Published for SISSA by Springer
Received: May 12, 2016
Revised: June 10, 2016
Accepted: June 14, 2016
Published: June 21, 2016

$A_4 \times \mathrm{U}(1)_{\mathrm{PQ}}$ model for the lepton flavor structure and the strong CP problem

Takaaki Nomura,[a] Yusuke Shimizu[a,b] and Toshifumi Yamada[a]

[a] *School of Physics, KIAS, Seoul 130-722, Republic of Korea*

[b] *Graduate School of Science, Hiroshima University, Higashi-Hiroshima, 739-8526, Japan*

E-mail: nomura@kias.re.kr, yu-shimizu@hiroshima-u.ac.jp, toshifumi@kias.re.kr

Abstract: We present a model with $A_4 \times \mathrm{U}(1)_{\mathrm{PQ}}$ lepton flavor symmetry which explains the origin of the lepton flavor structure and also solves the strong CP problem. Standard model gauge singlet fields, so-called "flavons", charged under the $A_4 \times \mathrm{U}(1)_{\mathrm{PQ}}$ symmetry are introduced and are coupled with the lepton and the Higgs sectors. The flavon vacuum expectation values (VEVs) trigger spontaneous breaking of the $A_4 \times \mathrm{U}(1)_{\mathrm{PQ}}$ symmetry. The breaking pattern of the A_4 accounts for the tri-bimaximal neutrino mixing and the deviation from it due to the non-zero θ_{13} angle, and the breaking of the $\mathrm{U}(1)_{\mathrm{PQ}}$ gives rise to a pseudo-Nambu-Goldstone boson, axion, whose VEV cancels the QCD θ term. We investigate the breaking of the $A_4 \times \mathrm{U}(1)_{\mathrm{PQ}}$ symmetry through an analysis on the scalar potential and further discuss the properties of the axion in the model, including its decay constant, mass and coupling with photons. It is shown that the axion decay constant is related with the right-handed neutrino mass through the flavon VEVs. Experimental constraints on the axion and their implications are also studied.

Keywords: Beyond Standard Model, Discrete Symmetries, Global Symmetries, Neutrino Physics

ArXiv ePrint: 1604.07650

doi:10.1007/JHEP06(2016)125

JHEP06(2016)125

Contents

1 Introduction

The standard model (SM) is the most attractive model which explains almost all the results of collider and low-energy experiments, and the last piece of the SM, the Higgs particle, has been discovered very recently. However there remains a mystery about the origin of the structures of quark and lepton flavors. As to the lepton sector, neutrino oscillation experiments have provided interesting information about the flavor structure in the form of the two neutrino mass squared differences and the two large mixing angles. The reactor neutrino experiments [1]–[3] have also reported a non-zero θ_{13}, which was the last mixing angle to be measured in the lepton sector. Deriving the neutrino mass and mixing angles theoretically is a challenge in the quest of physics beyond the SM. The non-Abelian discrete symmetry is one of the candidates for the origin of the flavor structures. Many authors have proposed models with non-Abelian discrete flavor symmetries in the lepton sector as well as in the quark sector (see for review [4]–[7]). Those models contain new gauge singlet scalar fields, so-called "flavons", in addition to the SU(2) doublet SM Higgs field. In order to obtain vacuum expectation values (VEVs) and VEV alignments of flavon scalar fields, one usually introduces so-called "driving fields" in the framework of the supersymmetry (SUSY) with $\mathrm{U}(1)_R$ symmetry.

The strong CP problem is another mystery of the SM. In QCD theory, a non-zero QCD vacuum angle θ is allowed which leads to the violation of CP symmetry. Searches for neutron electric dipole moment report an experimental bound of $|\theta| \lesssim 10^{-11}$ [8]. Such an unnaturally small θ requires a theoretical explanation. The most elegant solution to the strong CP problem is to introduce the $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry [9]. The $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry is anomalous in QCD, and when spontaneously broken, it yields a pseudo-Nambu-Goldstone boson called axion. The axion field develops a VEV that cancels the θ term. The axion decay constant, which is the scale of $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry breaking, should be much higher than the electroweak scale to evade the constraint from supernova cooling [10–12]. Hence the breaking of $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry is associated with the VEV of some new scalar field with a $\mathrm{U}(1)_{\mathrm{PQ}}$ charge. The pervasive models of the axion are KSVZ model [13, 14] and DFSZ model [15, 16]. In the former, the new scalar couples with new vector-like quarks, while in the latter, two Higgs doublets are introduced and are coupled with the new scalar.

In this paper, we present a model with $A_4 \times \mathrm{U}(1)_{\mathrm{PQ}}$ symmetry that explains the origin of the lepton flavor structure and also solves the strong CP problem. We show that one can successfully assign A_4 and $\mathrm{U}(1)_{\mathrm{PQ}}$ charges to the leptons, quarks, Higgs fields and flavons. Note that the quarks and Higgs fields are A_4 trivial singlets, which allows the same quark Yukawa couplings as in the minimal SUSY SM. A_4 flavor symmetry regulates the flavor structure of the lepton sector. Since flavon fields have $\mathrm{U}(1)_{\mathrm{PQ}}$ charges, non-zero VEVs of flavons realize the spontaneous breaking of $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry, yielding an axion field. The Higgs fields have $\mathrm{U}(1)_{\mathrm{PQ}}$ charges and the axion in our model is of DFSZ-type. What is unique in our model is that the axion decay constant is tied with flavon VEVs, which are also related with the right-handed Majorana neutrino mass. Another appeal is that, in conventional A_4 flavor symmetry models, additional Z_N symmetry is often imposed to forbid certain terms, while in our model, the $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry plays the role of the Z_N symmetry and no ad-hoc Z_N symmetry is necessary. In ref. [17], another model with $A_4 \times \mathrm{U}(1)_{\mathrm{PQ}}$ symmetry has been presented. There, flavon fields with $\mathrm{U}(1)_{\mathrm{PQ}}$ charges couple directly with the quark sector and the axion has a property different from the DFSZ and KSVZ axions. In our model, on the other hand, they couple to quarks only through the two Higgs fields and the axion is the DFSZ axion. Hence the two models are distinctively different.[1]

This paper is organized as follows. In section 2, we present our model of $A_4 \times \mathrm{U}(1)_{\mathrm{PQ}}$ lepton flavor symmetry with SUSY and $\mathrm{U}(1)_R$ symmetry, and discuss the potential of the flavon and driving fields. We also comment on the origin of the Higgs μ-term in the model. In section 3, we study the properties of the axion in the model and its connection to the flavon properties. Also, experimental constraints on the axion are discussed. The section 4 is devoted to summary. In appendix A, we review the multiplication rule of A_4 group.

2 $A_4 \times \mathrm{U}(1)_{\mathrm{PQ}}$ model

We construct an $A_4 \times \mathrm{U}(1)_{\mathrm{PQ}}$ model for describing the quark and lepton sectors. In this model, we require that the left-handed lepton doublets $l = (l_e, l_\mu, l_\tau)$ transform as a triplet under the A_4, while the right-handed charged leptons are assigned to the singlets as $\mathbf{1}$, $\mathbf{1''}$, and $\mathbf{1'}$ for e^c_R, μ^c_R, and τ^c_R. The right-handed Majorana neutrinos are introduced as an A_4 triplet $\nu^c_R = (\nu^c_{eR}, \nu^c_{\mu R}, \nu^c_{\tau R})$. On the other hand, the left-handed quark doublets q_1, q_2, q_3 and the right-handed up- and down-type quarks u^c_R, c^c_R, t^c_R, d^c_R, s^c_R, b^c_R transform as trivial singlets under the A_4. For the Higgs sector, we introduce the Higgs doublets h_u and h_d which are assigned to the A_4 trivial singlet. The setup accommodates the same Yukawa

[1] In ref. [18], a different type of model has been presented which explains the quark flavor structure with discrete flavor symmetries and also solves the strong CP problem. Unlike our model, that model's solution to the strong CP problem is based on the idea that CP is a fundamental symmetry of Nature and the CP phase of the SM is provided by spontaneous CP symmetry breaking.

	l	e_R^c	μ_R^c	τ_R^c	ν_R^c	q_1	q_2	q_3	u_R^c	c_R^c	t_R^c	d_R^c	s_R^c	b_R^c
SU(2)	2	1			1	2			1			1		
A_4	**3**	**1**	**1**$''$	**1**$'$	**3**	**1**			**1**			**1**		
$\mathrm{U(1)_{PQ}}$	$\frac{1}{4}(1-4q_u)$	$\frac{1}{4}(8q_u-5)$			$-\frac{1}{4}$	r			$-q_u-r$			q_u-r-1		
$\mathrm{U(1)}_R$	1	1			1	1			1			1		

	h_u	h_d	ϕ_T	ϕ_S	ξ	ξ'	ϕ_0^T	ϕ_0^S	ξ_0
SU(2)	2		1	1			1	1	
A_4	**1**		**3**	**3**	**1**	**1**$'$	**3**	**3**	**1**
$\mathrm{U(1)_{PQ}}$	q_u	$(1-q_u)$	0	$\frac{1}{2}$			0	-1	
$\mathrm{U(1)}_R$	0		0	0			2	2	

Table 1. The assignments of leptons, quarks, Higgs, flavons, and driving fields. The $\mathrm{U(1)_{PQ}}$ charges are normalized in such a way that the charges of h_u and h_d sum to 1. q_u and r are arbitrary constants.

couplings for the quarks as in the minimal SUSY SM. The gauge singlet flavons ϕ_T, ϕ_S, ξ, and ξ' are added where $\phi_T = (\phi_{T1}, \phi_{T2}, \phi_{T3})$ and $\phi_S = (\phi_{S1}, \phi_{S2}, \phi_{S3})$ are triplets, ξ is trivial singlet, and ξ' is singlet-prime under the A_4 symmetry, respectively. We introduce a $\mathrm{U(1)_{PQ}}$ symmetry to forbid irrelevant couplings in the lepton sector. The $A_4 \times \mathrm{U(1)_{PQ}}$ charge assignments are shown in table 1, where the $\mathrm{U(1)_{PQ}}$ charges are normalized in such a way that the sum of the charges of the Higgs doublets is 1. In addition, we introduce the so-called "driving fields" $\phi_0^T = (\phi_{01}^T, \phi_{02}^T, \phi_{03}^T)$ and $\phi_0^S = (\phi_{01}^S, \phi_{02}^S, \phi_{03}^S)$ which are A_4 triplets, and ξ_0 which is A_4 trivial singlet, in order to obtain VEVs and VEV alignments for the flavons. Then the VEV alignments can be generated through F-terms which couples flavons to driving fields and carries the R charge $+2$ under $\mathrm{U(1)}_R$ symmetry. For leptons and quarks, we also assign R charge $+1$. The charge assignments of driving fields are also shown in table 1. In the setup, the superpotential w respecting $A_4 \times \mathrm{U(1)_{PQ}}$ symmetry at the leading order is written as[2]

$$
\begin{aligned}
w &\equiv w_d + w_Y \,, \\
w_d &\equiv w_d^T + w_d^S \,, \\
w_d^T &= -M\phi_0^T\phi_T + g\phi_0^T\phi_T\phi_T \,, \\
w_d^S &= g_1\phi_0^S\phi_S\phi_S - g_2\phi_0^S\phi_S\xi + g_2'\phi_0^S\phi_S\xi' + g_3\xi_0\phi_S\phi_S - g_4\xi_0\xi\xi + \lambda\xi_0 h_u h_d \,, \\
w_Y &\equiv w_\ell + w_D + w_N + w_{Y_u} + w_{Y_d} \,, \\
w_\ell &= y_e\phi_T l e_R^c h_d/\Lambda + y_\mu\phi_T l\mu_R^c h_d/\Lambda + y_\tau\phi_T l\tau_R^c h_d/\Lambda \,, \\
w_D &= y_D l\nu_R^c h_u \,, \\
w_N &= y_{\phi_S}\phi_S\nu_R^c\nu_R^c + y_\xi\xi\nu_R^c\nu_R^c + y_{\xi'}\xi'\nu_R^c\nu_R^c \,, \\
w_{Y_u} &= y_{i\alpha}q_i\alpha_R^c h_u \quad (i=1,2,3,\ \alpha=u,c,t) \,, \\
w_{Y_d} &= y_{i\beta}q_i\beta_R^c h_d \quad (i=1,2,3,\ \beta=d,s,b) \,,
\end{aligned} \tag{2.1}
$$

[2]This model is same charge assignments of ref. [19] except for Z_3 charges because we introduce $\mathrm{U(1)_{PQ}}$ symmetry instead of Z_3 symmetry.

where M is generally complex mass parameter, g's are trilinear couplings which are also complex parameters,[3] λ is trilinear coupling for SU(2) doublet Higgs and driving field, y's are complex Yukawa couplings, and Λ is the A_4 cutoff scale. Note that it is consistent with the SM in the quark sector, since we assign A_4 charges to quark fields as trivial singlets. From this superpotential, we discuss the potential analysis including flavons and driving fields in the next subsection.

2.1 Potential analyses including flavons and driving fields

Let us discuss the potential for scalar fields including flavons and driving fields. The superpotential w_d^T and w_d^S in eq. (2.1) are obtained as

$$\begin{aligned}
w_d^T &= -M\phi_0^T\phi_T + g\phi_0^T\phi_T\phi_T \\
&= -M(\phi_{01}^T\phi_{T1} + \phi_{02}^T\phi_{T3} + \phi_{03}^T\phi_{T2}) \\
&\quad + \frac{2g}{3}\left[\phi_{01}^T(\phi_{T1}^2 - \phi_{T2}\phi_{T3}) + \phi_{02}^T(\phi_{T2}^2 - \phi_{T1}\phi_{T3}) + \phi_{03}^T(\phi_{T3}^2 - \phi_{T1}\phi_{T2})\right], \\
w_d^S &= g_1\phi_0^S\phi_S\phi_S - g_2\phi_0^S\phi_S\xi + g_2'\phi_0^S\phi_S\xi' + g_3\xi_0\phi_S\phi_S - g_4\xi_0\xi\xi + \lambda\xi_0 h_u h_d \\
&= \frac{2g_1}{3}\left[\phi_{01}^S(\phi_{S1}^2 - \phi_{S2}\phi_{S3}) + \phi_{02}^S(\phi_{S2}^2 - \phi_{S1}\phi_{S3}) + \phi_{03}^S(\phi_{S3}^2 - \phi_{S1}\phi_{S2})\right] \\
&\quad - g_2(\phi_{01}^S\phi_{S1} + \phi_{02}^S\phi_{S3} + \phi_{03}^S\phi_{S2})\xi + g_2'(\phi_{01}^S\phi_{S3} + \phi_{02}^S\phi_{S2} + \phi_{03}^S\phi_{S1})\xi' \\
&\quad + g_3\xi_0(\phi_{S1}^2 + 2\phi_{S2}\phi_{S3}) - g_4\xi_0\xi^2 + \lambda\xi_0 h_u h_d \,.
\end{aligned} \tag{2.2}$$

In order to discuss the VEVs and VEV alignments of flavons, we consider the scalar potential except for driving fields, which is given as

$$\begin{aligned}
V &\equiv V_T + V_S\,, \\
V_T &= \sum_i \left|\frac{\partial w_d^T}{\partial \phi_{0i}^T}\right|^2 + \text{h.c.} \\
&= 2\left| -M\phi_{T1} + \frac{2g}{3}(\phi_{T1}^2 - \phi_{T2}\phi_{T3})\right|^2 + 2\left| -M\phi_{T3} + \frac{2g}{3}(\phi_{T2}^2 - \phi_{T1}\phi_{T3})\right|^2 \\
&\quad + 2\left| -M\phi_{T2} + \frac{2g}{3}(\phi_{T3}^2 - \phi_{T1}\phi_{T2})\right|^2, \\
V_S &= \sum \left|\frac{\partial w_d^S}{\partial X}\right|^2 + \text{h.c.} \quad (X = \phi_{0i}^S,\, \xi_0) \\
&= 2\left|\frac{2g_1}{3}(\phi_{S1}^2 - \phi_{S2}\phi_{S3}) - g_2\phi_{S1}\xi + g_2'\phi_{S3}\xi'\right|^2 + 2\left|\frac{2g_1}{3}(\phi_{S2}^2 - \phi_{S1}\phi_{S3}) - g_2\phi_{S3}\xi + g_2'\phi_{S2}\xi'\right|^2 \\
&\quad + 2\left|\frac{2g_1}{3}(\phi_{S3}^2 - \phi_{S1}\phi_{S2}) - g_2\phi_{S2}\xi + g_2'\phi_{S1}\xi'\right|^2 + 2\left|g_3(\phi_{S1}^2 + 2\phi_{S2}\phi_{S3}) - g_4\xi^2 + \lambda h_u h_d\right|^2.
\end{aligned} \tag{2.3}$$

[3]In order to obtain the positive number of v_T, v_S, u, and u' for eqs. (2.4) and (2.5), we take negative sign for several terms in eq. (2.1).

Applying the condition of the potential minimum ($V_T = 0$), we derive the VEV alignment of ϕ_T as

$$\langle \phi_T \rangle = v_T(1,0,0)\,, \qquad v_T = \frac{3M}{2g}\,, \tag{2.4}$$

where v_T is generally complex number since M and g are complex. By using VEV and VEV alignment of eq. (2.4), we obtain diagonal mass matrix for the charged leptons. Then the phase of v_T can be removed and we take M/g as real parameter without loss of generality. In the following analysis, we adopt M and g as real parameters for simplicity. On the other hand, we derive the VEV alignment of ϕ_S and VEVs of ξ, ξ', h_u and h_d from the condition of the potential minimum ($V_S = 0$) in eq. (2.3) as

$$\begin{aligned} &\langle h_u \rangle = \langle h_d \rangle = 0\,, && \langle \xi \rangle = u\,, && \langle \xi' \rangle = u', \\ &\langle \phi_S \rangle = v_S(1,1,1)\,, && v_S^2 = \frac{g_4}{3g_3}u^2, && u' = \frac{g_2}{g_2'}u\,. \end{aligned} \tag{2.5}$$

As a result, we can take VEVs u and u' as free parameters. Applying the VEVs and VEV alignment of eq. (2.5), the neutrino mass matrix induces the lepton mixing as 1-3 rotation from tri-bimaximal mixing (TBM) [20, 21]. Furthermore, we obtain the non-zero θ_{13} which comes from A_4 singlet flavon VEV ratio u'/u (see refs. [22]–[26]).

On the other hand the scalar potential including driving fields is given as

$$\begin{aligned} V_d &\equiv V_d^T + V_d^S\,, \\ V_d^T &= \sum_i \left|\frac{\partial w_d^T}{\partial \phi_{Ti}}\right|^2 + \text{h.c.} \\ &= 2\left|\phi_{01}^T\left(-M + \frac{4g}{3}\phi_{T1}\right) - \frac{2g}{3}(\phi_{02}^T\phi_{T3} + \phi_{03}^T\phi_{T2})\right|^2 \\ &\quad + 2\left|-\phi_{03}^T\left(M + \frac{2g}{3}\phi_{T1}\right) + \frac{2g}{3}(-\phi_{01}^T\phi_{T3} + 2\phi_{02}^T\phi_{T2})\right|^2 \\ &\quad + 2\left|-\phi_{02}^T\left(M + \frac{2g}{3}\phi_{T1}\right) + \frac{2g}{3}(-\phi_{01}^T\phi_{T2} + 2\phi_{03}^T\phi_{T3})\right|^2, \\ V_d^S &= \sum_i \left|\frac{\partial w_d^S}{\partial X_i}\right|^2 + \text{h.c.} \quad (X_i = \phi_{Si},\, \xi,\, \xi') \\ &= 2\left|\phi_{01}^S\left(\frac{4g_1}{3}\phi_{S1} - g_2\xi\right) - \frac{2g_1}{3}\phi_{02}^S\phi_{S3} - \phi_{03}^S\left(\frac{2g_1}{3}\phi_{S2} + g_2'\xi'\right) + 2g_3\xi_0\phi_{S1}\right|^2 \\ &\quad + 2\left|-\frac{2g_1}{3}\phi_{01}^S\phi_{S3} + \phi_{02}^S\left(\frac{4g_1}{3}\phi_{S2} + g_2'\xi'\right) - \phi_{03}^S\left(\frac{2g_1}{3}\phi_{S1} + g_2\xi\right) + 2g_3\xi_0\phi_{S3}\right|^2 \\ &\quad + 2\left|-\phi_{01}^S\left(\frac{2g_1}{3}\phi_{S2} - g_2'\xi'\right) - \phi_{02}^S\left(\frac{2g_1}{3}\phi_{S1} + g_2\xi\right) + \frac{4g_1}{3}\phi_{03}^S\phi_{S3} + 2g_3\xi_0\phi_{S2}\right|^2 \\ &\quad + 2\left|-g_2(\phi_{01}^S\phi_{S1} + \phi_{02}^S\phi_{S3} + \phi_{03}^S\phi_{S2}) - 2g_4\xi_0\xi\right|^2 + 2\left|g_2'(\phi_{01}^S\phi_{S3} + \phi_{02}^S\phi_{S2} + \phi_{03}^S\phi_{S1})\right|^2. \end{aligned} \tag{2.6}$$

JHEP06(2016)125

Taking VEVs and VEV alignments in eqs. (2.4) and (2.5), the scalar potential including driving fields of eq. (2.6) are rewritten as

$$
\begin{aligned}
V_d^T &= 2|M\phi_{01}^T|^2 + 8|M\phi_{03}^T|^2 + 8|M\phi_{02}^T|^2, \\
V_d^S &= 2\left|\left[\phi_{01}^S\left(\frac{4g_1}{3}c_S - g_2\right) - \frac{2g_1}{3}c_S\phi_{02}^S - \phi_{03}^S\left(\frac{2g_1}{3}c_S + g_2\right) + 2g_3c_S\xi_0\right]u\right|^2 \\
&\quad + 2\left|\left[-\frac{2g_1}{3}c_S\phi_{01}^S + \phi_{02}^S\left(\frac{4g_1}{3}c_S + g_2\right) - \phi_{03}^S\left(\frac{2g_1}{3}c_S + g_2\right) + 2g_3c_S\xi_0\right]u\right|^2 \\
&\quad + 2\left|\left[-\phi_{01}^S\left(\frac{2g_1}{3}c_S - g_2\right) - \phi_{02}^S\left(\frac{2g_1}{3}c_S + g_2\right) + \frac{4g_1}{3}c_S\phi_{03}^S + 2g_3c_S\xi_0\right]u\right|^2 \\
&\quad + 2\left|\left[-g_2c_S(\phi_{01}^S + \phi_{02}^S + \phi_{03}^S) - 2g_4\xi_0\right]u\right|^2 + 2\left|g_2'c_S(\phi_{01}^S + \phi_{02}^S + \phi_{03}^S)u\right|^2,
\end{aligned}
\tag{2.7}
$$

where we define $c_S^2 = g_4/(3g_3)$ and we eliminate trilinear coupling g and VEV of flavon v_T, v_S, and u' in eqs. (2.4) and (2.5). Then, the VEVs of driving fields ϕ_0^T, ϕ_0^S, and ξ_0 are zeros which are derived from the condition of the potential minimum ($V_d^T = 0$ and $V_d^S = 0$) in eq. (2.7) such as trivial solution;

$$
\langle\phi_{0i}^T\rangle = \langle\phi_{0i}^S\rangle = \langle\xi_0\rangle = 0\,. \tag{2.8}
$$

Therefore flavons take VEVs in eqs. (2.4) and (2.5), which break A_4 and driving fields take zero VEVs in eq. (2.8). Thus in A_4 breaking scale, Higgs μ term does not have mass scale such as $\mu = \lambda\langle\xi_0\rangle = 0$. In the next subsection, we will discuss how to get non-zero μ term from soft SUSY breaking.

2.2 Higgs μ term from soft SUSY breaking

Let us discuss the non-zero Higgs μ term in this subsection. After A_4 breaking, $\mu = \lambda\langle\xi_0\rangle$ is zero because VEV of driving field $\langle\xi_0\rangle = 0$, which we discussed previous subsection. Then, we consider soft SUSY breaking term in order to obtain Higgs μ term. The Lagrangian including soft SUSY breaking terms are written as

$$
\mathcal{L}_{\rm soft} \supset g_3A_{\phi_S}\xi_0\phi_S\phi_S - g_4A_\xi\xi_0\xi\xi + \text{h.c.}\,, \tag{2.9}
$$

where A_{ϕ_S} and A_ξ are trilinear soft SUSY breaking A-terms and we assume that the A-terms are proportional to the trilinear couplings g_3 and g_4, respectively. Taking VEVs of flavons ϕ_S and ξ as $\langle\phi_S\rangle = v_S(1,1,1)$ and $\langle\xi\rangle = u$, the Lagrangian including driving field ξ_0 is written as

$$
\begin{aligned}
\mathcal{L}_{\xi_0} &\supset 3g_3A_{\phi_S}v_S^2\xi_0 - g_4A_\xi u^2\xi_0 - 12|g_3v_S\xi_0|^2 - 4|g_4u\xi_0|^2 + \text{h.c.} \\
&= (A_{\phi_S} - A_\xi)g_4u^2\xi_0 - 4|g_4|\big(|g_3| - |g_4|\big)|u\xi_0|^2 + \text{h.c.}\,.
\end{aligned}
\tag{2.10}
$$

Then after soft SUSY breaking, the driving field ξ_0 gets VEV as

$$
\langle\xi_0\rangle = \frac{(A_{\phi_S} - A_\xi)g_4u^2}{4|g_4|\big(|g_3| - |g_4|\big)|u|^2} \simeq \mathcal{O}(A_{\not{\rm SUSY}})\ \text{TeV}, \tag{2.11}
$$

where $A_{\not{\rm SUSY}}$ is soft SUSY breaking scale. Therefore we obtain Higgs μ term as

$$
\mu = \lambda\langle\xi_0\rangle \simeq \mathcal{O}(A_{\not{\rm SUSY}})\ \text{TeV}. \tag{2.12}
$$

JHEP06(2016)125

3 Axion physics

The $U(1)_{PQ}$ symmetry in our model gives rise to an axion field whose VEV cancels the QCD θ term and thereby solves the strong CP problem [9]. In the model, the two Higgs fields have $U(1)_{PQ}$ charges as in the DFSZ model [15, 16] and the axion has properties similar to the DFSZ axion. We in this section describe the formulas for the axion decay constant, axion mass and axion coupling with photons in the model and discuss their connection with the flavon sector. Also, the current experimental bounds on axion properties are studied.

3.1 Axion properties

In our model, the $U(1)_{PQ}$ symmetry is spontaneously broken at the flavon VEV scale by the VEVs of fields ϕ_S, ξ, ξ'. We hereafter assume that the flavon VEV scale is much higher than the electroweak scale. The axion field a is then given by a linear combination of the phase components of ϕ_S, ξ, ξ',

$$\begin{aligned} \phi_{Si} &= \left(v_S + h_{Si}/\sqrt{2}\right) e^{i a_{Si}/(\sqrt{2} v_S)} \qquad (i=1,2,3)\,, \\ \xi &= \left(u + h_\xi/\sqrt{2}\right) e^{i a_\xi/(\sqrt{2}u)}, \qquad \xi' = \left(u' + h_{\xi'}/\sqrt{2}\right) e^{i a_{\xi'}/(\sqrt{2}u')}, \end{aligned} \tag{3.1}$$

in the following way:

$$a = \frac{\sqrt{2} v_S (a_{S1} + a_{S2} + a_{S3}) + \sqrt{2} u a_\xi + \sqrt{2} u' a_{\xi'}}{\sqrt{6 v_S^2 + 2u^2 + 2u'^2}}\,. \tag{3.2}$$

The axion decay constant, F_a, is defined in the basis where the Dirac masses of all quarks and leptons are set to be real through axial phase transformations on fermions, $\psi \to e^{i\gamma_5 \alpha}\psi$. In this basis, the couplings of the axion to SM fields other than derivative interactions are given by

$$\mathcal{L}_{\text{axion couplings}} = \frac{a}{F_a}\frac{g_C^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^{a\,\mu\nu} + c_{a\gamma\gamma}\frac{a}{F_a}\frac{e^2}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{3.3}$$

where g_C and e are the QCD and electromagnetic coupling constants, respectively. Here the axion decay constant F_a is given by

$$F_a = \frac{\sqrt{6 v_S^2 + 2u^2 + 2u'^2}}{N_{\rm DW}}\,, \tag{3.4}$$

where $N_{\rm DW}$ is the domain wall number and $N_{\rm DW} = 2N_{qg} = 6$ in the current model, with N_{qg} denoting the number of quark generations. It is remarkable that the axion decay constant is predicted to be of the same order as the flavon VEVs. In the same basis, the ratio of the axion couplings to gluons and photons, $c_{a\gamma\gamma}$, is calculated by summing up-type quark, down-type quark and charged lepton contributions to the axion-photon coupling as

$$c_{a\gamma\gamma} = 2\frac{N_{qg} N_c q_u Q_u^2 + N_{qg} N_c (1-q_u) Q_d^2 + N_{lg}(1-q_u) Q_e^2}{N_{qg}} = \frac{8}{3}\,, \tag{3.5}$$

where the number of colors is $N_c = 3$, the number of lepton generations is $N_{lg} = 3$, and the electric charges of up-type quarks, down type-quarks and charged leptons are $Q_u = 2/3$, $Q_d = -1/3$ and $Q_e = -1$, respectively.

JHEP06(2016)125

We next estimate the axion mass and the effective axion-photon coupling below the QCD confinement scale. This is easily done by imposing the following axial phase transformations on the three light quarks u, d, s to set the axion-gluon coupling to be zero:

$$\begin{aligned} &u \to e^{i\gamma_5\alpha_u} u\,, \qquad d \to e^{i\gamma_5\alpha_d} d\,, \qquad s \to e^{i\gamma_5\alpha_s} s \\ &\text{with } \frac{a}{F_a} - 2\alpha_u - 2\alpha_d - 2\alpha_s = 0\,. \end{aligned} \tag{3.6}$$

Note that α_u, α_d, α_s are not constants, but are dynamical fields that share the physical degree of freedom of the axion. A convenient choice for α_u, α_d, α_s is

$$\begin{aligned} &\alpha_u = \frac{a}{2F_a}\frac{1}{1+z+w}\,, \qquad \alpha_d = \frac{a}{2F_a}\frac{z}{1+z+w}\,, \qquad \alpha_s = \frac{a}{2F_a}\frac{w}{1+z+w} \\ &\text{where } z \equiv \frac{m_u}{m_d}\,,\ w \equiv \frac{m_u}{m_s}\,. \end{aligned} \tag{3.7}$$

Eq. (3.7) leads to $\alpha_u m_u = \alpha_d m_d = \alpha_s m_s$, with which the axion does not mix with π^0 and η' mesons in the chiral Lagrangian. With eq. (3.7), the mass term for the axion, π^0 meson and η' meson in the chiral Lagrangian is given by

$$\mathcal{L}_{\text{chiral}} = \frac{1}{2} v_\chi^3 \,\text{tr}\left\{\Sigma M + M^\dagger \Sigma^\dagger\right\}, \tag{3.8}$$

where

$$\begin{aligned} \Sigma &= \exp\left[2i\,\text{diag}\left(\frac{\pi^0}{2f_\pi} + \frac{\eta'}{2\sqrt{3}f_{\eta'}},\ -\frac{\pi^0}{2f_\pi} + \frac{\eta'}{2\sqrt{3}f_{\eta'}},\ -\frac{\eta'}{\sqrt{3}f_{\eta'}}\right)\right], \\ M &= \text{diag}(m_u e^{i2\alpha_u},\ m_d e^{i2\alpha_d},\ m_s e^{i2\alpha_s})\,, \end{aligned} \tag{3.9}$$

and v_χ corresponds to the scale of chiral symmetry breaking. Note that M contains the physical degree of freedom of the axion through α_u, α_d, α_s. After diagonalizing the mass term eq. (3.8), the axion mass, the π^0 mass and the η' mass are given by

$$\begin{aligned} m_a^2 &= \frac{v_\chi^3 m_u}{F_a^2}\frac{1}{1+z+w}\,, \\ m_{\pi^0}^2 &= \frac{v_\chi^3 m_u}{24 f_\pi^2 f_{\eta'}^2 zw}\Big[3f_{\eta'}^2(1+z)w + f_\pi^2(4z+w+wz) \\ &\qquad - \sqrt{-48 f_\pi^2 f_{\eta'}^2 zw(1+z+w) + \{3f_{\eta'}^2(1+z)w + f_\pi^2(4z+w+wz)\}^2}\,\Big]\,, \\ m_{\eta'}^2 &= \frac{v_\chi^3 m_u}{24 f_\pi^2 f_{\eta'}^2 zw}\Big[3f_{\eta'}^2(1+z)w + f_\pi^2(4z+w+wz) \\ &\qquad + \sqrt{-48 f_\pi^2 f_{\eta'}^2 zw(1+z+w) + \{3f_{\eta'}^2(1+z)w + f_\pi^2(4z+w+wz)\}^2}\,\Big]\,. \end{aligned} \tag{3.10}$$

Using eq. (3.10), we relate the axion mass with the π^0 mass in terms of $z = m_u/m_d$, $w = m_u/m_s$, F_a, f_π, and $f_{\eta'}$ by eliminating $v_\chi^3 m_u$. Substituting the experimental values $m_{\pi^0} = 135\,\text{MeV}$, $f_\pi = 93\,\text{MeV}$, $f_{\eta'} = 86\,\text{MeV}$, $z = 0.56$, and $w = 0.028$ [27], we obtain

$$m_a = 6.0 \times 10^{-6}\,\text{eV}\left(\frac{10^{12}\,\text{GeV}}{F_a}\right). \tag{3.11}$$

In the same basis, the axion coupling to photons is given by

$$\begin{aligned}\mathcal{L}_{\text{axion couplings}} &= \left\{c_{a\gamma\gamma} - 2\frac{2\alpha_u N_c q_u Q_u^2 + (2\alpha_d + 2\alpha_s)N_c(1-q_u)Q_d^2}{N_{qg}}\right\}\frac{a}{F_a}\frac{e^2}{32\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu} \\ &= \left(\frac{8}{3} - \frac{2}{3}\frac{4+z+w}{1+z+w}\right)\frac{a}{F_a}\frac{e^2}{32\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}. \end{aligned} \tag{3.12}$$

3.2 Experimental constraints

The observation of globular-cluster stars puts a bound on the axion-photon coupling. Defining the axion-photon coupling $g_{a\gamma\gamma}$ by $\mathcal{L} = (1/4)g_{a\gamma\gamma}aF_{\mu\nu}\tilde{F}^{\mu\nu}$, we find that the bound is expressed as [28]

$$|g_{a\gamma\gamma}| < 6.6 \times 10^{-11}\,\text{GeV}^{-1}, \tag{3.13}$$

at 95% C.L. The cooling of supernova 1987A puts a bound on the axion decay constant F_a as [10–12]

$$F_a \gtrsim 3.7 \times 10^8\,\text{GeV}. \tag{3.14}$$

Eq. (3.14) gives a more severe bound than eq. (3.13) on the axion decay constant. In the model, this also gives a lower bound of the flavon VEV scale and the right-handed Majorana neutrino mass scale. For example, if $v_S = u = u'$ and the Majorana Yukawa coupling is $O(0.1)$, the right-handed Majorana neutrino mass M_R is bounded from below as

$$M_R \gtrsim 10^8\,\text{GeV}. \tag{3.15}$$

If the axion is the dominant constituent of the dark matter, further experimental constraints apply. The ADMX Collaboration reports a bound on the ratio of the axion mass and the axion-photon coupling in the axion dark matter scenario [29], but the DFSZ axion is not yet constrained by this experiment. Since the ratio of the axion mass and the axion-photon coupling is the same in our model and in the DFSZ model, our model also evades the ADMX bound.

4 Summary

We have constructed a model based on the $A_4 \times \mathrm{U}(1)_{\mathrm{PQ}}$ flavor symmetry. The A_4 symmetry induces TBM in the lepton sector to explain large mixing angles, while it is consistent with the SM in the quark sector, since all of quark fields are assigned as A_4 trivial singlets. On the other hands, the $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry is introduced to forbid unwanted couplings in the lepton sector and to solve the strong CP problem. In the model, we have introduced the A_4 triplet and singlet flavons with $\mathrm{U}(1)_{\mathrm{PQ}}$ charges to break the symmetry and to obtain observed masses and mixing angles in the lepton sector. Furthermore, Majorana mass of right-handed neutrinos is generated after the symmetry breaking by flavon VEVs.

We have analyzed the scalar potential including flavons and driving fields in the framework with SUSY and $\mathrm{U}(1)_R$ symmetry. We have shown that the alignment of flavon VEVs is relevant to obtain the correct masses and mixing angles in the lepton sector. In addition, an additional A_4 singlet-prime flavon breaks TBM to obtain non-zero θ_{13}. Moreover non-zero Higgs μ term can be obtained from soft SUSY breaking term which induces non-zero

JHEP06(2016)125

VEV of a driving field ξ_0. As a result, μ term is given by the VEV of the driving field which is at the soft SUSY breaking scale.

We have studied the axion in the model, which has properties similar to the DFSZ axion, since our Higgs fields have $\mathrm{U}(1)_{\mathrm{PQ}}$ charges. The axion field a is given by a linear combination of the phase components of flavons which have non-zero VEVs, and the axion decay constant F_a is obtained in terms of the VEVs of flavons. Remarkably, the axion decay constant is of the same order as the flavon VEVs in the model. We have also derived the axion mass and axion-photon coupling as functions of the axion decay constant. As to experimental constraints on the axion decay constant, it should be larger than $O(10^8)$ GeV due to the constraint from cooling of supernova 1987A, and this can be interpreted as a lower bound on the flavon VEV scale in the model. We note that the Majorana mass of right-handed neutrino ν_R is related to F_a in our model since both of them are determined by the flavon VEVs. The Majorana mass should satisfy $M_R \gtrsim 10^8$ GeV when we assume that the Yukawa coupling for ν_R is $O(0.1)$ and that all flavon VEVs are of the same order. The axion in our model can be the dominant constituent of dark matter. In such a case, still, the axion is not constrained by the ADMX experiment, just like the DFSZ axion.

Acknowledgments

YS is supported by Grant-in-Aid for JSPS Fellows No. 16J05332. This work is also supported in part by the National Research Foundation of Korea (NRF) Research Grant NRF-2015R1A2A1A05001869 (YS and TY).

A Multiplication rule of A_4 group

We show the multiplication of A_4 group. The multiplication rule of the triplet is written as follows,

$$
\begin{aligned}
\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{\mathbf{3}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}} &= (a_1 b_1 + a_2 b_3 + a_3 b_2)_{\mathbf{1}} \\
&\oplus (a_3 b_3 + a_1 b_2 + a_2 b_1)_{\mathbf{1}'} \oplus (a_2 b_2 + a_1 b_3 + a_3 b_1)_{\mathbf{1}''} \\
&\oplus \frac{1}{3} \begin{pmatrix} 2a_1 b_1 - a_2 b_3 - a_3 b_2 \\ 2a_3 b_3 - a_1 b_2 - a_2 b_1 \\ 2a_2 b_2 - a_1 b_3 - a_3 b_1 \end{pmatrix}_{\mathbf{3}} \\
&\oplus \frac{1}{2} \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_1 b_2 - a_2 b_1 \\ a_1 b_3 - a_3 b_1 \end{pmatrix}_{\mathbf{3}} .
\end{aligned} \tag{A.1}
$$

More details are shown in the review [4]–[7].

References

[1] Daya Bay collaboration, F.P. An et al., *Observation of electron-antineutrino disappearance at Daya Bay*, Phys. Rev. Lett. **108** (2012) 171803 [arXiv:1203.1669] [INSPIRE].

[2] RENO collaboration, J.K. Ahn et al., *Observation of reactor electron antineutrino disappearance in the RENO experiment*, Phys. Rev. Lett. **108** (2012) 191802 [arXiv:1204.0626] [INSPIRE].

[3] Double CHOOZ collaboration, Y. Abe et al., *Background-independent measurement of θ_{13} in Double CHOOZ*, Phys. Lett. **B 735** (2014) 51 [arXiv:1401.5981] [INSPIRE].

[4] H. Ishimori et al., *Non-Abelian discrete symmetries in particle physics*, Prog. Theor. Phys. Suppl. **183** (2010) 1 [arXiv:1003.3552] [INSPIRE].

[5] H. Ishimori et al., *An introduction to non-Abelian discrete symmetries for particle physicists*, Lect. Notes Phys. **858** (2012) 1 [INSPIRE].

[6] H. Ishimori et al., *Non-Abelian discrete symmetry for flavors*, Fortschr. Phys. **61** (2013) 441 [INSPIRE].

[7] S.F. King, A. Merle, S. Morisi, Y. Shimizu and M. Tanimoto, *Neutrino mass and mixing: from theory to experiment*, New J. Phys. **16** (2014) 045018 [arXiv:1402.4271] [INSPIRE].

[8] C.A. Baker et al., *An improved experimental limit on the electric dipole moment of the neutron*, Phys. Rev. Lett. **97** (2006) 131801 [hep-ex/0602020] [INSPIRE].

[9] R.D. Peccei and H.R. Quinn, *CP conservation in the presence of instantons*, Phys. Rev. Lett. **38** (1977) 1440 [INSPIRE].

[10] R. Mayle et al., *Constraints on axions from SN 1987a*, Phys. Lett. **B 203** (1988) 188 [INSPIRE].

[11] G. Raffelt and D. Seckel, *Bounds on exotic particle interactions from SN 1987a*, Phys. Rev. Lett. **60** (1988) 1793 [INSPIRE].

[12] M.S. Turner, *Axions from SN 1987a*, Phys. Rev. Lett. **60** (1988) 1797 [INSPIRE].

[13] J.E. Kim, *Weak interaction singlet and strong CP invariance*, Phys. Rev. Lett. **43** (1979) 103 [INSPIRE].

[14] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Can confinement ensure natural CP invariance of strong interactions?*, Nucl. Phys. **B 166** (1980) 493 [INSPIRE].

[15] M. Dine, W. Fischler and M. Srednicki, *A simple solution to the strong CP problem with a harmless axion*, Phys. Lett. **B 104** (1981) 199 [INSPIRE].

[16] A.R. Zhitnitsky, *On possible suppression of the axion hadron interactions* (in Russian), *Sov. J. Nucl. Phys.* **31** (1980) 260 [*Yad. Fiz.* **31** (1980) 497] [INSPIRE].

[17] Y.H. Ahn, *Flavored Peccei-Quinn symmetry*, Phys. Rev. **D 91** (2015) 056005 [arXiv:1410.1634] [INSPIRE].

JHEP06(2016)125

[18] S. Antusch, M. Holthausen, M.A. Schmidt and M. Spinrath, *Solving the strong CP problem with discrete symmetries and the right unitarity triangle*, Nucl. Phys. **B 877** (2013) 752 [arXiv:1307.0710] [INSPIRE].

[19] Y. Muramatsu, T. Nomura and Y. Shimizu, *Mass limit for light flavon with residual Z_3 symmetry*, JHEP **03** (2016) 192 [arXiv:1601.04788] [INSPIRE].

[20] P.F. Harrison, D.H. Perkins and W.G. Scott, *Tri-bimaximal mixing and the neutrino oscillation data*, Phys. Lett. **B 530** (2002) 167 [hep-ph/0202074] [INSPIRE].

[21] P.F. Harrison and W.G. Scott, *Symmetries and generalizations of tri-bimaximal neutrino mixing*, Phys. Lett. **B 535** (2002) 163 [hep-ph/0203209] [INSPIRE].

[22] B. Brahmachari, S. Choubey and M. Mitra, *The A_4 flavor symmetry and neutrino phenomenology*, Phys. Rev. **D 77** (2008) 073008 [*Erratum ibid.* **D 77** (2008) 119901] [arXiv:0801.3554] [INSPIRE].

[23] J. Barry and W. Rodejohann, *Deviations from tribimaximal mixing due to the vacuum expectation value misalignment in A_4 models*, Phys. Rev. **D 81** (2010) 093002 [*Erratum ibid.* **D 81** (2010) 119901] [arXiv:1003.2385] [INSPIRE].

[24] Y. Shimizu, M. Tanimoto and A. Watanabe, *Breaking tri-bimaximal mixing and large θ_{13}*, Prog. Theor. Phys. **126** (2011) 81 [arXiv:1105.2929] [INSPIRE].

[25] I. de Medeiros Varzielas and D. Pidt, *UV completions of flavour models and large θ_{13}*, JHEP **03** (2013) 065 [arXiv:1211.5370] [INSPIRE].

[26] B. Karmakar and A. Sil, *Nonzero θ_{13} and leptogenesis in a type-I seesaw model with A_4 symmetry*, Phys. Rev. **D 91** (2015) 013004 [arXiv:1407.5826] [INSPIRE].

[27] Particle Data Group collaboration, K.A. Olive et al., *Review of particle physics*, Chin. Phys. **C 38** (2014) 090001 [INSPIRE].

[28] A. Ayala, I. Domínguez, M. Giannotti, A. Mirizzi and O. Straniero, *Revisiting the bound on axion-photon coupling from globular clusters*, Phys. Rev. Lett. **113** (2014) 191302 [arXiv:1406.6053] [INSPIRE].

[29] ADMX collaboration, S.J. Asztalos et al., *A SQUID-based microwave cavity search for dark-matter axions*, Phys. Rev. Lett. **104** (2010) 041301 [arXiv:0910.5914] [INSPIRE].