Mass limit for light flavon with residual Z_3 symmetry

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ABSTRACT: We present a modified Altarelli and Feruglio A_4 model where an additional A_4 singlet-prime flavon is introduced. In this model, non-zero θ_{13} is given by this additional A_4 singlet-prime flavon which breaks tri-bimaximal mixing. In the framework of the supersymmetry with $U(1)_R$ symmetry, we obtain vacuum expectation values (VEVs) and VEV alignments of flavons through driving fields. It is considered that flavon induces distinctive flavor violating process if flavon mass is light. Assuming mass of SUSY particles are sufficiently heavy so that the SUSY contributions can be negligible, we discuss the flavor violating Yukawa interaction through flavon exchange in the charged lepton sector. According to the potential analysis, the VEV of flavon breaks A_4 down to Z_3 in the charged lepton sector and relation among flavon masses is determined. Thanks for the residual Z_3 symmetry, many lepton flavor violating decay modes are forbidden except for $\tau \to \mu\mu\bar{e}$ and $\tau \to ee\bar{\mu}$. A mass limit of the flavon from these three-body decay modes is $60\,\mathrm{GeV}$ taking into account the current experimental lower bounds at the Belle experiment. In our model, we predict a ratio of the branching ratios $\tau \to \mu\mu\bar{e}$ and $\tau \to ee\bar{\mu}$ by using known charged lepton masses. We also find that the production cross section for the flavon can be $\mathcal{O}(1)\,\mathrm{fb}$. Thus the flavon would be found at the LHC run 2 by searching for 4-tau lepton signal.

KEYWORDS: Discrete Symmetries, Beyond Standard Model, Neutrino Physics

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Contents

1   Introduction

The neutrino experiments [1]–[5] are one of the most attractive experiments for the evidence of the beyond the standard model (SM). Actually, the neutrino oscillation experiments provide us that there are two neutrino mass squared differences and two large mixing angles. The reactor neutrino experiments also observed non-zero θ_{13}, which is the last mixing angle of the lepton sector [1, 2, 4]. The T2K experiment reported the first stage of the CP violating Dirac phase $\delta_{\rm CP}$ through the electron neutrino appearance in a muon neutrino beam [5]. If the neutrinos are Majorana particles, there are Majorana phases which are also sources of the CP violation. The neutrino-less double beta ($0\nu\beta\beta$) decay experiments are looking for the evidence of the Majorana particles and give the upper-bound of the effective neutrino mass m_{ee}. Thus the neutrino experiments provide us a new window of the beyond the SM in the theoretical point of view.

The non-Abelian discrete flavor symmetry can easily explain the large mixing of the lepton sector e.g. tri-bimaximal mixing (TBM) [6, 7], which is a simple mixing paradigm in the lepton sector. Actually, Altarelli and Feruglio (AF) proposed A_4 model of leptons [8, 9] which contains new gauge singlet scalar fields, so-called "flavons" in addition to the SU(2) doublet SM Higgs field. There are many authors who study flavor structure to derive TBM by using non-Abelian discrete flavor symmetry not only A_4 group but also many other groups. (See refs. of [10]–[13].) After reactor experiments reported non-zero θ_{13}, it is important to study the deviation from TBM precisely [14]–[24] or study other flavor paradigms, e.g. tri-bimaximal-Cabibbo mixing [25, 26].

Flavor models using non-Abelian discrete flavor symmetry have not been confirmed in the experimental point of view. Of course many authors discuss, under the framework of

the flavor symmetry models, the lepton flavor violation (LFV) e.g. $\mu \to e\gamma$ in addition to the prediction of the Dirac CP phase, Majorana phases, and effective mass of the $0\nu\beta\beta$ decay. In refs. [27]–[30], they discussed the mass restriction on the flavons, which are related to the SM Higgs, from the LFV and collider physics. However experimental constraints for gauge singlet flavons are not investigated although neutrino experimental data can be explained by models with gauge singlet flavons. Then, we discuss a mass restriction on the gauge singlet flavons. In general, the mass scale of the SM gauge singlet flavons and cutoff scale of the non-Abelian discrete flavor symmetry are taken very high scale. Actually, in refs. [8, 9], if the Dirac neutrino Yukawa couplings are $\mathcal{O}(1)$, the cutoff scale should be high scale such as $\mathcal{O}(10^{13})$–$\mathcal{O}(10^{15})$ GeV. They assumed that the magnitude of the flavon vacuum expectation values (VEVs) are almost same and tau lepton Yukawa coupling is perturbative. Then, the ratio of the flavon VEVs and cutoff scale should be lager than 0.0022. Therefore, the flavon mass is much heavier than the electro weak (EW) scale. However this requirement can be relaxed, if we take Dirac neutrino Yukawa couplings to be much less than $\mathcal{O}(1)$.[1] Then the flavon mass can be light without theoretical contradiction. Therefore we discuss the lower limit of the flavon mass from the experimental data in our paper.

We present a modified AF A_4 model which introduces an additional A_4 singlet-prime flavon breaking TBM [32]–[36], and calculate a potential of the flavon scalar fields. As well known, we need VEVs of flavons with specific alignments in order to obtain the correct masses and mixing angles in the lepton sector. In the framework of the supersymmetry (SUSY) with $U(1)_R$ symmetry, we obtain the VEVs of flavons and the alignments of them. Because the SUSY particles have not been found, we assume the mass of SUSY particles are sufficiently heavy so that the SUSY contributions can be negligible. Then we discuss the flavor violating Yukawa interaction through flavon exchange in the charged lepton sector. According to the potential analysis, the VEV of flavon breaks A_4 down to Z_3 in the charged lepton sector and relation among flavon masses is determined. Thanks for the residual Z_3 symmetry, many lepton flavor violating decay modes are forbidden except for $\tau \to \mu\mu\bar{e}$ and $\tau \to ee\bar{\mu}$ [37]. These three-body decay modes are mediated by the flavons. Therefore a mass limit of the flavon is $60\,\text{GeV}$ taking into account the current experimental lower bounds at the Belle experiment [38]. In addition, we predict a ratio of the branching ratios $\tau \to \mu\mu\bar{e}$ and $\tau \to ee\bar{\mu}$ by using known charged lepton masses.

In our model, the contribution of the muon $g-2$ is small and constraint from the LEP data [39] is also less stringent than the constraint given by flavor violating τ decay. Then we discuss the production at the hadron collider through radiation from charged leptons as other candidates for collider signatures. We find that the production cross section for the flavon can be $\mathcal{O}(1)\,\text{fb}$. Thus the flavon would be found at the LHC run 2 by searching for 4-tau lepton signal.

In section 2, we show the modified AF A_4 model and discuss the potential of the flavons. In section 3, we present the numerical analysis of the flavor physics and collider

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[1]If we introduce the Froggatt-Nielsen (FN) mechanism [31] as an additional $U(1)_{FN}$ flavor symmetry, the Yukawa couplings can be $\mathcal{O}(1)$.

	l	e_R^c	μ_R^c	τ_R^c	ν_R^c	$h_{u,d}$	ϕ_T	ϕ_S	ξ	ξ'	ϕ_0^T	ϕ_0^S	ξ_0
SU(2)	2	1	1	1	1	2	1	1	1	1	1	1	1
A_4	**3**	**1**	$\mathbf{1''}$	$\mathbf{1'}$	**3**	**1**	**3**	**3**	**1**	$\mathbf{1'}$	**3**	**3**	**1**
Z_3	ω	ω^2	ω^2	ω^2	ω^2	1	1	ω^2	ω^2	ω^2	1	ω^2	ω^2
U(1)$_R$	1	1	1	1	1	0	0	0	0	0	2	2	2

Table 1. Assignments of leptons, Higgs, flavons, and driving fields.

physics from flavon exchange. The section 4 is devoted to discussions and summary. In appendix A, we show the multiplication rule of the A_4 group. We show a full scalar potential of the relevant flavon in appendix B.

2 A_4 flavor model

In this section, we present a modified AF A_4 model which introduces an additional A_4 singlet-prime flavon [34]. Under the A_4 group, the left-handed lepton doublet $l = (l_e, l_\mu, l_\tau)$ are assumed to transform as the triplet, while the right-handed charged leptons are assigned to the singlets as **1**, $\mathbf{1''}$, and $\mathbf{1'}$ for e_R^c, μ_R^c, and τ_R^c, respectively. In ref. [34], the neutrino Majorana masses come from the Weinberg operator [40]. In this paper, we introduce the right-handed neutrinos which are gauge singlet and assigned to the triplet $\nu_R^c = (\nu_{eR}^c, \nu_{\mu R}^c, \nu_{\tau R}^c)$. These right-handed neutrinos can be origin of this Weinberg operator through the seesaw mechanism [41]–[46] in our model. Z_3 charges are assigned relevantly to the leptons, here we define $\omega^3 = 1$. On the other hand, the Higgs doublet h_u and h_d are assigned to the A_4 trivial singlet. We add gauge singlet flavons ϕ_T, ϕ_S, ξ, and ξ' which are assigned to the triplet for $\phi_T = (\phi_{T1}, \phi_{T2}, \phi_{T3})$ and $\phi_S = (\phi_{S1}, \phi_{S2}, \phi_{S3})$ and the trivial singlet for ξ, and the singlet-prime for ξ' under A_4 group, respectively. These flavons have different Z_3 charges as seen in table 1. In order to obtain VEVs and VEV alignments, we also add so-called "driving fields" $\phi_0^T = \left(\phi_{01}^T, \phi_{02}^T, \phi_{03}^T\right)$ and $\phi_0^S = \left(\phi_{01}^S, \phi_{02}^S, \phi_{03}^S\right)$ which are assigned to the triplet and ξ_0 which is assigned to the trivial singlet under A_4 group, respectively. We can generate the VEV alignments through F-terms by coupling flavons to driving fields, which carry the R charge $+2$ under U(1)$_R$ symmetry. We also assign R charge $+1$ to the lepton doublets, right-handed charged leptons, and right-handed Majorana neutrinos. The charge assignments of driving fields are also shown in table 1. Note that in the original AF model, they introduced A_4 trivial singlet flavon $\tilde{\xi}$ which has the same quantum numbers of ξ and was necessary to obtain a non-trivial VEV structure from the minimization of the potential. However A_4 singlet-prime flavon ξ' gives VEV structure without extra flavon $\tilde{\xi}$ [35]. In these setup, the superpotential for respecting $A_4 \times Z_3$ symmetry at the leading order in terms of the A_4 cutoff scale Λ is written as

$$w \equiv w_Y + w_d + \text{h.c.},$$
$$w_Y \equiv w_\ell + w_D + w_N,$$
$$w_\ell = y_e \phi_T l e_R^c h_d / \Lambda + y_\mu \phi_T l \mu_R^c h_d / \Lambda + y_\tau \phi_T l \tau_R^c h_d / \Lambda,$$

$$w_D = y_D l \nu_R^c h_u,$$
$$w_N = y_{\phi_S} \phi_S \nu_R^c \nu_R^c + y_\xi \xi \nu_R^c \nu_R^c + y_{\xi'} \xi' \nu_R^c \nu_R^c,$$
$$w_d \equiv w_d^T + w_d^S,$$
$$w_d^T = -M \phi_0^T \phi_T + g \phi_0^T \phi_T \phi_T,$$
$$w_d^S = g_1 \phi_0^S \phi_S \phi_S - g_2 \phi_0^S \phi_S \xi + g_2' \phi_0^S \phi_S \xi' + g_3 \xi_0 \phi_S \phi_S - g_4 \xi_0 \xi \xi, \tag{2.1}$$

where y's are complex Yukawa couplings, M is generally complex mass parameter, and g's are trilinear couplings which are also complex parameters.[2] From this superpotential, we discuss the potential analysis in the next subsection.

2.1 Potential analysis

In this subsection, we discuss the potential for scalar fields including flavons and driving fields. Let us write down the superpotential w_d^T and w_d^S in eq. (2.1) as

$$
w_d^T = -M \left(\phi_{01}^T \phi_{T1} + \phi_{02}^T \phi_{T3} + \phi_{03}^T \phi_{T2} \right)
$$
$$
+ \frac{2g}{3} \left[\phi_{01}^T \left(\phi_{T1}^2 - \phi_{T2}\phi_{T3} \right) + \phi_{02}^T \left(\phi_{T2}^2 - \phi_{T1}\phi_{T3} \right) + \phi_{03}^T \left(\phi_{T3}^2 - \phi_{T1}\phi_{T2} \right) \right],
$$
$$
w_d^S = \frac{2g_1}{3} \left[\phi_{01}^S \left(\phi_{S1}^2 - \phi_{S2}\phi_{S3} \right) + \phi_{02}^S \left(\phi_{S2}^2 - \phi_{S1}\phi_{S3} \right) + \phi_{03}^S \left(\phi_{S3}^2 - \phi_{S1}\phi_{S2} \right) \right]
$$
$$
- g_2 \left(\phi_{01}^S \phi_{S1} + \phi_{02}^S \phi_{S3} + \phi_{03}^S \phi_{S2} \right) \xi + g_2' \left(\phi_{01}^S \phi_{S3} + \phi_{02}^S \phi_{S2} + \phi_{03}^S \phi_{S1} \right) \xi'
$$
$$
+ g_3 \xi_0 \left(\phi_{S1}^2 + 2\phi_{S2}\phi_{S3} \right) - g_4 \xi_0 \xi^2. \tag{2.2}
$$

Then, the scalar potential is given as

$$V \equiv V_T + V_S,$$
$$V_T = \sum_i \left| \frac{\partial w_d^T}{\partial \phi_{0i}^T} \right|^2 + \text{h.c.}$$
$$= 2 \left| -M\phi_{T1} + \frac{2g}{3} \left(\phi_{T1}^2 - \phi_{T2}\phi_{T3} \right) \right|^2 + 2 \left| -M\phi_{T3} + \frac{2g}{3} \left(\phi_{T2}^2 - \phi_{T1}\phi_{T3} \right) \right|^2$$
$$+ 2 \left| -M\phi_{T2} + \frac{2g}{3} \left(\phi_{T3}^2 - \phi_{T1}\phi_{T2} \right) \right|^2,$$
$$V_S = \sum \left| \frac{\partial w_d^S}{\partial X} \right|^2 + \text{h.c.}$$
$$= 2 \left| \frac{2g_1}{3} \left(\phi_{S1}^2 - \phi_{S2}\phi_{S3} \right) - g_2 \phi_{S1}\xi + g_2' \phi_{S3}\xi' \right|^2 + 2 \left| \frac{2g_1}{3} \left(\phi_{S2}^2 - \phi_{S1}\phi_{S3} \right) - g_2 \phi_{S3}\xi + g_2' \phi_{S2}\xi' \right|^2$$
$$+ 2 \left| \frac{2g_1}{3} \left(\phi_{S3}^2 - \phi_{S1}\phi_{S2} \right) - g_2 \phi_{S2}\xi + g_2' \phi_{S1}\xi' \right|^2 + 2 \left| g_3 \left(\phi_{S1}^2 + 2\phi_{S2}\phi_{S3} \right) - g_4 \xi^2 \right|^2, \tag{2.3}$$

---

[2]In order to obtain the positive number of v_T, v_S, u, and u' for eqs. (2.4) and (2.5), we take negative sign for several terms in eq. (2.1).

where $X = \phi_{0i}^S$, ξ_0. Therefore, VEV alignment of ϕ_T is derived from the condition of the potential minimum ($V_T = 0$) in eq. (2.3) as

$$\langle \phi_T \rangle = v_T(1,0,0), \qquad v_T = \frac{3M}{2g}, \qquad (2.4)$$

here v_T is generally complex number because M and g are complex. Using the VEV and the VEV alignment of eq. (2.4), we obtain that the charged lepton mass matrix is diagonal. Then, we can remove the phase of v_T and take M/g as real parameter without loss of generality. Hereafter we take M and g as real parameters for simplicity. On the other hand, VEV alignment of ϕ_S and VEVs of ξ and ξ' are derived from the condition of the potential minimum ($V_S = 0$) in eq. (2.3) as

$$\langle \phi_S \rangle = v_S(1,1,1), \qquad \langle \xi \rangle = u, \qquad \langle \xi' \rangle = u', \qquad v_S^2 = \frac{g_4}{3g_3}u^2, \qquad u' = \frac{g_2}{g_2'}u. \qquad (2.5)$$

Therefore, we can take the VEVs u and u' as arbitrary numbers. Using the VEVs and the VEV alignment of eq. (2.5), the neutrino mass matrix derives the lepton mixing by taking an additional rotation of 1–3 generations of neutrinos in the TBM. Then, we obtain the non-zero θ_{13} which comes from A_4 singlet flavon VEV ratio u'/u. (See refs. [32]–[36].)

Before closing this subsection, we discuss the A_4 breaking patterns. A_4 is the symmetry group of a tetrahedron or even permutation of four elements. The number of elements is 12. The irreducible representations of A_4 are $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$, and $\mathbf{3}$. Also A_4 can be defined as the group generated by two elements S and T which satisfy the algebraic relations as

$$S^2 = T^3 = (ST)^3 = \mathbf{1}. \qquad (2.6)$$

On the one-dimensional representations, these generators are represented by

$$\begin{array}{llll} \mathbf{1}: & S = 1, & T = 1, \\ \mathbf{1}': & S = 1, & T = e^{4\pi i/3} \equiv \omega^2, \\ \mathbf{1}'': & S = 1, & T = e^{2\pi i/3} \equiv \omega. \end{array} \qquad (2.7)$$

On the three-dimensional representation, these generators are represented by

$$\mathbf{3}: \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \qquad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}. \qquad (2.8)$$

After A_4 is broken by taking the VEVs and the VEV alignments of flavons ϕ_T and ϕ_S, there are two breaking patterns $A_4 \to G_T$ and $A_4 \to G_S$ where G_T and G_S are subgroups of $A_4 = (Z_2 \times Z_2) \rtimes Z_3$. $\langle \phi_T \rangle = v_T(1,0,0)$ breaks A_4 down to $G_T = Z_3$, while $\langle \phi_S \rangle = v_S(1,1,1)$ breaks A_4 down to $G_S = Z_2$. Therefore, the LFV is restricted by residual Z_3 symmetry in the charged lepton sector. Hereafter, we focus on flavon ϕ_T which couples to the charged lepton sector, since we will discuss LFV and collider physics in section 3. In the next subsection, we discuss the mass of flavon ϕ_T.

2.2 Mass of the flavon

In this subsection, we discuss the mass of flavon ϕ_T which couples to the charged lepton sector. We expand the flavon field around the VEV v_T as

$$\phi_T = (\phi_{T1}, \phi_{T2}, \phi_{T3}) \rightarrow (v_T + \varphi_{T1}, \varphi_{T2}, \varphi_{T3}), \tag{2.9}$$

where φ_{Ti} are complex scalar fields. Then, we rewrite the scalar potential V_T in eq. (2.3) as $V_T = V_T^{\text{mass}} + \text{(other terms)},$[3] and V_T^{mass} is the mass term of flavon ϕ_T as

$$V_T^{\text{mass}} = 2M^2 \left(|\varphi_{T1}|^2 + 4 |\varphi_{T2}|^2 + 4 |\varphi_{T3}|^2 \right), \tag{2.10}$$

here we eliminate g by using eq. (2.4). Therefore, masses of scalar fields $m_{\varphi_{Ti}}$ are obtained as

$$(m_{\varphi_{T1}}^2, m_{\varphi_{T2}}^2, m_{\varphi_{T3}}^2) = (2M^2, 8M^2, 8M^2), \tag{2.11}$$

and the scalar fields φ_{Ti} do not mix each other in the mass term. Taking into account these masses, we discuss the flavor phenomenology and collider physics in the next section.

3 Flavor phenomenology and collider physics from flavon exchange

In this section we discuss flavor phenomenology and collider physics from flavon exchange. First of all, we assumed masses of SUSY particles to be heavy because we have not found any SUSY particles so that the SUSY contributions can be negligible. Then we show flavon Yukawa interactions in the charged lepton sector. Next we show flavor physics from flavon exchange and constraint for flavon mass. Finally, we show some predictions for flavor phenomenology and collider physics.

3.1 Flavon Yukawa interactions in charged lepton sector

In our model SM Yukawa interactions and flavon Yukawa interactions in the charged lepton sector come from following Lagrangian in eq. (2.1);

$$\mathcal{L}_\ell = y_e \left(\phi_T \bar{l} \right) e_R h_d / \Lambda + y_\mu \left(\phi_T \bar{l} \right)' \mu_R h_d / \Lambda + y_\tau \left(\phi_T \bar{l} \right)'' \tau_R h_d / \Lambda + \text{h.c.}, \tag{3.1}$$

where we use the same notation for superfields and SM fields. After expanding flavon field ϕ_T around VEV v_T in eq. (2.9) and taking the VEV of SU(2) doublet Higgs h_d as v_d, Lagrangian for charged lepton mass terms $\mathcal{L}_\ell^{\text{mass}}$ is written as

$$
\begin{aligned}
\mathcal{L}_\ell^{\text{mass}} &= \left(\bar{e}_L \ \bar{\mu}_L \ \bar{\tau}_L \right) \begin{pmatrix} \frac{y_e v_d}{\Lambda} v_T & 0 & 0 \\ 0 & \frac{y_\mu v_d}{\Lambda} v_T & 0 \\ 0 & 0 & \frac{y_\tau v_d}{\Lambda} v_T \end{pmatrix} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} + \text{h.c.} \\
&\equiv \left(\bar{e}_L \ \bar{\mu}_L \ \bar{\tau}_L \right) \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} + \text{h.c.}.
\end{aligned} \tag{3.2}
$$

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[3]In appendix B, we show the full scalar potential V_T.

In our model, charged leptons in the interaction basis is equal to those in the mass basis. Therefore, there is no mixing in the charged lepton sector in the leading level. Let us discuss the Lagrangian of the charged lepton and flavon interaction which induces flavon Yukawa interactions in the charged lepton sector $\mathcal{L}_\ell^{\mathrm{FY}}$ as

$$\mathcal{L}_\ell^{\mathrm{FY}} = \begin{pmatrix} \bar{e}_L & \bar{\mu}_L & \bar{\tau}_L \end{pmatrix} \begin{pmatrix} \frac{m_e}{v_T} & 0 & 0 \\ 0 & \frac{m_\mu}{v_T} & 0 \\ 0 & 0 & \frac{m_\tau}{v_T} \end{pmatrix} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \varphi_{T1}$$

$$+ \begin{pmatrix} \bar{e}_L & \bar{\mu}_L & \bar{\tau}_L \end{pmatrix} \begin{pmatrix} 0 & \frac{m_\mu}{v_T} & 0 \\ 0 & 0 & \frac{m_\tau}{v_T} \\ \frac{m_e}{v_T} & 0 & 0 \end{pmatrix} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \varphi_{T2}$$

$$+ \begin{pmatrix} \bar{e}_L & \bar{\mu}_L & \bar{\tau}_L \end{pmatrix} \begin{pmatrix} 0 & 0 & \frac{m_\tau}{v_T} \\ \frac{m_e}{v_T} & 0 & 0 \\ 0 & \frac{m_\mu}{v_T} & 0 \end{pmatrix} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \varphi_{T3} + \mathrm{h.c.}. \tag{3.3}$$

We find that φ_{T1} exchange does not induce flavor violation, while the other flavon exchanges induce flavor violation. The most interesting feature of these flavon interactions is that couplings are almost fixed by charged lepton masses except for A_4 triplet flavon VEV v_T.

Next, we discuss the lepton radiative flavor violating decays $\mu \to e\gamma$, $\tau \to \mu\gamma$, and $\tau \to e\gamma$. In the charged lepton sector, because we take the VEV alignment of ϕ_T as $v_T(1,0,0)$, A_4 breaks down to $G_T = Z_3$, which we discussed in section 2.1. Then flavons φ_{Ti} are transformed by eq. (2.8) as

$$\varphi_{T1} \to \varphi_{T1}, \qquad \varphi_{T2} \to \omega^2 \varphi_{T2}, \qquad \varphi_{T3} \to \omega \varphi_{T3}, \tag{3.4}$$

under the residual symmetry $G_T = Z_3$. The left-handed charged leptons are transformed by eq. (2.8) as

$$e_L \to e_L, \qquad \mu_L \to \omega \mu_L, \qquad \tau_L \to \omega^2 \tau_L. \tag{3.5}$$

On the other hand, the right-handed charged leptons are transformed by eq. (2.7) as

$$e_R \to e_R, \qquad \mu_R \to \omega \mu_R, \qquad \tau_R \to \omega^2 \tau_R. \tag{3.6}$$

Therefore many lepton flavor violating decay modes are forbidden by the residual symmetry $G_T = Z_3$ e.g. $\mu \to e\gamma$ except for several lepton flavor violating three-body decays. The muon $g - 2$ obtains contribution from one-loop diagram where only φ_{T1} propagates inside loop without flavor change. However the contribution is small since corresponding Yukawa coupling constant is m_μ/v_T. We thus discuss the lepton flavor violating three-body decay in the next subsection.

3.2 Lepton flavor violating three-body decay

In our model many lepton flavor violating three-body decay modes are forbidden by the residual symmetry $G_T = Z_3$. Dominant flavor changing decay modes are $\tau \to \mu\mu\bar{e}$ and

$\tau \to ee\bar{\mu}$ [37]. These decay modes are induced by flavon exchange at tree-level and these branching ratios are given by

$$\mathrm{BR}\left(\tau \to \mu\mu\bar{e}\right) = \tau_\tau \frac{m_\tau^5}{3072\pi^3} \left(\left| \frac{m_\tau m_\mu}{v_T^2 m_{\varphi T2}^2} \right|^2 + \left| \frac{m_\mu m_e}{v_T^2 m_{\varphi T3}^2} \right|^2 \right)$$

$$\simeq \frac{2.9 \times 10^6 \ \mathrm{GeV}^8}{v_T^4 (2\sqrt{2}M)^4},$$

$$\mathrm{BR}\left(\tau \to ee\bar{\mu}\right) = \tau_\tau \frac{m_\tau^5}{3072\pi^3} \left(\left| \frac{m_\mu m_e}{v_T^2 m_{\varphi T2}^2} \right|^2 + \left| \frac{m_\tau m_e}{v_T^2 m_{\varphi T3}^2} \right|^2 \right)$$

$$\simeq \frac{68 \ \mathrm{GeV}^8}{v_T^4 (2\sqrt{2}M)^4}, \tag{3.7}$$

where τ_τ is the lifetime of tau lepton and we use the magnitude of the charged lepton masses by the particle data group (PDG) [47]. In our model the decay width for $\tau \to \mu\mu\bar{e}$ is much larger than that for $\tau \to ee\bar{\mu}$. Current experimental lower bounds at the Belle experiment are $\mathrm{BR}\left(\tau \to \mu\mu\bar{e}\right) < 1.7 \times 10^{-8}$ and $\mathrm{BR}\left(\tau \to ee\bar{\mu}\right) < 1.5 \times 10^{-8}$ [38]. To realize these constraints, $2\sqrt{2}M \gtrsim 60$ GeV is required when we assume that VEV v_T is equal to flavon masses $m_{\varphi T2}$, $m_{\varphi T3}$. In addition, we can predict a ratio of these branching ratios r by using known charged lepton masses as

$$r = \frac{\mathrm{BR}\left(\tau \to \mu\mu\bar{e}\right)}{\mathrm{BR}\left(\tau \to ee\bar{\mu}\right)} = \frac{m_\mu^2 \left(m_\tau^2 + m_e^2 \right)}{m_e^2 \left(m_\mu^2 + m_\tau^2 \right)} \simeq \frac{m_\mu^2}{m_e^2}, \tag{3.8}$$

because of $m_{\varphi T2} = m_{\varphi T3}$. Then the ratio of $\mathrm{BR}\left(\tau \to \mu\mu\bar{e}\right)$ and $\mathrm{BR}\left(\tau \to ee\bar{\mu}\right)$ is $r \simeq 4.3 \times 10^4$. As a result of the calculations, we find that flavon mass can be very light. Therefore, we discuss the flavon collider phenomenology in the next subsection.

3.3 Flavon collider phenomenology

After obtaining the constraint on the flavon mass from flavor violating lepton decay, we discuss collider physics of the flavon where the flavon exchanging and production processes could be experimentally tested since the flavon can be light as $m_{\varphi Ti} \simeq \mathcal{O}(100)\,\mathrm{GeV}$.

We first discuss constraints on the flavon mass from the t-channel processes $e\bar{e} \to \mu\bar{\mu}$ and $\tau\bar{\tau}$ at the LEP experiment. The relevant 4-Fermi interactions are obtained via φ_{T2}, φ_{T3} exchange such that

$$\mathcal{L}_{4-\mathrm{fermi}} \supset \frac{m_\mu^2}{64M^4}(\bar{\mu}_R e_L)(\bar{e}_L \mu_R) + \frac{m_\tau^2}{64M^4}(\bar{\tau}_R e_L)(\bar{e}_L \tau_R) + \mathrm{h.c.}, \tag{3.9}$$

where $m_{\varphi T2} = m_{\varphi T3} = 2\sqrt{2}M$ and $v_T = 2\sqrt{2}M$ is adopted, and we only show the interactions which induce the processes $e\bar{e} \to \mu\bar{\mu}$ and $e\bar{e} \to \tau\bar{\tau}$. We thus find that $e\bar{e} \to \tau\bar{\tau}$ process is dominant. Then we obtain the constraint on the flavon mass from the LEP data [39];

$$(2\sqrt{2}M)^2 \gtrsim 620 m_\tau \ \mathrm{GeV}. \tag{3.10}$$

Final state	$\varphi_{T1}\tau\bar{\tau}$	$\varphi_{T2}\tau\bar{\mu}$	$\varphi_{T3}\tau\bar{e}$	$\varphi_{T2}\tau\bar{\nu}_\mu$	$\varphi_{T3}\tau\bar{\nu}_e$
Cross section [fb]	0.59	0.017	0.017	0.040	0.040

Table 2. Dominant flavon production cross sections at the LHC 14 TeV where $v_T = 2m_{\varphi_{T1}} = m_{\varphi_{T2}} = m_{\varphi_{T3}} = 65$ GeV is adopted. The values of cross sections are sum of shown final states and its charge conjugation.

Therefore this constraint requires $2\sqrt{2}M \gtrsim 33$ GeV which is less stringent than the constraint given by flavor violating τ decay in previous subsection.

Flavons φ_{Ti} can be produced at the hadron collider through radiation from charged leptons, i.e. $pp \to \varphi_{Ti}\bar{\ell}\ell'$ and $pp \to \varphi_{Ti}\bar{\ell}\nu(\ell\bar{\nu})$ where $\ell = e, \mu$ and τ. Taking into account the flavon-lepton Yukawa coupling proportional to m_τ, the dominant final states in flavon production processes are summarized in the first low of table 2. Produced flavons then decay into lepton pair where the dominant decay modes are $\varphi_{T1} \to \tau\bar{\tau}$, $\varphi_{T2} \to \mu\bar{\tau}$ and $\varphi_{T3} \to e\bar{\tau}$. Thus the signals are four-leptons including at least two τ leptons. The production cross sections are calculated using CalcHEP [48] with CTEQ6L PDF [49], which are shown in the second low of the table 2. Here we adopted $v_T = 2m_{\varphi_{T1}} = m_{\varphi_{T2}} = m_{\varphi_{T3}} = 65$ GeV which is close to the lower limit from flavor violating lepton decay. We find that the φ_{T1} production cross section can be $\mathcal{O}(1)$ fb while those of φ_{T2}, φ_{T3} are $\mathcal{O}(10^{-2})$ fb since the mass of φ_{T1} is half of the others. Thus φ_{T1} would be found at the LHC run 2 by searching for 4-tau lepton signal since SM background is not large. Moreover φ_{T2}, φ_{T3} provide peak of invariant mass distribution for $\mu\tau$ and $e\tau$ pair respectively which are significant signal of flavor violating interaction and SM background also could be highly reduced with relevant kinematical cuts. Thus φ_{T2}, φ_{T3} will be also important target at the High-Luminosity LHC, which could be tested with large amount of integrated luminosity although the production cross sections are small. Furthermore the flavon-lepton Yukawa interactions can be tested at the lepton colliders like ILC [50]–[54] where detailed analysis is left as future work.

We can calculate these cross sections as a function of the product of the flavon mass and the VEV because other couplings are determined by charged lepton masses. Branching ratio of the lepton flavor violating three-body decay modes is also a function of same one. Then, we can predict relation between collider signature and flavor physics. Therefore if we measure one of them, the rest one can be signature of our model.

4 Discussions and summary

Flavor models which introduce gauge singlet flavons using non-Abelian discrete flavor symmetry have not been confirmed in the experimental point of view. In general, the mass scale of the SM gauge singlet flavons and cutoff scale of the non-Abelian discrete flavor symmetry are assumed to be very high scale because many authors take Yukawa couplings to be $\mathcal{O}(1)$ so that the flavon masses are much heavier than the EW scale. However this requirement can be relaxed, if we take Dirac neutrino Yukawa couplings to be much less than $\mathcal{O}(1)$ in our model. Then the flavon mass can be light without theoretical contradic-

tion. Therefore we discussed the lower limit of the flavon mass from the experimental data in our paper.

We presented the modified AF A_4 model which introduces an additional A_4 singlet-prime flavon breaking TBM, and calculated the potential of the flavon scalar fields. Note that sometimes the contributions from the next-to-leading order are added to realize non-zero θ_{13}. However we ignored the contributions from the next-to-leading order because the non-zero θ_{13} has been already derived by the A_4 singlet flavon VEV ratio u'/u and such contributions can be eliminated in specific UV completions of the flavor models in refs. [35, 55]. As well known, we need the VEVs of flavons with specific alignments in order to obtain the correct masses and mixing angles in the lepton sector. In the framework of the SUSY with $U(1)_R$ symmetry, we obtained the VEVs of flavons and the alignments of them. Because the SUSY particles have not been found, we assume the mass of SUSY particles are sufficiently heavy so that the SUSY contributions can be negligible. Then we discussed the flavor violating Yukawa interaction through flavon exchange in the charged lepton sector. According to the potential analysis, the VEV of flavon ϕ_T breaks A_4 down to $G_T = Z_3$ in the charged lepton sector and the masses of flavons $m_{\varphi_{T2}}$ and $m_{\varphi_{T3}}$ are same and twice as heavy as $m_{\varphi_{T1}}$. Thanks for the residual Z_3 symmetry, many lepton flavor violating decay modes are forbidden except for $\tau \to \mu\mu\bar{e}$ and $\tau \to ee\bar{\mu}$. These three-body decay modes are mediated by flavons φ_{T2} or φ_{T3}. Therefore the mass limit of flavons φ_{T2}, φ_{T3} is $60\,\mathrm{GeV}$ taking into account the current experimental lower bounds at the Belle experiment. Then if we assume that the magnitude of the flavon VEV is same as the mass of flavon such as $v_T = 2\sqrt{2}M$, tau lepton Yukawa coupling y_τ is $\mathcal{O}(1)$, and $\tan\beta = 3$, the cutoff scale Λ should be at least $\mathcal{O}(10)\,\mathrm{TeV}$ to realize tau lepton mass. Therefore we should take $\tan\beta$ to be small and we will find the new physics in the near future. In addition, we predicted the ratio of the branching ratios $\tau \to \mu\mu\bar{e}$ and $\tau \to ee\bar{\mu}$ such as $r \simeq 4.3 \times 10^4$ by using known charged lepton masses.

The contribution of the muon $g - 2$ is small and constraint from the LEP data is also less stringent than the constraint given by flavor violating τ decay in our model. Then we discussed the production at the hadron collider through radiation from charged leptons as other candidates for collider signatures. We found that the φ_{T1} production cross section can be $\mathcal{O}(1)\,\mathrm{fb}$ while those of φ_{T2}, φ_{T3} are $\mathcal{O}(10^{-2})\,\mathrm{fb}$ since the mass of φ_{T1} is half of the others. Thus φ_{T1} would be found at the LHC run 2 by searching for 4-tau lepton signal. The φ_{T2}, φ_{T3} provide peak of invariant mass distribution for $\mu\tau$ and $e\tau$ pair respectively which are significant signal of flavor violating interaction and SM background also could be highly reduced with relevant kinematical cuts. Thus φ_{T2}, φ_{T3} will be also important target at the High-Luminosity LHC, which could be tested with large amount of integrated luminosity although the production cross sections are small. Furthermore the flavon-lepton Yukawa interactions can be tested at the lepton colliders like ILC.

In our model, we can predict relation between flavor physics and collider signature because flavor symmetry fixes many couplings and the residual Z_3 symmetry makes flavon mass limit light as accessible in collider search. Therefore if we measure one of them, the rest one can be signature of our model. In other flavor models, flavon mass limit is not light as accessible in collider search if there is no residual symmetry. On the other hand,

there are rich signatures of flavor physics, then we can predict many relations between these signatures. Therefore it is important to study phenomenology from flavon exchange in flavor models.

Acknowledgments

A Multiplication rule of A_4 group

In this appendix A, we show the multiplication of A_4 group. The multiplication rule of the triplet is written as follow;

$$
\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{\mathbf{3}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}} = (a_1 b_1 + a_2 b_3 + a_3 b_2)_{\mathbf{1}} \oplus (a_3 b_3 + a_1 b_2 + a_2 b_1)_{\mathbf{1'}}
$$

$$
\oplus (a_2 b_2 + a_1 b_3 + a_3 b_1)_{\mathbf{1''}}
$$

$$
\oplus \frac{1}{3} \begin{pmatrix} 2a_1 b_1 - a_2 b_3 - a_3 b_2 \\ 2a_3 b_3 - a_1 b_2 - a_2 b_1 \\ 2a_2 b_2 - a_1 b_3 - a_3 b_1 \end{pmatrix}_{\mathbf{3}} \oplus \frac{1}{2} \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_1 b_2 - a_2 b_1 \\ a_1 b_3 - a_3 b_1 \end{pmatrix}_{\mathbf{3}} . \tag{A.1}
$$

More details are shown in the review [10]–[13].

B Full scalar potential V_T

We show the full potential V_T as follow;

$$
V_T = 2M^2 \left(|\varphi_{T1}|^2 + 4\,|\varphi_{T2}|^2 + 4\,|\varphi_{T3}|^2 \right)
$$

$$
+ \frac{2M^2}{v_T} \Big[(\varphi_{T1} + \varphi_{T1}^*) \left(|\varphi_{T1}|^2 + 2\,|\varphi_{T2}|^2 + 2\,|\varphi_{T3}|^2 \right) - (\varphi_{T1} \varphi_{T2}^* \varphi_{T3}^* + \varphi_{T1}^* \varphi_{T2} \varphi_{T3})
$$

$$
- 2\left(\varphi_{T2}^2 \varphi_{T3}^* + \varphi_{T2}^{*2} \varphi_{T3} \right) - 2\left(\varphi_{T2} \varphi_{T3}^{*2} + \varphi_{T2}^* \varphi_{T3}^2 \right) \Big]
$$

$$
+ \frac{2M^2}{v_T^2} \Big[|\varphi_{T1}|^4 + |\varphi_{T2}|^4 + |\varphi_{T3}|^4 + |\varphi_{T1}|^2\,|\varphi_{T2}|^2 + |\varphi_{T2}|^2\,|\varphi_{T3}|^2 + |\varphi_{T3}|^2\,|\varphi_{T1}|^2
$$

$$
- \left(\varphi_{T1}^2 \varphi_{T2}^* \varphi_{T3}^* + \varphi_{T1}^{*2} \varphi_{T2} \varphi_{T3} \right) - \left(\varphi_{T1} \varphi_{T2}^{*2} \varphi_{T3} + \varphi_{T1}^* \varphi_{T2}^2 \varphi_{T3}^* \right)
$$

$$
- \left(\varphi_{T1} \varphi_{T2} \varphi_{T3}^{*2} + \varphi_{T1}^* \varphi_{T2}^* \varphi_{T3}^2 \right) \Big], \tag{B.1}
$$

where we eliminate g by using eq. (2.4).

References

[1] DAYA BAY collaboration, F.P. An et al., *Observation of electron-antineutrino disappearance at Daya Bay*, *Phys. Rev. Lett.* **108** (2012) 171803 [arXiv:1203.1669] [INSPIRE].

[2] RENO collaboration, J.K. Ahn et al., *Observation of Reactor Electron Antineutrino Disappearance in the RENO Experiment*, *Phys. Rev. Lett.* **108** (2012) 191802 [arXiv:1204.0626] [INSPIRE].

[3] MINOS collaboration, P. Adamson et al., *Measurement of Neutrino and Antineutrino Oscillations Using Beam and Atmospheric Data in MINOS*, *Phys. Rev. Lett.* **110** (2013) 251801 [arXiv:1304.6335] [INSPIRE].

[4] DOUBLE CHOOZ collaboration, Y. Abe et al., *Background-independent measurement of θ_{13} in Double CHOOZ*, *Phys. Lett.* **B 735** (2014) 51 [arXiv:1401.5981] [INSPIRE].

[5] T2K collaboration, K. Abe et al., *Observation of Electron Neutrino Appearance in a Muon Neutrino Beam*, *Phys. Rev. Lett.* **112** (2014) 061802 [arXiv:1311.4750] [INSPIRE].

[6] P.F. Harrison, D.H. Perkins and W.G. Scott, *Tri-bimaximal mixing and the neutrino oscillation data*, *Phys. Lett.* **B 530** (2002) 167 [hep-ph/0202074] [INSPIRE].

[7] P.F. Harrison and W.G. Scott, *Symmetries and generalizations of tri-bimaximal neutrino mixing*, *Phys. Lett.* **B 535** (2002) 163 [hep-ph/0203209] [INSPIRE].

[8] G. Altarelli and F. Feruglio, *Tri-bimaximal neutrino mixing from discrete symmetry in extra dimensions*, *Nucl. Phys.* **B 720** (2005) 64 [hep-ph/0504165] [INSPIRE].

[9] G. Altarelli and F. Feruglio, *Tri-bimaximal neutrino mixing, A_4 and the modular symmetry*, *Nucl. Phys.* **B 741** (2006) 215 [hep-ph/0512103] [INSPIRE].

[10] H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada and M. Tanimoto, *Non-Abelian Discrete Symmetries in Particle Physics*, *Prog. Theor. Phys. Suppl.* **183** (2010) 1 [arXiv:1003.3552] [INSPIRE].

[11] H. Ishimori, T. Kobayashi, H. Ohki, H. Okada, Y. Shimizu and M. Tanimoto, *An introduction to non-Abelian discrete symmetries for particle physicists*, *Lect. Notes Phys.* **858** (2012) 1 [INSPIRE].

[12] H. Ishimori, T. Kobayashi, Y. Shimizu, H. Ohki, H. Okada and M. Tanimoto, *Non-Abelian discrete symmetry for flavors*, *Fortsch. Phys.* **61** (2013) 441 [INSPIRE].

[13] S.F. King, A. Merle, S. Morisi, Y. Shimizu and M. Tanimoto, *Neutrino Mass and Mixing: from Theory to Experiment*, *New J. Phys.* **16** (2014) 045018 [arXiv:1402.4271] [INSPIRE].

[14] P.S. Bhupal Dev, R.N. Mohapatra and M. Severson, *Neutrino Mixings in SO(10) with Type II Seesaw and θ_{13}*, *Phys. Rev.* **D 84** (2011) 053005 [arXiv:1107.2378] [INSPIRE].

[15] P.S. Bhupal Dev, B. Dutta, R.N. Mohapatra and M. Severson, *θ_{13} and Proton Decay in a Minimal SO(10) × S_4 model of Flavor*, *Phys. Rev.* **D 86** (2012) 035002 [arXiv:1202.4012] [INSPIRE].

[16] Y.H. Ahn and S.K. Kang, *Non-zero θ_{13} and CP-violation in a model with A_4 flavor symmetry*, *Phys. Rev.* **D 86** (2012) 093003 [arXiv:1203.4185] [INSPIRE].

[17] H. Ishimori, S. Khalil and E. Ma, *CP Phases of Neutrino Mixing in a Supersymmetric $B - L$ Gauge Model with T_7 Lepton Flavor Symmetry*, *Phys. Rev.* **D 86** (2012) 013008 [arXiv:1204.2705] [INSPIRE].

[18] H. Ishimori and E. Ma, *New Simple A_4 Neutrino Model for Nonzero θ_{13} and Large δ_{CP}*, *Phys. Rev.* **D 86** (2012) 045030 [arXiv:1205.0075] [INSPIRE].

[19] C. Hagedorn, S.F. King and C. Luhn, *SUSY $S_4 \times$ SU(5) revisited*, *Phys. Lett.* **B 717** (2012) 207 [`arXiv:1205.3114`] [INSPIRE].

[20] A. Damanik, *Nonzero θ_{13} and Neutrino Masses from the Modified Tribimaximal Neutrino Mixing Matrix*, `arXiv:1206.0987` [INSPIRE].

[21] J.A. Acosta, A. Aranda, M.A. Buen-Abad and A.D. Rojas, *Non-diagonal charged lepton mass matrix and non-zero θ_{13}*, *Phys. Lett.* **B 718** (2013) 1413 [`arXiv:1207.6093`] [INSPIRE].

[22] W. Rodejohann and H. Zhang, *Simple two Parameter Description of Lepton Mixing*, *Phys. Rev.* **D 86** (2012) 093008 [`arXiv:1207.1225`] [INSPIRE].

[23] Y. Shimizu, M. Tanimoto and K. Yamamoto, *Predicting CP-violation in Deviation from Tri-bimaximal mixing of Neutrinos*, *Mod. Phys. Lett.* **A 30** (2015) 1550002 [`arXiv:1405.1521`] [INSPIRE].

[24] Y. Shimizu and M. Tanimoto, *Testing the minimal S_4 model of neutrinos with the Dirac and Majorana phases*, *JHEP* **12** (2015) 132 [`arXiv:1507.06221`] [INSPIRE].

[25] S.F. King, *Tri-bimaximal-Cabibbo Mixing*, *Phys. Lett.* **B 718** (2012) 136 [`arXiv:1205.0506`] [INSPIRE].

[26] Y. Shimizu, R. Takahashi and M. Tanimoto, *Minimal Neutrino Texture with Neutrino Mass Ratio and Cabibbo Angle*, *Prog. Theor. Exp. Phys.* **2013** (2013) 063B02 [`arXiv:1212.5913`] [INSPIRE].

[27] M. Holthausen, M. Lindner and M.A. Schmidt, *Lepton flavor at the electroweak scale: A complete A_4 model*, *Phys. Rev.* **D 87** (2013) 033006 [`arXiv:1211.5143`] [INSPIRE].

[28] I. de Medeiros Varzielas, O. Fischer and V. Maurer, $\mathbb{A}_4$ *symmetry at colliders and in the universe*, *JHEP* **08** (2015) 080 [`arXiv:1504.03955`] [INSPIRE].

[29] T. Kobayashi, Y. Omura, F. Takayama and D. Yasuhara, *Study of lepton flavor violation in flavor symmetric models for lepton sector*, *JHEP* **10** (2015) 042 [`arXiv:1505.07636`] [INSPIRE].

[30] I. Medeiros Varzielas and O. Fischer, *Non-Abelian family symmetries as portals to dark matter*, *JHEP* **01** (2016) 160 [`arXiv:1512.00869`] [INSPIRE].

[31] C.D. Froggatt and H.B. Nielsen, *Hierarchy of Quark Masses, Cabibbo Angles and CP-violation*, *Nucl. Phys.* **B 147** (1979) 277 [INSPIRE].

[32] B. Brahmachari, S. Choubey and M. Mitra, *The A_4 flavor symmetry and neutrino phenomenology*, *Phys. Rev.* **D 77** (2008) 073008 [*Erratum ibid.* **D 77** (2008) 119901] [`arXiv:0801.3554`] [INSPIRE].

[33] J. Barry and W. Rodejohann, *Deviations from tribimaximal mixing due to the vacuum expectation value misalignment in A_4 models*, *Phys. Rev.* **D 81** (2010) 093002 [*Erratum ibid.* **D 81** (2010) 119901] [`arXiv:1003.2385`] [INSPIRE].

[34] Y. Shimizu, M. Tanimoto and A. Watanabe, *Breaking Tri-bimaximal Mixing and Large θ_{13}*, *Prog. Theor. Phys.* **126** (2011) 81 [`arXiv:1105.2929`] [INSPIRE].

[35] I. de Medeiros Varzielas and D. Pidt, *UV completions of flavour models and large θ_{13}*, *JHEP* **03** (2013) 065 [`arXiv:1211.5370`] [INSPIRE].

[36] B. Karmakar and A. Sil, *Nonzero θ_{13} and leptogenesis in a type-I seesaw model with A_4 symmetry*, *Phys. Rev.* **D 91** (2015) 013004 [`arXiv:1407.5826`] [INSPIRE].

[37] E. Ma, *Quark and Lepton Flavor Triality*, *Phys. Rev.* **D 82** (2010) 037301 [`arXiv:1006.3524`] [INSPIRE].

[38] K. Hayasaka et al., *Search for Lepton Flavor Violating Tau Decays into Three Leptons with 719 Million Produced $\tau^+\tau^-$ Pairs*, *Phys. Lett.* **B 687** (2010) 139 [arXiv:1001.3221] [inSPIRE].

[39] DELPHI, LEP, ALEPH, OPAL and L3 collaborations, SLD Electroweak Group, SLD Heavy Flavor Group and LEP Electroweak Working Group, D. Abbaneo et al., *A Combination of preliminary electroweak measurements and constraints on the standard model*, hep-ex/0312023 [inSPIRE].

[40] S. Weinberg, *Baryon and Lepton Nonconserving Processes*, *Phys. Rev. Lett.* **43** (1979) 1566 [inSPIRE].

[41] P. Minkowski, $\mu \to e\gamma$ *at a Rate of One Out of* 10^9 *Muon Decays?*, *Phys. Lett.* **B 67** (1977) 421 [inSPIRE].

[42] T. Yanagida, *Horizontal gauge symmetry and masses of neutrinos*, in proceedings of the *Workshop on the Baryon Number of the Universe and Unified Theories*, Tsukuba, Japan, February 13–14 1979, O. Sawada and A. Sugamoto eds., National Lab for High Energy Physics (1979) [KEK-79-18].

[43] M. Gell-Mann, P. Ramond and R. Slansky, *Complex Spinors and Unified Theories*, in *Supergravity*, P. van Nieuwenhuizen and D.Z. Freedman eds., North Holland, Amsterdam The Netherlands (1979).

[44] R.N. Mohapatra and G. Senjanović, *Neutrino Mass and Spontaneous Parity Violation*, *Phys. Rev. Lett.* **44** (1980) 912 [inSPIRE].

[45] J. Schechter and J.W.F. Valle, *Neutrino Masses in* SU(2) × U(1) *Theories*, *Phys. Rev.* **D 22** (1980) 2227 [inSPIRE].

[46] J. Schechter and J.W.F. Valle, *Neutrino Decay and Spontaneous Violation of Lepton Number*, *Phys. Rev.* **D 25** (1982) 774 [inSPIRE].

[47] PARTICLE DATA GROUP collaboration, K.A. Olive et al., *Review of Particle Physics*, *Chin. Phys.* **C 38** (2014) 090001 [inSPIRE].

[48] A. Belyaev, N.D. Christensen and A. Pukhov, *CalcHEP 3.4 for collider physics within and beyond the Standard Model*, *Comput. Phys. Commun.* **184** (2013) 1729 [arXiv:1207.6082] [inSPIRE].

[49] P.M. Nadolsky et al., *Implications of CTEQ global analysis for collider observables*, *Phys. Rev.* **D 78** (2008) 013004 [arXiv:0802.0007] [inSPIRE].

[50] T. Behnke et al., *The International Linear Collider Technical Design Report — Volume 1: Executive Summary*, arXiv:1306.6327 [inSPIRE].

[51] H. Baer et al., *The International Linear Collider Technical Design Report — Volume 2: Physics*, arXiv:1306.6352 [inSPIRE].

[52] C. Adolphsen et al., *The International Linear Collider Technical Design Report — Volume 3.I: Accelerator & in the Technical Design Phase*, arXiv:1306.6353 [inSPIRE].

[53] C. Adolphsen et al., *The International Linear Collider Technical Design Report — Volume 3.II: Accelerator Baseline Design*, arXiv:1306.6328 [inSPIRE].

[54] T. Behnke et al., *The International Linear Collider Technical Design Report — Volume 4: Detectors*, arXiv:1306.6329 [inSPIRE].

[55] I. de Medeiros Varzielas and L. Merlo, *Ultraviolet Completion of Flavour Models*, *JHEP* **02** (2011) 062 [arXiv:1011.6662] [inSPIRE].