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Nonrelativistic limit of the abelianized ABJM model and the ADS/CMT correspondence

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Abstract: We consider the nonrelativistic limit of the abelian reduction of the massive ABJM model proposed in [1], obtaining a supersymmetric version of the Jackiw-Pi model. The system exhibits an $\mathcal{N} = 2$ Super-Schrödinger symmetry with the Jackiw-Pi vortices emerging as BPS solutions. We find that this $(2+1)$-dimensional abelian field theory is dual to a certain (3+1)-dimensional gravity theory that differs somewhat from previously considered abelian condensed matter stand-ins for the ABJM model. We close by commenting on progress in the top-down realization of the AdS/CMT correspondence in a critical string theory.

Keywords: AdS-CFT Correspondence, Holography and condensed matter physics (AdS/CMT), Solitons Monopoles and Instantons, Supersymmetric gauge theory

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Contents

1 Introduction

Due in no small part to its role in the AdS/CFT correspondence [2], 4-dimensional, $\mathcal{N}=4$ supersymmetric Yang-Mills theory (SYM) has provided a remarkable new window into the physics of strongly coupled gauge theories, resulting in a veritable spectrum of insights that range from the phenomenology of the quark-gluon plasma to the structure of scattering amplitudes in quantum field theories. Its 3-dimensional counterpart, an $\mathcal{N}=6$ superconformal Chern-Simons-matter theory called the ABJM model [3], has furnished an equally impressive laboratory within which to understand planar field theories and promises a powerful toolbox with which to attack various condensed matter systems, again via the gauge/gravity duality.

This Anti de-Sitter/Condensed-Matter-Theory (AdS/CMT) correspondence is usually defined somewhat phenomenologically, by building gravity duals with the required fields and symmetries to describe relevant physics in (typically) abelian condensed matter models and certainly much of the progress in the field has been made in this *bottom-up* approach [4]. While this progress is certainly remarkable, we felt it unsatisfactory and found the need to ask whether it was possible to realize such a correspondence in a critical string theory, like the type IIA $AdS_4 \times \mathbb{CP}^3$.

This article is a continuation of the *top-down* program for the construction of an AdS/CMT correspondence initiated in [1, 5]. There it was demonstrated that a (fully quantum) consistent truncation of (a massive deformation of) the ABJM model [6, 7], reduces to a relativistic version of the Landau-Ginzburg model. The latter of course, plays a key role in many condensed matter phenomena, for example, quantum critical phases [8, 9]. Then, in [10], we extended the truncation to the supersymmetric case, demonstrating its consistency with the supersymmetry of the parent (m)ABJM model and its utility in planar condensed matter systems.[1]

However, while there are certainly condensed matter systems that are *effectively* relativistic, most are, in fact, nonrelativistic. The primary purpose of this paper is therefore to take the next logical step, and perform a nonrelativistic limit on the abelian reduction found in [10]. We find a reduction to the so-called Jackiw-Pi model of [12, 13], with the well studied Jackiw-Pi vortices arising as solutions of the reduced (m)ABJM model. Further, we also find a supersymmetric version of the model defined in [14], with an $\mathcal{N}=2$ supersymmetric Schrödinger symmetry. This particular reduction allows us to describe regular systems with Schrödinger symmetry, via the usual AdS/CFT holography, in terms of a $(d+1)$-dimensional gravity dual. This is to be contrasted with the previously known standard example of holography for Schrödinger symmetry, between an unusual dipole theory and a gravity dual in $(d+2)$ dimensions, obtained from the discrete light cone quantization (DLCQ) of usual AdS/CFT dualities [15–17]. We also compare our model with the abelian nonrelativistic model in [18, 19], that was used as a stand-in for ABJM to describe compressible Fermi surfaces.

The paper is organized as follows. In section 2 we consider the nonrelativistic limit of the abelian reduction of the massive ABJM model, and explore two choices for the supersymmetry transformation rules. In section 3 we further truncate the model to obtain the supersymmetric version of the model of [14], whose symmetry is coded in an $\mathcal{N}=2$ Super-Schrödinger algebra. In section 4 we describe applications to the AdS/CMT correspondence, first by comparing with systems with Schrödinger symmetry previously used in this context, and then by comparing with the nonrelativistic model of [18], previously used to understand the physics of compressible Fermi surfaces. We conclude with a discussion in section 5.

2 A nonrelativistic limit of abelianized ABJM

Our starting point for this study is the abelian reduction of the mass-deformed ABJM model proposed in [10]. The action for the supersymmetric abelian reduction of massive ABJM was found in eq. 4.6 of that paper, and reads[2]

$$\begin{aligned}S = -\frac{N(N-1)}{2}\int d^3x\Big\{&\frac{k}{4\pi}\epsilon^{\mu\nu\lambda}\big(a_\mu^{(2)}f_{\nu\lambda}^{(1)}+a_\mu^{(1)}f_{\nu\lambda}^{(2)}\big)+|D_\mu\phi_i|^2+|D_\mu\chi_i|^2\\ &+i\sum_{i=1,2}\Big[\bar\eta_i(\not{D}+\mu)\eta_i+\bar{\tilde\eta}_i(\not{D}-\mu)\tilde\eta_i\Big]\\ &-\frac{2\pi i}{k}\Big[(|\phi_1|^2+|\chi_1|^2)(\bar\eta_2\eta_2+\bar{\tilde\eta}_2\tilde\eta_2)+(|\phi_2|^2+|\chi_2|^2)(\bar\eta_1\eta_1+\bar{\tilde\eta}_1\tilde\eta_1)\Big]\end{aligned}$$

[1] For an extension of these methods to $\mathcal{N}=4$ SYM, see [11].

[2] Note that this version differs from that found in [10] by an i, a minus sign and replacing the $\eta^\dagger$'s with $\bar\eta$'s.

$$+\left(\frac{2\pi}{k}\right)^2\Big[(|\phi_1|^2+|\chi_1|^2)\big(|\chi_2|^2-|\phi_2|^2-c^2\big)^2+(|\phi_2|^2+|\chi_2|^2)\big(|\chi_1|^2-|\phi_1|^2-c^2\big)^2$$
$$+4|\phi_1|^2|\phi_2|^2(|\chi_1|^2+|\chi_2|^2)+4|\chi_1|^2|\chi_2|^2(|\phi_1|^2+|\phi_2|^2)\Big]\Bigg\}\,, \tag{2.1}$$

where $c^2\equiv k\mu/(2\pi)$, the abelian covariant derivative on the scalars $D_\mu\phi_i=\left(\partial_\mu-iA_\mu^{(i)}\right)\phi_i$, with a similar relation holding for fermions, and $\bar\eta\equiv\eta^\dagger\gamma^0$. Here $a_\mu^{(1)},a_\mu^{(2)}$ are gauge fields, ϕ_1,ϕ_2 and χ_1,χ_2 are complex scalars and η_1,η_2 and $\tilde\eta_1,\tilde\eta_2$ are complex 2-component Dirac spinors. This action is invariant under the following set of supersymmetry transformations rules[3]

$$\begin{aligned}
\delta\phi_1 &= i\bar\epsilon\tilde\eta_{\dot 1},\\
\delta\phi_2 &= -i\bar\epsilon\tilde\eta_{\dot 2},\\
\delta\chi_{\dot 1} &= -i\bar\epsilon\eta_1,\\
\delta\chi_{\dot 2} &= i\bar\epsilon\eta_2,\\
\delta a_\mu^{(1)} &= \frac{2\pi}{k}\left(\bar\epsilon\gamma_\mu[\phi_2\tilde\eta_{\dot 2}^*-\chi_{\dot 2}\eta_2^*]+\bar\epsilon^*\gamma[\phi_2^*\tilde\eta_{\dot 2}-\chi_{\dot 2}^*\eta_2]\right),\\
\delta a_\mu^{(2)} &= -\frac{2\pi}{k}\left(\bar\epsilon\gamma_\mu[\phi_1\tilde\eta_{\dot 1}^*-\chi_{\dot 1}\eta_1^*]+\bar\epsilon^*\gamma^\mu[\phi_1^*\tilde\eta_{\dot 1}-\chi_{\dot 1}^*\eta_1]\right),\\
\delta\eta_1 &= \gamma^\mu D_\mu\chi_{\dot 1}+\frac{2\pi}{k}\epsilon\chi_{\dot 1}(|\phi_2|^2+|\chi_{\dot 2}|^2)-\mu\epsilon\chi_{\dot 1},\\
\delta\eta_2 &= -\gamma^\mu\epsilon D_\mu\chi_{\dot 2}-\frac{2\pi}{k}\epsilon\chi_{\dot 2}(|\phi_1|^2+|\chi_{\dot 2}|^2)+\mu\epsilon\chi_{\dot 2},\\
\delta\tilde\eta_{\dot 1} &= -\gamma^\mu\epsilon D_\mu\phi_1-\frac{2\pi}{k}\epsilon\phi_1(|\phi_2|^2+|\chi_{\dot 2}|^2)-\mu\epsilon\phi_1,\\
\delta\tilde\eta_{\dot 2} &= \gamma^\mu\epsilon D_\mu\phi_2+\frac{2\pi}{k}\epsilon\phi_2(|\phi_1|^2+|\chi_{\dot 1}|^2)+\mu\epsilon\phi_2.
\end{aligned} \tag{2.2}$$

Since the parameter ϵ is complex, we have an SO(2) = U(1) R-symmetry, resulting in an $\mathcal{N}=2$ susy in three dimensions.

2.1 A nonrelativistic limit of the action

The nonrelativistic limit for the nonabelian $\mathcal{N}=6$ mass-deformed ABJM was first considered in [20, 21]. Here, we will focus on the abelianized ABJM. In order to take the nonrelativistic limit, we first need to restore the factors of $\hbar$ and c by dimensional analysis.[4] Writing also $((\partial_0,A_0)=(\partial_t,A_t)/c)$ and renaming the nonrelativistic μ as m, the scalar part of the Lagrangian becomes

$$-\frac{2}{N(N-1)}\mathcal{L}_{\rm scal}=-\frac{1}{c^2}(D_t\tilde\phi_j)\overline{(D_t\tilde\phi_j)}+(D_i\tilde\phi_j)\overline{(D_i\tilde\phi_j)}-\frac{1}{c^2}(D_t\tilde\chi_j)\overline{(D_t\tilde\chi_j)}$$

[3] Note that we have removed the 1/2 in front of the μ term in the susy rules and multiplied it with a minus sign everywhere, correcting the corresponding result in [10], as is easily checked. We have also used ϵ^* instead of ϵ in the second terms in $\delta a_\mu^{(i)}$.

[4] The dimensions of constants and fields in terms of mass M, length L and time T are: $[\hbar]=ML^2T^{-1}$, $[m]=M$, $[c]=LT^{-1}$, $[k]=L^{-1}T$, $[Z^{\hat A}]=[W^{\dagger\hat A}]=M^{1/2}L^{1/2}T^{-1/2}$, $[\psi_A]=M^{1/2}T^{-1/2}$, $[A_\mu]=[\hat A_\mu]=L^{-1}$, $[A_t]=T^{-1}$.

$$
\begin{aligned}
&+(D_i\tilde{\chi}_j)\overline{(D_i\tilde{\chi}_j)} + \frac{m^2c^2}{\hbar^2}\left(|\tilde{\phi}_1|^2 + |\tilde{\phi}_2|^2 + |\tilde{\chi}_1|^2 + |\tilde{\chi}_2|^2\right) \\
&-\frac{8\pi}{k}\frac{mc}{\hbar}\frac{1}{\hbar c}\left(|\tilde{\chi}_1|^2|\tilde{\chi}_2|^2 - |\tilde{\phi}_1|^2|\tilde{\phi}_2|^2\right) + \frac{4\pi^2}{(k\hbar c)^2}\Big[(|\tilde{\chi}_1|^2 + |\tilde{\phi}_1|^2)(|\tilde{\chi}_2|^2 + |\tilde{\phi}_2|^2) \\
&\times(|\tilde{\chi}_1|^2 + |\tilde{\chi}_2|^2 + |\tilde{\phi}_1|^2 + |\tilde{\phi}_2|^2)\Big], \qquad (2.3)
\end{aligned}
$$

whereas the pure gauge (Chern-Simons) part of the Lagrangian is topological, so it is unchanged, up to the fact that now there is a relative c between the spatial and temporal parts of the action,

$$
S_{CS}^{NR} = -\frac{N(N-1)}{2}\int d^3x\left\{\frac{k\hbar}{4\pi}\epsilon^{\mu\nu\lambda}\left(A_\mu^{(2)}F_{\nu\lambda}^{(1)} + A_\mu^{(1)}F_{\nu\lambda}^{(2)}\right)\right\}. \qquad (2.4)
$$

Note that the overall factor of c is cancelled by the re-definition of A_0 in the nonrelativistic, $c \to \infty$ limit, which also eliminates the sextic terms in the scalar potential. For the remaining terms, we must replace the fields with their nonrelativistic versions. In principle, a complex scalar field $\tilde{\phi}$ would be written as

$$
\tilde{\phi} = \frac{\hbar}{\sqrt{2m}}\left[\phi e^{-i\frac{mc^2}{\hbar}t} + \hat{\phi}^* e^{+i\frac{mc^2}{\hbar}t}\right], \qquad (2.5)
$$

with ϕ and $\hat{\phi}^*$ complex fields representing particles and anti-particles respectively, separately conserved. However, we will be working in the zero antiparticle sector, where we drop the second term so that

$$
(\tilde{\phi}, \tilde{\chi}) \longrightarrow \left(\frac{\hbar}{\sqrt{2m}}\phi(x,t)e^{-imc^2t/\hbar}, \frac{\hbar}{\sqrt{2m}}\chi(x,t)e^{-imc^2t/\hbar}\right). \qquad (2.6)
$$

From the kinetic (time derivative) term, the purely scalar part is

$$
-\frac{\hbar^2}{2mc^2}\left(\partial_t\phi\partial_t\bar{\phi} + \frac{i2mc^2}{\hbar}\bar{\phi}\partial_t\phi + \frac{m^2c^4}{\hbar^2}|\phi|^2\right). \qquad (2.7)
$$

Of the three terms present, the first does not survive the nonrelativistic limit, and the last one cancels the mass term. On the other hand, the terms containing the interaction with A_t are

$$
-\frac{\hbar^2}{2mc^2}\left(\frac{2mc^2}{\hbar}\phi A_t\bar{\phi} + iA_t\bar{\phi}\partial_t\phi - iA_t\phi\partial_t\bar{\phi} + A_t^2|\phi|^2\right). \qquad (2.8)
$$

Here only the first term survives so that the kinetic term for the scalar contributes in total

$$
-\bar{\phi}i\hbar D_t\phi. \qquad (2.9)
$$

Putting everything together, gives the nonrelativistic scalar action

$$
\begin{aligned}
S_{scal}^{NR} = -\frac{N(N-1)}{2}\int dx^3\Bigg\{ &-\bar{\phi}_i\left(i\hbar D_t + \frac{\hbar^2}{2m}D_i^2\right)\phi_i - \bar{\chi}_i\left(i\hbar D_t + \frac{\hbar^2}{2m}D_i^2\right)\chi_i \\
&+\frac{2\pi\hbar^2}{mk}\left(|\phi_1|^2|\phi_2|^2 - |\chi_1|^2|\chi_2|^2\right)\Bigg\}. \qquad (2.10)
\end{aligned}
$$

Note that for $k > 0$, the χ's have negative potential after the nonrelativistic limit. The positive sextic terms, which would have regulated this dependence and restored the positivity of the potential, do not survive the limit. Evidently then, for $k > 0$ we seem to need (at least one of the) $\chi_i = 0$ for the consistency of the theory, while $k < 0$ requires (at least one of the)$\phi_i = 0$.

Let us go now to the fermionic sector of the theory where, with the factors of $\hbar$ and c included, the Lagrangian reads

$$\begin{aligned} -\frac{2}{N(N-1)}\mathcal{L}_{\text{fer}} = {} & i\sum_{i=1,2}\left[\bar{\eta}_i\left(\not{D}+\frac{mc}{\hbar}\right)\eta_i+\bar{\tilde{\eta}}_i\left(\not{D}-\frac{mc}{\hbar}\right)\tilde{\eta}_i\right] \\ & -\frac{2\pi i}{k\hbar c}\Big[(|\phi_1|^2+|\chi_1|^2)(\bar{\eta}_2\eta_2+\bar{\tilde{\eta}}_2\tilde{\eta}_2)+(|\phi_2|^2+|\chi_2|^2)(\bar{\eta}_1\eta_1+\bar{\tilde{\eta}}_1\tilde{\eta}_1)\Big]. \end{aligned} \tag{2.11}$$

Again we should really write the nonrelativistic version of the fermion fields as

$$\tilde{\eta} = \sqrt{\hbar c}\left[\psi e^{-i\frac{mc^2}{\hbar}t}+\sigma_2\hat{\psi}^* e^{+i\frac{mc^2}{\hbar}t}\right] , \tag{2.12}$$

where ψ and $\hat{\psi}$ are complex fields corresponding to particles and anti-particles respectively. In the zero antiparticle sector then,

$$\eta_i = \sqrt{\hbar c}\psi_i(x,t)e^{-i\frac{mc^2}{\hbar}t}. \tag{2.13}$$

For the three-dimensional gamma matrices we choose the representation in which

$$\gamma^0 = i\tau^3\,,\ \gamma^1 = \tau^1\,,\ \gamma^2 = -\tau^2, \tag{2.14}$$

so that

$$i\bar{\eta}\not{D}\eta = i\eta^\dagger\gamma^0\left(\frac{1}{c}\gamma^0 D_t+\gamma^i D_i\right)\eta = \hbar c\psi^\dagger\left(-\frac{1}{c}D_t+i\frac{mc}{\hbar}+\gamma^0\gamma^i D_i\right)\psi \tag{2.15}$$

and $\gamma^0\gamma^1 = -\tau^2$, $\gamma^0\gamma^2 = -\tau^1$. Consequently,

$$i\gamma^0\gamma^i D_i = i\gamma^0\gamma^1 D_1 + i\gamma^0\gamma^2 D_2 = \begin{pmatrix} 0 & -D_+ \\ D_- & 0 \end{pmatrix} , \tag{2.16}$$

where $D_\pm \equiv D_1 \pm iD_2$. In the nonrelavistic limit only half of the fermion components remain dynamical. For brevity, we will analyze the term with positive mass with an analogous analysis holding for the negative mass case. Substituting (2.13) into the kinetic term in $-2\mathcal{L}/N(N-1)$, gives it the form

$$\begin{pmatrix}\psi^\dagger_{i,1} & \psi^\dagger_{i,2}\end{pmatrix}\begin{pmatrix} -i\hbar D_t - 2mc^2 & -\hbar c D_+ \\ \hbar c D_- & -i\hbar D_t \end{pmatrix}\begin{pmatrix}\psi_{i,1} \\ \psi_{i,2}\end{pmatrix}. \tag{2.17}$$

The ensuing equations of motion

$$i\hbar D_t\psi_{i,1} + 2mc^2\psi_{i,1} + \hbar c D_+\psi_{i,2} = 0$$

$$\hbar c D_- \psi_{i,1} - i\hbar D_t \psi_{i,2} = 0, \tag{2.18}$$

substantiate our claim above that only half of the fermion components are dynamical, since we can solve $\psi_{i,1}$ in terms of $\psi_{i,2}$ by taking only the leading order contribution:

$$\psi_{i,1} = -\frac{\hbar}{2mc} D_+ \psi_{i,2} - i\frac{\hbar}{2mc^2} D_t \psi_{i,1}. \tag{2.19}$$

The equation of motion for $\psi_{i,2}$ is the Pauli equation for nonrelativistic fermions:

$$-i\hbar D_t \psi_{i,2} - \frac{\hbar^2}{2m} D_- D_+ \psi_{i,2} + \mathcal{O}\left(\frac{1}{c}\right) = 0, \tag{2.20}$$

rewritten, using $D_- D_+ = D_1 D_1 + D_2 D_2 + i[D_1, D_2] = D_j D_j + F_{12}$, as

$$i\hbar D_t \psi_{i,2} = -\frac{\hbar^2}{2m} D_- D_+ \psi_{i,2} = -\frac{\hbar^2}{2m} \left(D_j D_j + F_{12}\right) \psi_{i,2}. \tag{2.21}$$

This equation should be obtained from the action for the fermions. In the action, the terms in $-2\mathcal{L}/N(N-1)$ are rewritten as

$$\begin{aligned}
&\psi_{i,1}^\dagger(-i\hbar D_t)\psi_{i,1} + \psi_{i2}^\dagger(-i\hbar D_t)\psi_{i,2} + \hbar c(\psi_{i,2}^\dagger D_- \psi_{i,1} - \psi_{i,1}^\dagger D_+ \psi_{i,2}) - 2mc^2 \psi_{i,1}^\dagger \psi_{i,1} \\
&\quad = \psi_{i,2}^\dagger \left(-i\hbar D_t - \frac{\hbar^2}{2m} D_- D_+\right) \psi_{i,2} \\
&\quad = -i\hbar \psi_{i,2}^\dagger D_t \psi_{i,2} + \frac{\hbar^2}{2m} |D_j \psi_{i,2}|^2 - F_{12} \psi_{i,2}^\dagger \psi_{i,2} \ ,
\end{aligned} \tag{2.22}$$

where in the second line we have substituted $\Delta_+ \psi_{i,2}$ from the first eq. in (2.18) and $\psi_{i,1}$ in $D_i \psi_{i,1}$ from the same equation, and dropped terms that vanish in the $c \to \infty$ limit, and in the third we have used $D_- D_+ = D_j D_j + F_{12}$ and partially integrated one D_j. The same analysis carried out for the $\tilde{\eta}$ fermions leads to the kinetic terms

$$\begin{pmatrix} \tilde{\psi}_{i,1}^\dagger & \tilde{\psi}_{i,2}^\dagger \end{pmatrix} \begin{pmatrix} -i\hbar D_t & -\hbar c D_+ \\ \hbar c D_- & -i\hbar D_t - 2mc^2 \end{pmatrix} \begin{pmatrix} \psi_{i,1} \\ \psi_{i,2} \end{pmatrix} \ , \tag{2.23}$$

which give the similar result,

$$\begin{aligned}
\tilde{\psi}_{i,2} &= \frac{\hbar}{2mc} D_- \tilde{\psi}_{i,1}, \\
i\hbar D_t \tilde{\psi}_{i,1} &= -\frac{\hbar^2}{2m} D_+ D_- \tilde{\psi}_{i,1} = -\frac{\hbar^2}{2m} \left(D_j D_j - F_{12}\right) \tilde{\psi}_{i,1}.
\end{aligned} \tag{2.24}$$

All in all, in the nonrelativistic limit, the fermions are written as

$$\tilde{\eta}_i \longrightarrow \sqrt{\hbar c} e^{-i\frac{mc^2}{\hbar} t} \begin{pmatrix} \tilde{\psi}_{i1} \\ \frac{\hbar}{2mc} D_- \tilde{\psi}_{i1} \end{pmatrix} \ , \ \eta_i \longrightarrow \sqrt{\hbar c} e^{-i\frac{mc^2}{\hbar} t} \begin{pmatrix} -\frac{\hbar}{2mc} D_+ \psi_{i2} \\ \psi_{i2} \end{pmatrix}. \tag{2.25}$$

For the "Yukawa" terms (scalar coupling to fermions) on the other hand, we have the replacement

$$-i\bar{\eta}_i \eta_i = \eta_i^\dagger \tau_3 \eta_i \to -\psi_i^\dagger \psi_i$$

$$-i\tilde{\bar{\eta}}_i\tilde{\eta}_i = \tilde{\eta}_i^\dagger\tau_3\tilde{\eta}_i \to +\tilde{\psi}_i^\dagger\tilde{\psi}_i. \tag{2.26}$$

In what follows, we will drop the 1 and 2 indices on the fermions with the understanding that only one component survives the nonrelativistic limit.

After replacing the fields in the action with their nonrelativistic avatars, the fermionic part of the action takes the form

$$\begin{aligned} S_{\text{fer}}^{NR} = -\frac{N(N-1)}{2}\int dx^3 \Bigg\{ \sum_{j=1,2}\Bigg[&-\psi_j^\dagger\left(i\hbar D_t + \frac{1}{2m}\left(D_i^2 + F_{12}^{(j)}\right)\right)\psi_j \\ &-\tilde{\psi}_j^\dagger\left(i\hbar D_t + \frac{1}{2m}\left(D_i^2 - F_{12}^{(j)}\right)\right)\tilde{\psi}_j\Bigg] \\ &-\frac{\pi\hbar^2}{km}\Big[(|\phi_1|^2+|\chi_1|^2)(\psi_2^\dagger\psi_2 - \tilde{\psi}_2^\dagger\tilde{\psi}_2) \\ &\quad +(|\phi_2|^2+|\chi_2|^2)(\psi_1^\dagger\psi_1 - \tilde{\psi}_1^\dagger\tilde{\psi}_1)\Big].\Bigg\}. \end{aligned} \tag{2.27}$$

Together, the equations (2.4), (2.10), and (2.27) furnish the full nonrelativistic abelianized massive ABJM action.

2.2 Nonrelativistic limit of the full susy rules

As a check, we now attempt to take the same nonrelativistic limit at the level of the supersymmetry transformation rules (2.3). Reintroducing $\hbar$ and c, by replacing μ with $mc/\hbar$, k with kc and D_μ with $D_t/c + D_i$, we get

$$\begin{aligned} \delta\phi_1 &= i\bar{\epsilon}\tilde{\eta}_{\dot{1}}, \\ \delta\phi_2 &= -i\bar{\epsilon}\tilde{\eta}_{\dot{2}}, \\ \delta\chi_{\dot{1}} &= -i\bar{\epsilon}\eta_1, \\ \delta\chi_{\dot{2}} &= i\bar{\epsilon}\eta_2, \\ \delta A_\mu^{(1)} &= \frac{2\pi}{kc}\left(\bar{\epsilon}\gamma_\mu[\phi_2\tilde{\eta}_{\dot{2}}^* - \chi_{\dot{2}}\eta_2^*] + \bar{\epsilon}^*\gamma[\phi_2^*\tilde{\eta}_{\dot{2}} - \chi_{\dot{2}}^*\eta_2]\right), \\ \delta A_\mu^{(2)} &= -\frac{2\pi}{kc}\left(\bar{\epsilon}\gamma_\mu[\phi_1\tilde{\eta}_{\dot{1}}^* - \chi_{\dot{1}}\eta_1^*] + \bar{\epsilon}^*\gamma^\mu[\phi_1^*\tilde{\eta}_{\dot{1}} - \chi_{\dot{1}}^*\eta_1]\right), \\ \delta\eta_1 &= \gamma^\mu\epsilon D_\mu\chi_{\dot{1}} + \frac{2\pi}{kc}\epsilon\chi_{\dot{1}}(|\phi_2|^2 + |\chi_{\dot{2}}|^2) - \frac{mc}{\hbar}\epsilon\chi_{\dot{1}}, \\ \delta\eta_2 &= -\gamma^\mu\epsilon D_\mu\chi_{\dot{2}} - \frac{2\pi}{kc}\epsilon\chi_{\dot{2}}(|\phi_1|^2 + |\chi_{\dot{2}}|^2) + \frac{mc}{\hbar}\epsilon\chi_{\dot{2}}, \\ \delta\tilde{\eta}_{\dot{1}} &= -\gamma^\mu\epsilon D_\mu\phi_1 - \frac{2\pi}{kc}\epsilon\phi_1(|\phi_2|^2 + |\chi_{\dot{2}}|^2) - \frac{mc}{\hbar}\epsilon\phi_1, \\ \delta\tilde{\eta}_{\dot{2}} &= \gamma^\mu\epsilon D_\mu\phi_2 + \frac{2\pi}{kc}\epsilon\phi_2(|\phi_1|^2 + |\chi_{\dot{1}}|^2) + \frac{mc}{\hbar}\epsilon\phi_2, \end{aligned} \tag{2.28}$$

where here δA_μ is understood as $(\frac{1}{c}\delta A_0, \delta A_i)$, and $\gamma^\mu D_\mu = \frac{1}{c}\gamma^0 D_0 + \gamma^i D_i$. Since in the nonrelativistic limit, one of the components of the fermions goes to zero, the same has to happen in the susy transformation rules: the variation of the component that goes to zero should also go to zero, and only the variation of the other component should be finite.

Note that ϵ is a complex 2-component spinor. Since a minimal spinor in 3 dimensions is Majorana, with only one independent complex (or two real) component(s), these susy rules correspond to $\mathcal{N}=2$ supersymmetry. These components, which we denote by ϵ_1 (upper) and ϵ_2 (lower) respectively, are to be understood as the independent supersymmetries in the nonrelativistic limit. We first consider the transformation rule for the scalar ϕ_1,

$$\delta\phi_1 = -\sqrt{\frac{2mc}{\hbar}}\left(\epsilon_1^*\tilde{\psi}_{1,1} - \epsilon_2^*\frac{\hbar}{2mc}D_-\tilde{\psi}_{1,1}\right). \tag{2.29}$$

Since $c \to \infty$, both terms are singular in the nonrelativistic limit. In order to circumvent this behaviour, we need to rescale the supersymmetry parameters. This rescaling is not unique. One possible choice for a rescaling of the susy parameters is

$$\epsilon_i \to \sqrt{\frac{\hbar}{2mc}}\epsilon_i; \quad i = 1,2. \tag{2.30}$$

In that case, for the variations of the scalars we obtain

$$\begin{aligned}\delta\phi_1 &= -\epsilon_1^*\tilde{\psi}_{1,1}\\ \delta\phi_2 &= +\epsilon_1^*\tilde{\psi}_{2,1}\\ \delta\chi_1 &= -\epsilon_2^*\psi_{1,2}\\ \delta\chi_2 &= +\epsilon_2^*\psi_{2,2},\end{aligned} \tag{2.31}$$

while for the fermion variations,

$$\begin{aligned}\delta\begin{pmatrix}-\frac{\hbar}{2mc}D_+\psi_{1,2}\\ \psi_{1,2}\end{pmatrix} &\simeq (\tau_3\epsilon-\epsilon)\chi_1\\ \delta\begin{pmatrix}-\frac{\hbar}{2mc}D_+\psi_{2,2}\\ \psi_{2,2}\end{pmatrix} &\simeq -(\tau_3\epsilon-\epsilon)\chi_2\\ \delta\begin{pmatrix}\tilde{\psi}_{1,1}\\ \frac{\hbar}{2mc}D_-\tilde{\psi}_{1,1}\end{pmatrix} &\simeq -(\tau_3\epsilon+\epsilon)\phi_1\\ \delta\begin{pmatrix}\tilde{\psi}_{2,1}\\ \frac{\hbar}{2mc}D_-\tilde{\psi}_{2,1}\end{pmatrix} &\simeq +(\tau_3\epsilon+\epsilon)\phi_1\end{aligned} \tag{2.32}$$

Clearly in the nonrelativistic limit the same half of the components vanish on the left hand side and on the right hand side, as it should be. The other half gives

$$\begin{aligned}\delta\psi_{1,2} &= \epsilon_2\chi_1\\ \delta\psi_{2,2} &= -\epsilon_2\chi_2\\ \delta\tilde{\psi}_{1,1} &= -\epsilon_1\phi_1\\ \delta\tilde{\psi}_{2,1} &= +\epsilon_1\phi_2.\end{aligned} \tag{2.33}$$

Finally, the variations of the gauge fields are expressed as

$$\delta A_0^{(1)} = \frac{2\pi\hbar^2}{2mk}(\epsilon_1^*\tilde{\psi}_{2,1}\phi_2^* - \epsilon_2^*\psi_{2,2}\chi_2^*) + c.c,$$

$$
\begin{aligned}
\delta A_i^{(1)} &= 0, \\
\delta A_0^{(2)} &= -\frac{2\pi\hbar^2}{2mk}(\epsilon_1^* \tilde{\psi}_{1,1}\phi_2^* - \epsilon_2^* \psi_{2,2}\chi_1^*) + c.c, \\
\delta A_i^{(2)} &= 0.
\end{aligned}
\tag{2.34}
$$

Certainly then, the reduction passes this check at the level of the supersymmetry transformations. However, as we have already seen, at the level of the action, when $k > 0$ we have a negative potential for χ and for $k < 0$, a negative potential for ϕ, signalling a possible instability.

At this juncture, it is worth noting that the two susies act on $(\phi, \tilde{\psi}, A_\mu^{(1,2)})$ and $(\chi, \psi, A_\mu^{(1,2)})$ respectively, with the action on the $A_\mu^{(1,2)}$ being specifically a nonlinear one only. At the level of the linearized action, the two supersymmetries evidently act on different fields. Therefore in some sense this corresponds to two different sets of $\mathcal{N} = 1$ invariant fields put together.

2.3 Truncating the susy rules and the action

If, however, we would like to keep *both* terms in the transformation (2.29) finite, another rescaing that is afforded to us is

$$
(\epsilon_1, \epsilon_2) \longrightarrow \left(\sqrt{\frac{\hbar}{2mc}}\epsilon_1, \sqrt{\frac{c}{2m\hbar}}\epsilon_2 \right). \tag{2.35}
$$

Then the transformation rule for ϕ_1 takes the form

$$
\delta\phi_1 = -\epsilon_1^* \tilde{\psi}_{1,1} + \frac{1}{2m}\epsilon_2^* D_- \tilde{\psi}_{1,1}. \tag{2.36}
$$

The first term on the right hand side is called *kinematical* supersymmetry transformation $\delta_K\phi_1$, and the second a *dynamical* one which we denote $\delta_D\phi_1$, with similar rules holding for ϕ_2. However, a problem appears when we consider $\chi_{1,2}$. For example, the transformation rule for χ_1,

$$
\delta\chi_1 = -\frac{c}{\hbar}\epsilon_2^* \psi_{1,2} - \frac{1}{2m}\frac{\hbar}{c}\epsilon_1^* D_- \psi_{1,2}. \tag{2.37}
$$

implies that, in order to have supersymmetry with *both* kinematical and dynamical terms in the nonrelativistic abelian case, we are forced to truncate the model by setting $\chi_i = \psi_i = 0$. Since in this case we will be left with only one set of ψs, we will remove the tilde for simplicity from now on. With this truncation and rescaling of supersymmetry parameters, the truncated action becomes

$$
\begin{aligned}
S^{NR} = -\frac{N(N-1)}{2} \int d^3x \Bigg\{ & \frac{k\hbar}{4\pi}\epsilon^{\mu\nu\lambda}(A_\mu^{(2)}F_{\nu\lambda}^{(1)} + A_\mu^{(1)}F_{\nu\lambda}^{(2)}) - \bar{\phi}_i \left(i\hbar D_t + \frac{\hbar^2}{2m}D_j^2 \right)\phi_i \\
& - \sum_{j=1,2} \left[\psi_j^\dagger \left(i\hbar D_t + \frac{1}{2m}\left(D_i^2 - F_{12}^{(j)} \right) \right) \psi_j \right] \\
& + \frac{\pi\hbar^2}{km}\left[(|\phi_1|^2)(\psi_2^\dagger\psi_2) + (|\phi_2|^2)(\psi_1^\dagger\psi_1) \right] + \frac{2\pi\hbar^2}{mk}\left(|\phi_1|^2|\phi_2|^2 \right) \Bigg\},
\end{aligned}
\tag{2.38}
$$

with the supersymmetry transformation rules,

$$\begin{aligned}
\delta\phi_1 &= -\epsilon_1^*\psi_{1,1} + \frac{1}{2m}\epsilon_2^* D_-\psi_{1,1},\\
\delta\phi_2 &= \epsilon_1^*\psi_{2,1} - \frac{1}{2m}\epsilon_2^* D_-\psi_{2,1},\\
\delta A_t^{(1)} &= +\frac{\pi\hbar}{mk}(\epsilon_1^*\phi_2\psi_{2,1}^* + \epsilon_1\phi_2^*\psi_{2,1}) + \frac{2\pi\hbar}{(2m)^2k}(\epsilon_2^*\phi_2 D_+\psi_{2,1}^* + \epsilon_2\phi_2^* D_-\psi_{2,1}),\\
\delta A_1^{(1)} &= -\frac{i\pi\hbar}{mk}(\epsilon_2^*\phi_2\psi_{2,1}^* + \epsilon_2\phi_2^*\psi_{2,1}),\\
\delta A_2^{(1)} &= -\frac{\pi\hbar}{mk}(\epsilon_2^*\phi_2\psi_{2,1}^* + \epsilon_2\phi_2^*\psi_{2,1}),\\
\delta A_t^{(2)} &= -\frac{\pi\hbar}{mk}(\epsilon_1^*\phi_1\psi_{1,1}^* + \epsilon_1\phi_1^*\psi_{1,1}) - \frac{2\pi\hbar}{(2m)^2k}(\epsilon_2^*\phi_1 D_+\psi_{1,1}^* + \epsilon_1\phi_1^* D_-\psi_{1,1}),\\
\delta A_1^{(2)} &= \frac{i\pi\hbar}{mk}(\epsilon_2^*\phi_1\psi_{1,1}^* + \epsilon_2\phi_1^*\psi_{,11}),\\
\delta A_2^{(1)} &= \frac{\pi\hbar}{mk}(\epsilon_2^*\phi_1\psi_{1,1}^* + \epsilon_2\phi_1^*\psi_{1,1}),\\
\delta\psi_{1,1} &= \frac{1}{2m}\epsilon_2 D_-\phi_1 - \epsilon_1\phi_1,\\
\delta\psi_{2,1} &= -\frac{1}{2m}\epsilon_2 D_-\phi_2 + \epsilon_1\phi_2,
\end{aligned} \tag{2.39}$$

We also note the intermediate result for the fermion variation

$$\delta\begin{pmatrix}\psi_{1,1}\\ \frac{\hbar^2}{2mc}D_-\psi_{1,1}\end{pmatrix} = \sqrt{\frac{\hbar}{2mc}}\left[-\frac{1}{c}\gamma^0\epsilon D_0\phi_1 - \gamma^i\epsilon D_i\phi_1 - \frac{mc}{\hbar}\phi_1(\tau_3\epsilon+\epsilon) - \frac{\pi\hbar^2}{mkc}|\phi_2|^2\phi_1\right]. \tag{2.40}$$

with a similar one for $\psi_{2,1}$, where

$$\epsilon = \begin{pmatrix}\sqrt{\frac{\hbar}{2mc}}\epsilon_1\\ \sqrt{\frac{c}{2m\hbar}}\epsilon_2\end{pmatrix}. \tag{2.41}$$

Then we see that the first and last terms vanish as $c\to\infty$, whereas the remaining $(\tau_3\epsilon+\epsilon)$ and the $\gamma^i\epsilon D_i$ terms correctly vanish for the lower component only, as it should be, by comparison with the left hand side.

3 The (supersymmetric) Jackiw-Pi model

3.1 The Jackiw-Pi model and its vortex solutions

In a remarkable series of papers in the early 1990's, beginning with [12], Jackiw and Pi undertook a systematic study of the classical and quantum properties of the gauged nonlinear Schrödinger equation[5]

$$iD_t\psi = -\frac{1}{2}D_i^2\psi - g\bar{\psi}\psi\psi, \tag{3.1}$$

[5] Our notation in this subsection only will match the original literature instead of the rest of this article.

for a charged scalar, ψ coupled to an abelian Chern-Simons gauge field whose dynamics is governed by

$$\frac{1}{2}\epsilon^{\mu\nu\lambda}F_{\nu\lambda} = \frac{1}{\kappa}j^{\mu}, \tag{3.2}$$

and with Chern-Simons coupling (or topological mass) κ. These equations derive from the Lagrangian density

$$\mathcal{L} = \frac{\kappa}{4}\epsilon^{\mu\nu\lambda}A_\mu F_{\nu\lambda} + i\bar{\psi}D_t\psi - \frac{1}{2}|D_i\psi|^2 + \frac{g}{2}(\bar{\psi}\psi)^2, \tag{3.3}$$

which defines the so-called Jackiw-Pi model, which has seen enormous development over the past twenty five years as much for its pedagogical value in teaching us about four dimensional field theories as for the role that it plays in planar condensed matter systems like the quantum Hall effect. For the specific value of the scalar coupling $g = 1/|\kappa|$, the theory takes on a "self-dual" structure with the (static) equations of motion descending to the first order set of Bogomolnyi-like equations

$$\begin{aligned} D_i\psi &= i\epsilon_{ij}D_j\psi, \\ \epsilon_{ij}\,\partial_i A_j &= -\frac{1}{\kappa}\bar{\psi}\psi, \end{aligned}$$

supplemented by the Chern-Simons Gauss law constraint that any solution carrying charge Q also possess a magnetic flux $\Phi = -Q/\kappa$. These equations are solved exactly by taking the ansatz $\psi = \sqrt{\rho}\,e^{i\omega}$, and writing the first order system as a Liouville equation

$$\nabla^2 \ln\rho = -\frac{2}{\kappa}\rho, \tag{3.4}$$

for the square modulus of the complex scalar. This equation admits a general solution in terms of a holomorphic function $f(z)$ of the complex coordinate $z = r\,e^{i\theta}$ on the plane as

$$\rho(r) = \frac{4\kappa|f'(z)|^2}{(1+|f(z)|^2)^2}. \tag{3.5}$$

As a specific example, the choice $f(z) = c_0 z^{-n}$ yields the axially symmetric solution

$$\psi(r) = \frac{2\sqrt{\kappa}n}{r}\left(\left(\frac{r_0}{r}\right)^n + \left(\frac{r}{r_0}\right)^n\right)^{-1} e^{i(1-n)\theta}, \tag{3.6}$$

where the integration constants r_0 and n are interpreted, respectively, as a scale parameter and a topological charge. This corresponds to an n-vortex solution located at the origin, the so-called Jackiw-Pi vortex.

3.2 Nonrelativistic vortices in ABJM

Returning now to the problem at hand,[6] we consider the bosonic part of the nonrelativistic action (2.38),

$$S_{bos}^{NR} = -\frac{N(N-1)}{2}\int d^3x\left\{\frac{k\hbar}{4\pi}\epsilon^{\mu\nu\lambda}\left(A_\mu^{(2)}F_{\nu\lambda}^{(1)} + A_\mu^{(1)}F_{\nu\lambda}^{(2)}\right) + \bar{\phi}_i\left(i\hbar D_t + \frac{\hbar^2}{2m}D_j^2\right)\phi_i\right.$$

[6] And reverting again to our usual notation.

$$+\frac{2\pi\hbar^2}{mk}\left(|\phi_1|^2|\phi_2|^2\right)\Bigg\}, \tag{3.7}$$

and take as an ansatz for a further reduction of the model,

$$\begin{aligned} A_\mu^{(1)} &= A_\mu^{(2)} = A_\mu, \\ \phi_1 &= \phi_2 = \phi. \end{aligned} \tag{3.8}$$

Substituting this into the action leads to

$$S_{JP} = -N(N-1)\int d^3x \left\{ \frac{k\hbar}{4\pi}\epsilon^{\mu\nu\lambda}A_\mu F_{\nu\lambda} - \bar{\phi}\left(i\hbar D_t + \frac{\hbar^2}{2m}D_j^2\right)\phi + \frac{\pi\hbar^2}{mk}\left(\phi\bar{\phi})^2\right)\right\}, \tag{3.9}$$

which, up to an overall factor of $N^2 - N$, is just the action for the Jackiw-Pi model (3.3) encountered a bove [12, 13] and, as such, clearly admits all of the latter's solitonic solutions including, the self-dual n-vortex Jackiw-Pi vortices (3.6). We now show how to understand these vortices in the present context.

The authors of [22] found a class of vortex solutions in the nonrelativistic limit of the massive ABJM action first considered by [20]. These are, in fact, nothing but the Jackiw-Pi vortex, embedded in the ABJM model via the abelianization ansatz in [10]. Indeed, their solution (eq. (68) of [22]) is in our notation

$$\begin{aligned} Q^\alpha(x) &= \phi(x)G^\alpha, \\ R^\alpha(x) &= \chi(x)G^\alpha, \\ A_\mu(x) &= a_\mu(x)G^\alpha G_\alpha^\dagger, \\ \hat{A}_\mu(x) &= a_\mu(x)G_\alpha^\dagger G^\alpha, \end{aligned} \tag{3.10}$$

which is just the abelian reduction ansatz in [10], together with the restriction $\phi_1 = \phi_2 = \phi$, $\chi_1 = \chi_2 = \chi$ and $a_\mu^{(1)} = a_\mu^{(2)} = a_\mu$. This is, of course, the same condition we administered for comparison with the Jackiw-Pi Lagrangian (for $\chi = 0$). In this case, the BPS equations reduce to

$$(D_1 - iD_2)\phi(x) = 0, \quad (D_1 + iD_2)\chi(x) = 0\,, \tag{3.11}$$

giving two different types of solutions (referred to as "BPS I" and "BPS II" in [22]) depending on whether either χ or ϕ is turned off. The BPS I vortex solutions are then found from the ansatz $\chi = 0$ with

$$\phi(x) = e^{i\theta(x)}\rho(x)^{1/2}\,, \tag{3.12}$$

which leads to

$$\begin{aligned} \rho(x) &= \frac{k}{2\pi}\nabla^2\ln(1 + |f(z)|^2) \\ \theta(x) &= -(n-1)\arctan(x_2/x_1)\,, \end{aligned} \tag{3.13}$$

where $f(z)$ is a holomorphic function of $z = x_1 + ix_2$. The BPS II solutions on the other hand, are given by $\phi = 0$ and

$$\chi(x) = e^{i\theta(x)}\rho(x)^{1/2}\,, \tag{3.14}$$

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and

$$\begin{aligned}\rho(x) &= -\frac{k}{2\pi}\nabla^2 \ln(1+|f(z)|^2)\\ \theta(x) &= (n-1)\arctan(x_2/x_1)\ .\end{aligned} \tag{3.15}$$

It was demonstrated in [22] that these vortex solutions are indeed BPS, i.e. they break one conformal, one dynamical and five kinematical supersymmetries, i.e. exactly half of the 2 conformal, 2 dynamical and 10 kinematical supersymmetries of the full theory. As we will see shortly, this remains true in our case. Since in our case (after the abelian reduction), we have only $\mathcal{N}=2$ supersymmetry, i.e. 4 supercharges (2 dynamical and 2 kinematical) and 2 conformal supercharges, the vortices will break half of those, i.e. one conformal, one dynamical and one kinematical supersymmetries.

3.3 BPS Chern-Simons matter vortices and Jackiw-Pi vortices

An $\mathcal{N}=2$ supersymmetric version of the Jackiw-Pi model was considered by Leblanc et al. [14]. Recently, in [23], the quantum Hall effect for this gauge theory was studied. The model possesses several remakable properties that will be explored in the next subsection. For now, we show that the same theory can be obtained from the ABJM model in our nonrelativistic abelian reduction, only with different couplings. Indeed, with the reduction ansatz $\phi_1=\phi_2\equiv\phi$, $\psi_1=-\psi_2\equiv\psi$, $A_\mu^{(1)}=A_\mu^{(2)}\equiv A_\mu$, a redefinition $k/(2\pi)\equiv\kappa$ and some partial integrations, it is straightforward to show that the action (2.38) reduces to

$$\begin{aligned}S = +N(N-1)\int d^3x\,\Bigg[&-\frac{\kappa\hbar}{2}\epsilon^{\mu\nu\rho}A_\mu F_{\nu\rho}+\phi^*(i\hbar D_t)\phi-\frac{\hbar^2}{2m}|D\phi|^2+\psi^*(i\hbar D_t)\psi\\ &-\frac{\hbar^2}{2m}|D\psi|^2+\frac{F_{12}}{2m}|\psi|^2-\frac{\hbar^2}{2\kappa m}|\phi|^2|\psi|^2-\frac{\hbar^2}{2m\kappa}|\phi|^4\Bigg]\,.\end{aligned} \tag{3.16}$$

Further, noting that $\epsilon^{\mu\nu\rho}A_\mu F_{\nu\rho}=\frac{2}{c}A_0F_{12}-\epsilon^{ij}A_i\frac{1}{c}\partial_0 A_j$, replacing our $(A_0)/c$ with A_0, and denoting $F_{12}=B$, we get precisely eq. (2.8) of [14]. The Yukawa term and scalar potential take the form

$$\lambda_1|\phi|^2|\psi|^2+\lambda_2|\phi|^4, \tag{3.17}$$

In particular, with $e\equiv 1$, we identify

$$\lambda_1=\lambda_2=-\frac{\hbar^2}{2m\kappa}. \tag{3.18}$$

Note that this combination of constants is *not* the one considered for the $\mathcal{N}=2$ supersymmetric case in [14], where rather

$$\lambda_1=+\frac{\hbar^2}{2mc\kappa};\qquad \lambda_2=3\lambda_1. \tag{3.19}$$

However, in *both* cases, the Yukawa and self-interaction couplings satisfy the condition

$$2\lambda_1-\lambda_2+\frac{1}{2m\kappa}=0\ , \tag{3.20}$$

a necessary condition for $\mathcal{N}=1$ supersymmetry. In [14], it was further claimed that the condition (3.19) is the only solution to the $\mathcal{N}=2$ supersymmetry invariance. We disagree. In fact, we obtain the same supersymmetry tranformation laws, with the identification

$$\epsilon_1^{\text{ours}} = -\sqrt{2m}\epsilon_1^{\text{theirs}}, \quad \epsilon_2^{\text{ours}} = i\sqrt{2m}\epsilon_2^{\text{theirs}}, \tag{3.21}$$

and claim that they have simply not considered the case $\kappa < 0$, which will result in our solution, as we now explain.

In fact, there are *two* possible Bogomolnyi bounds, which arise from being able to write the Hamiltonian in two ways as a sum of complete squares plus a topological term,

$$\begin{aligned}\frac{1}{N(N-1)}\mathcal{H} = \frac{\hbar^2}{2m}\left[|D_\pm\phi|^2 + |D_\pm\psi|^2\right] \pm \frac{\hbar^2}{2}\vec{\nabla}\times[\vec{j}_B+\vec{j}_F] \pm \frac{\hbar^2}{4m}\vec{\nabla}^\rho_F \\ -\left[\lambda_1 \pm \frac{\hbar^2}{2m\kappa}\right]\rho_B^2 - \left[\lambda_2 \pm \frac{\hbar^2}{m\kappa} - \frac{\hbar^2}{2m\kappa}\right]\rho_B\rho_F\,,\end{aligned} \tag{3.22}$$

with bosonic and fermionic currents

$$\begin{aligned}\vec{j}_B &= \frac{1}{2mi}[\phi^*\vec{D}\phi - (\vec{D}\phi)^*\phi] \\ \vec{j}_F &= \frac{1}{2mi}[\psi^*\vec{D}\psi - (\vec{D}\psi)^*\psi + i\vec{\nabla}\times\rho_F].\end{aligned} \tag{3.23}$$

If the fields are sufficiently well behaved, the integrals over the j_B, j_F and ρ_F terms vanish. If, in addition, the couplings

$$\lambda_1 = \mp\frac{\hbar^2}{2m\kappa}; \quad \lambda_2 = (1\mp 2)\frac{\hbar^2}{2m\kappa}\,, \tag{3.24}$$

the Hamiltonian reduces to

$$H = \int d^2x\ \frac{\hbar^2}{2m}[|D_\pm\phi|^2 + |D_\pm\psi|^2]\,, \tag{3.25}$$

which reaches its minimum value, zero, when the BPS equations

$$D_1\phi = \mp iD_2\phi; \quad D_1\psi = \mp iD_2\psi. \tag{3.26}$$

are satisfied. This choice of couplings clearly includes both our set, as well as that of [14]. Since, by the Olive-Witten theorem, these Bolgomolnyi equations are implied by the supersymmetry algebra in a supersymmetric theory, each BPS bound corresponds to specific set of supersymmetry transformations. This substantiates our claim above.

The vortex solutions of the BPS system (3.26) are easily extracted via the ansatz

$$\phi = e^{i\theta_B}\rho_B^{1/2}; \quad \psi = \eta\, e^{i\theta_F}\rho_F^{1/2}\,, \tag{3.27}$$

where η is a constant spinor. As in the usual Jackiw-Pi case, these equations can be combined (using the fact that the fermionic and bosonic densities must be proportional) to produce the Liouville equation

$$\nabla^2 \ln\rho = \pm\frac{2}{\kappa}\rho. \tag{3.28}$$

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This equation admits finite energy solutions only when the right hand side is negative as, for example when the lower sign is chosen with $\kappa > 0$, as in [14]. However, and this is the subtlety that was not fully appreciated in [14], it is also possible to have finite energy solutions by choosing the *upper sign* and $\kappa < 0$, as we have. At the level of the action, this corresponds to a parity transformation, which in turn leads to a supersymmetric theory with different couplings, BPS equations and solutions in a perfectly consistent way.

3.4 Symmetries

The symmetry algebra of our action, reduced to the supersymmetric Jackiw-Pi model is the same as in [14], even with the differing choice of couplings. Indeed, the algebra is independent of the values of the λ_i's. For completeness, we review it here.

The reduced theory is invariant under the following bosonic symmetry operators: the Hamiltonian H, momentum $\vec{P}$, Galilean boost $\vec{G}$, angular momentum J_{12}, dilation D, special conformal transformation K and number operator N. These symmetry operators satisfy the conformal Galilean algebra,

$$\begin{aligned}
[P^i,P^j] &= [P^i,H] = [J,H] = [G^i,G^j] = 0,\\
[J,P^i] &= \epsilon^{ij}P^j \Rightarrow [M_{ij},P_k] = i(\delta_{ik}P_j - \delta_{jk}P_i),\\
[J,G^i] &= \epsilon^{ij}G^j \Rightarrow [M_{ij},G_k] = i(\delta_{ik}G_k - \delta_{jk}G_k),\\
[P^i,G^j] &= \delta^{ij}mN \equiv i\delta_{ij}\tilde{N},\\
[H,G^i] &= P,\\
[D,H] &= -H,\\
[D,K] &= K,\\
[H,K] &= 2D,\\
[K,J] &= [K,G^i] = [D,J] = 0,\\
[K,P^i] &= -G^i,\\
[D,P^i] &= -\frac{1}{2}P^i,\\
[D,G^i] &= \frac{1}{2}G^i.
\end{aligned} \tag{3.29}$$

Here $i\tilde{N} \equiv mN$ is a mass operator that acts as a central charge. This is in excellent agreement with the conformal Galilean symmetry algebra considered in [24] (see also [25]) for $z = 2$, i.e. the *Schrödinger algebra*, with the identifications

$$D = \frac{1}{2}i\tilde{D}; \quad K = -C; \quad M_{12} = iJ; \quad G^i = iK^i\ , \tag{3.30}$$

where $\tilde{D}, C, K^i, M_{ij}$ are the operators in [24]. The more general relations

$$[\tilde{D},K_i] = (1-z)iK_i; \quad [\tilde{D},H] = ziH \tag{3.31}$$

reduce to the above for $z = 2$. Finally, the symmetry operators (in our notation, and for our action) are:

$$H = N(N-1)\int d^2x \left[\frac{\hbar^2}{2m}\left(|D\phi|^2 + |\Delta\psi|^2\right) - \frac{B}{2m}\rho_F + \frac{\hbar^3}{2m\kappa}\rho_B\rho_F + \frac{\hbar^2}{2m\kappa}\rho_B^2\right]$$

$$
\begin{aligned}
P^i &= \int d^2x\, \mathcal{P}^i = \frac{1}{2i}\int d^2x \left[\phi^* D^i\phi - (D^i\phi)^*\phi + \psi^* D^i\Psi - (D^i\psi)^*\psi\right] \\
J &= \int d^2x \left[\vec{r}\times\vec{\mathcal{P}} + \frac{\rho_F}{2}\right] \\
N &= \int d^2x \left[\rho_B + \rho_F\right] \\
G^i &= tP^i - m\int d^2x \left[r^i(\rho_B+\rho_F)\right] \\
D &= tH - \frac{1}{2}\int d^2x\, \vec{r}\cdot\vec{\mathcal{P}} \\
K &= -t^2H + 2tD + \frac{m}{2}\int d^2x\ \ r^2(\rho_B+\rho_F)\ ,
\end{aligned} \tag{3.32}
$$

and where $\rho_B = |\phi|^2$ and $\rho_F = |\psi|^2$. We note also that the system is also invariant under two supersymmetries, which, together with the above form a supergroup, of $\mathcal{N} = 2$ supersymmetric Schrödinger symmetry. Then the mass operator, which previously appeared only as a central charge, splits into bosonic and fermionic parts, N_B and N_F,

$$
N_B = \int d^2x\, \rho_B; \quad N_F = \int d^2x\, \rho_F\ , \tag{3.33}
$$

which, together with a new generator F coming from the commutator of the supercharge Q_2 with the generator of special conformal transformations, gives a total of 16 generators of the Super-Schrödinger algebra. In fact, since only the explicit form of the generators $H, \vec{P}, J, \vec{G}, N_B, N_F, D$ and K in terms of the fields are modified with respect to (3.32) in the full action *before* truncation to the $\mathcal{N} = 2$ Lagrangian of [14] and not their number, the symmetry algebra of the full theory with 4 complex scalars and 4 fermions is the same.

4 AdS/CMT applications

4.1 Comments on systems with super-Schrödinger symmetry

The appearance of this Super-Schrödinger symmetry is remarkable in the context of the AdS/Condensed matter correspondence in two ways:

- It is, as far as we are aware, the first explicit example with an action, of a system with Schrödinger (or in fact with any conformal Galilean) algebra derived from a well-defined AdS/CFT duality in a "top-down" way, i.e. embedded in a critical string background (compare this, for example, to the nonlocal dipole theory constructed in [15]) and that could be derived by reduction from 3+1 dimensions,[7] as is probably required for a good interpretation as a condensed matter model.
- It is also a concrete example of a nonrelativistic AdS/CFT duality where the gravity dual is, as usual, $(d+1)-$dimensional and not $(d+2)-$dimensional. Indeed, the other concrete example of nonrelativistic AdS/CFT derived from a known duality was constructed in [15–17] by taking a discrete light cone quantization (DLCQ) of a

[7] We have not shown that explicitly, however note that the symmetry algebra can be embedded in 3+1 dimensions, and the theory is abelian.

known AdS/CFT pair, and in so doing killing one more coordinate (say, x^+, leaving an x^-), in addition to the radial coordinate r, leaving a duality between "CFT_d" and "AdS_{d+2}". For instance, the relevant case addressed in those works is the limit of the canonical $AdS_5 \times S^5/\mathcal{N} = 4$ SYM duality, leading to a duality between a 5-dimensional gravity dual and a 3-dimensional field theory with Schrödinger symmetry.

To summarise then, here we have a duality between a (2+1)-dimensional condensed matter system with a well-defined action and a certain limit of a 4-dimensional gravity dual, corresponding to massive ABJM (a deformation of the AdS_4 dual of the pure ABJM). While it is true that we still do not understand fully the effect of the abelian reduction nor of the norelativistic limit on the gravity dual; since the starting point was a conventional 4-dimensional gravity dual, we do anticipate that it will remain true of the endpoint as well.[8,9,10]

Finally, one may question how was it possible to obtain a system with Schrödinger (conformal Galilean) symmetry, when we started from a system with a mass term (the non-conformal massive ABJM theory)? This is curious, but perfectly consistent even though after taking an abelian reduction and a nonrelativistic limit on the above and obtaining a theory with mass parameter m since, in the nonrelativistic limit, we can define units such that $\hbar$ *and* m are dimensionless. In other words, $[t] = [r^2]$ and the dilatation symmetry is defined as

$$\delta t = 2\alpha t; \quad \delta\vec{r} = \alpha\vec{r}. \tag{4.1}$$

4.2 Comparison with nonrelativistic abelian toy models for ABJM

In this penultimate section of the article, we will reflect on some more phenomenological aspects of the theory, keeping in mind our ultimate goal of building a concrete AdS/CMT correspondence embedded into a critical AdS/CFT duality. We will focus in particular on the physics of compressible quantum matter. In an interesting recent work [18], Huijse and Sachdev, initiated a study of compressible Fermi surfaces in as close to a "top-down" approach as we have yet encountered. Their models were drawn from the canonical AdS/CFT

[8] At this point, it is worth noting that a nonrelativistic limit of massive ABJM was taken, and a super-Schrödinger symmetry was found in [20, 21]. There too (see, for example, section 3.5 of [20]) it was noted that only an $\mathcal{N} = 2$ subset (as is ours) could be embedded in a *four dimensional* relativistic superconformal symmetry, via DLCQ (as in [15–17]). Their full action and symmetry group cannot be embedded in 3+1 dimensions.

[9] Also note that gravitational spacetimes with the $\mathcal{N} = 2$ Super-Schrödinger symmetry have been found in [26, 27], however they still could only be interpreted as holographic duals in *two* dimensions lower.

[10] It is a logical possibility that the gravity dual is stringy, though that seems highly unlikely given that the gravity dual to massive ABJM is not, and we are simply taking some limits on it. The same could be said about the possibility that singularities appear in the limit: it is hard to see how this can happen as an effect of the limit, though the possibility could not be discarded. It could also be that there is a gravity dual in Horava theory, like it was argued in [28] to happen in some cases, or in a Newton-Cartan theory [29] (though in that case only near the boundary the geometry was shown to become Newton-Cartan), but we also believe it unlikely. More likely, as in the usual cases in [15–17], the gravity dual background is modified by the limit, but not the gravitational theory itself, which remains Einstein. In any case, as we are familiar with from other examples like the pp wave limit, the study of a subset of the field theory implies a limit (a subset of) the gravity dual, so we can think of the gravity dual theory as simply a subset of the one dual to massive ABJM, and it is in that sense that we have a concrete example of AdS/CFT duality.

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duals (*viz* the 3-dimensional $\mathcal{N} = 6$ ABJM and 4-dimensional $\mathcal{N} = 4$ SYM theories) but even here, the paradigmic actions were taken only as a guide to developing a stable toy model. We would like to be able to do better.

To that end, and for comparison, we write here the expression for action for the toy model proposed in [18],

$$\begin{aligned} S = \int d^3x \Bigg[& f_+^\dagger \left((\partial_\tau - iA_\tau) - \frac{(\vec{\nabla} - i\vec{A})^2}{2m_f} - \mu \right) f_+ \\ & + f_-^\dagger \left((\partial_\tau + iA_\tau) - \frac{(\vec{\nabla} + i\vec{A})^2}{2m_f} - \mu \right) f_- \\ & + b_+^\dagger \left((\partial_\tau - iA_\tau) - \frac{(\vec{\nabla} - i\vec{A})^2}{2m_b} + \epsilon_1 - \mu \right) b_+ \\ & + b_-^\dagger \left((\partial_\tau + iA_\tau) - \frac{(\vec{\nabla} + i\vec{A})^2}{2m_b} + \epsilon_1 - \mu \right) b_- \\ & + \frac{u}{2}(b_+^\dagger b_+ + b_-^\dagger b_-)^2 + v b_+^\dagger b_-^\dagger b_- b_+ - g_1(b_+^\dagger b_-^\dagger f_- f_+ + h.c.) \\ & + c^\dagger \left(\partial_\tau - \frac{(\vec{\nabla})^2}{2m_c} + \epsilon_2 - \mu \right) c - g_2(c^\dagger(f_+ b_- + f_- b_+) + h.c.) \Bigg]. \end{aligned} \tag{4.2}$$

Here A_μ is an emergent gauge field (i.e., not the electromagnetic gauge field), corresponding to a local U(1) symmetry. Importantly, the theory also possesses a *global* U(1) symmetry with corresponding charge,

$$Q = f_+^\dagger f_+ + f_-^\dagger f_- + b_+^\dagger b_+ + b_-^\dagger b_- + 2c^\dagger c, \tag{4.3}$$

so both fundamental charged bosons $b_\pm$ and fermions $f_\pm$ as well as a neutral fermion c all couple to the gauge field.

Since the kinetic terms for the fields are guaranteed to be the correct ones, we will instead focus on the scalar potential and Yukawa terms, whose sum we will denote by V. Before the nonrelativistic limit,

$$\frac{2}{N(N-1)} V = V_{\text{mass}} + V_{\text{fer}} + V_{\text{quar}} + V_{\text{sext}}, \tag{4.4}$$

where (with $\tilde{m} \equiv mc/\hbar$, $\tilde{k} = k\hbar c$)

$$V_{\text{mass}} = \tilde{m} \sum_{i=1,2} \left[\bar{\eta}_i \eta_i + \bar{\tilde{\eta}}_i \tilde{\eta}_i \right] + \tilde{m}^2 \left[|\chi_1|^2 + |\chi_2|^2 + |\phi_1|^2 + |\phi_2|^2 \right], \tag{4.5}$$

$$V_{\text{fer}} = -\frac{2\pi i}{\tilde{k}} \left[(|\phi_1|^2 + |\chi_1|^2)(\bar{\eta}_2 \eta_2 + \bar{\tilde{\eta}}_2 \tilde{\eta}_2) + (|\phi_2|^2 + |\chi_2|^2)(\bar{\eta}_1 \eta_1 + \bar{\tilde{\eta}}_1 \tilde{\eta}_1) \right], \tag{4.6}$$

$$V_{\text{quar}} = \frac{8\pi \tilde{m}}{\tilde{k}} \left[|\phi_1|^2 |\phi_2|^2 - |\chi_1|^2 |\chi_2|^2 \right], \tag{4.7}$$

$$V_{\text{sext}} = \frac{4\pi^2}{\tilde{k}^2} \left[(|\chi_1|^2 + |\phi_1|^2)(|\chi_2|^2 + |\phi_2|^2)(|\chi_1|^2 + |\chi_2|^2 + |\phi_1|^2 + |\phi_2|^2) \right]. \tag{4.8}$$

The limit was defined by

$$\Phi_b \longrightarrow \frac{\hbar}{\sqrt{2m}} \Phi_b e^{-imc^2 t/\hbar}, \quad \Psi_f \longrightarrow \sqrt{\hbar c} \Psi_f e^{-imc^2 t/\hbar}, \tag{4.9}$$

where Φ_b, Ψ_f are generic bosonic and fermionic fields, respectively. Of these, the mass terms are cancelled by the contributions of the mass in the exponent of the fields, the sextic term goes to zero, and for the rest we get

$$\begin{aligned} V_{\text{fer}}^{NR} &= -\frac{\pi i\hbar^2}{mk}\Big[(|\phi_1|^2+|\chi_1|^2)(\eta_2^\dagger\eta_2+\tilde{\eta}_2^\dagger\tilde{\eta}_2)+(|\phi_2|^2+|\chi_2|^2)(\eta_1^\dagger\eta_1+\tilde{\eta}_1^\dagger\tilde{\eta}_1)\Big], \quad (4.10)\\ V_{\text{quar}}^{NR} &= \frac{2\pi\hbar^2}{km}(|\phi_1|^2|\phi_2|^2-|\chi_1|^2|\chi_2|^2), \end{aligned}$$

after the nonrelativistic limit. Now, comparing this with the toy model (4.2) we note that

1. we are unable to obtain objects of all 3 charges (+,− and 0) simultaneously.
2. we have $u = g_1 = g_2 = 0$ and $\epsilon_1 = \epsilon_2 = \mu$ and $m_b = m_f = m$, and
3. we obtain additional terms of the form $b^\dagger b f^\dagger f$.

With the truncation $A_\mu^{(1)} = -A_\mu^{(2)} = A_\mu$ and recalling that the covariant derivative $D_\mu = (\partial_\mu - iA_\mu^{(i)})$ acting on both ϕ_i and ψ_i, we obtain the action

$$\begin{aligned} S = N(N-1)\int d^3x \Bigg[&\frac{k\hbar}{4\pi}\epsilon^{\mu\nu\rho}A_\mu F_{\nu\rho} + f_+^\dagger\left((\partial_\tau - iA_\tau) - \frac{(\vec{\nabla}-i\vec{A})^2}{2m} - \frac{F_{12}}{2m}\right)f_+ \\ &+f_-^\dagger\left((\partial_\tau + iA_\tau) - \frac{(\vec{\nabla}+i\vec{A})^2}{2m} + \frac{F_{12}}{2m}\right)f_- + b_+^\dagger\left((\partial_\tau - iA_\tau) - \frac{(\vec{\nabla}-i\vec{A})^2}{2m}\right)b_+ \\ &+b_-^\dagger\left((\partial_\tau + iA_\tau) - \frac{(\vec{\nabla}+i\vec{A})^2}{2m}\right)b_- - \frac{2\pi\hbar^2}{mk}b_+^\dagger b_-^\dagger b_+ b_- \\ &-\frac{\pi\hbar^2}{km}[b_+^\dagger b_+ f_-^\dagger f_- + b_-^\dagger b_- f_+^\dagger f_+]\Bigg], \end{aligned} \quad (4.11)$$

with an extra CS term, and where to facilitate comparison to (4.2) we have denoted $\psi_1 = f_+, \psi_2 = f_-, \phi_1 = b_+, \phi_2 = b_-$ and changed to the conventions of [18]. In other words, $v = -2\pi\hbar^2/(mk)$, $c = 0$ and we also have some new couplings. Finally, it is worth noting that the matching is only consistent either for $f_- = 0$, $\mu = F_{12}/(2m)$, or for $\mu = F_{12} = 0$.

On the other hand, if we implemented the truncation by setting $A_\mu^{(2)} = 0$, we would have no CS term, and $f_- = b_- = 0$, producing the action,[11]

$$\begin{aligned} S = N(N-1)\int d^3x \Bigg[&f_+^\dagger\left((\partial_\tau - iA_\tau) - \frac{(\vec{\nabla}-i\vec{A})^2}{2m} - \frac{F_{12}}{2m}\right)f_+ + c^\dagger\left(\partial_\tau - \frac{\vec{\nabla}^2}{2m}\right)c \\ &+b_+^\dagger\left((\partial_\tau - iA_\tau) - \frac{(\vec{\nabla}-i\vec{A})^2}{2m}\right)b_+ - \frac{\pi\hbar^2}{km}c^\dagger c f_+^\dagger f_+ \\ &-\frac{2\pi\hbar^2}{mk}b_+^\dagger b_+ c^\dagger c\Bigg]. \end{aligned} \quad (4.12)$$

Either way, the top-down model that we obtain does *not* match perfectly that of [18]. Evidently then, while the mathematical structure of the two models are strikingly similar, their differences are sufficient to warrant further development, and it remains to be seen how much of the condensed matter physics can actually be reproduced.

[11] Here, we set also $\psi_2 = 0$ and denoting $\psi_1 = f_+, \phi_1 = b_+, \phi_2 = c$.

5 Conclusions

As part of a more ambitious progam aimed at a full *top-down* realization of the AdS/CMT correspondence in the AdS/CFT duality, this article details our analysis of the nonrelativistic limit of the abelian reduction of the massive ABJM model proposed in [10]. We have also checked that this limit commutes with our abelianization procedure (though we did not present the details here). Moreover, in our study of the supersymmetry laws governing the nonrelativistic limit, we found that the scaling of the supersymmetry parameters is not unique. Either we can keep all the fields, and the supersymmetry laws become rather simple and involve only a kinematical piece. However, the price we pay is that the resulting scalar potential is unbounded from below. Alternatively, we can truncate the theory to 2 complex fermions and 2 complex scalars, and obtain a system with $\mathcal{N} = 2$ supersymmetry with both kinematical and dynamical susy pieces. Further truncation to a single complex fermion and a single complex scalar yields a supersymmetric version of the Jackiw-Pi model considered in [14], although with novel values for the parameters.

The system we obtain has Super-Schrödinger symmetry and constitutes a concrete example of an interesting condensed matter model with an explicit action, obtained as a limit of a known AdS/CFT duality. Moreover, the holographic duality here is of the conventional type related to a $(d+1)$-dimensional gravity theory, instead of the previously constructed $(d+2)-$dimensional holographic dual.

Finally, on a more phenomenological note, we have compared our top-down construction with previously used abelian nonrelativistic condensed matter avatars for the ABJM model [18] that were explored in the context of compressible quantum matter, and found that there are certainly differences, the similarities between the models is striking. Indeed, it would be intriguing to push these similarities and see just how much of the physics of quantum matter can be teased from the nonrelativistic abelianized ABJM model. We leave this as an invitation to future work.

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