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Estimation of Directions of Arrival by Matching Pursuit (EDAMP)

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We propose a novel system architecture that employs a matching pursuit-based basis selection algorithm for directions of arrival estimation. The proposed system does not require a priori knowledge of the number of angles to be resolved and uses very small number of snapshots for convergence. The performance of the algorithm is not affected by correlation in the input signals. The algorithm is compared with well-known directions of arrival estimation methods with different branch-SNR levels, correlation levels, and different angles of arrival separations.

Keywords and phrases: directions of arrival estimation, adaptive antennas, matching pursuit algorithm, spatial resolution.

1. INTRODUCTION

In recent years, the impact of adaptive antennas and array processing to the system performance of wireless communication systems has gained intense attention. Adaptive (or smart) antennas consist of an antenna array combined with space and time processing. The processing of different antennas helps to improve system performance in terms of both capacity and quality, in particular by decreasing cochannel interference. A detailed overview of adaptive antennas can be found in [1, 2].

One of the most important problems for adaptive antenna systems in order to perform well is to have reliable reference inputs. These references include array element positions and characteristics, directions of arrivals, planar properties and dimensionality of the incoming signals. In this paper we investigate one of the most critical problems of adaptive antenna systems, namely directions of arrival (DOA) estimation.

For an adaptive system to be effective, it must have very accurate estimations of the DOA for the signal and the interferers. Once the directions are estimated accurately then processing in spatial, time, or other domains can be accomplished in order to improve the system performance.

There are many different approaches and algorithms for estimating DOA with various complexities and resolution properties such as ML [3], Bartlett [4], MVDR [1], MUSIC [5], and ESPRIT [6]. Variations to these models can also be found in the recent literature, some of which will be referred to in the following section.

For estimation of DOA, we consider a high-resolution basis selection algorithm, the flexible tree-search-based orthogonal matching pursuit (FTB-OMP) algorithm that is proposed in [7]. The FTB-OMP algorithm heuristically converges to the maximum likelihood solution. The algorithm selects a basis for signal decomposition by determining a small, possibly the smallest, subset of vectors chosen from a large redundant set of vectors to match the given data. This problem has various applications such as time/frequency representations [8], speech coding [9], and spectral estimation [10]. For the case of DOA, this set of vectors are modeled as possible outputs of the antenna array elements when the signal is arriving from a certain direction. The problem of selecting correct linear combination of these elements is equivalent to the problem of selecting correct DOA.

In DOA estimation, typically only a small number of directions contain the signal. Hence, the solution to the DOA estimation problem will be sparse. In this paper, we propose to use the FTB-OMP algorithm for DOA estimation, by exploiting the sparsity property of the DOA. The proposed technique is named as estimation of directions of arrival by matching pursuit (EDAMP). The main advantages of EDAMP are the flexibility and increased resolution at low signal-to-noise ratio (SNR) levels. It also does not require the a priori knowledge of the number of signals to be resolved, and it is not affected by the correlation of the signals arriving from different directions. The output of the algorithm is directly the angles of arrivals and their corresponding amplitudes; hence it does not require any postprocessing of output amplitudes at different angles as would be required in the case of conventional DOA estimators.

In the next section, the problem statement for the DOA estimation will be presented. In Section 3, the FTB-OMP algorithm employed in EDAMP structure will be summarized. In Section 4, the system model for estimating directions will be given. In Section 5, the simulation results will be presented for different scenarios. Finally in Section 6, the conclusions will be given.

2. PROBLEM STATEMENT

Consider an antenna array consisting of N elements. The output of these elements is a vector $\mathbf{x}$ of size $N \times 1$. Generally $\mathbf{x}$ corresponds to a linear combination of signals from different directions. If we consider ith and jth elements of $\mathbf{x}$, depending on DOA and the distance between them, x_i and x_j contain the same signals with different phase shifts. The problem is to identify each signal's DOA from $\mathbf{x}$ which is a weighted sum of the signals plus noise.

In the literature, different methods for achieving this goal are presented.

(i) The first one is the maximum likelihood (ML) approach [3]. Although it is the best one in terms of performance, it has formidable complexity. So other suboptimum algorithms which generally converge to ML performance at high SNR are proposed.

(ii) The second approach is finding the array response in the spectral domain for different angles, and recovering the local maximas as DOA [1, 4].

(iii) The third one is the eigenstructure method. In this method the space spanned by the eigenvectors is partitioned into signal subspace and noise subspace, hence they are referred to as subspace algorithms. After partitioning, signal subspace is investigated to recover DOA. The most popular subspace algorithms are ESPRIT [6] and MUSIC [5]. These algorithms are more complex than spectral domain algorithms since they require eigenvalue decomposition. However they have performances in between ML algorithm and spectral domain algorithms. On the other hand, they have poor performances in the low-SNR regions [1, 2].

Many different techniques, including independent component analysis [11], and many modified versions of these algorithms have been proposed in addition to the main ones mentioned above [1, 12, 13, 14].

In this paper we propose to use the EDAMP algorithm as a solution to the DOA estimation problem in order to achieve high resolution with low complexity. In EDAMP, we propose to use a high-resolution basis selection algorithm FTB-OMP. In the next section, the FTB-OMP algorithm will be described in detail.

3. BASIS SELECTION ALGORITHMS

The basis selection problem can be stated over $\mathbb{C}$ as follows. Let $\mathcal{D} = \{a_k\}_{k=1}^{n}$ be a set/dictionary of vectors which is highly redundant (i.e., $a_k \in \mathbb{C}^m$ and $m \ll n$ with $\mathbb{C}^m = \text{Span}(\mathcal{D})$).

The basis selection problem can be viewed as finding the most sparse solution to a linear system of equations. More precisely, if we form a matrix A from the columns of the dictionary $\mathcal{D}$, $A = [a_1, a_2, \ldots, a_n]$, the problem can be stated as finding an $\bar{\mathbf{x}}$, with at most r nonzero entries such that

$$\|\bar{\mathbf{x}} - \mathbf{x}\| \leq \epsilon \tag{1}$$

for $\epsilon \geq 0$, and $r > 1$.

Even though it would give the ML solution, finding the most sparse solution to (1) in an overcomplete dictionary using an exhaustive search is infeasible for large dimensions. In order to solve this problem, suboptimal methods based on sequential and parallel basis selection have been proposed. Due to high-complexity requirements of the parallel basis selection algorithms [15], sequential basis selection (SBS) methods are more frequently used for practical purposes [10, 16, 17].

In the following sections, we describe the orthogonal matching pursuit (OMP), and the tree-search-based OMP algorithms. There are several other decomposition algorithms such as best orthogonal basis [18] and method of frames [19], which are not considered here due to their low resolution and poor sparsity properties.

The algorithms are explained based on the notation in [20]. As mentioned before, basis selection in OMP algorithms is performed sequentially, that is, one at a time.

Let the residual vector after the pth iteration be denoted by b_p, with $b_0 = \mathbf{x}$. P_{S_p} denotes the orthogonal projection matrix onto the range space of S_p, and $P_{S_p}^{\perp} = I - P_{S_p}$ denotes its orthogonal complement with $P_{S_0} = 0$ and $P_{S_0}^{\perp} = I$. The projection matrix on the space spanned by a_k, with $\|a_k\| = 1$, is $P_{a_k} = a_k a_k^T$. The algorithm terminates after r iterations.

3.1. *Orthogonal matching pursuit algorithm*

The orthogonal matching pursuit (OMP) algorithm is proposed in [20, 21], independently. OMP is also called modified matching pursuit algorithm [20].

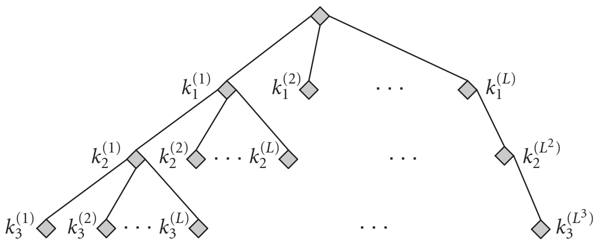

Figure 1: L-branch search tree.

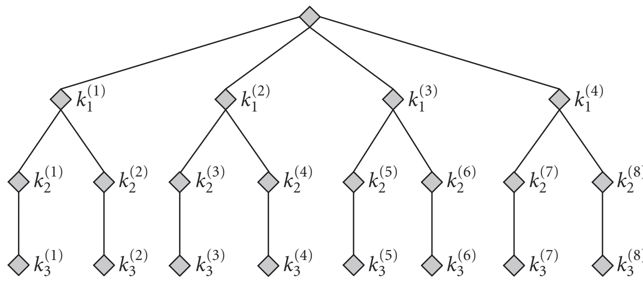

Figure 2: $L = 4$, $d = 2$ search tree for $r = 4$.

The OMP selects k_p in the pth iteration by finding the vector best aligned with the residual obtained by projecting b onto the orthogonal complement of the range space S_{p-1}, that is,

$$\begin{aligned} k_p &= \arg\max_l |a_l^T P_{S_{p-1}}^{\perp} b| \\ &= \arg\max_l |a_l^T b_{p-1}|, \quad l \notin I_{p-1}. \end{aligned} \tag{2}$$

With the initial values, $\hat{a}_{k_p}^0 = a_{k_p}$, $q_0 = 0$, we can write

$$P_{S_p} = P_{S_{p-1}} + q_p q_p^T, \tag{3}$$

where

$$\begin{aligned} \hat{a}_{k_p}^l &= \hat{a}_{k_p}^{l-1} - (q_{l-1}^T \hat{a}_{k_p}^{l-1}) q_{l-1}, \quad l = 1, 2, \ldots, p, \\ q_p &= \frac{\hat{a}_{k_p}^p}{||\hat{a}_{k_p}^p||}. \end{aligned} \tag{4}$$

The residual b_p is updated as follows:

$$b_p = P_{S_p}^{\perp} b_{p-1} = b_{p-1} - (q_p^T b_{p-1}) q_p. \tag{5}$$

The coefficients c_i change with each iteration and can be evaluated by taking the orthogonal projection of $\mathbf{x}$ onto S_p. The algorithm terminates when either $p = r$, or $\|b_p\| \le \epsilon$.

3.2. Tree-search-based orthogonal matching pursuit algorithm

Matching pursuit algorithms with tree-based search are proposed in [22]. We focus on TB-OMP algorithm.

In this algorithm, the best matching vector indices, $\{k_p^{(1)}, k_p^{(2)}, \ldots, k_p^{(L)}\}$ at the pth iteration are selected according to

$$\begin{aligned} k_p^{(i)} &= \arg\max_l |a_l^T P_{S_{p-1}}^{\perp} b|, \\ l &\ne \left\{k_p^{(1)}, k_p^{(2)}, \ldots, k_p^{(i-1)}\right\}, \quad i = 1, \ldots, L. \end{aligned} \tag{6}$$

At the end of r iterations, the search grows exponentially to a tree with L^r leaves as shown in Figure 1. The leaf corresponding to the smallest residual error vector yields the solution.

3.3. Flexible tree-search-based orthogonal matching pursuit

In [22], it is concluded that OMP algorithm offers a good compromise between performance and running time among the tree-search techniques, namely the matching pursuit and the order recursive matching pursuit algorithms.

In this section, we summarize the efficient tree-search-based OMP algorithms with branch pruning, the flexible tree-search-based OMP (FTB-OMP), that has been recently proposed in [7]. A maximum of L branches are searched at each partial solution. Thus, the resolution is adaptive, since it changes for different values of L in the algorithm. Note that TB-OMP (proposed in [22]) also has this adaptive nature, but has a prohibitive running time since it does not employ tree-pruning.

Our objective is to prune the tree branches that are heuristically believed to be unnecessary. Our heuristic is only to keep branches among $k_p^{(1)}, k_p^{(2)}, \ldots, k_p^{(L)}$ which are closely "aligned" with the OMP first choice branch $k_p^{(1)}$. We measure this alignment by the correlation between vectors which is defined as

$$\rho_{ij} = \frac{\langle a_i, a_j \rangle}{||a_i|| ||a_j||}. \tag{7}$$

In the algorithm, an input design parameter correlation threshold ξ is given. A branch is assumed to be unnecessary when the candidate vector is not aligned with $k_p^{(1)}$, that is, $|\rho_{k_p^{(1)}, k_p^{(i)}}| < \xi$.

In flexible tree-search-based OMP (FTB-OMP), the branching factor L is of variable size. In the first iteration $L = M$, where M is a parameter of the algorithm. At the ith iteration L is set to $\lceil M/d^i \rceil$, where $\lceil \cdot \rceil$ represents the ceiling function. The parameter $d > 0$, represents the speed of the decay in the branching factor of the search tree. The idea in this algorithm is to start the search with a large number of branches at the initial iteration, where an erroneous selection is more likely to appear, and to reduce the branching factor as the number of iterations increases. A search tree for $L = 4$, $d = 2$ is shown in Figure 2. For the special case $d = 1$, the algorithm keeps L as the branching factor.

Note that FTB-OMP is a generalization of both OMP and TB-OMP algorithms. By choosing $\xi = 1$, we require full alignment so that only $k_p^{(1)}$ is kept, reproducing OMP. By choosing $\xi = 0$, and $d = 1$, we place no restriction on

FTB-OMP $(d, p, r, L, \xi, \epsilon)$
Global $K = [k_1, k_2, \ldots]$, Best_res, Best_k
    Calculate b_{p-1} as in (5)
    If $\|b_{p-1}\| <$ Best_res
        Best_k $= [k_1, \ldots, k_{p-1}]$
        Best_res $\leftarrow \|b_{p-1}\|$
    end
    If $p > r$ or $\|b_{p-1}\| < \epsilon$, then *return*
    Calculate $\{k_p^{(1)}, k_p^{(2)}, \ldots, k_p^{(L)}\}$ as in (6)
    For each $i = 1$–L do
    If $|\rho_{k_p^{(1)}, k_p^{(i)}}| \geq \xi$
        $k_p = k_p^{(2)}$
        FTB-OMP $(d, p+1, r, \lceil L/d \rceil, \xi, \epsilon)$
    end
end

Algorithm 1: Pseudocode for FTB-OMP.

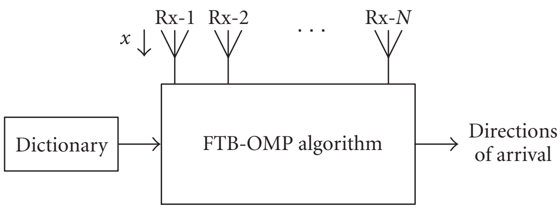

Figure 3: EDAMP estimation of DOA.

alignment, reproducing TB-OMP. A value $0 < \xi < 1$ represents a compromise between the number of nodes for OMP (r nodes), and for TB-OMP ($(L^{r+1} - 1)/(L - 1)$). Further reduction on the tree-size is achieved by using decay parameter d. This reduction makes the algorithm more competitive even without tree-pruning ($\xi = 0$). A pseudocode for FTB-OMP is given in Algorithm 1.

4. SYSTEM MODEL

In our system model for DOA estimation, we consider an adaptive antenna array of N elements as in Figure 3. The input signal is assumed to be a plane wave or equivalently it can be decomposed into plane waves.

Let $\mathbf{x}$ be the received vector formed by the received signal at each antenna element. For a uniform linear array the dictionary $\mathcal{D}$ can be obtained as

$$\mathcal{D} = \left\{ \begin{matrix} 1 & 1 & \cdots & 1 \\ e^{j\psi_1} & e^{j\psi_2} & \cdots & e^{j\psi_M} \\ \vdots & \vdots & \vdots & \vdots \\ e^{j(N-1)\psi_1} & e^{j(N-1)\psi_2} & \cdots & e^{j(N-1)\psi_M} \end{matrix} \right\}, \quad (8)$$

where ψ_i is the phase difference between elements of array, when the signal arrives from angle θ_i. The relation between ψ_i and θ_i is given as $\psi_i = (2\pi l/\lambda)\cos(\theta_i)$, where λ is the wavelength and l is the array spacing between the antenna elements. For the case in (8), the possible range of DOA is divided into M sections. These sections form the dictionary $\mathcal{D}$. Also for presentation purposes, we stick to the notation of [2] and define $u = \cos(\theta_i)$.

Depending on the DOA, the received signal vector of size $N \times 1$ will be a linear combination of the columns of $\mathcal{D}$ plus noise. Hence, detecting the DOA problem will reduce to finding correct linear combination of the columns of $\mathcal{D}$.

When the signal arrives from an individual angle only, the problem is straightforward and algorithm chooses the column of $\mathcal{D}$, which has the maximum inner product with the received vector $\mathbf{x}$. However when the signal arrives from more than one angle, $\mathbf{x}$ is a linear combination of columns of $\mathcal{D}$ and trying every possible linear combination would give the ML solution. On the other hand, this would bring formidable complexity to the system. By employing the FTB-OMP algorithm presented in the previous section, we propose a heuristic approximation to ML solution.

FTB-OMP algorithm selects the columns of $\mathcal{D}$ which are estimated to form $\mathbf{x}$, and these columns correspond to the DOA. FTB-OMP also returns to the coefficients of these columns, which represent the amplitude of the corresponding DOA.

There are three main advantages of the application of FTB-OMP.

(i) It does not require the number of directions to be estimated. By comparing the amplitude in $\mathbf{x}$ and amplitude of the resolved signals defined by the space spanned by the columns of $\mathcal{D}$, which have already been chosen by the algorithm, it is capable of deciding whether all the components are resolved or not. Considering that most of the spectral and subspace algorithms require the number of directions as an input, this is a very important advantage.

(ii) The algorithm allows flexibility between complexity and resolution property. By increasing the search depth, a closer solution to ML can be achieved, by decreasing the search depth algorithm running time can be decreased. But for both cases, it is computationally advantageous to the subspace-based algorithms, since it works on spectral domain and does not require eigenvalue decomposition.

(iii) In EDAMP, not the signal subspaces but the amplitudes of the received signals are used. As a result, system performance is robust to correlation between the inputs from different angles.

In the next section we support these advantages by simulation results.

5. SIMULATION RESULTS

In the simulations we consider a 10-element uniform linear array (ULA) that has element separation of $\lambda/2$ as shown in Figure 4. The SNR values correspond to the signal-to-noise ratios at the input of each antenna element and they are assumed to be the same. However the noise at each element is assumed to be independent identically distributed (i.i.d.) additive white Gaussian noise (AWGN). The system SNR is much higher than the SNR at each element. Hence, low-SNR results presented in the paper are of practical interest as well.

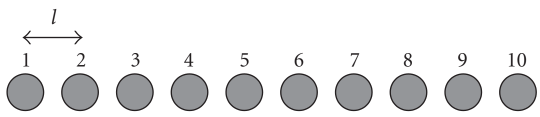

Figure 4: Array structure of ULA.

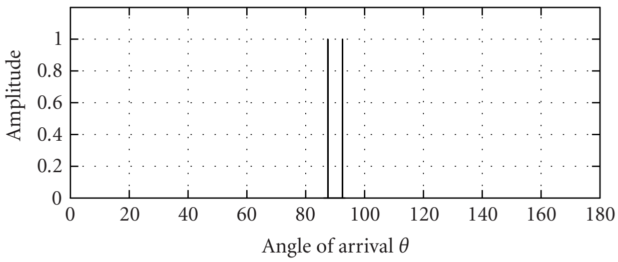

Figure 5: Arrival angles $\theta_1 = 87.52^\circ$, $\theta_2 = 92.48^\circ$.

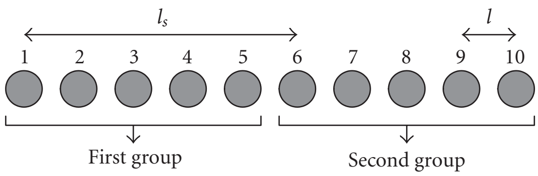

Figure 6: Subarrays for ESPRIT: first five elements of the original array form the first subarray, and last five elements of the original array form the second subarray.

Unless stated otherwise, two different signal directions with $u_1 = 0.0433$ and $u_2 = -0.0433$ (the minimum distance that can be resolved for a 10-element ULA [2]) are considered. The amplitudes in both directions are assumed to be the same. These u values correspond to 87.52° and 92.48°. As shown in Figure 5, the range of estimation is between 0° and 180°.

In the subspace-based algorithms, for the convergence of the eigenvalues, 100 independent snapshots are used. The results are averaged over 1000 Monte Carlo simulations.

Other than the proposed EDAMP algorithm as described in the previous section, Bartlett [4], MVDR [2], MUSIC [5], and ESPRIT [6] algorithms have also been considered. These algorithms have been simulated with the parameters defined above, and all of the results presented in this work about these algorithms have been calibrated with the results on their performances presented in the literature prior to this work [1, 2].

Bartlett algorithm is generated as a traditional beamformer with 10 elements, steered along different angles and acquiring the maximum amplitude points. Application of MVDR is simply using MVDR beamformer coefficients instead of uniform coefficients of Bartlett. For MUSIC, the parameters described in [2, 5] are employed for 10 antenna elements.

For the ESPRIT algorithm, the antenna array is divided into two subarrays, one being the shifted version of the other in space. The constant phase shift between two subarrays is employed for the resolution. For simulations, 5-element-shifted ESPRIT is considered as shown in Figure 6.

Table 1: Parameters of FTB-OMP algorithm used in EDAMP simulations.

Parameter	Value
Tree-pruning (ξ)	0.25
Number of branches (L)	100
Decaying parameter (d)	10
Maximum iteration (r)	3

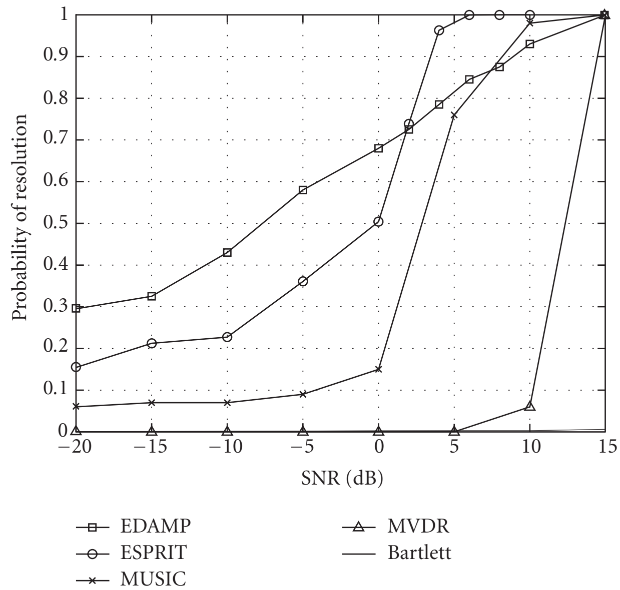

Figure 7: Probability of resolution versus SNR for uncorrelated inputs.

In Table 1, the parameters used for FTB-OMP algorithm employed in the simulations are given. With these parameters, EDAMP requires much less computational time when compared to ESPRIT and MUSIC. In terms of floating point operations in MATLAB simulation platform, EDAMP requires approximately half the number of flops required by ESPRIT, and one fourth the number of flops required by MUSIC.

5.1. *Uncorrelated inputs*

We first look at the case when the signals arriving from different angles are uncorrelated. In Figure 7, the novel EDAMP algorithm is compared with all four algorithms mentioned above. As can be seen in Figure 7, EDAMP performs well especially in the low-SNR region and the probability of resolution increases linearly with SNR. For uncorrelated channels at low SNR, EDAMP outperforms every other algorithm, and at high SNR, ESPRIT performs the best.

In Figure 8, root mean square error (RMSE) in the estimated angles is shown. RMSE is normalized by the null-to-null beamwidth (BW_{NN}) of the 10-element antenna array. As it is seen in Figure 8, at low SNR EDAMP outperforms ESPRIT and at high SNR, ESPRIT is better in terms of RMSE performance.

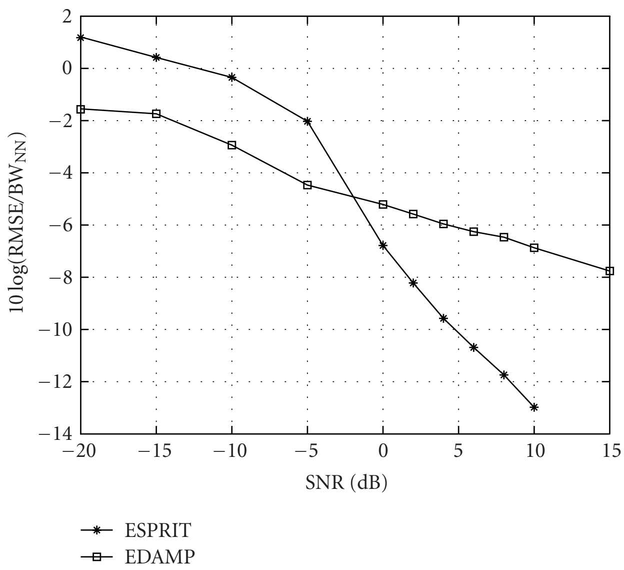

Figure 8: RMSE of DOA normalized by null-to-null beamwidth for uncorrelated inputs.

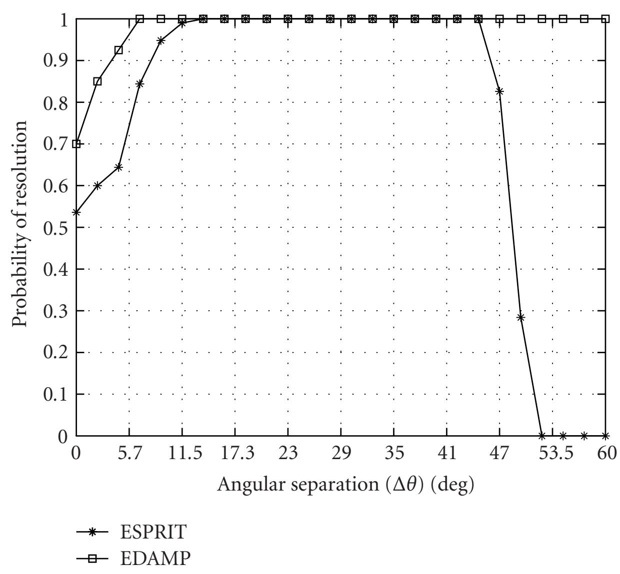

Figure 9: Probability of resolution versus angular separation for uncorrelated inputs for SNR = 3 dB.

Next, the effect of angular separation on the probability of resolution is investigated. In Figure 9, it is depicted that for SNR = 3 dB, EDAMP can resolve more closely separated signals when compared to ESPRIT. Also in Figure 9, we can see another limitation of ESPRIT. In ESPRIT algorithm, the antenna array is divided into two symmetric subarrays. The resolution property is highly dependent on the distance between the first element of the first array and first element of the second array, which is denoted by l_s [2]. The ESPRIT scheme that we employ in our simulations is the one with highest resolution available for a 10-element antenna array [2]. However, in ESPRIT algorithm, the resolvable angles are limited by the relation

$$-\frac{1}{l_s} < u < \frac{1}{l_s}. \tag{9}$$

For the scheme employed which is shown in Figure 6, $l_s = 5$. Since

$$-\frac{1}{5} < u < \frac{1}{5}, \tag{10}$$

the largest value of Δu, for resolution is $1/5 + 1/5 = 0.4$. It is clearly seen that for $u > 0.4$, the performance of ESPRIT degrades very fast. On the other hand, EDAMP has no such limitation. One could select an ESPRIT scheme with smaller l_s hence increasing the resolvable range, but this would result in lower probability of resolution and worse RMSE in the resolvable range [2, 6].

5.2. Correlated inputs

Above we considered the case when two signals arriving from different angles were uncorrelated. Here, we investigate the effect of correlation on the system performance. The performance of subspace algorithms, namely MUSIC and ESPRIT are highly dependent on the correlation between input signals arriving from different angles [1, 2, 5, 6]. This is a natural outcome of subspace algorithms making use of eigenspace decomposition in order to separate noise, signal, and interference.

On the other hand, the performance of EDAMP is independent of correlation in the signals, since its resolving power depends solely on the amplitudes in different directions. This is supported by the results of Figures 10 and 11. Even for 90% correlation, the performance of EDAMP is the same as its performance with uncorrelated channels. However, as shown in Figures 10 and 11, the performances of MUSIC and ESPRIT are severely degraded with increased correlation.

It is seen that for highly correlated signals EDAMP resolution performance is much better than subspace algorithms such as MUSIC and ESPRIT.

5.3. Effect of number of snapshots

In wireless communications, especially for real-time applications, delays in the system are very critical. In DOA estimation, a number of snapshots is required for the estimation to be accurate [1]. When the number of snapshots increases, the delay in the system increases. It is well known that with insufficient number of snapshots, traditional DOA algorithms perform poorly. In EDAMP, snapshots are only utilized for running the algorithm again and averaging the estimations. For known signals, the snapshots can be utilized to decrease the SNR by averaging the signals from different snapshots. The number of snapshots, therefore, is not very critical as in the case of subspace algorithms. Here we investigate the effect of number of snapshots by decreasing it from 100 to 10, and the effect of number of snapshots when the SNR is 15 dB.

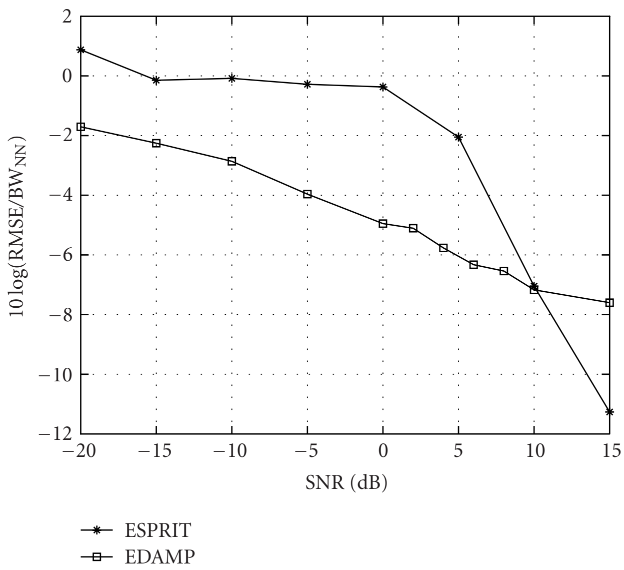

Figure 10: RMSE of DOA normalized by null-to-null beamwidth for 90%-correlated inputs.

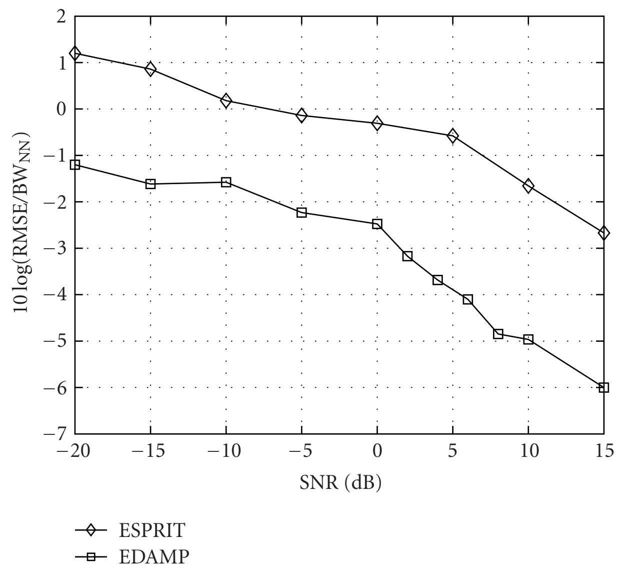

Figure 12: RMSE of DOA normalized by null-to-null beamwidth for 90%-correlated inputs with 10 snapshots.

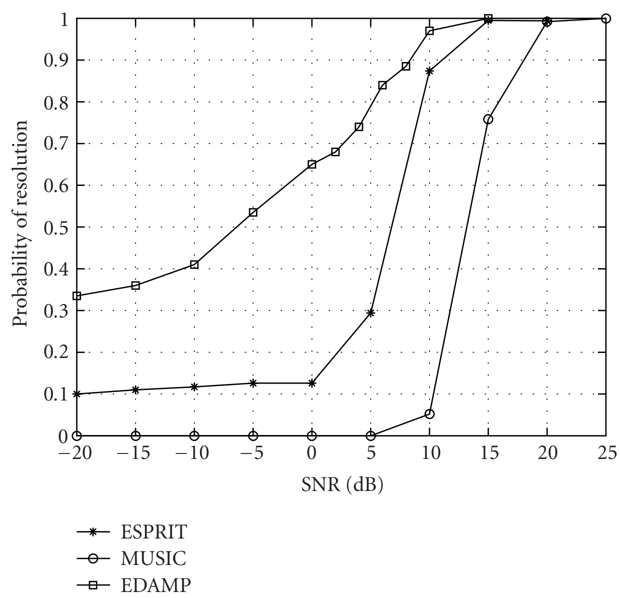

Figure 11: Probability of resolution versus SNR for 90%-correlated inputs.

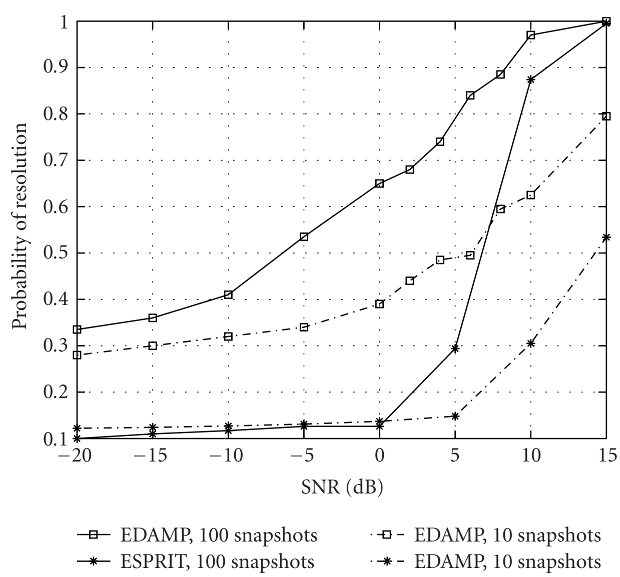

Figure 13: Comparison of probabilities of resolution of 90%-correlated inputs for 10 and 100 snapshots.

In Figures 12, 13, and 14 it is clearly depicted that EDAMP performs much better for low number of snapshots. Even at 10 snapshots, EDAMP shows acceptable performance, which makes EDAMP even more valuable for applications requiring short delays.

6. CONCLUSIONS

In this paper, we have presented a novel DOA estimator, EDAMP, which employs a based basis selection algorithm, namely FTB-OMP. Many advantages of EDAMP when compared to the traditional algorithms are presented, which can be summarized as follows.

The EDAMP algorithm gives directions of arrival and their corresponding amplitudes as output, so it does not require postprocessing to detect amplitudes after detecting directions. On the other hand, the algorithm does not need preprocessing since it does not require the number of DOA as input.

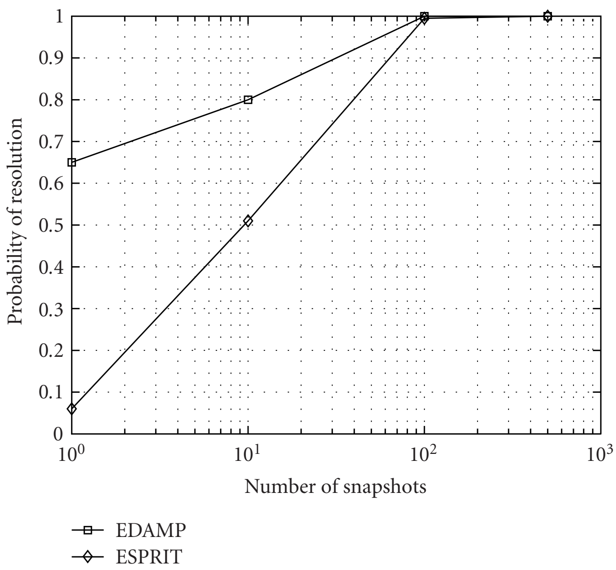

Figure 14: Probability of resolution versus number of snapshots for 90%-correlated inputs with SNR = 15 dB.

EDAMP is not affected by the correlations in the signals from different DOA, hence it is expected to perform better in multipath situations when compared to traditional techniques.

Since it is a heuristic approach to ML solution, it gives good resolution properties even at low-SNR situations. It also requires very few snapshots, when compared to subspace algorithms, thus decreasing processing time.

Many different variations of basis selection algorithms can be utilized for DOA estimation or similar estimation problems employing overcomplete sets and sparse solutions. Hence the idea presented in this paper promises many possible future research areas in several areas of signal processing, other than DOA estimation.

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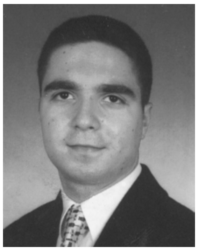

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