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Proton decay in a supersymmetric SO(10) model with missing partner mechanism

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Abstract: The extended supersymmetric SO(10) model with missing partner mechanism is studied. An intermediate vacuum expectation value is incorporated which corresponds to the see-saw scale. Gauge coupling unification is not broken explicitly. Proton decay is found to satisfy the present experimental limits at the cost of fine-tuning some parameters.

Keywords: GUT, Beyond Standard Model, Supersymmetric Standard Model

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Contents

1 Introduction

The supersymmetric version of SO(10) Grand Unified Theory (GUT) [1–18] is an interesting candidate for the new physics beyond the Standard Model (SM). First, the fermions in every of the three generations are contained in the same representation 16 of SO(10). These 16plets contain the right-handed neutrinos so that neutrino masses and mixings can be taken into account through the see-saw mechanism. Second, by including different Higgs multiplets, fermion masses and mixings can be described correctly. Third, once the two Higgs doublets for the charged fermion masses are arranged at the electro-weak scale, supersymmetry (SUSY) protects their smallness against quadratic divergences.

The supersymmetric SO(10) GUT has also problems to be solved. The contents of the Higgs multiplets are not fixed *a priori*. In the minimal version, the Higgs sector contains 10, 126, $\overline{126}$ and 210 [6–10]. This brings in several difficulties in the predictions. The model is highly constrained by the requirement of SUSY at high energy scales. To account for the neutrino masses and mixings, an intermediate scale given by the VEVs of $\overline{126}$ and 126 is needed which, however, breaks badly the unification of the gauge couplings. Also, there is the so-called doublet-triplet splitting (DTS) problem to explain that why the two Higgs doublets are light in the Minimal Supersymmetric SM (MSSM) while all the other multiplets, especially the triplets in the same minimal representation 10, are all superheavy.

The Missing Partner Mechanism (MPM) has been successfully used in supersymmetric GUT of SU(5) [19–23] to provide a natural realization of the DTS within the Higgs multiplets. This mechanism is then used in SO(10) model [24, 25]. To account for the charged fermion masses, this Minimal SO(10) model with MPM (Minimal SO10MPM) [25] needs to be extended to include more $\overline{126}$ and 126 while keeping the MPM [26]. However, there are still difficulties to overcome, including how to incorporate the see-saw mechanism and how to suppress proton decay in the model.

In the present work, we will study the extended SO10MPM. The SO(10) symmetry is broken directly to the SM gauge group, while an intermediate VEV to account for the

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see-saw scale is used which is only a small correction to the SO(10) symmetry breaking. The other $126-\overline{126}$ need to have large VEVs not only to keep unification, but also to suppress proton decay required by the experimental data.

In section 2 we will review the Minimal SO10MPM briefly. In section 3 the extended SO10MPM is constructed and solved explicitly. Mass matrices of doublets and triplets are shown in section 4. Fermion masses and proton decay are studied in section 5. In section 6 we will summarize.

2 Review of the minimal SO10MPM

In the SU(5) models, the MPM is realized by introducing a U(1) symmetry to forbid the $5-\overline{5}$ to have direct mass term. By coupling $5\,(\overline{5})$ Higgs to $\overline{50}(50)$, which contains a color triplet (anti-triplet) but no weak doublet, through 75 which breaks SU(5), the color triplets in $5-\overline{5}$ gain GUT scale masses while the weak doublets are still massless.

In SO(10), the 5 and $\overline{5}$ of SU(5) constitute a 10, and the $\overline{50}$ and 50 of SU(5) are contained in 126 and $\overline{126}$, respectively, as

$$\begin{aligned} 126 &= 1+\overline{5}+10+\overline{15}+45+\overline{50}, \\ \overline{126} &= 1+5+\overline{10}+15+\overline{45}+50. \end{aligned} \tag{2.1}$$

In total, a pair of $126-\overline{126}$ multiplets contains 2 pairs of doublets and 3 pairs of triplets. In addition, the SU(5)-breaking 75 is contained in 210 of SO(10) which breaks SO(10) symmetry. Then, in principle, the MPM of SU(5) can be embedded into SO(10). This needs to include more Higgs in the model and to impose an extra U(1) symmetry to eliminate unwanted bilinear and trilinear couplings from the superpotential, leaving some elements of the mass matrices to be zero. Denoting the multiplets which cannot(can) get mass through couplings with itself or its conjugate as "*light*"("*heavy*") fields, the Higgs fields

$$10\times 2+120+126/\overline{126}+210$$

are required in the minimal version [25], in which the 10s and 120 are *light* with $Q=1$, $126-\overline{126}$ are *heavy* with $Q=-1$, and 210 is also *heavy* with $Q=0$. The heavy fields $126-\overline{126}$ get their masses through couplings with a singlet X with $Q=2$, while the light fields get masses only through couplings with $126-\overline{126}$ and 210. Fermions are contained in the three 16plets with $Q=-\frac{1}{2}$, coupling only with the *light* fields. It is easy to count that there are 4 (4) pairs of *light* doublets (triplets), and 3 (4) pairs of *heavy* doublets (triplets). The mass matrices are

$$M_D = \begin{array}{c} \\ \overline{D}_{\rm light} \\ \overline{D}_{\rm heavy} \end{array} \begin{array}{c} \begin{array}{cc} D_{\rm light} & D_{\rm heavy} \end{array} \\ \begin{pmatrix} 0_{4\times 4} & O(G)_{4\times 3} \\ O(G)_{3\times 4} & O(G)_{3\times 3} \end{pmatrix} \end{array} \tag{2.2}$$

for the doublets, and

$$M_T = \begin{array}{c} \\ \overline{T}_{\rm light} \\ \overline{T}_{\rm heavy} \end{array} \begin{array}{c} \begin{array}{cc} T_{\rm light} & T_{\rm heavy} \end{array} \\ \begin{pmatrix} 0_{4\times 4} & O(G)_{4\times 4} \\ O(G)_{4\times 4} & O(G)_{4\times 4} \end{pmatrix} \end{array} \tag{2.3}$$

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	$\Psi_i(16)$	$H_1(10)$	$H_2(10)$	$D(120)$	$\Delta/\overline{\Delta}$	$\Delta_1/\overline{\Delta}_1$	$\Delta_2/\overline{\Delta}_2$	$\Phi(210)$	$X(1)$
Q	$-\frac{1}{2}$	1	1	1	1	-1	-1	0	2

Table 1. SO(10) multiplets and their U(1) charges. $\Psi_i (i = 1, 2, 3)$ represent three generations of fermions, $\Delta/\overline{\Delta}$, $\Delta_1/\overline{\Delta}_1$, $\Delta_2/\overline{\Delta}_2$ are all $126/\overline{126}$ multiplets, and X is an SO(10) singlet whose VEV breaks the U(1) symmetry. $H_{1,2}$ and D are 10plets and 120plet, respectively. Higgs multiplets with $Q = 1$ are *light* while the others are *heavy*.

for the triplets. Here, $0_{4\times4}$ stands for a 4×4 null matrix, while $O(G)$s mean matrices whose elements are of the GUT scale. Generally speaking, M_D has one zero (light) eigenvalue, and M_T has none. Because the determinant of M_D contains $7! = 5040$ terms, and each term inevitably has one factor that comes from the 4×4 null matrix part, this makes the determinant of M_D vanished. Since the heavy part of M_T is larger than that of M_D, all eigenvalues of M_T remain superheavy, and the DTS is thus achieved.

However, one cannot fit all fermion masses with only 10 and 120 fields, a *light* $\overline{126}$ must be introduced to couple with the fermions. Meanwhile, the *heavy* side of fields must be extended to keep the MPM [26]. We will study this extended SO10MPM in the next section which was not solved explicitly in [26].

3 Extended SO10MPM with 3 pairs of $126 - \overline{126}$

In the Minimal SO10MPM the see-saw mechanism is not included to explain the neutrino masses. In fact, SO(10) has a basic conflict between incorporating the see-saw mechanism to generate the neutrino masses and realizing the unification of gauge couplings. In the type-I see-saw models [27–30], the right-handed neutrinos need to have masses at the intermediate scale of the order of $O(10^{14})$GeV [31–33]. Similarly, in the type-II see-saw models, a weak triplet at the intermediate scale is needed. This intermediate scale, however, breaks the gauge coupling unification badly since fields at this scale will change dramatically the running behaviors of the coupling constants.

To incorporate the see-saw mechanism, we use the extended SO10MPM with three pairs of $126 - \overline{126}$. The GUT scale VEVs are assigned to the MSSM singlets of these $126 - \overline{126}$ except that of the $\overline{126}$ which couples to fermions has an intermediate VEV. The SO(10) symmetry is thus directly broken to MSSM symmetry, so that gauge coupling unification can be maintained hopefully as all particles other than the MSSM Higgs doublets are superheavy. All the three pairs of $126-\overline{126}$ are needed to keep SUSY unbroken at high energy, as the conditions for the F- and D-flatness are satisfied.

The model includes the following set of states in table 1, and their U(1) quantum numbers are also assigned to eliminate unwanted masses and couplings. The U(1) contains an anomaly which is canceled through the Green-Schwarz mechanism [34–37].

The general renormalizable Higgs superpotential is given by

$$W = \frac{1}{2} m_\Phi \Phi^2 + m_{\Delta i} \overline{\Delta}_i \Delta + \overline{m}_{\Delta i} \overline{\Delta} \Delta_i + \lambda \Phi^3 + \eta_{ij} X \overline{\Delta}_i \Delta_j + \left(\lambda_i \overline{\Delta}_i \Delta + \overline{\lambda}_i \overline{\Delta} \Delta_i\right) \Phi + \left(\beta_{ij} \Delta_i + \overline{\beta}_{ij} \overline{\Delta}_i\right) H_j \Phi + \left(\gamma_i \Delta_i + \overline{\gamma}_i \overline{\Delta}_i\right) D\Phi, \tag{3.1}$$

where $i, j = 1, 2$. Repeated indices stand for summation, as usually understood. Following the notation of [38], SO(10) breaks down to the MSSM when the MSSM singlets get VEVs as

$$\begin{aligned} \Phi_1 &= \langle \Phi(1,1,1) \rangle, \qquad \Phi_2 = \langle \Phi(15,1,1) \rangle, \qquad \Phi_3 = \langle \Phi(15,1,3) \rangle, \\ v_{(1,2)R} &= \langle \Delta(\overline{10},1,3) \rangle, \qquad \overline{v}_{(1,2)R} = \langle \overline{\Delta}(10,1,3) \rangle. \end{aligned} \tag{3.2}$$

Here the $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ indices are used to specify different singlets of MSSM.

Inserting these VEVs into (3.1), we get:

$$\begin{aligned} \langle W \rangle = & \frac{1}{2} m_\Phi \left(\Phi_1^2 + \Phi_2^2 + \Phi_3^2 \right) + m_{\Delta i} \overline{v}_{iR} v_R + \overline{m}_{\Delta i} \overline{v}_R v_{iR} + \lambda \left(\frac{1}{9\sqrt{2}} \Phi_2^3 + \frac{1}{2\sqrt{6}} \Phi_1 \Phi_3^2 + \frac{1}{3\sqrt{2}} \Phi_2 \Phi_3^2 \right) \\ & + \left(\lambda_i \overline{v}_{iR} v_R + \overline{\lambda}_i \overline{v}_R v_{iR} \right) \Phi_0 + \eta_{ij} X \overline{v}_{iR} v_{jR}, \end{aligned} \tag{3.3}$$

where we have defined

$$\Phi_0 \equiv \left[\Phi_1 \frac{1}{10\sqrt{6}} + \Phi_2 \frac{1}{10\sqrt{2}} + \Phi_3 \frac{1}{10} \right] \tag{3.4}$$

for further convenience.

To preserve SUSY at high energy, the F- and D-flatness conditions are required. The D-flatness condition requires

$$|v_R|^2 + |v_{1R}|^2 + |v_{2R}|^2 = |\overline{v}_R|^2 + |\overline{v}_{1R}|^2 + |\overline{v}_{2R}|^2, \tag{3.5}$$

while the F-flatness conditions are

$$\left\{ \frac{\partial}{\partial \Phi_1}, \frac{\partial}{\partial \Phi_2}, \frac{\partial}{\partial \Phi_3}, \frac{\partial}{\partial v_R}, \frac{\partial}{\partial v_{1R}}, \frac{\partial}{\partial v_{2R}}, \frac{\partial}{\partial \overline{v}_R}, \frac{\partial}{\partial \overline{v}_{1R}}, \frac{\partial}{\partial \overline{v}_{2R}}, \frac{\partial}{\partial X} \right\} \langle W \rangle = 0, \tag{3.6}$$

which are explicitly

$$0 = m_\Phi \Phi_1 + \frac{\lambda \Phi_3^2}{2\sqrt{6}} + \frac{1}{10\sqrt{6}} \left(\lambda_i \overline{v}_{iR} v_R + \overline{\lambda}_i \overline{v}_R v_{iR} \right), \tag{3.7}$$

$$0 = m_\Phi \Phi_2 + \frac{\lambda \Phi_2^2}{3\sqrt{2}} + \frac{\lambda \Phi_3^2}{3\sqrt{2}} + \frac{1}{10\sqrt{2}} \left(\lambda_i \overline{v}_{iR} v_R + \overline{\lambda}_i \overline{v}_R v_{iR} \right), \tag{3.8}$$

$$0 = m_\Phi \Phi_3 + \frac{\lambda \Phi_1 \Phi_3}{\sqrt{6}} + \frac{\sqrt{2} \lambda \Phi_2 \Phi_3}{3} + \frac{1}{10} \left(\lambda_i \overline{v}_{iR} v_R + \overline{\lambda}_i \overline{v}_R v_{iR} \right), \tag{3.9}$$

$$0 = M_1 \overline{v}_{1R} + M_2 \overline{v}_{2R}, \tag{3.10}$$

$$0 = \overline{M}_1 \overline{v}_R + \eta_{i1} X \overline{v}_{iR}, \tag{3.11}$$

$$0 = \overline{M}_2 \overline{v}_R + \eta_{i2} X \overline{v}_{iR}, \tag{3.12}$$

$$0 = \overline{M}_1 v_{1R} + \overline{M}_2 v_{2R}, \tag{3.13}$$

$$0 = M_1 v_R + \eta_{1i} X v_{iR}, \tag{3.14}$$

$$0 = M_2 v_R + \eta_{2i} X v_{iR}, \tag{3.15}$$

$$0 = \eta_{ij} \overline{v}_{iR} v_{jR}. \tag{3.16}$$

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Here we have defined:

$$M_i \equiv m_{\Delta i} + \lambda_i \Phi_0, \qquad \overline{M}_i \equiv \overline{m}_{\Delta i} + \overline{\lambda}_i \Phi_0. \tag{3.17}$$

Now that there are 10 equations for the F-flatness and 1 for the D-flatness in total, but only 9 of these constraints are independent. The situation here is comparable with that of the minimal SUSY SO(10) model in [6, 8], where there are 5 equations for the F-flatness and 1 for the D-flatness, but only 5 are independent. We can solve the above equations as follows. The first three equations in the F-flatness conditions, (3.7)–(3.9), are solved in the same way as in the minimal SUSY SO(10) model which gives

$$\Phi_1 = -\frac{\sqrt{6} m_\Phi}{\lambda} \frac{x\left(1-5x^2\right)}{(1-x)^2}, \tag{3.18}$$

$$\Phi_2 = -\frac{3\sqrt{2} m_\Phi}{\lambda} \frac{\left(1-2x-x^2\right)}{(1-x)}, \tag{3.19}$$

$$\Phi_3 = \frac{6 m_\Phi}{\lambda} x, \tag{3.20}$$

$$\left(\lambda_i \overline{v}_{iR} v_R + \overline{\lambda}_i \overline{v}_R v_{iR}\right) = \frac{60 m_\Phi^2}{\lambda} \frac{x(1-3x)\left(1+x^2\right)}{(1-x)^2}. \tag{3.21}$$

For x is not to be taken special values to generate accidental intermediate symmetries, all the VEVs $\Phi_{1,2,3}$ are at GUT scale [6, 8].

The next three equations, (3.10)–(3.12), are linear homogeneous equations about $\overline{v}_R$, $\overline{v}_{1R}$ and $\overline{v}_{2R}$, so they have non-zero solutions only when

$$\det \begin{pmatrix} 0 & M_1 & M_2 \\ \overline{M}_1 & \eta_{11} X & \eta_{21} X \\ \overline{M}_2 & \eta_{12} X & \eta_{22} X \end{pmatrix} = 0. \tag{3.22}$$

Then, among $\overline{v}_R$, $\overline{v}_{1R}$ and $\overline{v}_{2R}$, only one can be considered as free which is chosen to be $\overline{v}_{1R}$ without loss of generality. Similarly, equations (3.13)–(3.15) require

$$\det \begin{pmatrix} 0 & \overline{M}_1 & \overline{M}_2 \\ M_1 & \eta_{11} X & \eta_{12} X \\ M_2 & \eta_{21} X & \eta_{22} X \end{pmatrix} = 0 \tag{3.23}$$

to have non-zero solutions for v_R, v_{1R} and v_{2R}, and v_{1R} is taken to be free among them. Note that the condition (3.23) is exactly the same as (3.22). However, the transpose relation of the matrices in (3.22) and in (3.23) means that the two sets of equations (3.10)–(3.12) and (3.13)–(3.15) have different solutions. The last equation, (3.16), holds automatically after taking (3.7)–(3.15) into account.

When all the superpotential parameters are fixed, this condition (3.22) or (3.23) determines the value of Φ_0, and thus x can be extracted from (3.4) and (3.18)–(3.20). The ratio of $\overline{v}_{1R}$ to v_{1R} is determined by the D-flatness condition (3.5). With x being solved already, all these VEVs of $126 - \overline{126}$ are given.

To realize the see-saw mechanism, an intermediate value for $\overline{v}_R$ is taken. All the other VEVs are taken at the GUT scale. Consequently, there is no Higgs existing at the intermediate scale. The only exceptions are the right-handed neutrinos needed in the type-I see-saw.

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4 Mass matrices for doublets and triplets

In the Extended SO10MPM model, there are 6 (7) pairs of *light* doublets (triplets) and 5 (7) pairs of *heavy* doublets (triplets). The mass matrix for the doublets is written as

$$M_D = \begin{pmatrix} A_{11(6\times6)} & A_{12(6\times5)} \\ A_{21(5\times6)} & A_{22(5\times5)} \end{pmatrix}, \tag{4.1}$$

where the bases are

$$H^{(1,2,\frac{1}{2})}_{1(1,2,2)}, H^{(1,2,\frac{1}{2})}_{2(1,2,2)}, D^{(1,2,\frac{1}{2})}_{(1,2,2)}, D^{(1,2,\frac{1}{2})}_{(15,2,2)}, \Delta^{(1,2,\frac{1}{2})}_{(15,2,2)}, \overline{\Delta}^{(1,2,\frac{1}{2})}_{(15,2,2)}$$

for the first 6 columns,

$$H^{(1,2,-\frac{1}{2})}_{1(1,2,2)}, H^{(1,2,-\frac{1}{2})}_{2(1,2,2)}, D^{(1,2,-\frac{1}{2})}_{(1,2,2)}, D^{(1,2,-\frac{1}{2})}_{(15,2,2)}, \overline{\Delta}^{(1,2,-\frac{1}{2})}_{(15,2,2)}, \Delta^{(1,2,-\frac{1}{2})}_{(15,2,2)}$$

for the first 6 rows,

$$\Delta^{(1,2,\frac{1}{2})}_{1(15,2,2)}, \overline{\Delta}^{(1,2,\frac{1}{2})}_{1(15,2,2)}, \Delta^{(1,2,\frac{1}{2})}_{2(15,2,2)}, \overline{\Delta}^{(1,2,\frac{1}{2})}_{2(15,2,2)}, \Phi^{(1,2,\frac{1}{2})}_{(\overline{10},2,2)}$$

for the last 5 columns, and

$$\overline{\Delta}^{(1,2,-\frac{1}{2})}_{1(15,2,2)}, \Delta^{(1,2,-\frac{1}{2})}_{1(15,2,2)}, \overline{\Delta}^{(1,2,-\frac{1}{2})}_{2(15,2,2)}, \Delta^{(1,2,-\frac{1}{2})}_{2(15,2,2)}, \Phi^{(1,2,-\frac{1}{2})}_{(10,2,2)}$$

for the last 5 rows. In these bases, the subscripts and the superscripts label the $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ and the SM representations, respectively. In the M_D, A_{11} is a 6×6 null matrix, while

$$A_{12} = \begin{pmatrix} \beta_{11}\Phi^{(D)}_{H\Delta} & \overline{\beta}_{11}\Phi^{(D)}_{H\overline{\Delta}} & \beta_{21}\Phi^{(D)}_{H\Delta} & \overline{\beta}_{21}\Phi^{(D)}_{H\overline{\Delta}} & -\frac{\overline{\beta}_{i1}\overline{v}_{iR}}{\sqrt{5}} \\ \beta_{12}\Phi^{(D)}_{H\Delta} & \overline{\beta}_{12}\Phi^{(D)}_{H\overline{\Delta}} & \beta_{22}\Phi^{(D)}_{H\Delta} & \overline{\beta}_{22}\Phi^{(D)}_{H\overline{\Delta}} & -\frac{\overline{\beta}_{i2}\overline{v}_{iR}}{\sqrt{5}} \\ \frac{\gamma_1\Phi_3}{4\sqrt{30}} & \frac{\overline{\gamma}_1\Phi_3}{4\sqrt{30}} & \frac{\gamma_2\Phi_3}{4\sqrt{30}} & \frac{\overline{\gamma}_2\Phi_3}{4\sqrt{30}} & -\frac{\overline{\gamma}_i\overline{v}_{iR}}{2\sqrt{30}} \\ \gamma_1\Phi^{(D)}_{D\Delta} & \overline{\gamma}_1\Phi^{(D)}_{D\overline{\Delta}} & \gamma_2\Phi^{(D)}_{D\Delta} & \overline{\gamma}_2\Phi^{(D)}_{D\overline{\Delta}} & -\frac{\overline{\gamma}_i\overline{v}_{iR}}{2\sqrt{10}} \\ \overline{m}_{\Delta_1} + \overline{\lambda}_1\Phi^{(D)}_{\overline{\Delta}\Delta} & 0 & \overline{m}_{\Delta_2} + \overline{\lambda}_2\Phi^{(D)}_{\overline{\Delta}\Delta} & 0 & 0 \\ 0 & m_{\Delta_1} + \lambda_1\Phi^{(D)}_{\Delta\overline{\Delta}} & 0 & m_{\Delta_2} + \lambda_2\Phi^{(D)}_{\Delta\overline{\Delta}} & \frac{\lambda_i\overline{v}_{iR}}{10} \end{pmatrix}, \tag{4.2}$$

$$A_{21} = \begin{pmatrix} \overline{\beta}_{11}\Phi^{(D)}_{H\Delta} & \overline{\beta}_{12}\Phi^{(D)}_{H\Delta} & \frac{\overline{\gamma}_1\Phi_3}{4\sqrt{30}} & \overline{\gamma}_1\Phi^{(D)}_{D\Delta} & m_{\Delta_1} + \lambda_1\Phi^{(D)}_{\Delta\overline{\Delta}} & 0 \\ \beta_{11}\Phi^{(D)}_{H\overline{\Delta}} & \beta_{12}\Phi^{(D)}_{H\overline{\Delta}} & \frac{\gamma_1\Phi_3}{4\sqrt{30}} & \gamma_1\Phi^{(D)}_{D\overline{\Delta}} & 0 & \overline{m}_{\Delta_1} + \overline{\lambda}_1\Phi^{(D)}_{\overline{\Delta}\Delta} \\ \overline{\beta}_{21}\Phi^{(D)}_{H\Delta} & \overline{\beta}_{22}\Phi^{(D)}_{H\Delta} & \frac{\overline{\gamma}_2\Phi_3}{4\sqrt{30}} & \overline{\gamma}_2\Phi^{(D)}_{D\Delta} & m_{\Delta_2} + \lambda_2\Phi^{(D)}_{\Delta\overline{\Delta}} & 0 \\ \beta_{21}\Phi^{(D)}_{H\overline{\Delta}} & \beta_{22}\Phi^{(D)}_{H\overline{\Delta}} & \frac{\gamma_2\Phi_3}{4\sqrt{30}} & \gamma_2\Phi^{(D)}_{D\overline{\Delta}} & 0 & \overline{m}_{\Delta_2} + \overline{\lambda}_2\Phi^{(D)}_{\overline{\Delta}\Delta} \\ -\frac{\beta_{i1}v_{iR}}{\sqrt{5}} & -\frac{\beta_{i2}v_{iR}}{\sqrt{5}} & -\frac{\gamma_i v_{iR}}{2\sqrt{30}} & -\frac{\gamma_i v_{iR}}{2\sqrt{10}} & 0 & \frac{\overline{\lambda}_i v_{iR}}{10} \end{pmatrix}, \tag{4.3}$$

and

$$A_{22} = \begin{pmatrix} \eta_{11}X & 0 & \eta_{12}X & 0 & 0 \\ 0 & \eta_{11}X & 0 & \eta_{21}X & \frac{\overline{\lambda}_1\overline{v}_R}{10} \\ \eta_{21}X & 0 & \eta_{22}X & 0 & 0 \\ 0 & \eta_{12}X & 0 & \eta_{22}X & \frac{\overline{\lambda}_2\overline{v}_R}{10} \\ 0 & \frac{\lambda_1 v_R}{10} & 0 & \frac{\lambda_2 v_R}{10} & m_\Phi + \frac{\lambda\Phi_2}{\sqrt{2}} + \frac{\lambda\Phi_3}{2} \end{pmatrix}, \tag{4.4}$$

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where for simplicity we have defined

$$\begin{aligned}
\Phi^{(D)}_{H\Delta} &= \frac{\Phi_2}{\sqrt{10}} - \frac{\Phi_3}{2\sqrt{5}}, & \Phi^{(D)}_{H\overline{\Delta}} &= -\frac{\Phi_2}{\sqrt{10}} - \frac{\Phi_3}{2\sqrt{5}}, \\
\Phi^{(D)}_{D\Delta} &= \frac{\Phi_1}{4\sqrt{15}} - \frac{\Phi_3}{6\sqrt{10}}, & \Phi^{(D)}_{D\overline{\Delta}} &= \frac{\Phi_1}{4\sqrt{15}} + \frac{\Phi_3}{6\sqrt{10}}, \\
\Phi^{(D)}_{\Delta\overline{\Delta}} &= \frac{\Phi_2}{15\sqrt{2}} - \frac{\Phi_3}{30}, & \Phi^{(D)}_{\overline{\Delta}\Delta} &= \frac{\Phi_2}{15\sqrt{2}} + \frac{\Phi_3}{30}.
\end{aligned} \tag{4.5}$$

It is easy to see that there is one zero-valued eigen-state in the row and another one in the column, which are the MSSM Higgs doublets. It can be also noted that these light doublets contain no components from the doublets in 210 which break $B-L$ quantum numbers. Consequently, the $\mathrm{SU}(2)_L$ triplets in $\overline{126}$s has no VEV, which excludes the type-II see-saw mechanism [39] in the model.

The mass matrix for the Higgs triplets are divided into four 7×7 blocks as

$$M_T = \begin{pmatrix} B_{11(7\times7)} & B_{12(7\times7)} \\ B_{21(7\times7)} & B_{22(7\times7)} \end{pmatrix}, \tag{4.6}$$

where the bases are

$$H^{(3,1,-\frac{1}{3})}_{1(6,1,1)}, H^{(3,1,-\frac{1}{3})}_{2(6,1,1)}, D^{(3,1,-\frac{1}{3})}_{(6,1,3)}, D^{(3,1,-\frac{1}{3})}_{(10,1,1)}, \overline{\Delta}^{(3,1,-\frac{1}{3})}_{(6,1,1)}, \overline{\Delta}^{(3,1,-\frac{1}{3})}_{(10,1,3)}, \Delta^{(3,1,-\frac{1}{3})}_{(6,1,1)}$$

for the first 7 columns,

$$H^{(\overline{3},1,\frac{1}{3})}_{1(6,1,1)}, H^{(\overline{3},1,\frac{1}{3})}_{2(6,1,1)}, D^{(\overline{3},1,\frac{1}{3})}_{(6,1,3)}, D^{(\overline{3},1,\frac{1}{3})}_{(\overline{10},1,1)}, \overline{\Delta}^{(\overline{3},1,\frac{1}{3})}_{(6,1,1)}, \Delta^{(\overline{3},1,\frac{1}{3})}_{(6,1,1)}, \Delta^{(\overline{3},1,\frac{1}{3})}_{(\overline{10},1,3)}$$

for the first 7 rows,

$$\Delta^{(3,1,-\frac{1}{3})}_{1(6,1,1)}, \overline{\Delta}^{(3,1,-\frac{1}{3})}_{1(6,1,1)}, \overline{\Delta}^{(3,1,-\frac{1}{3})}_{1(10,1,3)}, \Delta^{(3,1,-\frac{1}{3})}_{2(6,1,1)}, \overline{\Delta}^{(3,1,-\frac{1}{3})}_{2(6,1,1)}, \overline{\Delta}^{(3,1,-\frac{1}{3})}_{2(10,1,3)}, \Phi^{(3,1,-\frac{1}{3})}_{(15,1,3)}$$

for the last 7 columns, and

$$\overline{\Delta}^{(\overline{3},1,\frac{1}{3})}_{1(6,1,1)}, \Delta^{(\overline{3},1,\frac{1}{3})}_{1(6,1,1)}, \Delta^{(\overline{3},1,\frac{1}{3})}_{1(\overline{10},1,3)}, \overline{\Delta}^{(\overline{3},1,\frac{1}{3})}_{2(6,1,1)}, \Delta^{(\overline{3},1,\frac{1}{3})}_{2(6,1,1)}, \Delta^{(\overline{3},1,\frac{1}{3})}_{2(\overline{10},1,3)}, \Phi^{(\overline{3},1,\frac{1}{3})}_{(15,1,3)}$$

for the last 7 rows. In the M_T, B_{11} is a 7×7 null matrix, the rest are

$$B_{12} = \begin{pmatrix}
\beta_{11}\Phi^{(T)}_{H\Delta} & \overline{\beta}_{11}\Phi^{(T)}_{H\overline{\Delta}} & -\overline{\beta}_{11}\frac{\sqrt{2}\Phi_3}{\sqrt{15}} & \beta_{21}\Phi^{(T)}_{H\Delta} & \overline{\beta}_{21}\Phi^{(T)}_{H\overline{\Delta}} & -\overline{\beta}_{21}\frac{\sqrt{2}\Phi_3}{\sqrt{15}} & \frac{\overline{\beta}_{i1}\overline{v}_{iR}}{\sqrt{5}} \\
\beta_{12}\Phi^{(T)}_{H\Delta} & \overline{\beta}_{12}\Phi^{(T)}_{H\overline{\Delta}} & -\overline{\beta}_{12}\frac{\sqrt{2}\Phi_3}{\sqrt{15}} & \beta_{22}\Phi^{(T)}_{H\Delta} & \overline{\beta}_{22}\Phi^{(T)}_{H\overline{\Delta}} & -\overline{\beta}_{22}\frac{\sqrt{2}\Phi_3}{\sqrt{15}} & \frac{\overline{\beta}_{i2}\overline{v}_{iR}}{\sqrt{5}} \\
\frac{\gamma_1\Phi_3}{12\sqrt{5}} & \frac{\overline{\gamma}_1\Phi_3}{12\sqrt{5}} & \frac{\overline{\gamma}_1\Phi_2}{6\sqrt{5}} & \frac{\gamma_2\Phi_3}{12\sqrt{5}} & \frac{\overline{\gamma}_2\Phi_3}{12\sqrt{5}} & \frac{\overline{\gamma}_2\Phi_2}{6\sqrt{5}} & \frac{\overline{\gamma}_i\overline{v}_{iR}}{2\sqrt{15}} \\
-\frac{\gamma_1\Phi_2}{6\sqrt{10}} & \frac{\overline{\gamma}_1\Phi_2}{6\sqrt{10}} & \frac{\overline{\gamma}_1\Phi_3}{6\sqrt{10}} & -\frac{\gamma_2\Phi_2}{6\sqrt{10}} & \frac{\overline{\gamma}_2\Phi_2}{6\sqrt{10}} & \frac{\overline{\gamma}_2\Phi_3}{6\sqrt{10}} & \frac{\overline{\gamma}_i\overline{v}_{iR}}{2\sqrt{15}} \\
\overline{m}_{\Delta_1} & 0 & 0 & \overline{m}_{\Delta_2} & 0 & 0 & 0 \\
0 & m_{\Delta_1} & \frac{\lambda_1\Phi_3}{15\sqrt{2}} & 0 & m_{\Delta_2} & \frac{\lambda_2\Phi_3}{15\sqrt{2}} & -\frac{\lambda_i\overline{v}_{iR}}{10\sqrt{3}} \\
0 & \frac{\lambda_1\Phi_3}{15\sqrt{2}} & m_{\Delta_1}+\lambda_1\Phi^{(T)}_{\Delta} & 0 & \frac{\lambda_2\Phi_3}{15\sqrt{2}} & m_{\Delta_2}+\lambda_2\Phi^{(T)}_{\Delta} & -\frac{\lambda_i\overline{v}_{iR}}{5\sqrt{6}}
\end{pmatrix}, \tag{4.7}$$

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$$B_{21}=\begin{pmatrix} \overline{\beta}_{11}\Phi^{(T)}_{H\Delta} & \overline{\beta}_{12}\Phi^{(T)}_{H\Delta} & \frac{\overline{\gamma}_1\Phi_3}{12\sqrt{5}} & -\frac{\overline{\gamma}_1\Phi_2}{6\sqrt{10}} & 0 & 0 & m_{\Delta_1} \\ \beta_{11}\Phi^{(T)}_{H\overline{\Delta}} & \beta_{12}\Phi^{(T)}_{H\overline{\Delta}} & \frac{\gamma_1\Phi_3}{12\sqrt{5}} & \frac{\gamma_1\Phi_2}{6\sqrt{10}} & \overline{m}_{\Delta_1} & \frac{\overline{\lambda}_1\Phi_3}{15\sqrt{2}} & 0 \\ -\beta_{11}\frac{\sqrt{2}\Phi_3}{\sqrt{15}} & -\beta_{12}\frac{\sqrt{2}\Phi_3}{\sqrt{15}} & \frac{\gamma_1\Phi_2}{6\sqrt{5}} & \frac{\gamma_1\Phi_3}{6\sqrt{10}} & \frac{\overline{\lambda}_1\Phi_3}{15\sqrt{2}} & \overline{m}_{\Delta_1}+\overline{\lambda}_1\Phi^{(T)}_{\Delta} & 0 \\ \overline{\beta}_{21}\Phi^{(T)}_{H\Delta} & \overline{\beta}_{22}\Phi^{(T)}_{H\Delta} & \frac{\overline{\gamma}_2\Phi_3}{12\sqrt{5}} & -\frac{\overline{\gamma}_2\Phi_2}{6\sqrt{10}} & 0 & 0 & m_{\Delta_2} \\ \beta_{21}\Phi^{(T)}_{H\overline{\Delta}} & \beta_{22}\Phi^{(T)}_{H\overline{\Delta}} & \frac{\gamma_2\Phi_3}{12\sqrt{5}} & \frac{\gamma_2\Phi_2}{6\sqrt{10}} & \overline{m}_{\Delta_2} & \frac{\overline{\lambda}_2\Phi_3}{15\sqrt{2}} & 0 \\ -\beta_{21}\frac{\sqrt{2}\Phi_3}{\sqrt{15}} & -\beta_{22}\frac{\sqrt{2}\Phi_3}{\sqrt{15}} & \frac{\gamma_2\Phi_2}{6\sqrt{5}} & \frac{\gamma_2\Phi_3}{6\sqrt{10}} & \frac{\overline{\lambda}_2\Phi_3}{15\sqrt{2}} & \overline{m}_{\Delta_2}+\overline{\lambda}_2\Phi^{(T)}_{\Delta} & 0 \\ \frac{\beta_{i1}v_{iR}}{\sqrt{5}} & \frac{\beta_{i2}v_{iR}}{\sqrt{5}} & \frac{\gamma_i v_{iR}}{2\sqrt{15}} & \frac{\gamma_i v_{iR}}{2\sqrt{15}} & -\frac{\overline{\lambda}_i v_{iR}}{10\sqrt{3}} & -\frac{\overline{\lambda}_i v_{iR}}{5\sqrt{6}} & 0 \end{pmatrix}, \tag{4.8}$$

and

$$B_{22}=\begin{pmatrix} \eta_{11}X & 0 & 0 & \eta_{12}X & 0 & 0 & 0 \\ 0 & \eta_{11}X & 0 & 0 & \eta_{21}X & 0 & -\frac{\overline{\lambda}_1\overline{v}_R}{10\sqrt{3}} \\ 0 & 0 & \eta_{11}X & 0 & 0 & \eta_{21}X & -\frac{\overline{\lambda}_1\overline{v}_R}{5\sqrt{6}} \\ \eta_{21}X & 0 & 0 & \eta_{22}X & 0 & 0 & 0 \\ 0 & \eta_{12}X & 0 & 0 & \eta_{22}X & 0 & -\frac{\overline{\lambda}_2\overline{v}_R}{10\sqrt{3}} \\ 0 & 0 & \eta_{12}X & 0 & 0 & \eta_{22}X & -\frac{\overline{\lambda}_2\overline{v}_R}{5\sqrt{6}} \\ 0 & -\frac{\lambda_1 v_R}{10\sqrt{3}} & -\frac{\lambda_1 v_R}{5\sqrt{6}} & 0 & -\frac{\lambda_2 v_R}{10\sqrt{3}} & -\frac{\lambda_2 v_R}{5\sqrt{6}} & m_\Phi+\lambda(\frac{\Phi_1}{\sqrt{6}}+\frac{\Phi_2}{3\sqrt{2}}+\frac{2\Phi_3}{3}) \end{pmatrix}, \tag{4.9}$$

where

$$\begin{aligned} \Phi^{(T)}_{H\Delta} &= -\frac{\Phi_1}{\sqrt{10}}+\frac{\Phi_2}{\sqrt{30}}, \\ \Phi^{(T)}_{H\overline{\Delta}} &= -\frac{\Phi_1}{\sqrt{10}}-\frac{\Phi_2}{\sqrt{30}}, \\ \Phi^{(T)}_{\Delta} &= \frac{\Phi_1}{10\sqrt{6}}+\frac{\Phi_2}{30\sqrt{2}}. \end{aligned} \tag{4.10}$$

We can observe that there is no zero eigenvalue of the triplet mass matrix. Also, all the other Higgs are massive except those Goldstone modes. This justifies the realization of the MPM in the extended model.

5 Fermion masses and proton decay

In the present model, the fermion sector is described by the superpotential

$$W = Y^{ij}_{10_1}\Psi_i\Psi_j H_1 + Y^{ij}_{10_2}\Psi_i\Psi_j H_2 + Y^{ij}_{120}\Psi_i\Psi_j D + Y^{ij}_{126}\Psi_i\Psi_j\overline{\Delta}. \tag{5.1}$$

In [40, 41] the fermion masses can be fitted by using only one 10_H and one $\overline{126}_H$. As in [40] for example, the resultant fermion masses are in accord with their experimental values except the electron mass. In present, although all the Higgs $H_{1,2}$, D and $\overline{\Delta}$ contribute to the fermion masses, we can also use the H_1 and $\overline{\Delta}$ as in [40], taking contributions from H_2 and D as small corrections to the electron mass. These corrections are suppressed by a factor of

$$\frac{Y^{ij}_{10_2}}{Y^{ij}_{126}} \sim \frac{Y^{ij}_{120}}{Y^{ij}_{126}} \sim \frac{m_e}{m_\tau} \sim 10^{-4}$$

and are negligible in studying proton decay.

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At the GUT scale the fermion masses are taken from [42] for $\tan\beta(M_{\rm SUSY}) = 10$, $\mu = 2\times 10^{16}$GeV. After numerical fitting, we get the Yukawa couplings $Y^{ij}_{10_1}$ and Y^{ij}_{126} in the u-diagonal basis

$$Y^{ij}_{10_1} = \begin{pmatrix} 0.000281 & -0.000784 - 0.000103i & 0.00760 + 0.00270i \\ -0.000784 - 0.000103i & 0.00174 & -0.0337 + 0.000650i \\ 0.00760 + 0.00270i & -0.0337 + 0.000650i & 0.999 \end{pmatrix}, \tag{5.2}$$

$$Y^{ij}_{126} = \begin{pmatrix} 0.0000651 & -0.0000535 + 7.06\times 10^{-6}i & -0.000519 - 0.000184i \\ -0.0000535 + 7.06\times 10^{-6}i & -0.00180 & 0.00231 - 4.45\times 10^{-5}i \\ -0.000519 - 0.000184i & 0.00231 - 4.45\times 10^{-5}i & -0.000958 \end{pmatrix}. \tag{5.3}$$

One can easily verify these couplings by calculating the mass eigenvalues of the following matrices

$$\begin{aligned} M_u &= \left(\alpha^u Y_{10_1} + \beta^u Y_{126}\right) v_u, \\ M_d &= \left(\alpha^d Y_{10_1} + \beta^d Y_{126}\right) v_d, \\ M_e &= \left(\alpha^d Y_{10_1} - 3\beta^d Y_{126}\right) v_d, \end{aligned} \tag{5.4}$$

where $v_u = 123.8$GeV and $v_d = 17.87$GeV (see table 5 of [42]). $\alpha^u = 0.6647$, $\beta^u = -0.7471$, $\alpha^d = 0.06816$ and $\beta^d = -0.9977$ come from numerical fitting. We also noticed that the normalizations $(\alpha^u)^2 + (\beta^u)^2 = 1$ and $(\alpha^d)^2 + (\beta^d)^2 = 1$ are not accurate due to the existence of H_2 and $D(120)$, but the deviations are small to be neglected reasonably.

In studying proton decay via dimension-5 operators, we limit our analysis to $LLLL$ operators only, although contributions from $RRRR$ operators are also sizable [43, 44]. The resultant operators are contained in the superpotential

$$W_5 = C^{ijkl}(Q_i Q_j)(Q_k L_l), \tag{5.5}$$

where the contractions of the indices are understood as

$$(Q_i Q_j)(Q_k L_l) = \epsilon_{\alpha\beta\gamma}\left(u_i^\alpha d'^\beta_j - d'^\alpha_i u_j^\beta\right)\left(u_k^\gamma e_l - d'^\gamma_k \nu_l\right). \tag{5.6}$$

In (5.5) we have defined

$$\begin{aligned} C^{ijkl} &= C^{ijkl}_{11} + C^{ijkl}_{12} + C^{ijkl}_{21} + C^{ijkl}_{22}, \\ C^{ijkl}_{11} &= Y^{ij}_{10_1}\left(M_T^{-1}\right)_{11} Y^{kl}_{10_1}, & C^{ijkl}_{12} &= Y^{ij}_{10_1}\left(M_T^{-1}\right)_{15} Y^{kl}_{126}, \\ C^{ijkl}_{21} &= Y^{ij}_{126}\left(M_T^{-1}\right)_{51} Y^{kl}_{10_1}, & C^{ijkl}_{22} &= Y^{ij}_{126}\left(M_T^{-1}\right)_{55} Y^{kl}_{126}, \end{aligned} \tag{5.7}$$

where M_T^{-1} is the inverse of the triplet mass matrix M_T in (4.6), and only $\left(M_T^{-1}\right)_{11}$, $\left(M_T^{-1}\right)_{15}$, $\left(M_T^{-1}\right)_{51}$ and $\left(M_T^{-1}\right)_{55}$ contribute to the $LLLL$-type proton decay because of the $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ basis in the triplet mass matrix. The relevant up-left block in M_T^{-1} equals to inverse of the effective triplet mass matrix which is got by integrating out the down-right block B_{22} in (4.6),

$$M_{\rm eff} = -B_{12}\cdot B_{22}^{-1}\cdot B_{21}. \tag{5.8}$$

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Running the dimension-5 operators down to the SUSY scale and dressing them by wino-loops, we get the four-fermion operators. For the dominant decay modes $p \to K^+\nu_l$, the coefficients are

$$C_{sudv_l} = 2\,\frac{\alpha_2}{2\pi} f_\Delta\; A_L\; A_S\; \left(\sum_{j,k=1}^{3} C^{1jkl} V_{j2} V_{k1} + \sum_{i,j=1}^{3} C^{ij1l} V_{j2} V_{i1}\right), \tag{5.9}$$

$$C_{dusv_l} = 2\,\frac{\alpha_2}{2\pi} f_\Delta\; A_L\; A_S\; \left(\sum_{j,k=1}^{3} C^{1jkl} V_{j1} V_{k2} + \sum_{i,j=1}^{3} C^{ij1l} V_{j1} V_{i2}\right), \tag{5.10}$$

where $\frac{\alpha_2}{2\pi} f_\Delta \sim \frac{\alpha_2}{2\pi}\frac{M_{\rm wino}}{M_{\rm SUSY}^2} \sim \frac{\alpha_2}{2\pi} \times 2.5 \times 10^{-5}{\rm GeV}^{-1}$ is the triangle diagram factor for $M_{\rm wino} = 400{\rm GeV}$ and $M_{\rm SUSY} = 4{\rm TeV}$. $A_L = 0.22$ is the long-distance renormalization factor [45], and V_{ij}s are the CKM matrix elements. The short-distance renormalization factor A_S is

$$A_S = \left(\frac{\alpha_1(m_Z)}{\alpha_{10}(M_{\rm GUT})}\right)^{-\frac{1}{33}} \left(\frac{\alpha_2(m_Z)}{\alpha_{10}(M_{\rm GUT})}\right)^{-3} \left(\frac{\alpha_3(m_Z)}{\alpha_{10}(M_{\rm GUT})}\right)^{\frac{4}{3}} \sim 7.16, \tag{5.11}$$

if we take $\alpha_{10}(M_{\rm GUT}) = \frac{1}{25}$. The decay rates for $p \to K^+\nu_l$ are

$$\Gamma\left(p \to K^+\nu_l\right) = \frac{{\beta_p}^2\left({m_p}^2 - {m_K}^2\right)^2}{32\pi {m_p}^3 {f_\pi}^2} \left|\frac{2m_p}{3m_B} D C_{sudv_l} + \left(1 + \frac{m_p}{3m_B}(D+3F)\right) C_{dusv_l}\right|^2, \tag{5.12}$$

where $m_p = 0.938{\rm GeV}$, $m_K = 0.494{\rm GeV}$, $f_\pi = 0.131{\rm GeV}$, $D = 0.81$, $F = 0.44$ [45], and $\beta_p = 0.012{\rm GeV}^3$ [46] are hadronic parameters.

To estimate proton decay rates, we need to analyze the effective triplet mass matrix $M_{\rm eff}$ in (5.8) in some details. In getting the right-handed neutrino masses, the VEV $\overline{v}_R$ is taken a small value $10^{14}{\rm GeV}$ compared to the GUT scale $2 \times 10^{16}{\rm GeV}$. In the limit of neglecting $\overline{v}_R$, there are three zero eigenvalues in the matrix B_{22}, since in this limit equations (3.8)–(3.9) require $\eta_{11}/\eta_{21} \sim \eta_{12}/\eta_{22}$ and thus B_{22} has rank-4 instead of rank-7. Consequently, the elements in $M_{\rm eff}$ are all divergent. However, these elements are correlated, thus elements in the inverse of $M_{\rm eff}$ are not small even in this limit. Proton decay proceeds fastly whenever the eigenvalues in $M_{\rm eff}$ are not all very big, being irrelevant to the appearance of all the large entries.

To suppress proton decay, we need to keep all the eigenvalues in B_{12} and B_{21} at GUT scale while suppressing those in B_{22}. The later can be achieved only by fine-tuning slightly the parameters $\eta_{ij}(i,j = 1,2)$ and $m_\Phi + \lambda(\frac{\Phi_1}{\sqrt{6}} + \frac{\Phi_2}{3\sqrt{2}} + \frac{2\Phi_3}{3})$. The former, to keep the eigenvalues in B_{12} and B_{21} all large, requires at least one pair of large VEVs in $\overline{v}_{iR}$ (see the last row in the matrix of (4.7)) and v_{iR} (see the last column in the matrix of (4.8)). These large $\overline{v}_{iR}$ and v_{iR}, now required by suppressing proton decay, correspond to the direct breaking of SO(10) into MSSM, and explicitly breaking of the unification is avoided as all the Higgs particles beyond MSSM are at GUT scale.

Numerically, there are too many parameters in the model to analyze. The typical dimensional and dimensionless parameters are taken as in table 2 and 3, respectively. They lead to the VEVs in table 4.

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parameter	m_Φ	m_{Δ_1}	m_{Δ_2}	$\overline{m}_{\Delta_1}$	$\overline{m}_{\Delta_2}$	X
value	1	3	2.96	5	2	30

Table 2. Dimensional parameters (in 10^{16}GeV).

parameter	x	λ	λ_1	λ_2	$\overline{\lambda}_1$	$\overline{\lambda}_2$	γ_1	γ_2	$\overline{\gamma}_1$	$\overline{\gamma}_2$	η_{11}
value	0.17	0.25	0.6	-0.62	1.8	1.32	1.5	1.28	1.05	0.89	0.02
parameter	η_{12}	η_{21}	η_{22}	β_{11}	β_{12}	β_{21}	β_{22}	$\overline{\beta}_{11}$	$\overline{\beta}_{12}$	$\overline{\beta}_{21}$	$\overline{\beta}_{22}$
value	0.015	0.025	0.0188	-1	1.2	1.5	1.35	-1.5	1.24	1.35	1.35

Table 3. Dimensionless parameters.

VEV	Φ_1	Φ_2	Φ_3	v_R	$\overline{v}_R$	v_{1R}	$\overline{v}_{1R}$	v_{2R}	$\overline{v}_{2R}$
value	2.1	12.9	4.08	1.82	0	5.66	18.23	15.0	11.94

Table 4. The VEVs (in 10^{16}GeV).

channel	lifetime
$\tau(p \to K^+\nu_e)$	$= 3.81 \times 10^{36}$yrs
$\tau(p \to K^+\nu_\mu)$	$= 1.08 \times 10^{34}$yrs $\times \left\vert \frac{0.012\text{GeV}^3}{\beta_p} \left(\frac{\alpha_{10}^{-1}}{25}\right)^{\frac{56}{33}} \frac{2.5\times10^{-5}\text{GeV}^{-1}}{f_\Delta} \right\vert^2$
$\tau(p \to K^+\nu_\tau)$	$= 1.60 \times 10^{34}$yrs

Table 5. Proton partial lifetime /years.

Note that the VEVs v_Rs and $\bar{v}_R$s are normalized to $\frac{1}{\sqrt{120}}$ [38], thus the appearance of their large values is artificial. Indeed, all the gauge superfields get masses at the GUT scale through the VEVs listed in table 4. Equations (3.5)–(3.16) are satisfied. $\overline{v}_R = 0$ is taken to study proton decay; in analyzing neutrino masses, however, it should take its practical values like 10^{14}GeV which are still negligible compared to the GUT scale.

We have also checked numerically the entire Higgs spectrum, confirming the absence of intermediate state which may otherwise spoil the gauge coupling unification explicitly. There is the problem, however, that the total $\beta-$function will be a large number after unification, so the coupling constant of SO(10) will blow up quickly.

The numerical results of proton partial lifetimes are listed in table 5. As we can see, they can be well above the current experimental limit, although at the cost of fine-tuning some parameters. According to the recent discussion of [47], by taking decoupling effects of SUSY particles into account, the triangle diagram factors can be even smaller, leading to longer proton lifetime.

6 Summary and conclusions

The extended SO10MPM is analyzed with the following results. First, type-I see-saw can be realized by introducing an intermediate VEV which couples to fermions. Second, SUSY is maintained at high energy. Third, unification is hopefully realized although a fully

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adjustment of the parameters are not carried out. Fourth, proton lifetime is consistent with the data if fine-tuning is used slightly. Works we have not done here include a fully numerical calculation of gauge coupling unification including GUT scale threshold effects, and a fully analysis with electron mass corrected by the 120plet Higgs effect. These are big challenges in future researches. We thank Jun-hui Zheng for useful discussions.

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