D}}/(4\pi) \sim 10^8$ GeV. They behave as very light and almost decoupled ALPs.
- Finally, in section 4.4, we shall outline the general conditions under which the EFT can have more than one light axion with an intermediate scale decay constant.

4.1 A GUT-like model with a QCD axion with large decay constant

Here we shall study the model of section 4 of [25] and derive the low-energy properties of the closed string axions. This type IIB model involves intersecting magnetised D7-branes, tadpole and Freed-Witten anomaly cancellation and all the geometric moduli stabilised in a way compatible with the presence of chirality. The internal space is an orientifold of a K3 fibred Calabi-Yau three-fold with $h_+^{1,1} = 4$ and $h_-^{1,1} = 0$ realised as a hypersurface embedded in a toric variety. The relevant divisors for model building are the K3 fibre D_1 , two intersecting rigid four-cycles D_4 and D_5 and a del Pezzo divisor D_7 . Expanding the Kähler form J in this basis as:

$$J = t_1 \hat{D}_1 + t_4 \hat{D}_4 + t_5 \hat{D}_5 + t_7 \hat{D}_7, \tag{4.1}$$

the only non-vanishing intersection numbers are:

$$k_{145} = k_{447} = k_{777} = 2, \quad k_{155} = k_{444} = k_{455} = k_{477} = -2, \quad k_{555} = 4, \tag{4.2}$$

and so the overall volume reads:

$$\mathcal{V} = \frac{1}{6} \sum_{i,j,k} k_{ijk} t_i t_j t_k = (t_1 - t_5) (2t_4 - t_5) t_5 + t_4 t_5^2 - \frac{1}{3} t_5^3 - \frac{1}{3} (t_4 - t_7)^3. \tag{4.3}$$

The volume of the basis divisors, which are the real part of the Kähler moduli $T_j = \tau_j + i c_j$, are given by:

$$\begin{aligned} \tau_1 &= (2t_4 - t_5) t_5, & \tau_4 &= 2t_1 t_5 - t_5^2 - (t_4 - t_7)^2, \\ \tau_5 &= 2(t_1 - t_5) (t_4 - t_5), & \tau_7 &= (t_4 - t_7)^2. \end{aligned} \tag{4.4}$$

The \mathbb{P}^1 base of the fibration is given by the intersection of the two rigid cycles D_4 and D_5 :

$$t_{\text{base}} = \text{Vol}(\mathbb{P}^1) = \text{Vol}(D_4 \cap D_5) = \sum_i k_{45i} t_i = 2(t_1 - t_5), \tag{4.5}$$

and so the expression for the volume (4.3) can also be rewritten as:

$$\mathcal{V} = \frac{1}{2} t_{\text{base}} \tau_1 + t_4 t_5^2 - \frac{1}{3} t_5^3 - \frac{1}{3} \tau_7^{3/2}. \tag{4.6}$$

The GUT-like model is realised by wrapping $N_a = 5$ D7-branes around D_4 and $N_b = 2$ D7-branes around D_5 (this number has to be even in order to cancel the K-theory charges). Given that each divisor is transversally invariant under the orientifold, we obtain symplectic gauge groups $\text{Sp}(2N_a)$ and $\text{Sp}(2N_b)$ which are broken down to unitary groups by switching on appropriate magnetic fluxes \mathcal{F}_a and \mathcal{F}_b on the world-volume of the D7-branes, so that $\text{Sp}(10) \rightarrow \text{SU}(5) \times \text{U}(1)$ and $\text{Sp}(4) \rightarrow \text{U}(1) \times \text{U}(1)$. As explained in the previous sections, the anomalous $\text{U}(1)$ s become massive via the Stückelberg mechanism by eating up a closed string axion. The $\text{U}(1)$ charges of the Kähler moduli are given by:

$$q_{a1} = 2, \quad q_{a4} = 1, \quad q_{a5} = -4, \quad q_{a7} = q_{b4} = -1, \quad q_{b1} = 9, \quad q_{b5} = -8, \quad q_{b7} = 0,$$

and so the combinations of two-forms \tilde{c}_i dual to the axions c_i which are eaten are:

$$\begin{aligned} \tilde{c}_a &= \sum_i q_{ai} \tilde{c}_i = 2\tilde{c}_1 + \tilde{c}_4 - 4\tilde{c}_5 - \tilde{c}_7 \\ \tilde{c}_b &= 9\tilde{c}_1 - \tilde{c}_4 - 8\tilde{c}_5. \end{aligned} \tag{4.7}$$

Therefore the visible sector gauge group at low-energy is $\text{SU}(5) \times \text{U}(1)$ where the massless $\text{U}(1)$ factor turns out to be very weakly coupled, and so it behaves as a *dark force* [21]. The gauge fluxes \mathcal{F}_a and \mathcal{F}_b induce also moduli-dependent Fayet-Iliopoulos (FI) terms which fix two Kähler moduli at:

$$\tau_4 = \frac{3}{19} \tau_1 - \tau_7, \quad \tau_5 = \frac{18}{19} \tau_1. \tag{4.8}$$

Notice that the combinations of axions that are eaten up are the same as the combinations of Kähler moduli which are fixed by the FI-terms:

$$c_a = \frac{3}{19} c_1 - c_4 - c_7, \quad c_b = \frac{18}{19} c_1 - c_5. \tag{4.9}$$

Given that the two moduli fixed by the D-terms obtain an $\mathcal{O}(M_s)$ mass, the EFT can be studied just in terms of the two remaining Kähler moduli τ_1 and τ_7 . Thus we can simply integrate out the fixed moduli and work with an ‘effective’ volume form to determine the masses of the light axions. This is particularly pertinent given that D-term corrections to the potential typically dominate over all others.

The D-term stabilisation (4.8) gives a volume of the form:

$$\mathcal{V} = \alpha \left(\tau_1^{3/2} - \gamma \tau_7^{3/2} \right), \tag{4.10}$$

where $\alpha = 86/(57\sqrt{19})$ and $\gamma = 1/(3\alpha)$. The form of the volume (4.10) is very similar to (2.10), and so the canonical normalisation takes the same form as the simplest Swiss-cheese example of section 2.3.1:

$$\frac{c_1}{\tau_1} \simeq \mathcal{O}(1) a_1 + \mathcal{O} \left(\tau_7^{3/4} \mathcal{V}^{-1/2} \right) a_7, \tag{4.11}$$

$$\frac{c_7}{\tau_7} \simeq \mathcal{O}(1) a_1 + \mathcal{O} \left(\tau_7^{-3/4} \mathcal{V}^{1/2} \right) a_7. \tag{4.12}$$

In order to stabilise τ_1 and τ_7 , the authors of [25] introduced also $N_{gc} = 4$ D7-branes around the del Pezzo divisor D_7 which give rise to a pure $\mathcal{N} = 1$ Sp(8) hidden sector that undergoes gaugino condensation. The interplay between this non-perturbative effect and the leading order α' correction to the Kähler potential fixes both τ_1 and τ_7 following the standard LVS procedure. The minimum is at:

$$\tau_7 \sim g_s^{-1} \quad \text{and} \quad \mathcal{V} \sim W_0 e^{\frac{2\pi\tau_7}{5}} \sim W_0 e^{\frac{2\pi}{5} \frac{1}{g_s}}. \quad (4.13)$$

For values of W_0 of order unity, the volume, and so τ_1 , would be exponentially large resulting in an exponentially small value of the visible sector gauge coupling since from (4.8) we realise that $\alpha_{\text{GUT}}^{-1} = \tau_4$ is proportional to τ_1 . Thus this phenomenological requirement tells us that we need to fine tune $W_0 \ll 1$. This tuning is actually also needed to obtain TeV-scale SUSY since, for $\mathcal{V} \sim \mathcal{O}(10^4)$, the gravitino mass (and the soft terms generated via gravity mediation) turns out to be of the order the TeV scale only if $W_0 \sim \mathcal{O}(10^{-10})$. Moreover the non-perturbative effects on D_7 generate a potential for the axion c_7 , and so the canonically normalised axion a_7 obtains a mass of the order $m_{3/2}$. This axion is definitely too heavy for playing the rôle of the QCD axion, and so we shall focus on the remaining axion a_1 which remains massless since the corresponding saxion, i.e. τ_1 , is fixed purely perturbatively via α' corrections.

The couplings of a_1 to the gauge bosons living on D_4 and D_5 scale as:

$$\begin{aligned} \mathcal{L} \supset & \frac{\left(\frac{3}{19}c_1 - c_7\right)}{M_P} g_{\text{SU}(5)}^2 \text{tr}(F_{\text{SU}(5)} \wedge F_{\text{SU}(5)}) + \frac{18}{19} \frac{c_1}{M_P} g_{\text{U}(1)}^2 \text{tr}(F_{\text{U}(1)} \wedge F_{\text{U}(1)}) \\ & \simeq \mathcal{O}\left(\frac{1}{M_P}\right) a_1 \text{tr}(F_{\text{SU}(5)} \wedge F_{\text{SU}(5)}) + \mathcal{O}\left(\frac{1}{M_P}\right) a_1 \text{tr}(F_{\text{U}(1)} \wedge F_{\text{U}(1)}). \end{aligned} \quad (4.14)$$

The volume scaling of these couplings can again be understood in terms of the locality argument of (2.9). In fact, the two rigid divisors D_4 and D_5 are non-local effects since they control the size of the overall volume by defining the \mathbb{P}^1 base of the fibration: $t_{\text{base}} = \text{Vol}(D_4 \cap D_5)$. Hence we correctly found that the massless axion couples to these gauge bosons with Planck strength.

The axion decay constant instead scale as the 10D KK scale:

$$f_{a_1} \simeq \frac{M_P}{4\pi\tau_1} \simeq \frac{M_{\text{KK}}^{10\text{D}}}{4\pi} \simeq 5 \cdot 10^{14} \text{ GeV}, \quad (4.15)$$

while the anomaly coefficient is of order unity, $C_{11} \simeq \mathcal{O}(1)$, and so a_1 can play the rôle of the QCD axion. As explained in section 3.3, the abundance of the axion a_1 will be diluted by the late out-of-equilibrium decay of τ_7 and a_7 .

4.2 An SU(3) × SU(2) model with a QCD axion with large decay constant plus a light ALP

Here we study the second example of [25] which has two massless axions. This is another globally consistent chiral model with magnetised D7-branes and moduli stabilisation within type IIB Calabi-Yau compactifications. The internal manifold is the same described in

section 4.1. However this time the gauge group is not GUT-like since the brane fluxes are chosen in such a way to give rise to just one D-term condition, so that the volume of the Calabi-Yau is no longer proportional to an inverse power of the visible sector gauge coupling. In turn, W_0 can be set of order unity and the volume can be exponentially large lowering the string scale to the intermediate value $M_s \simeq 5 \cdot 10^{10}$ GeV and obtaining TeV-scale supersymmetry without any fine-tuning. Thus there is no space anymore for GUT theories but only for an MSSM-like construction. This is achieved by wrapping $N_a = 3$ D7-branes around D_4 and $N_{K3} = 1$ D7-branes around the K3 fibre D_1 . The corresponding gauge groups are $\text{Sp}(6)$, which is broken down to $\text{SU}(3) \times \text{U}(1)$ by switching on appropriate magnetic flux \mathcal{F}_a on D_4 , and $\text{Sp}(2) \cong \text{SU}(2)$. Notice that the total gauge flux on D_1 is zero.

The anomalous $\text{U}(1)$ becomes massive via the Stückelberg mechanism by eating up a closed string axion. The $\text{U}(1)$ charges of the Kähler moduli are given by:

$$q_{a1} = 2, \quad q_{a4} = 1, \quad q_{a5} = -4, \quad q_{a7} = -1,$$

and so the combination of two-forms (dual to the axions) which is eaten up is:

$$\tilde{c}_a = \sum_i q_{ai} \tilde{c}_i = 2\tilde{c}_1 + \tilde{c}_4 - 4\tilde{c}_5 - \tilde{c}_7.$$

Therefore the visible sector gauge group at low-energy is $\text{SU}(3) \times \text{SU}(2)$. The gauge flux \mathcal{F}_a induces also a moduli-dependent FI term which fixes one Kähler modulus at:

$$\tau_4 = 3(\tau_1 - \tau_5) - \tau_7, \tag{4.16}$$

and so the combination of axions eaten is:

$$c_a = 3c_1 - c_4 - 3c_5 - c_7. \tag{4.17}$$

Given that the modulus fixed by the D-term has an $\mathcal{O}(M_s)$ mass, the EFT can be studied just in terms of the three remaining Kähler moduli τ_1 , τ_5 and τ_7 . The D-term stabilisation (4.16) gives a volume of the form:

$$\mathcal{V} = \frac{1}{3} \left(\sqrt{\tau_s} \tau_b - \tau_f^{3/2} \right), \tag{4.18}$$

where the variables τ_1 and τ_5 have been traded for τ_s and τ_b defined as:

$$\tau_s \equiv \tau_1 - \tau_5, \quad \tau_b \equiv \frac{10\tau_1 - \tau_5}{2}, \tag{4.19}$$

where the label s stays for ‘small’ and the label b for ‘big’. This redefinition is suggested by the D-term condition (4.16) which implies that the combination $(\tau_1 - \tau_5)$ has to be fixed small in order to obtain a visible sector gauge coupling $\alpha_{\text{vis}}^{-1} = \tau_4$ which is not too small. The form of the volume (4.18) is very similar to (2.18) once we identify the base t_b with $\tau_b/\sqrt{\tau_s}$ and the fibre τ_f with τ_s . Given that we are interested in the regime where τ_s is much smaller than τ_b , we shall consider the anisotropic limit where the size of the base $\sqrt{t_b}$

is much bigger than the size of the fibre $\tau_f^{1/4}$. Hence the canonical normalisation takes the same form as the fibred example of section 2.3.2:

$$\frac{c_s}{\tau_s} \simeq \mathcal{O}(1) a_s + \mathcal{O}\left(\tau_7^{3/2} \mathcal{V}^{-1}\right) a_b + \mathcal{O}\left(\tau_7^{3/4} \mathcal{V}^{-1/2}\right) a_7, \quad (4.20)$$

$$\frac{c_b}{\tau_b} \simeq \mathcal{O}\left(\tau_7^{3/2} \mathcal{V}^{-1}\right) a_s + \mathcal{O}(1) a_b + \mathcal{O}\left(\tau_7^{3/4} \mathcal{V}^{-1/2}\right) a_7, \quad (4.21)$$

$$\frac{c_7}{\tau_7} \simeq \mathcal{O}(1) a_s + \mathcal{O}(1) a_b + \mathcal{O}\left(\tau_7^{-3/4} \mathcal{V}^{1/2}\right) a_7. \quad (4.22)$$

In order to stabilise $\mathcal{V} \simeq \sqrt{\tau_s \tau_b}$ and τ_7 , the authors of [25] introduced also $N_{gc} = 3$ D7-branes around the del Pezzo divisor D_7 which give rise to a pure $\mathcal{N} = 1$ Sp(6) hidden sector that undergoes gaugino condensation. The interplay between this non-perturbative effect and the leading order α' correction to the Kähler potential fixes both the combination $\mathcal{V} \simeq \sqrt{\tau_s \tau_b}$ and τ_7 following the standard LVS procedure. The minimum is at:

$$\tau_7 \sim g_s^{-1} \quad \text{and} \quad \mathcal{V} \sim W_0 e^{\frac{\pi \tau_7}{2}} \sim W_0 e^{\frac{\pi}{2} \frac{1}{g_s}}. \quad (4.23)$$

For natural values of W_0 of order unity and a volume of order $\mathcal{V} \sim \mathcal{O}(10^{14})$, the gravitino mass (and the soft terms generated via gravity mediation) is around the TeV scale and the string scale is intermediate: $M_s \simeq 5 \cdot 10^{10}$ GeV.

Moreover the non-perturbative effects on D_7 generate a potential for the axion c_7 , and so the canonically normalised axion a_7 obtains a mass of the order $m_{3/2}$. This axion is definitely too heavy for playing the rôle of the QCD axion, and so we shall focus on the remaining massless axions a_s and a_f . These axions remain massless since the corresponding saxions are fixed purely perturbatively via α' and string loop corrections to the Kähler potential. The couplings of a_s and a_f to the gauge bosons living on D_4 and D_1 scale as:

$$\begin{aligned} \mathcal{L} \supset & \frac{(3c_s - c_7)}{M_P} g_{\text{SU}(3)}^2 \text{tr}(F_{\text{SU}(3)} \wedge F_{\text{SU}(3)}) + \frac{1}{9} \frac{(2c_b - c_s)}{M_P} g_{\text{SU}(2)}^2 \text{tr}(F_{\text{SU}(2)} \wedge F_{\text{SU}(2)}) \\ & \simeq \mathcal{O}\left(\frac{1}{M_P}\right) a_s \text{tr}(F_{\text{SU}(3)} \wedge F_{\text{SU}(3)}) + \mathcal{O}\left(\frac{1}{M_P}\right) a_b \text{tr}(F_{\text{SU}(3)} \wedge F_{\text{SU}(3)}) \\ & + \mathcal{O}\left(\frac{\tau_7^{3/2}}{\mathcal{V} M_P}\right) a_s \text{tr}(F_{\text{SU}(2)} \wedge F_{\text{SU}(2)}) + \mathcal{O}\left(\frac{1}{M_P}\right) a_b \text{tr}(F_{\text{SU}(2)} \wedge F_{\text{SU}(2)}). \end{aligned} \quad (4.24)$$

The volume scaling of these couplings can again be understood in terms the locality argument of (2.9). In fact, both the K3 fibre τ_1 and the rigid divisor D_4 are non-local effects since they control the size of the overall volume (recall that D_4 controls the size of the \mathbb{P}^1 base of the fibration). Hence we correctly found that the massless axions couple to these gauge bosons with Planck strength or weaker.

In the anisotropic limit $\tau_b \gg \tau_s \sim \tau_7 \simeq \mathcal{O}(10)$, the axion decay constants become:

$$f_{a_s} \simeq \frac{M_P}{4\pi\tau_s} \simeq 10^{16} \text{ GeV}, \quad f_{a_b} \simeq \frac{M_P}{4\pi\tau_b} \simeq \frac{M_{\text{KK}}^{6D}}{4\pi} \simeq 5 \text{ TeV}, \quad (4.25)$$

while the anomaly coefficients look like (see (2.28)):

$$C_{s\text{SU}(3)} \sim C_{s\text{SU}(2)} \sim C_{b\text{SU}(2)} \simeq \mathcal{O}(1), \quad C_{b\text{SU}(3)} \simeq \mathcal{O}(\mathcal{V}^{-1}).$$

As described in section 3.3, a_s can be the QCD axion with an $\mathcal{O}(M_{\text{GUT}})$ decay constant. This value is not in the allowed window but it might still be viable in the presence of the late out-of-equilibrium decay of the heavy moduli τ_7 and a_7 which would dilute the axion a_s [22]. The other axion a_b is instead a light ALP with a very low decay constant which does not create any problem with dark matter overproduction. We finally mention that the SU(2) gauge group is a hyperweak interaction since it is realised via D7-branes wrapping an exponentially large four-cycle.

4.3 A chiral model with a QCD axion with intermediate decay constant plus two light ALPs

In this section we shall present an explicit type IIB model with the presence of a closed string modulus which is a perfect QCD axion candidate since it is not eaten up by any anomalous U(1), does not develop any potential by non-perturbative effects in g_s and has a decay constant at the intermediate scale.

The model is very similar to the one used in [53] to obtain a modulated reheating scenario with the production of detectable non-Gaussianities of local form. It is based on an orientifold of a Calabi-Yau three-fold with a K3 or a T^4 fibration structure and $h_+^{1,1} = 5$ and $h_-^{1,1} = 0$. Again, this internal space can be realised as a hypersurface embedded in a toric variety [32]. The relevant divisors for model building are the K3 or T^4 fibre D_1 , the divisor D_2 controlling the \mathbb{P}^1 base of the fibration, a del Pezzo four-cycle D_3 and two rigid divisors D_4 and D_5 which intersect each other but have no intersections with the other four-cycles. The Kähler form J can be expanded in this basis as:

$$J = t_1 \hat{D}_1 + t_2 \hat{D}_2 - t_3 \hat{D}_3 - t_4 \hat{D}_4 - t_5 \hat{D}_5, \tag{4.26}$$

where we took a minus sign for the rigid cycles in order to have positive dual two-cycles. All the non-vanishing intersection numbers are positive and are:

$$k_{122}, \quad k_{333}, \quad k_{444}, \quad k_{445}, \quad k_{455}, \quad k_{555}, \tag{4.27}$$

with the relation $k_{455}^2 = k_{445} k_{555}$. Thus the overall volume reads:

$$\mathcal{V} = \alpha \left[\sqrt{\tau_1} \tau_2 - \gamma_3 \tau_3^{3/2} - \gamma_5 \tau_5^{3/2} - \gamma_4 (\tau_4 - x \tau_5)^{3/2} \right], \tag{4.28}$$

where $\alpha, \gamma_3, \gamma_4, \gamma_5$ and x are $\mathcal{O}(1)$ constants which depend on the intersection numbers (in particular $x = k_{445}/k_{455}$). In [53], the authors consider a particular set-up suitable to build a phenomenologically viable model of modulated reheating. The visible sector is built via two stacks of intersecting D7-branes wrapping two different combinations of the rigid divisors D_4 and D_5 . Moreover an E3-instanton wrapped around D_5 gives rise to poly-instanton corrections to the superpotential since D_5 is assumed to be a rigid four-cycle with Wilson lines, i.e. $h^{2,0}(D_5) = 0$ while $h^{1,0}(D_5) = 1$ [42]. The gauge fluxes are chosen in such a way to yield chirality at the intersection between the two visible sector stacks but with the generation of just one FI-term. Furthermore, no chiral matter at the intersection of the visible sector with the E3-instanton is induced. This set-up allows to keep the field

τ_5 light since it develops a potential only via tiny poly-instanton effects. This is a crucial condition to obtain a viable modulated reheating scenario. More precisely, three effects develop a potential for τ_4 , τ_5 and the absolute value of an open string field $|\phi|$ which is a visible sector singlet:

1. D-terms fix $|\phi|$,
2. String loop corrections to the Kähler potential fix the combination $(\tau_4 - x \tau_5)$,
3. Poly-instanton corrections to the superpotential fix τ_5 .

Moreover, as in standard LVS, τ_3 is fixed non-perturbatively, the volume by the interplay of non-perturbative effects on D_3 and α' corrections to the Kähler potential, while the fibre modulus τ_1 is stabilised by string loops.

Due to the locality argument of (2.9) only the axions of τ_4 , τ_5 and ϕ (the corresponding phase) couple to the visible sector as $1/M_s$, and so can be good QCD axion candidates if the string scale is intermediate. However, the axion of τ_5 cannot be the QCD axion since it develops a non-perturbative superpotential. On the other hand, a combination of the axion of τ_4 and the phase of ϕ will be eaten up by the anomalous U(1) while the other can play the rôle of the QCD axion. If one is not interested in keeping τ_5 very light, then one could drop the assumption that D_5 has Wilson lines resulting in the absence of poly-instanton corrections. Then also this divisor would be frozen by string loop effects, and the corresponding axion would also remain massless and have the right coupling to QCD.

In the analysis of [53], the open string field ϕ plays a crucial rôle because in its absence it is impossible to obtain a light modulating field. In fact, a combination of τ_4 and τ_5 would be fixed by D-terms while another would be stabilised by string loops which beat the tiny poly-instanton effects and generate a mass which is not below the Hubble constant (a crucial condition to find a viable modulated reheating). However, given that we are not interested in modulated reheating, we shall explore exactly this simpler possibility since it involves no open string fields, and so we consider it to be more model-independent.

In our set-up the visible sector is realised by two stacks N_a and N_b of intersecting D7-branes wrapped around the divisors D_4 and D_5 : $D_a = D_4$ and $D_b = D_5$. We also turn on the following world-volume fluxes:

$$F_a = \left(f_4 + \frac{1}{2}\right) \hat{D}_4 + f_5 \hat{D}_5, \quad F_b = g_4 \hat{D}_4 + \left(g_5 + \frac{1}{2}\right) \hat{D}_5, \quad (4.29)$$

where the half-integer contributions come from the cancellation of the Freed-Witten anomalies.⁷ The U(1)-charges of the Kähler moduli induced by these fluxes can be written one in terms of the other as:

$$x q_{a5} = q_{a4} - \left(k_{444} - \frac{k_{445}^2}{k_{455}}\right) \left(f_4 + \frac{1}{2}\right), \quad x q_{b5} = q_{b4}. \quad (4.30)$$

⁷The value of the B-field is fixed in order to cancel the total flux $\mathcal{F}_3 = F_3 - B$ on D_3 so that the instanton wrapped on this del Pezzo contributes to the superpotential. Given that D_3 does not intersect with D_4 and D_5 , the gauge flux F and the total flux $\mathcal{F} = F - B$ on D_4 and D_5 give rise to the same phenomenological features.

The moduli-dependent FI-terms take the form:

$$\xi_a = \frac{1}{4\pi\mathcal{V}} \int J \wedge F_a \wedge \hat{D}_a = \frac{1}{4\pi} \left(q_{a4} \frac{\partial K}{\partial \tau_4} + q_{a5} \frac{\partial K}{\partial \tau_5} \right), \quad (4.31)$$

$$\xi_b = \frac{1}{4\pi\mathcal{V}} \int J \wedge F_b \wedge \hat{D}_b = \frac{1}{4\pi} \left(q_{b4} \frac{\partial K}{\partial \tau_4} + q_{b5} \frac{\partial K}{\partial \tau_5} \right). \quad (4.32)$$

The chiral intersections between the stacks of D7-branes on D_4 and D_5 depend on the gauge fluxes in the following way:

$$I_{ab} = \int (F_a - F_b) \wedge \hat{D}_a \wedge \hat{D}_b = q_{a5} - q_{b4}.$$

Setting $g_4 = -k_{555}/(2k_{455})$ and $g_5 = 0$, we obtain $q_{b4} = q_{b5} = 0$ which implies $\xi_b = 0$, and so we are left over with just one FI-term. Consequently, just one closed string axion will be eaten up. Moreover, we obtain $I_{ab} = q_{a5}$ which implies $q_{a5} \neq 0$ in order to have chiral matter.

From (4.30) we notice that we can choose the fluxes in such a way to have $q_{a5} \neq 0$ and $q_{a4} = 0$ at the same time, so that the FI-term (4.31) simplifies to:⁸

$$\xi_a = \frac{q_{a5}}{4\pi} \frac{\partial K}{\partial \tau_5} = \frac{q_{a5}}{4\pi} \frac{3\alpha (\gamma_5 \sqrt{\tau_5} - \gamma_4 x \sqrt{\hat{\tau}_4})}{\mathcal{V}}, \quad (4.33)$$

where we introduced the combination $\hat{\tau}_4$ defined as $\hat{\tau}_4 \equiv \tau_4 - x \tau_5$. In the absence of open string singlets which can develop a non-zero VEV, the vanishing of the D-terms implies $\xi_a = 0$, and so one combination of τ_4 and τ_5 is fixed at:

$$\tau_5 = \lambda \hat{\tau}_4 \quad \text{where} \quad \lambda \equiv \left(\frac{\gamma_4 x}{\gamma_5} \right)^2. \quad (4.34)$$

This combination can be fixed within the regime of validity of the EFT due to the intersection between D_4 and D_5 . In fact, in the absence of this intersection for $x = 0$, (4.34) would imply the shrinking of the rigid divisor D_5 (see also [25] for related issues). Moreover, (4.34) implies that the combination of axions eaten is:

$$c_a = c_5 - \lambda \hat{c}_4,$$

where $\hat{c}_4 \equiv c_4 - x c_5$. Given that the modulus fixed by the D-term receives an $\mathcal{O}(M_s)$ mass, the EFT can be studied in terms of the four remaining Kähler moduli τ_1 , τ_2 , τ_3 and $\hat{\tau}_4$. Hence, after the D-term stabilisation (4.34), the volume (4.28) simplifies to:

$$\mathcal{V} = \alpha \left(\sqrt{\tau_1} \tau_2 - \gamma_3 \tau_3^{3/2} - \lambda_4 \hat{\tau}_4^{3/2} \right), \quad \text{where} \quad \lambda_4 \equiv \gamma_5 \lambda^{3/2} + \gamma_4. \quad (4.35)$$

The form of the volume (4.35) is very similar to (2.18) once we identify the base t_b with $\tau_2/\sqrt{\tau_1}$ and the fibre τ_f with τ_1 (setting $\alpha = 1$ without loss of generality). Hence the

⁸The choice of fluxes to produce $q_{a4} = 0$ is just meant to simplify the system since the same considerations would apply also to the more general case with $q_{a4} \neq 0$.

canonical normalisation, in the limit $\mathcal{V} \simeq \sqrt{\tau_1} \tau_2 \gg \tau_3 \sim \hat{\tau}_4$, takes the same form as the fibred example of section 2.3.2:

$$\frac{c_1}{\tau_1} \simeq \mathcal{O}(1) a_1 + \mathcal{O}\left(\frac{\hat{\tau}_4^{3/2}}{\mathcal{V}}\right) a_2 + \mathcal{O}\left(\frac{\tau_3^{3/4}}{\mathcal{V}^{1/2}}\right) a_3 + \mathcal{O}\left(\frac{\hat{\tau}_4^{3/4}}{\mathcal{V}^{1/2}}\right) a_4, \quad (4.36)$$

$$\frac{c_2}{\tau_2} \simeq \mathcal{O}\left(\frac{\hat{\tau}_4^{3/2}}{\mathcal{V}}\right) a_1 + \mathcal{O}(1) a_2 + \mathcal{O}\left(\frac{\tau_3^{3/4}}{\mathcal{V}^{1/2}}\right) a_3 + \mathcal{O}\left(\frac{\hat{\tau}_4^{3/4}}{\mathcal{V}^{1/2}}\right) a_4, \quad (4.37)$$

$$\frac{c_3}{\tau_3} \simeq \mathcal{O}(1) a_1 + \mathcal{O}(1) a_2 + \mathcal{O}\left(\frac{\mathcal{V}^{1/2}}{\tau_3^{3/4}}\right) a_3 + \mathcal{O}\left(\frac{\hat{\tau}_4^{3/4}}{\mathcal{V}^{1/2}}\right) a_4. \quad (4.38)$$

$$\frac{\hat{c}_4}{\hat{\tau}_4} \simeq \mathcal{O}(1) a_1 + \mathcal{O}(1) a_2 + \mathcal{O}\left(\frac{\tau_3^{3/4}}{\mathcal{V}^{1/2}}\right) a_3 + \mathcal{O}\left(\frac{\mathcal{V}^{1/2}}{\hat{\tau}_4^{3/4}}\right) a_4. \quad (4.39)$$

Notice that the mixing terms between the axions a_3 and a_4 associated to the blow-up modes τ_3 and $\hat{\tau}_4$ are very suppressed due to the fact that these two rigid divisors do not intersect each other, and so they correspond to the resolution of singularities which are localised in different regions of the Calabi-Yau background [45, 46].

The overall volume and τ_3 are fixed by the leading order F-term scalar potential following the standard LVS procedure which requires non-perturbative effects on D_3 and the leading order α' correction to the Kähler potential. The minimum is at:

$$\tau_3 \sim g_s^{-1} \quad \text{and} \quad \mathcal{V} \sim W_0 e^{\frac{2\pi\tau_3}{N_3}}, \quad (4.40)$$

where $N_3 = 1$ in the case of an E3-instanton whereas N_3 is the rank of the condensing gauge group in the case of gaugino condensation on D7-branes. For natural values of $W_0 \simeq \mathcal{O}(1)$ and a volume of order $\mathcal{V} \simeq \mathcal{O}(10^{14})$, the gravitino mass (and the soft terms generated via gravity mediation) is around the TeV scale and the string scale is intermediate: $M_s \simeq 5 \cdot 10^{10}$ GeV.

The non-perturbative effects on D_3 generate a potential for the axion c_3 , and so the canonically normalised axion a_3 obtains a mass of the order $m_{3/2}$. This axion is definitely too heavy for playing the rôle of the QCD axion, and so we shall focus on the remaining massless axions a_1 , a_2 and a_4 . These axions remain massless since the corresponding saxions, i.e. \mathcal{V} , τ_1 and $\hat{\tau}_4$, are fixed purely perturbatively via α' and string loop corrections to the Kähler potential [24, 25]. As explained in [24], in order to fix τ_1 via g_s effects, two stacks of D7-branes have to be wrapped around D_1 and D_2 , whereas the string loops which stabilise $\hat{\tau}_4$ are sourced by the visible sector stacks on D_4 and D_5 . In fact, following the systematic analysis of string loop corrections performed in [40], the leading order behaviour of the effective scalar potential generated by g_s effects is:

$$V_{(g_s)} = \left(\frac{\mu_1}{\sqrt{\hat{\tau}_4}} - \frac{\mu_2}{\sqrt{\hat{\tau}_4} - \mu_3} \right) \frac{W_0^2}{\mathcal{V}^3} + \mathcal{O}\left(\frac{1}{\mathcal{V}^4}\right), \quad (4.41)$$

where μ_1 and μ_2 are positive constants which depend on the complex structure moduli, while μ_3 is a positive constant that depends on the VEV of stabilised moduli like τ_3 once we include the intersection of the branes wrapping D_4 and D_5 with the O7-planes

and additional Whitney-type branes for tadpole cancellation (see [25]). For natural $\mathcal{O}(1)$ values of the coefficients of the string loop corrections, the potential (4.41) can develop a minimum at:

$$\hat{\tau}_4 = \frac{\mu_1 \mu_3^2}{(\sqrt{\mu_1} + \sqrt{\mu_2})^2} \sim \mathcal{O}(10). \quad (4.42)$$

This minimum is indeed located at small $\hat{\tau}_4$ since for example for $\mu_1 = \mu_2 = 1$ and $\mu_3 = 10$, we obtain $\hat{\tau}_4 = 25$.

The couplings of a_1 , a_2 and a_4 to the gauge bosons living on D_1 , D_2 , D_4 and D_5 scale as (recalling that $\tau_4 = \hat{\tau}_4 + x \tau_5$ and $\tau_5 = \lambda \hat{\tau}_4 \Leftrightarrow \tau_4 = (1 + x \lambda) \hat{\tau}_4$):⁹

$$\begin{aligned} \mathcal{L} \supset & \frac{c_1}{M_P} g_1^2 \text{tr}(F_1 \wedge F_1) + \frac{c_2}{M_P} g_2^2 \text{tr}(F_2 \wedge F_2) \\ & + (1 + x \lambda) \frac{\hat{c}_4}{M_P} g_4^2 \text{tr}(F_4 \wedge F_4) + \lambda \frac{\hat{c}_4}{M_P} g_5^2 \text{tr}(F_5 \wedge F_5) \\ \simeq & \left[\mathcal{O}\left(\frac{1}{M_P}\right) a_1 + \mathcal{O}\left(\frac{\hat{\tau}_4^{3/2}}{\mathcal{V} M_P}\right) a_2 + \mathcal{O}\left(\frac{\hat{\tau}_4^{3/4}}{\mathcal{V}^{1/2} M_P}\right) a_4 \right] \text{tr}(F_1 \wedge F_1) \\ & + \left[\mathcal{O}\left(\frac{\hat{\tau}_4^{3/2}}{\mathcal{V} M_P}\right) a_1 + \mathcal{O}\left(\frac{1}{M_P}\right) a_2 + \mathcal{O}\left(\frac{\hat{\tau}_4^{3/4}}{\mathcal{V}^{1/2} M_P}\right) a_4 \right] \text{tr}(F_2 \wedge F_2) \\ & + \sum_{i=4}^5 \left[\mathcal{O}\left(\frac{1}{M_P}\right) a_1 + \mathcal{O}\left(\frac{1}{M_P}\right) a_2 + \mathcal{O}\left(\frac{\mathcal{V}^{1/2}}{\hat{\tau}_4^{3/4} M_P}\right) a_4 \right] \text{tr}(F_i \wedge F_i). \end{aligned}$$

We stress that we managed to obtain a closed string axion, a_4 , which is not eaten up by any anomalous U(1) and couples to the visible sector on D_4 and D_5 as $1/M_s$ opposed to the standard $1/M_P$ gravitational couplings.

The axion decay constants instead become:

$$f_{a_1} = \frac{M_P}{4\pi\tau_1}, \quad f_{a_2} \simeq \frac{M_P}{4\pi\tau_2}, \quad f_{a_4} \simeq \frac{M_s}{\sqrt{4\pi}} \simeq 10^{10} \text{ GeV},$$

whereas the anomaly coefficients look like:

$$\begin{aligned} C_{11} &\simeq \mathcal{O}(1), & C_{21} &\simeq \mathcal{O}\left(\frac{(\hat{\tau}_4 \tau_1)^{3/2}}{\mathcal{V}^2}\right), & C_{41} &\simeq \mathcal{O}\left(\frac{\tau_1 \sqrt{\hat{\tau}_4}}{\mathcal{V}}\right) \\ C_{12} &\simeq \mathcal{O}\left(\left(\frac{\hat{\tau}_4}{\tau_1}\right)^{3/2}\right), & C_{22} &\simeq \mathcal{O}(1), & C_{42} &\simeq \mathcal{O}\left(\sqrt{\frac{\hat{\tau}_4}{\tau_1}}\right), \\ C_{1i} &\simeq \mathcal{O}\left(\frac{\hat{\tau}_4}{\tau_1}\right), & C_{2i} &\simeq \mathcal{O}\left(\frac{\hat{\tau}_4 \sqrt{\tau_1}}{\mathcal{V}}\right), & C_{4i} &\simeq \mathcal{O}(1), \quad \forall i = 4, 5. \end{aligned} \quad (4.43)$$

The field a_4 is a perfect QCD axion candidate since its decay constant is intermediate and $C_{4i} \simeq \mathcal{O}(1) \forall i = 4, 5$. Therefore we have managed to build an explicit example where a closed string axion with these features [15] survives in the EFT since it is not eaten up by any anomalous U(1). As pointed out in section 3.3, this axion will not form a relevant

⁹We neglect the axionic coupling to hidden sector gauge bosons living on D_3 in the case of gaugino condensation.

component of dark matter since it will be diluted by the decay of $\hat{\tau}_4$. Thus the constraints from isocurvature fluctuations do not apply to this case.

Moreover, there are two further ALPs, a_1 and a_2 , whose behaviour is different in the case of a isotropic or anisotropic Calabi-Yau.

- *Isotropic case:* in the isotropic limit $\tau_1 \sim \tau_2 \sim \mathcal{V}^{2/3}$, the decay constants of a_1 and a_2 scale as the 10D KK scale:

$$f_{a_1} \simeq f_{a_2} \simeq \frac{M_P}{4\pi\mathcal{V}^{2/3}} \simeq \frac{M_{\text{KK}}^{10\text{D}}}{4\pi} \simeq 10^8 \text{ GeV} , \quad (4.44)$$

and the anomaly coefficients simplify to:

$$C_{11} \simeq C_{22} \simeq \mathcal{O}(1), \quad C_{21} \simeq C_{12} \simeq \mathcal{O}(\mathcal{V}^{-1}), \quad C_{1i} \simeq C_{2i} \simeq \mathcal{O}(\mathcal{V}^{-2/3}).$$

The two non-local axions a_1 and a_2 correspond to two light and almost decoupled ALPs that do not create any problem with dark matter overproduction. Moreover the gauge theories on τ_1 and τ_2 are hyperweak.

- *Anisotropic case:* in the anisotropic limit $\tau_2 \gg \tau_1 \sim \hat{\tau}_4$, the decay constants of a_1 and a_2 become:

$$f_{a_1} \simeq \frac{M_P}{4\pi\hat{\tau}_4} \simeq 10^{16} \text{ GeV}, \quad f_{a_2} \simeq \frac{M_P}{4\pi\tau_2} \simeq \frac{M_{\text{KK}}^{6\text{D}}}{4\pi} \simeq 5 \text{ TeV}, \quad (4.45)$$

while the anomaly coefficients take the simplified form:

$$C_{11} \simeq C_{12} \simeq C_{22} \simeq C_{1i} \simeq \mathcal{O}(1), \quad C_{2i} \simeq \mathcal{O}(\mathcal{V}^{-1}), \quad C_{21} \simeq \mathcal{O}(\mathcal{V}^{-2}).$$

Now the anomaly coefficient C_{1i} is of order unity, and so also a_f can play the rôle of the QCD axion but with a GUT-scale decay constant. However, there are no problems with dark matter overproduction since, as explained in section 3.3, the abundance of the axion a_1 will be diluted by the decay of $\hat{\tau}_4$. Notice that in this case $\tau_1 \sim \hat{\tau}_4$, and so the visible sector could also be placed on the fibre divisor. However, the gauge theory on the base divisor τ_2 is still a hyperweak interaction. Finally the non-local axion a_2 is a light (almost decoupled) ALP with a very low decay constant of the order the TeV-scale.

4.4 A chiral model with a QCD axion and many ALPs with intermediate decay constants

The previous example has shown how it is possible to obtain models with a QCD axion in the axion window. However, it also shows how there can be additional ALPs coupling to the visible sector with a very similar decay constant. For this, we simply need additional local cycles intersecting the visible branes, but without additional D-term conditions. The same method of moduli stabilisation will apply: for n intersecting local cycles, d of them are fixed by D-terms (and so d axions are eaten up) while the remaining $n - d$ moduli are frozen by loop corrections, leading to $n - d$ light axions with intermediate scale decay

constant. We stress that the number of D-term conditions can be reduced either by not wrapping some of the intersecting cycles or by setting some world-volume fluxes to zero.

To be concrete, consider the blow-up of a \mathbb{Z}_7 singularity which has no non-compact exceptional divisors. Thus it will not be altered by the space in which it is embedded, and so it can be studied locally. This is not necessary: a much larger class of examples could be found, for example, via the blow-ups of del-Pezzo singularities, but then we would need to take into account the global embedding; we leave the construction of these rich and fully realistic examples to future work. We denote the overall volume $\mathcal{V}_0 \simeq \mathcal{V}$ (which could be a Swiss-cheese or a fibred Calabi-Yau) with a small cycle τ_s stabilised at $\tau_s \sim \ln \mathcal{V}$ by a gaugino condensate wrapping D_s . The volume form is then:

$$\mathcal{V} = \mathcal{V}_0 + \frac{1}{6} [8(t_1^3 + t_2^3 + t_3^3) - 6(t_1^2 t_3 + t_1 t_2^2 + t_2 t_3^2) + 6t_1 t_2 t_3], \quad (4.46)$$

and the Kähler cone is given by:

$$t_1 - 2t_3 > 0, \quad t_3 - 2t_2 > 0, \quad t_2 - 2t_1 > 0.$$

We wrap a ‘visible’ stack on $D_a = D_1$, and another on $D_b = D_2$, turning a flux of $\mathcal{F}_a = 3\hat{D}_3 + \frac{1}{2}\hat{D}_1$, leading to $I_{ab} = 3$ and no other chiral intersections. The D-term generated stabilises τ_3 in terms of τ_1 and τ_2 at:

$$\tau_3 = \frac{\tau_2}{\tau_1} (2\tau_2 - \tau_1), \quad (4.47)$$

or equivalently:

$$t_1 = -\frac{5\tau_1 + 2\tau_2}{7\sqrt{2}\sqrt{\tau_1}}, \quad t_2 = -\frac{3\tau_1 + 4\tau_2}{7\sqrt{2}\sqrt{\tau_1}}, \quad t_3 = -\frac{8\tau_2 - \tau_1}{7\sqrt{2}\sqrt{\tau_1}}.$$

Hence we can write the volume form effectively as:

$$\mathcal{V} = \mathcal{V}_0 - \frac{1}{21\sqrt{2}\tau_1^{3/2}} (5\tau_1^3 + 6\tau_1^2\tau_2 - 6\tau_1\tau_2^2 + 16\tau_2^3). \quad (4.48)$$

\mathcal{V}_0 is stabilised in the usual LVS way; string loops then stabilise the remaining moduli τ_1 and τ_2 via terms in the potential of the form (4.41) (the full loop potential is rather complicated even in this simple case so we neglect to present it here).

The axions corresponding to moduli τ_1 and τ_2 are thus left massless at this level, and couple with string strength to branes a and b , since the effective mixing matrix is (restricting indices to $i, j = 1 \dots 3$, since the mixing with τ_s and τ_b is negligible):

$$\mathcal{K}_{ij} = \frac{(6\tau_2^2 + 2\tau_1\tau_2 - \tau_1^2)^{-1}}{14\sqrt{2}\mathcal{V}\sqrt{\tau_1}} \begin{pmatrix} 28\tau_2^2 + 12\tau_1\tau_2 - 5\tau_1^2 & \tau_1(10\tau_2 - 3\tau_1) & \tau_1(\tau_1 + 6\tau_2) \\ \tau_1(10\tau_2 - 3\tau_1) & \tau_1(48\tau_2 - 13\tau_1) & \tau_1(12\tau_2 - 5\tau_1) \\ \tau_1(\tau_1 + 6\tau_2) & \tau_1(12\tau_2 - 5\tau_1) & \tau_1(11\tau_1 + 24\tau_2) \end{pmatrix}.$$

This is non-degenerate with full mixing between the axions. For $\tau_1 = \tau_2 = \tau_*$, the D-term relation (4.47) gives also $\tau_3 = \tau_*$ and so the mixing matrix simplifies to:

$$\mathcal{K}_{ij} = \frac{1}{14\sqrt{2}\mathcal{V}\sqrt{\tau_*}} \begin{pmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{pmatrix}.$$

Therefore the gauge couplings of groups a and b are equal, $g_a^2 = g_b^2 \equiv g^2 \sim \tau_*^{-1}$. Taking for example $\tau_* = 10$, the axionic couplings turn out to be:

$$\mathcal{L} \supset \frac{g^2 \sqrt{\mathcal{V}}}{2\pi M_P} [(3.1 a_1 - 1.8 a_2) \text{tr}(F_a \wedge F_a) - (2.3 a_1 - 2.8 a_2) \text{tr}(F_b \wedge F_b)], \quad (4.49)$$

giving a ‘QCD axion’ $a_{\text{QCD}} = (0.86 a_1 - 0.5 a_2)$ together with an ALP $a_{\text{ALP}} = (0.5 a_1 + 0.86 a_2)$ and:

$$\mathcal{L} \supset \frac{g^2 \sqrt{\mathcal{V}}}{2\pi M_P} [3.6 a_{\text{QCD}} \text{tr}(F_a \wedge F_a) - (3.6 a_{\text{ALP}} + 0.6 a_{\text{QCD}}) \text{tr}(F_b \wedge F_b)]. \quad (4.50)$$

Hence if we interpret the cycle ‘ b ’ as a U(1) stack, the ALP will couple with approximately equal strength to the photon as the QCD axion does. We believe this is a generic prediction of this class of models: it is highly non-generic to have an ALP that couples parametrically more strongly to the photon than the QCD axion does. In order to achieve this, it would be necessary for there to be a local stack which carries a U(1) which contributes to the hypercharge, but which does not intersect the QCD stack, and has no mass mixing (through D-terms) with the axion of the QCD stack. This implies a construction with at least four stacks of branes; it would be an interesting scenario to realise.

4.4.1 Explaining astrophysical anomalies with a stringy ALP

Since the above construction is built from intersecting branes in the geometric regime, the coupling of the local axion to matter fields should be given by the first expression in (2.36). Hence we expect the ALP coupling to electrons to be suppressed only by a factor of the gauge coupling squared relative to the photon coupling. This is perfectly consistent with the astrophysical hints for an ALP, — an anomalous energy loss of white dwarfs (see section B.1.1 in the appendix) and an anomalous transparency of the Universe for TeV photons (see section B.1.2) — which would be explained by:

$$C_{i\gamma}/C_{ie} \simeq 10, \quad f_{a_i}/C_{i\gamma} \simeq 10^8 \text{ GeV}, \quad m_{\text{ALP}} \lesssim 10^{-9} \div 10^{-10} \text{ eV}. \quad (4.51)$$

For models in this class - i.e. built from local constructions of typical size τ_* - we find generically:

$$\frac{C_{i\gamma}}{C_{ie}} \sim \frac{8\pi\tau_*}{3}, \quad \frac{f_{a_i}}{C_{i\gamma}} = \frac{1}{8\pi N_{i\gamma} \tau_*^{1/4}} \frac{M_P}{\sqrt{\mathcal{V}}} = \frac{1}{8\pi N_{i\gamma} \tau_*^{1/4}} \sqrt{\frac{\sqrt{2} M_P m_{3/2}}{g_s^{1/2} W_0}}. \quad (4.52)$$

where $N_{i\gamma}$ is a purely numerical factor coming from the canonical normalisation of the ALP. For $m_{3/2} = 10 \text{ TeV}$, $g_s \simeq 0.1$ and $W_0 \sim 10$ we can easily arrive at the desired figure for $N_{i\gamma} \sim 10$.

The ALP mass could be generated, for example, by a single Kähler potential instanton, which for $m_{\text{ALP}} \sim 10^{-10} \text{ eV}$ would require:

$$\tau_* \sim \frac{1}{\pi} \ln \left(\frac{g_s m_{3/2}}{m_{\text{ALP}}} \right) \sim 16. \quad (4.53)$$

Note that there is then a small amount of tension with the ratio $C_{i\gamma}/C_{ie}$, but this is the least well predicted quantity. Hence it is possible to explain these astrophysical hints for an ALP via a natural intermediate string-scale scenario with stable moduli.

Note that such an ALP would constitute only a tiny ($\mathcal{O}(10^{-9})$) fraction of dark matter, and would satisfy all constraints - interestingly being just one or two orders of magnitude in mass heavier than the black hole super-radiance constraint [19, 54] (see section B.2) and within the reach of future experiments (see figure 1 and appendix B).

5 Conclusions

String phenomenology holds the promise of an axiverse — the QCD axion plus a (possibly large) number of further ultralight axion-like particles, possibly populating each decade of mass down to the Hubble scale, 10^{-33} eV. However, although a plenitude of axions is a generic prediction of string theory, there may be few or no light axions remaining once constraints such as tadpole cancellation and moduli stabilisation are taken into account (for example in the KKLT scenario each axion would be made massive by an exponential term in the superpotential) or, particularly for D-brane models, when considering which axions are eaten in making U(1) symmetries massive. This last issue is particularly important, for example, in GUT models where the diagonal U(1) \subset U(5) is made massive in this way; we have shown that in such cases (and more generally for any model where the diagonal U(1) \subset U(3) is made massive like this) if the Standard Model is built on a local cycle, the non-local axions cannot solve the strong CP problem, and particular care must be taken to ensure that there are additional light local axions that can do the job.

We have shown that the promise of an axiverse is fulfilled in the LARGE Volume Scenario of IIB string compactifications. In fact, the least fine-tuned (i.e. $W_0 \sim \mathcal{O}(1)$) models of globally consistent constructions with magnetised D-branes and chirality have at least two light axions. In general, the closed string axions separate into local and non-local classes, where the non-local axions may have small decay constants but also small couplings and vanishingly small masses, and so are unlikely to overproduce dark matter (unless the Standard Model also wraps a non-local cycle but in this case the axions would be diluted by the decay of heavy Kähler moduli).

We have also determined for the LVS axiverse the couplings of axions to matter fields, both at tree level and one loop. The results at one loop generalise the existing non-supersymmetric formulae to supersymmetric models. However, these rely upon the conjectured form of the Kähler metric for matter fields; it would be interesting to calculate these couplings in specific examples.

Moreover, we have shown how to build models that exhibit a QCD axion with a decay constant $f_a \sim M_P/(4\pi\sqrt{\mathcal{V}}) \sim 10^{10} \div 10^{12}$ GeV, and an additional axion-like particle (or even more than one) having the same decay constant and coupling to the photon¹⁰ $C_{i\gamma} \sim C_{a\gamma} \sim \mathcal{O}(1)$ (see section 4.4). At both ends of the above range of the decay constant there are exciting phenomenological opportunities. For $f_a \sim M_P/(4\pi\sqrt{\mathcal{V}}) \sim 10^{12}$ GeV, the QCD axion can be the dominant part of dark matter and be detected in haloscopes

¹⁰With potentially many much more weakly coupled axions.

exploiting microwave cavities, cf. appendix B.2. For $f_a \sim M_P/(4\pi\sqrt{V}) \sim 10^{10}$ GeV, the additional ALPs could explain astrophysical anomalies (see appendix B) and be searched for in the next generation of helioscopes (see section B.1.1) or in the next-to-next generation of light-shining-through-a-wall experiments (see section B.3).

The particular model of inflation is also relevant for the cosmological constraints. In the LARGE Volume Scenario there are well-defined models of inflation [55, 56], and so these questions can be readily addressed.

If the Hubble scale during inflation is larger than the gravitino mass, the latter can be of the order the TeV scale giving rise to an intermediate string scale which allows couplings for the axions compatible with the astrophysical hints. However, reheating at the end of inflation is problematic (since in this case the inflaton is the lightest modulus, i.e. the volume mode) and it is an interesting question as to how this can be achieved.

Alternatively, if the Hubble scale during inflation is lower than the gravitino mass, the latter has to be much above the TeV scale. This singles out sequestered models where the soft terms can be around the TeV scale even if the gravitino mass is at the intermediate scale [35]. The string scale turns out to be around the GUT-scale, allowing again for good QCD axion candidates. Even in this case there can be heavy particles that decay and dilute other relics, but now, due to a higher string scale, the axion will be a component of non-thermal dark matter. Contrary to the situation with $H_{\text{inf}} > m_{3/2}$, inflation and reheating are better understood, and so we can have more control over all the cosmological evolution of the axions and the moduli from the early Universe till today.

Since the phenomenology of axions is so rich, with astrophysical constraints and hints, and many experiments searching for them, it is highly worthwhile to investigate what values of the couplings can be predicted from string theory. We have compiled a comprehensive analysis of these in the LVS context where both local and global questions can be asked about the models in a consistent way. We hope that this will be of use to experimentalists and model builders alike.

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A Phenomenology of theories with many axions

A.1 Effective Lagrangian at the weak scale

For the low energy phenomenology of axions, their couplings to gauge bosons other than gluons, specifically to photons, and to SM fermions, specifically to light quarks and electrons, are of uttermost importance. It is known that for the QCD axion a great deal about its low energy couplings can be learned from effective field theory techniques, which allow to take electro-weak symmetry breaking and QCD effects into account. In this spirit, generalising the considerations in [57] to the case of several axion-like fields, a_i , we write down

the most general effective Lagrangian at the weak scale, to first order in the fields a_i , which consists of anomalous couplings to gauge bosons and derivative couplings to matter:

$$\begin{aligned}
 \mathcal{L} \supset & \frac{1}{2} \partial_\mu a_i \partial^\mu a_i - \frac{g_3^2}{32\pi^2} \left(\theta_0 + C_{i3} \frac{a_i}{f_{a_i}} \right) F_{3,\mu\nu}^b \tilde{F}_3^{b,\mu\nu} \\
 & - \frac{g_2^2}{32\pi^2} C_{iW} \frac{a_i}{f_{a_i}} F_{W,\mu\nu}^b \tilde{F}_W^{b,\mu\nu} - \frac{g_Y^2}{32\pi^2} C_{iY} \frac{a_i}{f_{a_i}} F_{Y,\mu\nu} \tilde{F}_Y^{\mu\nu} \\
 & - \sum_{\psi_L, \psi_R} (\psi_R M \psi_L + \bar{\psi}_L M^\dagger \bar{\psi}_R) + \sum_{\psi_L} \frac{X_{\psi_L}^i}{f_{a_i}} \psi_L \sigma^\mu \bar{\psi}_L \partial_\mu a_i + \sum_{\psi_R} \frac{X_{\psi_R}^i}{f_{a_i}} \psi_R \sigma^\mu \bar{\psi}_R \partial_\mu a_i \\
 & - \sum_{\psi_L, \psi_R} \left(i \psi_R g^i \psi_L \frac{a_i}{f_{a_i}} + \text{h.c.} \right) - V(a_i), \tag{A.1}
 \end{aligned}$$

where θ_0 is an initial theta-angle, F_W and F_Y are the SU(2) and hypercharge field strengths, respectively, g_2 and g_Y the corresponding gauge couplings, M is the mass matrix of the fermions, written as Weyl fermions ψ_L and ψ_R . Note that in general the couplings to fermions need not conserve parity (see [58] for a related discussion on this issue). Furthermore, the Yukawa couplings on the last line are the leading terms in the expansion of Yukawa couplings of the form:

$$\mathcal{L} \supset -e^{-ib\phi} H \psi_R \psi_L + \text{h.c.} \tag{A.2}$$

since we expect the full theory to respect the axionic shift symmetries, only broken by non-perturbative effects. Those contributed other than from QCD are encapsulated in the potential $V(a_i)$, which we expect to give a mass to all ALPs at some scale - except for those that are gauged, and obtain their mass through being eaten (these shall be discussed in section 3.2). When an ALP couples to a gaugino condensate or D-brane instanton, we expect the mass to be large, and we can simply exclude it from the analysis to follow. These are leading contributions in the superpotential. On the other hand, for ALPs that do not obtain masses in that way, the masses induced are many orders of magnitude smaller, coming from multi-instanton effects or Kähler potential corrections, and have been argued to be many orders of magnitude below the QCD axion mass [15]. Therefore in the following we shall neglect ALP masses and comment on their inclusion at the end.

In the text of the paper we have derived the predictions for the dimensionless couplings C_{i3} , C_{iW} , C_{iY} , $X_{\psi_L}^i$, $X_{\psi_R}^i$, and g^i , and the decay constants f_{a_i} from particular string compactifications. Here, in the next subsection, we will further scale the theory down through electro-weak symmetry breaking, heavy quark thresholds and QCD chiral symmetry breaking to arrive at the low energy Lagrangian involving only the QCD axion, ALPs, photons and electrons, which are then compared to astrophysical and cosmological observations, as well as to laboratory experiments, in section B.

A.2 Effective Lagrangian below chiral symmetry breaking

After electro-weak symmetry breaking, we diagonalise the fermion mass matrix, transforming to the physical basis with real masses m_Ψ , $M \rightarrow U_R^\dagger M U_L = m_\Psi$, $\tilde{X}_{\psi_L}^i \equiv$

$U_L^T X_{\Psi_L}^i U_L^*, \tilde{X}_{\Psi_R}^i \equiv U_R^\dagger X_{\Psi_R}^i U_R$ (noting that this can lead to flavour-changing axionic couplings) and write $\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$:

$$\begin{aligned} \mathcal{L} \supset & \frac{1}{2} \partial_\mu a_i \partial^\mu a_i - \frac{g_3^2}{32\pi^2} \left(\theta_0 + \arg \det(M) + C_{i3} \frac{a_i}{f_{a_i}} \right) F_{3,\mu\nu}^b \tilde{F}_3^{b,\mu\nu} \\ & - \frac{e^2}{32\pi^2} (C_{iW} + C_{iY}) \frac{a_i}{f_{a_i}} F_{em,\mu\nu} \tilde{F}_{em}^{\mu\nu} \\ & + \sum_\Psi m_\Psi \bar{\Psi} \Psi + \sum_\Psi \left[\frac{1}{2} (\tilde{X}_{\psi_R}^i + \tilde{X}_{\psi_L}^i) \bar{\Psi} \gamma^\mu \gamma_5 \Psi + \frac{1}{2} (\tilde{X}_{\psi_R}^i - \tilde{X}_{\psi_L}^i) \bar{\Psi} \gamma^\mu \Psi \right] \frac{\partial_\mu a_i}{f_{a_i}}. \end{aligned} \quad (\text{A.3})$$

The mass of the QCD axion is generated by the coupling to QCD, but is complicated by the presence of the light quarks - if there were none then its mass would be that the η' acquires. One can however use chiral perturbation theory to calculate it: with chiral rotations of the quarks one may eliminate the $F_3 \tilde{F}_3$ term and consider the effective pion Lagrangian. In the approximation $m_s \gg m_u, m_d$, one can consider only the π^0 and its mixing with the axions (one combination of η and η' is made massive by the strange quark mass, the other by the anomaly). Making the rotations $\Psi \rightarrow \exp(-i\alpha_\Psi \gamma_5) \Psi$, with:

$$\alpha_u + \alpha_d = -\frac{c_u + c_d}{2} \left(\theta + C_{i3} \frac{a_i}{f_{a_i}} \right), \quad c_u + c_d = 1, \quad \theta \equiv \theta_0 + \arg \det(M), \quad (\text{A.4})$$

will eliminate the $F_3 \tilde{F}_3$ term and couple the axions to the quarks.

We are now ready to define the QCD axion a , with decay constant f_a and setting $C_{a3} = 1$, cf. eq. (1.1), and the remaining ALP fields a'_i , in terms of linear combinations of the original axion-like fields a_i :

$$\begin{aligned} \frac{a}{f_a} & \equiv \theta + C_{i3} \frac{a_i}{f_{a_i}}, \quad a_i = R_{ij} a'_j, \quad \sum_i \frac{C_{i3}}{f_{a_i}} R_{ij} = \frac{\delta_{aj}}{f_a}, \quad \alpha_{u,d} = -\frac{c_{u,d}}{2} a \\ a & = \hat{a} + f_a \arg \det(M), \quad (\tilde{X}_{\psi_R}^i + \tilde{X}_{\psi_L}^i) \rightarrow (\tilde{X}_{\psi_R}^j + \tilde{X}_{\psi_L}^j) R_{ji} - c_\psi \delta_{ia} \equiv \hat{X}_\psi^i - c_\psi \delta_{ia}, \end{aligned}$$

resulting in the effective Lagrangian:

$$\begin{aligned} \mathcal{L} \supset & -\frac{1}{2} \partial_\mu \pi^0 \partial^\mu \pi^0 - \frac{1}{2} \partial_\mu a_i \partial^\mu a_i \\ & - \frac{e^2}{32\pi^2} \left[\frac{4}{3} (4\alpha_u + \alpha_d) + (C_{iW} + C_{iY}) \frac{a_i}{f_{a_i}} - 2 \frac{\pi^0}{f_\pi} \right] F_{em,\mu\nu} \tilde{F}_{em}^{\mu\nu} \\ & - m_u v^2 \cos \left(\frac{\pi^0}{f_\pi} + c_u \frac{a}{f_a} \right) - m_d v^2 \cos \left(-\frac{\pi^0}{f_\pi} + c_d \frac{a}{f_a} \right) \\ & + \frac{1}{4} (\hat{X}_d^i - \hat{X}_u^i + (c_u - c_d) \delta_{ia}) \frac{f_\pi}{f_{a_i}} \partial^\mu \pi^0 \partial_\mu a'_i. \end{aligned} \quad (\text{A.5})$$

The kinetic mixing term on the last line between the QCD axion, ALPs and pion can be ignored provided that $\hat{X}_u - \hat{X}_d \ll f_{a_i}/f_\pi$; this is because the non-unitary transformation that diagonalises the kinetic terms can be chosen so that the pion merely rescales $\pi^0 \rightarrow \pi^0 (1 - \mathcal{O}(\frac{f_\pi^2}{f_{a_i}^2}))$ while the other fields shift by factors $a'_i \rightarrow a'_i + R'_{ij} a'_j \mathcal{O}(\frac{f_\pi}{f_{a_i}})$; these

lead to subleading (and therefore negligible, for the expected sizes of f_{a_i} compared to f_π) corrections to the physical masses and couplings. In fact, the all-orders diagonalisation is performed by:

$$\chi_i \equiv \frac{1}{4}(\hat{X}_d^i - \hat{X}_u^i + (c_u - c_d)\delta_{ia})\frac{f_\pi}{f_a}, \quad F \equiv \sqrt{1 - \sum_i \chi_i^2},$$

$$\pi^0 \rightarrow \frac{\pi^0}{F}, \quad a'_i \rightarrow a'_i - \frac{\chi_i}{F} \pi^0,$$

and thus there are no new couplings induced between the ALPs and the photon at any order.

A.2.1 Coupling to photons

Neglecting the mixing terms χ_i , the mass of the pion, the mass of the QCD axion and its coupling to photons are then found as usual by choosing the coefficients $c_u = \frac{1}{1+z}$, $c_d = 1 - c_u$, where $z \equiv m_u/m_d$, to eliminate the mass mixing term above; this gives: $m_a = m_\pi \frac{f_\pi}{f_a} \frac{\sqrt{z}}{1+z}$ and the QCD axion two-photon coupling:

$$\mathcal{L} \supset -\frac{1}{4}g_{a\gamma}aF_{\text{em},\mu\nu}\tilde{F}_{\text{em}}^{\mu\nu},$$

$$g_{a\gamma} \equiv \frac{\alpha}{2\pi f_a} \left(C_{aW} + C_{aY} + \Delta C_{a\gamma} \right),$$

$$\Delta C_{a\gamma} \equiv -\frac{2}{3} \frac{4+z}{1+z} \approx -1.95, \tag{A.6}$$

while the couplings of the ALPs a'_i to photons are just:

$$\mathcal{L} \supset -\frac{1}{4}g_{i\gamma}a'_iF_{\text{em},\mu\nu}\tilde{F}_{\text{em}}^{\mu\nu}, \quad g_{i\gamma} \equiv \frac{\alpha}{2\pi} \sum_j \left(\frac{C_{jW} + C_{jY}}{f_{a_j}} \right) R_{ji}. \tag{A.7}$$

The upshot is that the QCD axion has a ‘universal’ coupling to photons $\propto \Delta C_{a\gamma}$, but the ALPs do not; their coupling to photons and electrons comes entirely from the high-energy theory.

Up to this stage, we did not consider mass terms for ALPs. These will have effects on the ‘universal’ couplings to photons. In fact, we shall treat any ALP heavier than the pion as being already integrated out and ignore it for these purposes, and for ALPs of comparable mass to the QCD axion or heavier the analysis is necessarily involved. However, in the case that additional ALPs are *lighter* than the QCD axion, we can neglect the kinetic mixing terms as before, and, introducing ALP masses m_i^2 , the QCD axion is approximately shifted by:

$$a \rightarrow a - \sum_{i,j \neq a} \frac{m_j^2}{m_a^2} R_{ja} R_{ji} a_i, \tag{A.8}$$

and so the ALPs a_j acquire universal couplings, too, which are, however, suppressed in comparison to the universal coupling of the axion, by the ratio m_i^2/m_a^2 :

$$g_{i\gamma} \approx \frac{\alpha}{2\pi f_a} \left(C_{iW} + C_{iY} - \Delta C_{a\gamma} \sum_{j \neq a} \frac{m_j^2}{m_a^2} R_{ja} R_{ji} \right). \tag{A.9}$$

Conversely, since the ALPs are shifted by a similar amount, we expect the QCD axion to photon coupling to be shifted by an amount:

$$\Delta g_{a\gamma} \sim \frac{\alpha}{2\pi} \sum_i \frac{C_{jW} + C_{jY}}{f_{a_i}} \sum_{j \neq a} \frac{m_j^2}{m_a^2} R_{ja} R_{ji}. \quad (\text{A.10})$$

This is relevant in the case that an ALP couples much more strongly than the QCD axion to the photon *and* has some coupling to QCD.

A.2.2 Couplings to electrons

Finally, we consider the coupling to electrons which is also crucial for low energy phenomenology. Other than the direct coupling, there will also be one generated through the two-photon coupling at one loop. In supersymmetric theories, the two-photon coupling is accompanied by a coupling to gauginos:

$$\mathcal{L} \supset - \int d^2\theta (i a_i) \frac{g_{i\gamma}}{4} W^\alpha W_\alpha \supset \frac{1}{4} g_{i\gamma} a_i F_{\text{em},\mu\nu} \tilde{F}_{\text{em}}^{\mu\nu} + \frac{1}{2} g_{i\gamma} \partial_\mu a_i \lambda^\alpha \sigma^\mu \bar{\lambda}, \quad (\text{A.11})$$

and so we have a quantitatively different result. We also have a qualitatively different result, in that we find parity violating vector-like terms arising from the differences in squark masses. We can parameterise the coupling as:

$$\begin{aligned} \mathcal{L} \supset & \frac{C_{ie}^A}{2f_{a_i}} \bar{e} \gamma^\mu \gamma_5 e \partial_\mu a_i + \frac{C_{ie}^V}{2f_{a_i}} \bar{e} \gamma^\mu e \partial_\mu a_i, \\ C_{ie}^{A,V} = & \hat{X}_e^{A,V j} R_{ji} + \Delta_{i\gamma\gamma}[C_{ie}^{A,V}] + \delta_{ai} \Delta_{\text{QCD}}[C_{ae}^A], \end{aligned} \quad (\text{A.12})$$

where $\hat{X}_e^{A,V j} \equiv \frac{1}{2} (\tilde{X}_{e_R}^j \pm \tilde{X}_{e_L}^j)$ and $\Delta_{\text{QCD}}[C_{ae}^A] = \frac{3\alpha^2}{4\pi} \Delta C_{a\gamma} \ln(\Lambda_{\text{QCD}}/m_a)$ is a correction from pion loops to the QCD axion to electron coupling, unchanged in SUSY theories. We shall calculate the full contribution $\Delta_{i\gamma\gamma} C_{ie}^{A,V}$ in section C, but we note that to a rough approximation we can take:

$$\begin{aligned} \Delta_{i\gamma\gamma}[C_{ie}^A] & \approx \frac{3\alpha^2}{4\pi^2} C_{i\gamma} \ln\left(\frac{M_{\text{soft}}}{m_e}\right) + \frac{2\alpha^2}{4\pi^2} C_{i\gamma} \ln\left(\frac{\Lambda}{M_{\text{soft}}}\right), \\ \Delta_{i\gamma\gamma}[C_{ie}^V] & \approx \frac{2\alpha^2}{4\pi^2} C_{i\gamma} \ln\left(\frac{\Lambda}{M_{\text{soft}}}\right), \end{aligned} \quad (\text{A.13})$$

where M_{soft} is the scale of superpartner masses, and Λ the cutoff of the theory, of the order of the string scale.

B Axion searches in astrophysics, cosmology, and laboratory

The couplings of QCD axions and ALPs to photons and electrons give rise to a number of effects which can be searched for in astrophysics, cosmology, and in laboratory experiments. In the remainder of this section, we will summarise the present knowledge about the dimensionless couplings and the decay constants. Some astrophysical observations seem to point to a preferred scale of order $\sim 10^9$ GeV.

B.1 Axions from astrophysical sources

B.1.1 Stellar emission

Light and weakly interacting particles such as the QCD axion or ALPs are produced in hot astrophysical plasmas via their coupling to photons or via their coupling to electrons or nucleons. One of the strictest bounds on the coupling of axions to photons¹¹ arise from either the non-observation of an anomalous energy loss of Horizontal Branch (HB) stars due to axion emission [59] or (slightly stronger) from the non-observation of solar axions in the CAST experiment [60]:

$$\mathcal{L} \supset -\frac{1}{4}g_{a\gamma}aF_{\mu\nu}\tilde{F}^{\mu\nu}, \text{ with } g_{a\gamma} \equiv \frac{\alpha}{2\pi} \frac{C_{i\gamma}}{f_{a_i}} < 10^{-10} \text{ GeV}^{-1}. \quad (\text{B.1})$$

This translates into a bound

$$\frac{f_{a_i}}{C_{i\gamma}} \gtrsim \frac{\alpha}{2\pi} 10^{10} \text{ GeV} \gtrsim 10^7 \text{ GeV}. \quad (\text{B.2})$$

This is usually used to provide a lower bound for the decay constant of the QCD axion; due to the ‘universal’ contribution to its two-photon coupling, for a generic field-theoretical model, $C_{a\gamma} \sim 1$. On the other hand, there are some models with $C_{a\gamma} \ll 1$ where the axion decay constant can be lowered. The CAST bounds (B.1) and (B.2) can be improved in the next generation of helioscopes to $g_{i\gamma} \sim 10^{-12} \text{ GeV}^{-1} \rightarrow f_{a_i}/C_{i\gamma} \sim 10^9 \text{ GeV}$ [5].

Turning now to the axial-vector couplings to electrons, let us first note that, in many tree-level interactions, an axial-vector vertex is equivalent to a scalar one:

$$\mathcal{L} \supset \frac{g_{i\psi}^A}{2m_\psi} \bar{\psi}\gamma^\mu\gamma_5\psi\partial_\mu a_i \rightarrow -\frac{g_{i\psi}^A}{2m_\psi} a_i\partial_\mu(\bar{\psi}\gamma^\mu\gamma_5\psi) = -ig_{i\psi}^A a_i\bar{\psi}\psi, \quad (\text{B.3})$$

where we have used the equations of motion. Of course, this only applies for tree-level processes involving the fermions as on-shell external states.

Excessive axion emission from red giants places the following constraint on axion couplings to electrons [59]:

$$\begin{aligned} \mathcal{L} \supset \frac{C_{ei}}{2f_{a_i}} \bar{e}\gamma^\mu\gamma_5e\partial_\mu a_i &\rightarrow C_{ie} = \hat{X}_e^i, \\ g_{ie}^A \equiv \frac{C_{ie}m_e}{f_{a_i}} \lesssim 2.5 \times 10^{-13} &\rightarrow \frac{f_{a_i}}{C_{ie}} \gtrsim 2.0 \times 10^9 \text{ GeV}. \end{aligned} \quad (\text{B.4})$$

A slightly better limit arises from pulsationally unstable white dwarfs (ZZ Ceti stars), where the period increase provides a measure of the cooling speed. From the measured period increase of the star G117-B15A, it was found [61–64]:

$$g_{ie}^A < 1.3 \times 10^{-13} \rightarrow \frac{f_{a_i}}{C_{ie}} > 3.9 \times 10^9 \text{ GeV}. \quad (\text{B.5})$$

Hence, for $C_{ie} \gtrsim 10^{-2} C_{i\gamma}$, the limits from electron couplings are more constraining than the direct two-photon coupling. Noting that the ‘universal’ coupling of the QCD axion to

¹¹For a recent update on the bounds on this coupling, see [6].

electrons is suppressed by $\mathcal{O}(10^{-4})$, we see that the direct couplings could be important, as we stressed in their discussion in the context of the LARGE Volume Scenario.

However — and intriguingly — reanalysis of the same white dwarf with improvements in the accuracy and precision of the pulsational data identifies a non-standard energy loss [12, 65, 66]. Improvements in the luminosity distribution of white dwarf stars were also found to require additional energy loss, an absence thereof predicting an excess of white dwarfs at luminosities $\log L/L_\odot \simeq -2$. The observed excess in cooling speed is consistent with the existence of an axion with mass $m_i \lesssim$ keV and a coupling to electrons of strength:

$$g_{ie}^A = (2.0 \div 7.0) \times 10^{-13} \rightarrow \frac{f_{a_i}}{C_{ie}} \simeq (0.7 \div 2.6) \times 10^9 \text{ GeV} \quad (\text{B.6})$$

which is compatible, within the uncertainties, with the limits from red giants.

On parity violating couplings to the electron, there is a limit [59, 67] on a scalar Yukawa coupling $g_{Yie}^V \bar{e}e a_i$ of the order:

$$g_{Yie}^V \lesssim 10^{-14}. \quad (\text{B.7})$$

These would arise only from non-perturbative corrections to the Yukawa couplings of the form $W \supset e^{-b \frac{a_i}{f_{a_i}}} h_d e_L e_R$; they are difficult to determine in a string model without knowing the flavour structure of the Yukawa couplings but generically we could estimate $g_{Yie}^V \sim \frac{m_e}{f_{a_i}}$ - if they are present at all - which would translate into a bound of the order $f_{a_i} \gtrsim 5 \times 10^{10}$ GeV.

On the other hand, we will generically find in string models parity-violating vector-like couplings

$$\mathcal{L} \supset g_{i\psi}^V \bar{\psi} \gamma^\mu \psi \partial_\mu a_i. \quad (\text{B.8})$$

Here however the integration by parts yields a coupling to the divergence of the electron number current; hence we expect the simplest amplitudes involving these couplings to vanish, and the constraints upon them to be very weak.

The limits on couplings to nucleons C_{aN} are most strongly constrained by (lack of) energy loss from supernova SN1987A, and applies for the range $0.002 \text{ eV} \lesssim C_{aN} m_a \lesssim 2 \text{ eV}$ [59]:

$$3 \times 10^9 \text{ GeV} \lesssim f_a / C_{aN}. \quad (\text{B.9})$$

B.1.2 Conversion of astrophysical photon fluxes

Due to the $F_{\text{em}} \tilde{F}_{\text{em}} \propto \mathbf{E} \cdot \mathbf{B}$ coupling, photons can oscillate into axions in large-scale magnetic fields. These oscillations can lead to observable effects in astrophysical photon spectra, but with high sensitivity only for very small masses. Correspondingly, these observations typically constrain only ALPs, but not the QCD axion.

The absence of a γ -ray burst in coincidence with SN 1987A neutrinos provides, for $m_i \lesssim 10^{-9} \text{ eV}$, the most restrictive limit on the two-photon coupling of ALPs [68, 69]:

$$g_{a\gamma} \equiv \frac{\alpha}{2\pi} \frac{C_{i\gamma}}{f_{a_i}} \lesssim 10^{-11} \text{ GeV}^{-1} \rightarrow \frac{f_{a_i}}{C_{i\gamma}} \gtrsim \frac{\alpha}{2\pi} 10^{10} \text{ GeV} \gtrsim 10^8 \text{ GeV}, \quad (\text{B.10})$$

an order of magnitude better than the one inferred from the lifetime of HB stars and from CAST.

This constraint challenges the ALP interpretation of recent measurements of γ -ray spectra from distant active galactic nuclei (AGN), extending to energies \gtrsim TeV — a puzzling observation, since the γ -ray absorption due to e^+/e^- pair production off the extragalactic background light is expected to cut-off the spectra at very high energies.¹² Intriguingly, apart from conventional explanations, such a high transparency of the Universe may be explained by photon \leftrightarrow ALP oscillations: the conversion of gamma rays into ALPs in the magnetic fields around the AGNs or in the intergalactic medium, followed by their unimpeded travel towards our galaxy and the consequent reconversion into photons in the galactic/intergalactic magnetic fields, requiring a very light, $m_i \lesssim 10^{-9}$ eV, ALP coupling to two photons with strength [7–9]:

$$g_{i\gamma} \sim 10^{-11} \text{ GeV}^{-1} \rightarrow f_{a_i}/C_{i\gamma} \sim 10^8 \text{ GeV}, \quad (\text{B.11})$$

in the same ballpark as the constraint from the non-observation of γ -rays from SN 1987A.

A similar effect can occur when gamma ray-ALP conversions are induced by intergalactic magnetic fields B , which have a strength of $0.1 \div 1$ nG. In this case, steps in the power spectrum can be observed at a critical energy [71]:

$$\frac{E_{\text{crit}}}{\text{GeV}} = \left(\frac{m_{\text{ALP}}}{\mu\text{eV}} \right)^2 \frac{10^{-11} \text{ GeV}^{-1} \text{ Gauss}}{0.4 g_{a\gamma}} \frac{1}{B}. \quad (\text{B.12})$$

It has been reported in [10] that such steps are indeed seen at a critical energy of ~ 100 GeV, with the spectrum consistent with $g_{i\gamma} \sim 10^{-11} \text{ GeV}^{-1}$, giving:

$$m_{\text{ALP}} \sim 10^{-9} \div 10^{-10} \text{ eV}. \quad (\text{B.13})$$

Very recently, it was pointed out [11] that a similar value of the photon coupling is required for a possible explanation of the rapidly varying very high energy ($E > 50$ GeV) emission from the flat spectrum radio quasar PKS 1222+216, which represents a challenge for standard blazar scenarios: in the latter one is forced to invoke the existence of a very compact ($r \sim 10^{14}$ cm) emitting region at a large distance ($R > 10^{18}$ cm) from the jet base, in order to avoid absorption of gamma rays in the dense ultraviolet radiation field of the broad line region (BLR). In ref. [11] it was shown that one can also use a standard blazar model for PKS 1222+216 where gamma rays are produced close to the central engine, if one assumes that inside the source photons can oscillate into ALPs, with a similar coupling corresponding to $f_{a_i}/C_{i\gamma} \sim 10^8$ GeV.

Remarkably, the values of f_{a_i}/C_{ie} and $f_{a_i}/C_{i\gamma}$, inferred from very different astronomical observations, turn out to be in the same region.¹³ However, there is one crucial difference: while the WD value (B.6) can be realised both with the QCD axion or with a generic ALP, the anomalous transparency value (B.11) can not be realised with the QCD axion, since the mass of the latter, $m_a \sim m_\pi f_\pi / f_a \sim 10 \text{ meV} (10^9 \text{ GeV} / f_a)$, is much higher than the phenomenologically required values, which are in the sub-neV range.

¹²For a recent compilation of the available data and comparison with the expectation, see [70].

¹³There are also other hints of signals for ALPs in other parameter regions, for example in [72, 73].

B.1.3 Black hole super-radiance

Any axion that has a Compton wavelength comparable to the radius of black hole event horizons can cause the spin-down of the black hole. Since black holes with masses between $2M_\odot$ and $10^{10}M_\odot$ have been observed, this corresponds to masses of the order 10^{-10} to 10^{-21} eV. The more precise upper bound, independent of the cosmological abundance of the axions, is then [19, 54]:

$$m_a > 3 \times 10^{-11} \text{eV}. \tag{B.14}$$

Interestingly, this is just below the range required for the astrophysical hints.

B.2 Cosmic axions

B.2.1 Dark matter

The QCD axion [74–76] and more generally ALPs [6, 19, 22, 77, 78] may constitute populations of cold dark matter (CDM), produced non-thermally via the misalignment mechanism. The latter relies on assuming that fields in the early Universe have a random initial state (arising from quantum fluctuations during inflation) which is fixed by the expansion of the Universe; fields with mass m_i evolve on timescales $t \sim m_i^{-1}$. After such a timescale, the fields respond by attempting to minimise their potential, and consequently oscillate around the minimum. If there is no significant damping via decays, these oscillations can behave as a cold dark matter fluid since their energy density is diluted by the expansion of the Universe as $\rho_i \propto a^{-3}$, where a is the Universe scale factor. Without fine-tuning of the initial conditions and without further dilution by e.g. late entropy production, the expected cosmic mass fraction in QCD axion CDM is then [79]:

$$\frac{\Omega_a h^2}{0.112} \approx 6.3 \times \left(\frac{f_a}{10^{12} \text{ GeV}} \right)^{7/6} \left(\frac{\Theta_a}{\pi} \right)^2 \tag{B.15}$$

where Θ_a is the initial misalignment angle, while the one in ALP CDM is [6]:

$$\frac{\Omega_{a_i} h^2}{0.112} \approx 1.4 \times \left(\frac{m_i}{\text{eV}} \right)^{1/2} \left(\frac{f_{a_i}}{10^{11} \text{ GeV}} \right)^2 \left(\frac{\Theta_i}{\pi} \right)^2 \tag{B.16}$$

where m_i is the mass of the ALP.

Galactic halo QCD axions, as well as ALPs — if they couple to photons, $C_{i\gamma} \neq 0$ — can be directly searched for via their resonant conversion into a quasi-monochromatic microwave signal with frequency $\nu = m_a/(2\pi) = 0.24 \text{ GHz} \times (m_a/\mu\text{eV})$, in a high-Q electromagnetic cavity permeated by a strong static magnetic field [80]. A number of experiments of this type have already been done [81–85] and further improvements are underway [86–88]. These haloscope searches reach currently sensitivities of order

$$g_{a\gamma} \sim g_{i\gamma} = \frac{\alpha}{2\pi} \frac{C_{i\gamma}}{f_{a_i}} \lesssim 10^{-(15 \div 13)} \text{ GeV}^{-1} \rightarrow \frac{f_{a_i}}{C_{i\gamma}} \gtrsim 10^{10 \div 12} \text{ GeV}, \tag{B.17}$$

in the $(1 \div 10) \mu\text{eV}$ mass range.

Recently, [13] reported the possibility of probing for QCD axion dark matter for decay constants between 10^{16} and 10^{19} GeV via molecular interferometry experiments. While it would not apply to ALPs - relying on the coupling to QCD - it would be a very interesting project to probe such weak couplings.

B.2.2 Isocurvature and tensor modes

If the axions constitute a significant proportion of dark matter, then they produce isocurvature and tensor fluctuations, which may be used to place bounds on the inflationary scale. These have been discussed for the case of many axions, in, for example [19]. The proportion of isocurvature fluctuations $\langle |S^2| \rangle$ to the total $A_S \equiv \langle |S^2(k_0)| \rangle + \langle |R^2(k_0)| \rangle \simeq \langle |R(k_0)|^2 \rangle$ (where R are the adiabatic fluctuations and k_0 is the pivot scale) is constrained by [89] to be less than 0.077; we have:

$$A_S = (2.430 \pm 0.091) \times 10^{-9}$$

$$\alpha \equiv \frac{\langle |S^2| \rangle}{\langle |S^2| \rangle + \langle |R^2| \rangle} = \sum_{i=1}^{n_{\text{ax}}} \frac{1}{A_S} \left(\frac{\Omega_{a_i}}{\Omega_m} \right)^2 \frac{2\sigma_{\Theta_i}^2 (2\Theta_i^2 + \sigma_{\Theta_i}^2)}{(\Theta_i^2 + \sigma_{\Theta_i}^2)^2} < 0.077 \quad (\text{B.18})$$

where $\sigma_{\Theta_i} = \frac{H_{\text{inf}}}{2\pi f_{a_i}}$ is the standard deviation of axion angle fluctuations and Ω_m is total matter fraction of the critical energy density. Assuming that these fluctuations are dominated by one axion with $\Theta_i \gg \sigma_{\Theta_i}$ (i.e. no tuning of the angle) we have

$$H_{\text{inf}} < 4.3 \times 10^{-5} \left(\frac{\Omega_m}{\Omega_{a_i}} \right) \Theta_i f_{a_i}. \quad (\text{B.19})$$

For the single-ALP model to explain the astrophysical anomalies (section B), this gives $H_{\text{inf}} \lesssim 10^4 \text{ GeV}$. Since local axions will dominate over non-local ones in terms of energy density (since they couple much more strongly to the non-perturbative effects) we can use $f_{a_i} \sim M_P/\sqrt{\mathcal{V}}$. Assuming that the dark matter is entirely one axion gives us the restriction $H_{\text{inf}} < m_{3/2} \sim \frac{M_P}{\sqrt{\mathcal{V}}}$ for $\mathcal{V} \gtrsim 10^9$, which restricts the possibility of entropy injection by large numbers of moduli for large volumes. Thus, if we insist upon axion dark matter and entropy injection, we are restricted to GUT-scale strings; but on the other hand, intermediate-scale strings produce axions only within the cosmologically allowed window, eliminating the need for entropy injection.

Tensor fluctuations of axions are typically negligible compared to those induced by metric fluctuations; the constraint upon the ratio of tensor to scalar fluctuations is $r \equiv A_T/A_S < 0.36$, where $A_T = \frac{16H_{\text{inf}}^2}{\pi(8\pi M_P^2)}$, giving:

$$H_{\text{inf}} < 1.6 \times 10^{14} \text{ GeV}, \quad (\text{B.20})$$

which is typically much weaker than the isocurvature bound.

B.2.3 Rotation of CMB polarisation

For very light axions/ALPs that begin to oscillate between the CMB era and today (corresponding to masses between 4×10^{-28} and 10^{-33} eV), that in addition have a significant coupling to photons, polarisation of the CMB may occur [19]. The limit is:

$$\frac{C_{i\gamma}\alpha}{2\sqrt{3}} \lesssim 3.5 \times 10^{-2}. \quad (\text{B.21})$$

This is only potentially constraining for *local* axions, since non-local ones will have a parametrically suppressed $C_{i\gamma}$. With a combination of different cosmological observations, including galaxy redshift and weak lensing surveys, it should be possible to detect a fraction of ultralight axions to dark matter of a few percent [90].

B.3 Laboratory searches for axions

Currently, pure laboratory searches for very light (mass $< \text{eV}$) axions can not compete with the constraints from astrophysics and cosmology. The best sensitivity is obtained in light-shining-through-a-wall experiments, which are searching for $\gamma \rightarrow a_i \rightarrow \gamma$ conversions in an external magnetic field and thus probing the axion to two-photon coupling. Recently, a number of such experiments were performed; among them, ALPS was the most sensitive one, improving the limit to [91]:

$$g_{i\gamma} < 6.5 \times 10^{-8} \text{ GeV}^{-1} \rightarrow \frac{f_{a_i}}{C_{i\gamma}} \gtrsim 1.7 \times 10^5 \text{ GeV}, \tag{B.22}$$

for masses below $m_i \ll \text{meV}$. The next generation of these experiments is designed to reach a sensitivity of $g_{i\gamma} \sim 10^{-11} \text{ GeV}^{-1} \rightarrow f_{a_i}/C_{i\gamma} \sim 10^8 \text{ GeV}$ [4], thus starting to probe the region of interest for the possible explanation of the anomalous features in AGN spectra reported above.

C Axionic couplings to matter at one-loop

In addition to the tree level axionic couplings computed in section 2.4, there will also be corrections to the Kähler metric at one loop. These have not yet been calculated for all cases of interest: the result is only known for the coupling of non-local axions to matter at a singularity [92, 93] and to adjoint matter [94]. It would be very interesting to calculate the corrections involving local axions (i.e. blow-up moduli) for matter at a singularity along the lines of [95] but we leave this for future work. Here we shall analyse the contributions from field theory, and how they relate to the existing and future results.

The couplings to matter fields at one loop are generated via the coupling to photons or other gauge groups. This is to be distinguished from renormalisation of the axion-photon coupling, since we do not perform a chiral rotation - which would introduce axion couplings to mass terms. In non-supersymmetric theories, the correction from a loop involving photons and electrons is $\Delta_{i\gamma\gamma}[C_{ie}^{A,V}] = \frac{3\alpha^2}{4\pi^2} C_{i\gamma} \ln\left(\frac{f_{a_i}}{m_e}\right)$ - which depends upon the axion decay constant as a cutoff in the loop, or can be interpreted as a renormalisation of the axion-matter coupling. However, in supersymmetric theories, the two-photon coupling is related to a coupling to gauginos:

$$\mathcal{L} \supset \int d^2\theta (ia_i) \frac{g_{i\gamma}}{4} W^\alpha W_\alpha \supset \frac{1}{4} g_{i\gamma} a_i F_{em,\mu\nu} \tilde{F}_{em}^{\mu\nu} - \frac{1}{2} g_{i\gamma} \partial_\mu a_i \lambda^\alpha \sigma^\mu \bar{\lambda},$$

and so we also have loops involving gauginos and selectrons which will change the result. The cutoff-dependent contributions arise even before supersymmetry is broken, and are related to the corrections to the Kähler metric at one loop, whereas there are also more complicated finite contributions that arise when supersymmetry is broken. Let us recall that the Kähler potential at one loop is given by [96]:

$$\begin{aligned} \Delta K = & \frac{\Lambda^2}{16\pi^2} \left[\ln \det \hat{K} - \ln \det \hat{H} \right] \\ & - \frac{1}{32\pi^2} \left[\text{tr} \left(\mathcal{M}_\Phi^2 \left(\ln \frac{\mathcal{M}_\Phi^2}{\Lambda^2} - 1 \right) \right) - 2\text{tr} \left(\mathcal{M}_V^2 \left(\ln \frac{\mathcal{M}_V^2}{\Lambda^2} - 1 \right) \right) \right], \end{aligned} \tag{C.1}$$

where \hat{K} is the Kähler metric, $\hat{K}^{-1/2}$ is the matrix that diagonalises \hat{K} : $\hat{K}^{-1/2} T \hat{K} \hat{K}^{-1/2} = \delta_{\bar{i}j}$, $\hat{w} \equiv \partial_{\bar{i}j}^2 W$ and:

$$\begin{aligned} \hat{H}_{gh} &\equiv \text{Re}(f_{gh}), & \mathcal{M}^2 &\equiv \left(\hat{K}^{-1/2} \bar{w} \hat{K}^{-1} T_w \hat{K}^{-1/2} \right) \\ (\mathcal{M}_V^2)_{gh} &\equiv \hat{H}_{gk}^{-1/2} \left[(\bar{\Phi} T_k)^{\bar{i}} K_{\bar{i}j} (T_l \Phi)^j + (\bar{\Phi} T_l)^{\bar{i}} K_{\bar{i}j} (T_k \Phi)^j \right] \hat{H}_{kh}^{-1/2}. \end{aligned} \quad (\text{C.2})$$

This is a global supersymmetry calculation, so we must be careful how we interpret the result. Considering a model where $f_{gh} = \frac{\delta_{gh}}{g_g^2} (1 + g_{ig} T_i)$ for superfields $T_i \supset i c_i$ containing the axions c_i , we can compute the gauge corrections to the Kähler potential involving quadratic terms in matter fields Φ_i at first order in $T(\theta, \bar{\theta})$:

$$\Delta K_{\text{gauge}} \Big|_T = \frac{4}{32\pi^2} g_g^2 (T_\alpha + \bar{T}_\alpha) \hat{K} \Phi_j^\dagger \Phi_j C_2(\Phi_j) \left[\partial_{T_\alpha} (\ln \hat{K}) - \frac{g_{\alpha g}}{2} \right] \ln \left(\frac{\Lambda^2}{m_V^2} \right),$$

where we have inserted a mass m_V for the gauge boson as an infra-red regulator. This is necessary in a theory without a superpotential and thus massless particles, but when the matter fields obtain masses these should appear instead. We thus find the physical coupling:

$$\Delta \mathcal{L}_{\text{gauge}} = \frac{4}{32\pi^2} g_g^2 K_{j\bar{j}} \psi_j \sigma^\mu \bar{\psi}_j \partial_\mu c_\alpha C_2(\Phi_j) \left[\partial_{T_\alpha} (\ln \hat{K}) - \frac{g_{\alpha g}}{2} \right] \ln \left(\frac{\Lambda^2}{m_V^2} \right).$$

The first term in square brackets is generated by loops involving the tree-level coupling and thus we expect should be corrected to $\ln \left(e^{-\frac{K_0}{2}} \hat{K} \right)$ in a supergravity calculation, whereas the second term is the one truly generated by the axionic coupling. The above is related to the dependence on the moduli of the anomalous dimension of the fields. If we add a superpotential $W \supset Y \Phi_1 \Phi_2 \Phi_3$, we have:

$$\Delta K_{\text{Yukawa}} \Big|_T = - \frac{2}{32\pi^2} |Y|^2 (T_\alpha + \bar{T}_\alpha) \frac{\Phi_j^\dagger \Phi_j}{\hat{K}^2} \partial_{T_\alpha} (\ln \hat{K}) \ln \left(\frac{\Lambda^2}{m_\Phi^2} \right).$$

These two terms are apparently quite different. However, we list it for completeness to compare with the results for the behaviour of models of branes at orbifold singularities, where the anomalous dimension vanishes. There the gauge coupling is given by the dilaton, we have $\hat{K} = e^{\frac{K_0}{3}}$ and we should take $Y = \sqrt{2} g e^{\frac{K_0}{2}}$ with $g^2 = (S + \bar{S})^{-1}$. Hence the dilaton-axion coupling cancels between the two terms. However, it does indicate the very interesting result that at one loop the corrections to the Kähler potential involving twisted (blow-up) moduli should not only not vanish but involve a large logarithmic factor. It would be interesting to confirm this with a calculation along the lines of [95] where we could learn about the cutoff scale Λ introduced in string theory but we leave this to future work.

Since the corrections at one loop due to the $\partial_{T_i} (\ln \hat{K})$ term should be smaller than the tree-level term, we can take the dominant loop contribution to be:

$$\begin{aligned} \mathcal{L} &\supset - \frac{g_{ig} \alpha_g}{2\pi} \partial_\mu a_i \psi_j \sigma^\mu \bar{\psi}_j C_2(\psi_j) \ln \left(\frac{\Lambda}{m_V} \right) \\ &\supset - \frac{\alpha_g^2}{4\pi^2 f_{a_i}} C_{ig} \partial_\mu a_i \psi_j \sigma^\mu \bar{\psi}_j C_2(\psi_j) \ln \left(\frac{\Lambda}{m_V} \right), \\ \Delta C_{ig} &= - \frac{2\alpha_g^2}{4\pi^2} C_{ig} C_2(\psi_j) \ln \left(\frac{\Lambda}{m_V} \right), \end{aligned} \quad (\text{C.3})$$

in the physical basis where the fields ψ_j are canonically normalised. Note that this is for a left-handed fermion, so to obtain the electron couplings we should add together the couplings for left- and right-handed components. If we take the approximation that the electron couples only non-chirally to the photon, then no parity-violating vector-like coupling is generated. However, when we take the chiral nature of the electro-weak interactions into account we obtain the approximation (A.13).

Once the supersymmetry breaking terms are included, there are also finite pieces that should in principle be included, even if they are subdominant to the above. Then the effect of electro-weak symmetry breaking must be taken into account. This leads to non-diagonal gaugino and gauge boson couplings, and the final result is rather complicated. We simply note for reference that the contribution to the coupling of a Weyl fermion (i.e. the left- or right-handed component of a matter fermion) will be derived from coupling terms of the form:

$$\mathcal{L} \supset \frac{g_{gh}}{4} a F_{\mu\nu}^g \tilde{F}^{h\mu\nu} - \frac{\tilde{g}_{12}}{2} \partial_\mu a \lambda_1 \sigma^\mu \bar{\lambda}_2 - \sqrt{2} g_1 [\Phi^*(\psi \lambda_1) + \text{h.c.}] - \sqrt{2} g_2 [\Phi^*(\psi \lambda_2) + \text{h.c.}],$$

where g_1, g_2, \tilde{g}_{12} involve gauge couplings and mixing matrices of the gaugino mass eigenstates. The result is:

$$\begin{aligned} \Delta \mathcal{L} &= \Delta_{\text{boson}} \mathcal{L} + \Delta_{\text{fermion}} \mathcal{L} \\ \Delta_{\text{boson}} \mathcal{L} &= -(\partial_\mu a) \psi \sigma^\mu \bar{\psi} \frac{3g_g g_h g_{gh}}{2(4\pi)^2} \left[f(m_\psi^2, m_g^2) + \frac{m_h^2}{m_g^2 - m_h^2} (f(m_\psi^2, m_g^2) - f(m_\psi^2, m_h^2)) \right] \\ \Delta_{\text{fermion}} \mathcal{L} &= (\partial_\mu a) \psi \sigma^\mu \bar{\psi} g_1 g_2 \tilde{g}_{12} \left[-i \int \frac{d^4 q}{(2\pi)^4} \frac{\frac{1}{2} q^2 + M_1 M_2}{(q^2 - M_1^2)(q^2 - M_2^2)(q^2 - M_\Phi^2)} \right] \\ &= (\partial_\mu a) \psi \sigma^\mu \bar{\psi} \frac{g_1 g_2 \tilde{g}_{12}}{(4\pi)^2} \times \\ &\quad \times \left[\frac{1}{2} f(M_1^2, M_2^2) + \frac{M_1 M_2 + \frac{1}{2} M_\Phi^2}{M_2^2 - M_\Phi^2} (f(M_1^2, M_2^2) - f(M_1^2, M_\Phi^2)) \right] \\ f(m_1^2, m_2^2) &\equiv 1 + \left[\frac{m_2^2 \ln\left(\frac{\Lambda^2}{m_2^2}\right) - m_1^2 \ln\left(\frac{\Lambda^2}{m_1^2}\right)}{m_2^2 - m_1^2} \right]. \end{aligned}$$

Here we have given expressions in terms of gauge boson masses m_g, m_h , fermion masses m_ψ , sfermion masses M_Φ and gaugino masses M_1, M_2 . In the case that $m_g = m_h = m_V$, $g_1 = g_2 = g_g = g_h$, the leading cutoff-dependent part recovers the diagonal supersymmetric result above. Note that in general we have parity-violating terms.

D Axion decay constants for general fibred Calabi-Yau manifolds

The analysis of the axion decay constants performed in section 2.3.2 is simple since the volume form can be written explicitly in terms of the τ moduli. However, when we have a more complicated model, this may not be the case: we may only have the expression in terms of the Kähler parameters t_α and the τ_α as quadratic expressions of these. In this

case, we must use these to examine the metrics \mathcal{K}_0 and \mathcal{K}_0^{-1} . An interesting example of this is a general K3 or T^4 fibration in the anisotropic limit (which may in principle involve many moduli), where some $t_\alpha \sim xt_\alpha^0$ become large; we can expand all of the two-cycles, four-cycles and the volume as:

$$t_A \equiv xt_A^0 + t_A^1, \quad \tau_A \equiv x\tau_A^0 + \tau_A^1, \quad \mathcal{V} = x\mathcal{V}_0 + \dots,$$

and thus:

$$\begin{aligned} (\mathcal{K}_0)^{AB} &\sim \mathcal{V} \left(-xK_{AB}^0 + x\frac{\tau_A^0\tau_B^0}{\mathcal{V}_0} - K_{AB}^1 + \frac{\tau_A^0\tau_B^1 + \tau_A^1\tau_B^0 - \frac{\mathcal{V}_1}{\mathcal{V}_0}\tau_A^0\tau_B^0}{\mathcal{V}_0} + \dots \right) \\ (\mathcal{K}_0)_{AB} &\sim \frac{1}{\mathcal{V}} \left(\frac{x t_A^0 t_B^0}{2 \mathcal{V}_0} + \frac{t_A^0 t_B^1 + t_A^1 t_B^0 - \frac{\mathcal{V}_1}{\mathcal{V}_0} t_A^0 t_B^0}{2\mathcal{V}_0} - (K^{AB})^0 - \frac{1}{x}(K^{AB})^1 + \dots \right). \end{aligned}$$

The matrix \mathcal{C}_{AB} diagonalises both of these, and we can determine it order by order by considering the two matrices separately. Since the leading matrix in the expansion of $(\mathcal{K}_0)_{AB}$ is of order one and rank one (it is proportional to $t_A^0 t_B^0$) clearly there is exactly *one* axion that has a decay constant of the order of the Planck mass. This is the axion corresponding to the fibre: it is proportional to $t_A^0 a_A$ so the associated modulus is proportional to $t_A^0 \tau_A$; since $t_A^0 \tau_A^0 = 0$ this must be a small cycle, and the coupling to matter is therefore Planck suppressed.

There are also several axions with string scale decay constants, but on the other hand, by examining $(\mathcal{K}_0)^{AB}$ we see that there are axions that have decay constants of order $M_P \mathcal{V}^{-1}$! These would be very interesting phenomenologically if they coupled strongly to small cycles; however, (2.9) tells us that they must correspond to cycles with very small gauge couplings, and since in this limit there are only cycles $\tau_A^0 \sim \mathcal{V}$ or $\tau_A \sim 1$ the former case must apply: hence they have Planck-suppressed couplings to gauge groups. They are given by the eigenvectors $c_B^i a_B$ of the matrix $-\mathcal{V}_0 \mathcal{K}_{AB}^0 + \tau_A^0 \tau_B^0$; note that \mathcal{K}_{AB} has signature $(+, -, -, -, \dots)$ and \mathcal{K}_{AB}^0 is the matrix of intersections that have large sizes. Hence there may be more than one large cycle: in this case, they must all be fixed relative to each other via D-terms to give an effective canonical fibration volume form for moduli stabilisation [21].

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