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Research Article

An Efficient Method for Proportional Differentiated Admission Control Implementation

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The admission control mechanism inspired in the framework of proportional differentiated services has been investigated. The mechanism provides a predictable and controllable network service for real-time traffic in terms of blocking probability. Implementation of proportional differentiated admission control is a complicated computational problem. Previously, asymptotic assumptions have been used to simplify the problem, but it is unpractical for real-world applications. We improve previous solutions of the problem and offer an efficient nonasymptotic method for implementation of proportional differentiated admission control.

1. Introduction

Efficient implementation of admission control mechanisms is a key point for next-generation wireless network development. Actually, over the last few years an interrelation between pricing and admission control in QoS-enabled networks has been intensively investigated. Call admission control can be utilized to derive optimal pricing for multiple service classes in wireless cellular networks [1]. Admission control policy inspired in the framework of proportional differentiated services [2] has been investigated in [3]. The proportional differentiated admission control (PDAC) provides a predictable and controllable network service for real-time traffic in terms of blocking probability. To define the mentioned service, proportional differentiated service equality has been considered and the PDAC problem has been formulated. The PDAC solution is defined by the inverse Erlang loss function. It requires complicated calculations. To reduce the complexity of the problem, an asymptotic approximation of the Erlang B formula [4] has been applied. However, even in this case, the simplified PDAC problem remains unsolved.

In this paper, we improve the previous results in [3] and withdraw the asymptotic assumptions of the used approximation. It means that for the desired accuracy of the approximate formula an offered load has to exceed a certain threshold. The concrete value of the threshold has been derived. Moreover, an explicit solution for the considered problem has been provided. Thus, we propose a method for practical implementation of the PDAC mechanism.

The rest of the paper is organized as follows. In the next section, we give the problem statement. In Section 3, we first present a nonasymptotic approximation of the Erlang B formula. We then use it for a proportional differentiated admission control implementation and consider some alternative problem statements for an admission control policy. In Section 4, we present the results of numerous experiments with the proposed method. Section 5 is a brief conclusion.

2. Problem Statement

Let us consider the concept of admission control inspired in the framework of proportional differentiated services. In the above paper [3], whose notation we follow, PDAC problem is defined as

$$\delta_1 B_1(\rho_1, n_1) = \delta_2 B_2(\rho_2, n_2) = \cdots = \delta_K B_K(\rho_K, n_K). \quad (1)$$

Here,

(i) K: is a number of traffic classes. $K \geq 2$;

(ii) δ_i: is the weight of class $i, i = 1, \ldots, K$. This parameter reflects the traffic priority. By increasing the weight, we also increase the admittance priority of corresponding traffic class;

(iii) ρ_i: is the offered load of class i traffic;

(iv) $n_i = \lfloor C_i/b_i \rfloor$, C_i is an allotted partition of the link capacity, b_i is a bandwidth requirement of class i connections, and $\lfloor x \rfloor$ is the largest integer not greater than x;

(v) $B(\rho_i, n_i)$: is the Erlang loss function, that is, under the assumptions of exponential arrivals and general session holding times [5], it is the blocking probability for traffic of class $i, i = 1, \ldots, K$.

It needs to find $C_1, C_2, \ldots, C_K$ taking into account known $\delta_i, \rho_i, b_i, i = 1, \ldots, K$ and the restriction imposed by given link capacity, C:

$$\sum_{i=1}^{K} C_i = C. \quad (2)$$

Let us remark that variations of C_i imply a discrete changing of the function $B(\rho_i, n_i)$. Hence, it is practicably impossible to provide the strict equality in (1). It is reasonable to replace (1) by an approximate equality as follows:

$$\delta_1 B_1(\rho_1, n_1) \approx \delta_2 B_2(\rho_2, n_2) \approx \cdots \approx \delta_K B_K(\rho_K, n_K). \quad (3)$$

But, even in this case, the above problem is difficult and complex combinatorial problem. For its simplification, the following asymptotic approximation has been used [3]. If the capacity of link and the offered loads are increased together:

$$n \longrightarrow \infty, \qquad \rho \longrightarrow \infty, \quad (4)$$

and $\rho > n$, then the Erlang loss function

$$B(\rho, n) = \frac{\rho^n/n!}{\sum_{i=0}^{n} \rho^i/i!}, \quad (5)$$

can be approximated by

$$1 - \frac{n}{\rho}. \quad (6)$$

Taking into account the PDAC problem, the authors of [3] consider the limiting regime when

$$n_i \longrightarrow \infty, \qquad C_i \longrightarrow \infty, \quad (7)$$

and $\rho_i > C_i/b_i$, $i = 1, \ldots, K$. Under these conditions, the asymptotic approximation of the Erlang B formula has been used and (1) has been replaced by simplified equations as follows:

$$\delta_1\left(1 - \frac{C_1}{b_1\rho_1}\right) = \delta_2\left(1 - \frac{C_2}{b_2\rho_2}\right) = \cdots = \delta_K\left(1 - \frac{C_K}{b_K\rho_K}\right). \quad (8)$$

In practice, the limited regime (7) is not appropriate. But the simplification (8) can be used without the conditions (7). Actually, the approximation (6) can be applied without the condition (4). We prove it below.

3. Offered Technique

3.1. Approximate Erlang B Formula. We assert that for the desired accuracy of the approximation (6) an offered load has to exceed a certain threshold. The concrete value of the threshold is given by the following theorem.

Theorem 1. *For any small $\epsilon > 0$, if*

$$\rho \geq n + \frac{1}{\epsilon}, \quad (9)$$

then

$$1 - \frac{n}{\rho} < B(\rho, n) < 1 - \frac{n}{\rho} + \epsilon. \quad (10)$$

Proof. Here and below, we use the following designation:

$$\beta(\rho, n) = 1 - \frac{n}{\rho}. \quad (11)$$

Assume that $\rho > n$. First, we rewrite the Erlang B formula

$$B(\rho, n) = \left(\sum_{i=0}^{n} \frac{n!}{i!\rho^{n-i}}\right)^{-1}. \quad (12)$$

Remark that

$$\sum_{i=0}^{n} \frac{n(n-1)\cdots(i+1)}{\rho^{n-i}} \leq \sum_{i=0}^{n} \left(\frac{n}{\rho}\right)^{n-i}. \quad (13)$$

Taking into account properties of geometrical progression, we have

$$\frac{1}{B(\rho, n)} \leq \sum_{i=0}^{n} \left(\frac{n}{\rho}\right)^{n-i} < \frac{1}{\beta(\rho, n)}. \quad (14)$$

Hence

$$B(\rho, n) > 1 - \frac{n}{\rho}. \quad (15)$$

To prove the second inequality of the theorem, we use the following upper bound of the Erlang loss function [6]:

$$\text{UB} = \frac{n(1 - (\rho/n))^2 + 2(\rho/n) - 1}{2(\rho/n) - \rho(1 - (\rho/n))}. \quad (16)$$

Transform this as follows:

$$\text{UB} = \frac{\rho(\rho - n + 2) - n(\rho - n + 2) + n}{\rho(\rho - n + 2)}. \quad (17)$$

It implies

$$\text{UB} = 1 - \frac{n}{\rho} + \frac{n}{\rho(\rho - n + 2)}. \quad (18)$$

We have $n/\rho < 1$. Hence,

$$B(\rho, n) < UB < 1 - \frac{n}{\rho} + \frac{1}{\rho - n}. \quad (19)$$

Thus, for any ϵ such that

$$\epsilon > \frac{1}{\rho - n}, \tag{20}$$

it follows that

$$\text{UB} < 1 - \frac{n}{\rho} + \epsilon. \tag{21}$$

From the inequality (20), we obtain the condition (9). The proof is completed. □

Note that the approximate formula (6) can provide the required accuracy ϵ in the case of $\rho < n + 1/\epsilon$. Actually, if $\epsilon = 0.01$, $n = 200$, then the required accuracy is reached for $\rho = 270 < 300$. Thus, the condition (9) is sufficient but not necessary. It guarantees the desired accuracy of the approximation for any small ϵ and n.

3.2. PDAC Solution. Assume that the solution $(C_1, C_2, \dots, C_K)$ of the PDAC problem satisfies inequalities $\rho_i > C_i/b_i$, $i = 1, \dots, K$. Let us derive an analytical solution for the PDAC problem under the condition (8). Without reducing generality, assume that $\delta_1 \geq \delta_2 \geq \cdots \geq \delta_K$ and $\max_i \delta_i = \delta_1 = 1$, $i = 1, \dots, K$. Indeed, if $\delta_1 \neq 1$, then we define new weights $\hat{\delta}_i = \delta_i/\delta_1, i = 1, \dots, K$. Thus, the condition (8) can be reformulated as follows:

$$1 - \frac{C_1}{b_1\rho_1} = \delta_i \left(1 - \frac{C_i}{b_i\rho_i}\right), \quad i = 2, \dots, K. \tag{22}$$

According to the transitivity property, any solution of the PDAC problem under condition (8) is also a solution of the PDAC problem under condition (22). Therefore,

$$C_i = b_i\rho_i\left(1 + \frac{1}{\delta_i}\left(\frac{C_1}{b_1\rho_1} - 1\right)\right), \quad i = 2, \dots, K. \tag{23}$$

Using the equality (2), we get

$$C_1 = \frac{C + S_2}{1 + S_1}, \tag{24}$$

where

$$S_1 = \frac{1}{b_1\rho_1}\sum_{j=2}^{K}\frac{b_j\rho_j}{\delta_j}, \qquad S_2 = \sum_{j=2}^{K} b_j\rho_j\left(\frac{1}{\delta_j} - 1\right). \tag{25}$$

Thus, the formulas (23)–(25) provide the implementation of proportional differentiated admission control.

It is clear that for some values C, b_i, ρ_i, δ_i, we can obtain $C_1 > C$ in (24) or $C_i < 0$ in (23). Therefore, the problem is unsolvable and PDAC implementation is impossible for the given parameters.

More precisely, if $C_1 > C$, then we have from (24)

$$\begin{gathered}\frac{C + S_2}{1 + S_1} > C, \\ C < \frac{S_2}{S_1}.\end{gathered} \tag{26}$$

Using the following equality:

$$S_2 = b_1\rho_1 S_1 + \sum_{j=2}^{K} b_j\rho_j, \tag{27}$$

we derive

$$C < CL_1 = b_1\rho_1\left(1 - \frac{\sum_{j=2}^{K} b_j\rho_j}{\sum_{j=2}^{K} b_j\rho_j/\delta_j}\right). \tag{28}$$

From the inequality $C_i < 0$, we can write

$$\delta_i > 1 - \frac{C_1}{b_1\rho_1}, \quad \forall i = 2, \dots, K. \tag{29}$$

Therefore,

$$\delta_K = \min_i \delta_i > 1 - \frac{C_1}{b_1\rho_1}. \tag{30}$$

By substituting the expressions (24) for the C_1 into (30), we get after some manipulations the following inequality:

$$C > CL_2 = \sum_{j=1}^{K-1} b_j\rho_j\left(1 - \frac{\delta_K}{\delta_j}\right). \tag{31}$$

Note that the problem (22) has been formulated under the condition

$$C < \sum_{j=1}^{K} b_j\rho_j. \tag{32}$$

Actually, it implies

$$\frac{C_j}{b_j\rho_j} < 1, \quad \forall j = 1, \dots, K. \tag{33}$$

Thus, the region of acceptability for PDAC problem (22) is defined by

$$\max(CL_1, CL_2) < C < \sum_{j=1}^{K} b_j\rho_j. \tag{34}$$

It follows from the theorem that the approximation (6) is applicable even for $n = 1$ and any small $\epsilon > 0$ if $\rho > 1/\epsilon - 1$. In spite of this fact, the solution above cannot be useful for small values of the ratio C_i/b_i. In this case, the loss function $B(\rho_i, n_i)$ is sensitive to fractional part dropping under calculation $n_i = \lfloor C_i/b_i \rfloor$. For example, if $b_i = 128$ kb/s, $\rho_i = 2$, and we obtain $C_i = 255$ kb/s, then the approximate value of the blocking probability is about 0.004. But $n_i = \lfloor C_i/b_i \rfloor = 1$ and $B(1, 2) \approx 0.67$. Thus, the offered approximate formula is useful if the ratio C_i/b_i is relatively large.

3.3. Alternative Problem Statements. Let n_i be the number of channel assigned for class i traffic, $i = 1, \dots, K$. Each class i is characterized by a worst-case loss guarantee α_i [7, 8].

Consider the following optimization problem:

$$\min \sum_{i=1}^{K} n_i, \qquad (35)$$
$$B(\rho_i, n_i) \le \alpha_i, \quad \alpha_i \in (0,1], \ i = 1, \dots, K.$$

Assume that for all $i \in \{1,\dots,K\}$ $\exists n_i \in N : B(\rho_i, n_i) = \alpha_i$. It is well known that the Erlang loss function $B(\rho, n)$ is a decreasing function of n [9], that is, $B(\rho, n_1) < B(\rho, n_2)$ if $n_1 > n_2$. Therefore, the optimal solution $(n_1^*, n_2^*, \dots, n_K^*)$ of the problem (35) satisfies the mentioned condition

$$B(\rho_i, n_i^*) = \alpha_i. \qquad (36)$$

If we designate $\delta_i = 1/\alpha_i$, then we get

$$\delta_i B(\rho_i, n_i^*) = \delta_j B\left(\rho_j, n_j^*\right) = 1, \quad \forall i, j \in \{1,\dots,K\}. \qquad (37)$$

Thus, the optimization problem (35) is reduced to the problem (1).

Assume the approximation (6) is admissible. Therefore, the method from previous subsection is supposed to be used, but the optimal solution of the problem (35) can be computed by inverting the formula (36). Taking into account the approximation, we get

$$n_i^* = \rho_i(1 - \alpha_i). \qquad (38)$$

Note that in practice the solution n_i^* is not usually integer; thus, it has to be as follows:

$$\arg\min\{n_i \in N \mid n_i \ge \lfloor \rho_i(1-\alpha_i) \rfloor\}, \quad i = 1, \dots, K. \qquad (39)$$

We now consider the optimization of routing in a network through the maximization of the revenue generated by the network. The optimal routing problem is formulated as

$$\max \sum_{i=1}^{K} r_i \rho_i, \qquad (40)$$

$$B(\rho_i, n_i) \le \alpha_i, \quad \alpha_i \in (0,1], \ i = 1, \dots, K, \qquad (41)$$

where n_i is a fixed number of channels for class i traffic and r_i is a revenue rate of class i traffic. Obviously, the Erlang loss function $B(\rho, n)$ is an increasing function of ρ. Therefore, the optimal solution $(\rho_1^*, \rho_2^*, \dots, \rho_K^*)$ of the problem (40), (41) satisfies the following condition:

$$B(\rho_i^*, n_i) = \alpha_i. \qquad (42)$$

Hence, the problem (40), (41) can be reduced to the problem (1) as well. Under the approximation, the optimal solution takes the form

$$\rho_i^* = \frac{n_i}{1 - \alpha_i}, \quad i = 1, \dots, K, \qquad (43)$$

and the maximal total revenue is

$$\sum_{i=1}^{K} \frac{r_i n_i}{1 - \alpha_i}. \qquad (44)$$

Table 1

Class	C_i, kb/s	n_i	$B(\rho_i, n_i)$	$\delta_i B(\rho_i, n_i)$
1	130887	1022	0.0803	0.0803
2	129786	1013	0.0877	0.079
3	128409	1003	0.0961	0.0769
4	126639	989	0.108	0.0756
5	124279	970	0.1243	0.0746

4. Performance Evaluation

Let us illustrate the approximation quality. The difference $\Delta(\rho, n) = B(\rho, n) - \beta(\rho, n)$ is plotted as a function of offered load in Figure 1. If the number of channel n is relatively small then high accuracy of approximation is reached for heavy offered load. Let us remark that heavy offered load corresponds to high blocking probability. Generally, this situation is abnormal for general communication systems, but the blocking probability $B(n, \rho)$ decreases if the number of channels n increased relative accuracy ϵ. Let us designate $\rho^* = n + 1/\epsilon$. If the approximation (2) is admissible for ρ^* then it is also admissible for any $\rho > \rho^*$. In Figure 2, the behavior of losses function $B(n, \rho^*)$ according to different ϵ is shown. Thus, the provided approximation is attractive for a performance measure of queuing systems with a large number of devices.

Next, we consider a numerical example to evaluate the quality of a PDAC implementation based on the proposed method. Assume that $C = 640$ Mb/s, $K = 5$, $b_i = 128$ kb/s, $\rho_i = 1100$, $\delta_i = 1 - 0.1(i-1)$, $i = 1, \dots, 5$. In average, there are 1000 channels per traffic class. Following the theorem above, we conclude that the blocking probability can be replaced by the approximation (6) with accuracy about 0.01. Using (23)–(25), find a solution of the simplified PDAC problem and calculate the blocking probability for the obtained values. The results are shown in the Table 1.

Note that $\sum_{i=1}^{5} C_i = 640$ Mb/s and three channels per 128 kb/s have not been used. We get

$$\delta_i \left(1 - \frac{C_i}{b_i \rho_i}\right) = 0.0704, \quad i = 1, \dots, 5. \qquad (45)$$

It is easy to see that

$$\max_{i=1,\dots,5} \left\{ B(\rho_i, n_i) - \left(1 - \frac{C_i}{b_i \rho_i}\right) \right\} < 0.01,$$
$$\max_{i,j} \left| \delta_i B(\rho_i, n_i) - \delta_j B\left(\rho_j, n_j\right) \right| < 0.01. \qquad (46)$$

If $K = 10$, $\delta_i = 1 - 0.05(i-1)$, $i = 1, \dots, 10$, and other parameters are the same then

$$\max_{i,j} \left| \delta_i B(\rho_i, n_i) - \delta_j B\left(\rho_j, n_j\right) \right| < 0.001. \qquad (47)$$

If an obtained accuracy is not enough, then the formulas (23)–(25) provide efficient first approximation for numerical methods.

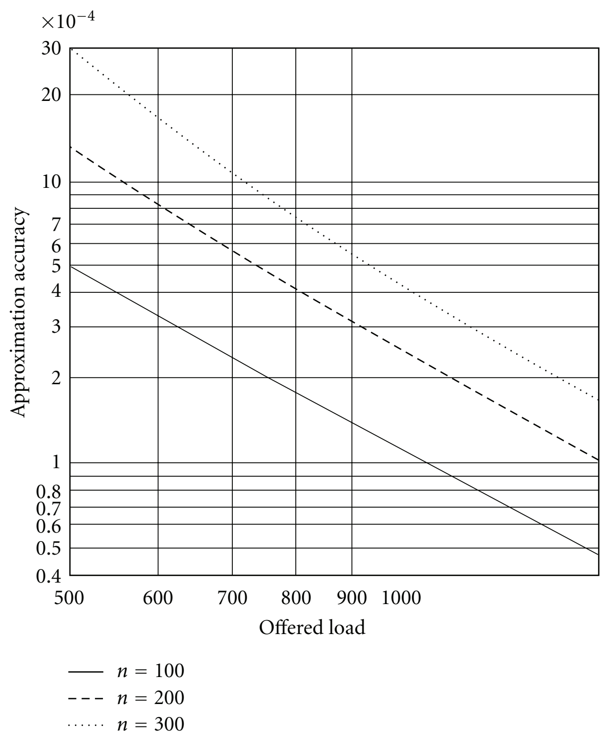

FIGURE 1: Approximation quality as a function of the offered load.

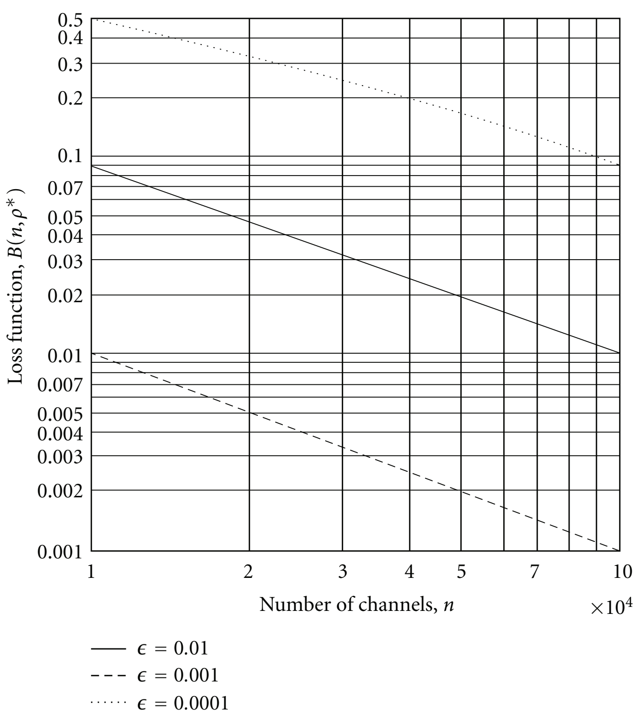

FIGURE 2: The behavior of losses function $B(n, \rho^*)$ according to different values of ϵ.

5. Conclusion

In this paper, a simple nonasymptotic approximation for the Erlang B formula is considered. We find the sufficient condition when the approximation is relevant. The proposed result allows rejecting the previously used limited regime and considers the proportional differentiated admission control under finite network resources. Following this way, we get explicit formulas for PDAC problem. The proposed formulas deliver high-performance computing of network resources assignment under PDAC requirements. Thus, an efficient method for proportional differentiated admission control implementation has been provided.

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