Hindawi Publishing Corporation
Journal of Inequalities and Applications
Volume 2010, Article ID 916806, 16 pages
doi:10.1155/2010/916806

Research Article

Approximation of Solutions to a System of Variational Inclusions in Banach Spaces

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Received 6 October 2009; Revised 9 January 2010; Accepted 29 January 2010

Academic Editor: Jong Kim

The purpose of this paper is to introduce an iterative method for finding solutions of a general system of variational inclusions with inverse-strongly accretive mappings. Strong convergence theorems are established in uniformly convex and 2-uniformly smooth Banach spaces.

1. Introduction and Preliminaries

Let $U_E = \{x \in E : \|x\| = 1\}$. A Banach space E is said to be uniformly convex if, for any $\epsilon \in (0,2]$, there exists $\delta > 0$ such that, for any $x, y \in U_E$,

$$\|x - y\| \geq \epsilon \quad \text{implies} \quad \left\| \frac{x+y}{2} \right\| \leq 1 - \delta. \tag{1.1}$$

It is known that a uniformly convex Banach space is reflexive and strictly convex. A Banach space E is said to be smooth if the limit

$$\lim_{t \to 0} \frac{\|x + ty\| - \|x\|}{t} \tag{1.2}$$

exists for all $x, y \in U_E$. It is also said to be uniformly smooth if the limit is attained uniformly for all $x, y \in U_E$. The norm of E is said to be Fréchet differentiable if, for any $x \in U_E$, the above limit is attained uniformly for all $y \in U_E$. The modulus of smoothness of E is defined by

$$\rho(\tau) = \sup\left\{\frac{1}{2}(\|x+y\| + \|x-y\|) - 1 : x, y \in E, \|x\| = 1, \|y\| = \tau\right\}, \tag{1.3}$$

where $\rho : [0,\infty) \to [0,\infty)$ is a function. It is known that E is uniformly smooth if and only if $\lim_{\tau\to 0}(\rho(\tau)/\tau) = 0$. Let q be a fixed real number with $1 < q \le 2$. A Banach space E is said to be q-uniformly smooth if there exists a constant $c > 0$ such that $\rho(\tau) \le c\tau^q$ for all $\tau > 0$.

From [1], we know the following property.

Let q be a real number with $1 < q \le 2$ and let E be a Banach space. Then E is q-uniformly smooth if and only if there exists a constant $K \ge 1$ such that

$$\|x+y\|^q + \|x-y\|^q \le 2(\|x\|^q + \|Ky\|^q), \quad \forall x, y \in E. \tag{1.4}$$

The best constant K in the above inequality is called the q-uniformly smoothness constant of E (see [1] for more details).

Let E be a real Banach space and E^* the dual space of E. Let $\langle\cdot,\cdot\rangle$ denote the pairing between E and E^*. For $q > 1$, the generalized duality mapping $J_q : E \to 2^{E^*}$ is defined by

$$J_q(x) = \left\{f \in E^* : \langle x, f\rangle = \|x\|^q, \|f\| = \|x\|^{q-1}\right\}, \quad \forall x \in E. \tag{1.5}$$

In particular, $J = J_2$ is called the normalized duality mapping. It is known that $J_q(x) = \|x\|^{q-2}J(x)$ for all $x \in E$. If E is a Hilbert space, then $J = I$ (the identity mapping). Note that

(1) E is a uniformly smooth Banach space if and only if J is single valued and uniformly continuous on any bounded subset of E,

(2) all Hilbert spaces, L^p (or l^p) spaces ($p \ge 2$) and the Sobolev spaces W_m^p ($p \ge 2$), are 2-uniformly smooth, while L^p (or l^p) and W_m^p spaces ($1 < p \le 2$) are p-uniformly smooth,

(3) typical examples of both uniformly convex and uniformly smooth Banach spaces are L^p, where $p > 1$. More precisely, L^p is $\min\{p, 2\}$-uniformly smooth for any $p > 1$.

Next, we assume that E is a smooth Banach space. Let T be a mapping from E into itself. In this paper, we use $F(T)$ to denote the set of fixed points of the mapping T.

Recall that the mapping T is said to be nonexpansive if

$$\|Tx - Ty\| \le \|x - y\|, \quad \forall x, y \in E. \tag{1.6}$$

T is said to be λ-strictly pseudocontractive if there exists a constant $\lambda \in (0,1)$ such that

$$\langle Tx - Ty, J(x-y)\rangle \le \|x-y\|^2 - \lambda\|(I-T)x - (I-T)y\|^2, \quad \forall x, y \in C. \tag{1.7}$$

Recall that an operator A of E into itself is said to be accretive if

$$\langle Ax - Ay, J(x-y)\rangle \ge 0, \quad \forall x, y \in E, \tag{1.8}$$

and, for any $\alpha > 0$, an operator A of E into itself is said to be α-inverse strongly accretive if

$$\langle Ax - Ay, J(x-y) \rangle \geq \alpha \| Ax - Ay \|^2, \quad \forall x, y \in E. \tag{1.9}$$

Evidently, the definition of the inverse-strongly accretive operator is based on that of the inverse-strongly monotone operator in real Hilbert spaces (see, e.g., [2]).

Next, we consider a system of quasivariational inclusions.

Find $(x^*, y^*) \in E \times E$ such that

$$\begin{aligned} &0 \in x^* - y^* + \rho_1 (A_1 y^* + M_1 x^*), \\ &0 \in y^* - x^* + \rho_2 (A_2 x^* + M_2 y^*), \end{aligned} \tag{1.10}$$

where $A_i : E \to E$ and $M_i : E \to 2^E$ are nonlinear mappings for each $i = 1, 2$.

As special cases of problem (1.10), we have the following.

(1) If $A_1 = A_2 = A$ and $M_1 = M_2 = M$, then problem (1.10) is reduced to the following.

Find $(x^*, y^*) \in E \times E$ such that

$$\begin{aligned} &0 \in x^* - y^* + \rho_1 (A y^* + M x^*), \\ &0 \in y^* - x^* + \rho_2 (A x^* + M y^*). \end{aligned} \tag{1.11}$$

(2) Further, if $x^* = y^*$ in problem (1.11), then problem (1.11) is reduced to the following.

Find $x^* \in E$ such that

$$0 \in Ax^* + Mx^*. \tag{1.12}$$

In 2006, Aoyama et al. [3] considered the following problem.

Find $u \in C$ such that

$$\langle Au, J(v - u) \rangle \geq 0, \quad \forall v \in C. \tag{1.13}$$

They proved that the variational inequality (1.13) is equivalent to a fixed point problem. The element $u \in C$ is a solution of the variational inequality (1.13) if and only if $u \in C$ satisfies the following equation:

$$u = P_C(u - \lambda Au), \tag{1.14}$$

where $\lambda > 0$ is a constant and P_C is a sunny nonexpansive retraction from E onto C, see the definition below.

Let D be a subset of C and P a mapping of C into D. Then P is said to be sunny if

$$P(Px + t(x - Px)) = Px, \tag{1.15}$$

whenever $Px + t(x - Px) \in C$ for $x \in C$ and $t \geq 0$. A mapping P of C into itself is called a retraction if $P^2 = P$. If a mapping P of C into itself is a retraction, then $Pz = z$ for all $z \in R(P)$, where $R(P)$ is the range of P. A subset D of C is called a sunny nonexpansive retract of C if there exists a sunny nonexpansive retraction from C onto D.

The following results describe a characterization of sunny nonexpansive retractions on a smooth Banach space.

Proposition 1.1 (see [4]). *Let E be a smooth Banach space and C a nonempty subset of E. Let $P : E \to C$ be a retraction and J the normalized duality mapping on E. Then the following are equivalent:*

(1) *P is sunny and nonexpansive;*

(2) $\langle x - Px, J(y - Px)\rangle \leq 0$, *for all* $x \in E$, $y \in C$.

Proposition 1.2 (see [5]). *Let C be a nonempty closed convex subset of a uniformly convex and uniformly smooth Banach space E and T a nonexpansive mapping of C into itself with $F(T) \neq \emptyset$. Then the set $F(T)$ is a sunny nonexpansive retract of C.*

For the class of nonexpansive mappings, one classical way to study nonexpansive mappings is to use contractions to approximate a nonexpansive mapping [6, 7]. More precisely, take $t \in (0,1)$ and define a contraction $T_t : C \to C$ by

$$T_t x = tu + (1-t)Tx, \quad \forall x \in C, \tag{1.16}$$

where $u \in C$ is a fixed point. Banach's contraction mapping principle guarantees that T_t has a unique fixed point x_t in C, that is,

$$x_t = tu + (1-t)Tx_t. \tag{1.17}$$

It is unclear, in general, what the behavior of x_t is as $t \to 0$, even if T has a fixed point. However, in the case of T having a fixed point, Browder [6] proved that if E is a Hilbert space, then x_t converges strongly to a fixed point of T. Reich [7] extended Browder's result to the setting of Banach spaces and proved that if E is a uniformly smooth Banach space, then x_t converges strongly to a fixed point of T and the limit defines the (unique) sunny nonexpansive retraction from C onto $F(T)$.

Reich [7] showed that, if E is uniformly smooth and D is the fixed point set of a nonexpansive mapping from C into itself, then there is a unique sunny nonexpansive retraction from C onto D and it can be constructed as follows.

Proposition 1.3. *Let E be a uniformly smooth Banach space and $T : C \to C$ a nonexpansive mapping with a fixed point. For each fixed $u \in C$ and every $t \in (0,1)$, the unique fixed point $x_t \in C$ of the contraction $C \ni x \mapsto tu + (1-t)Tx$ converges strongly as $t \to 0$ to a fixed point of T. Define $P : C \to D$ by $Pu = s - \lim_{t\to 0} x_t$. Then P is the unique sunny nonexpansive retract from C onto D, that is, P satisfies the property.*

$$\langle u - Pu, J(y - Pu)\rangle \leq 0, \quad \forall u \in C, \ y \in D. \tag{1.18}$$

Recently, many authors have studied the problems of finding a common element of the set of fixed points of a nonexpansive mapping and of the set of solutions to the variational inequality (1.13) by iterative methods (see, e.g., [3, 8–10]).

Aoyama et al. [3] proved the following theorem by using above propositions.

Theorem AIT. *Let E be a uniformly convex and 2-uniformly smooth Banach space and C a nonempty closed convex subset of E. Let P_C be a sunny nonexpansive retraction from E onto C, $\alpha > 0$ and A an α-inverse strongly-accretive operator of C into E with $S(C, A) \neq \emptyset$, where*

$$S(C, A) = \{x^* \in C : \langle Ax^*, J(x - x^*)\rangle \geq 0,\ x \in C\}. \tag{1.19}$$

If $\{\lambda_n\}$ and $\{\alpha_n\}$ are chosen such that $\lambda_n \in [a, \alpha/K^2]$ for some $a > 0$ and $\alpha_n \in [b, c]$ for some b, c with $0 < b < c < 1$, then the sequence $\{x_n\}$ defined by the following manners:

$$x_1 = x \in C, \qquad x_{n+1} = \alpha_n x_n + (1 - \alpha_n)P_C(x_n - \lambda_n A x_n) \tag{1.20}$$

converges weakly to some element z of $S(C, A)$, where K is the 2-uniformly smoothness constant of E.

Definition 1.4 (see [11]). Let $M : E \to 2^E$ be a multivalued maximal accretive mapping. The single valued mapping $J_{(M,\rho)} : E \to E$ defined by

$$J_{(M,\rho)}(u) = (I + \rho M)^{-1}(u), \quad \forall u \in E \tag{1.21}$$

is called the resolvent operator associated with M, where ρ is any positive number and I is the identity mapping.

Recently, Zhang et al. [11] considered problem (1.12) in Hilbert spaces. To be more precise, they proved the following theorem.

Theorem ZLC. *Let H be a real Hilbert space, $A : H \to H$ an α-cocoercive mapping, $M : H \to 2^H$ a maximal monotone mapping, and $S : H \to H$ a nonexpansive mapping. Suppose that the set $F(S) \cap \mathrm{VI}(H, A, M) \neq \emptyset$, where $\mathrm{VI}(H, A, M)$ is the set of solutions of variational inclusion* (1.12). *Suppose that $x_0 = x \in H$ and $\{x_n\}$ is the sequence defined by*

$$\begin{aligned} x_{n+1} &= \alpha_n x_0 + (1 - \alpha_n)Sy_n, \\ y_n &= J_{(M,\lambda)}(x_n - \lambda A x_n), \quad n \geq 0, \end{aligned} \tag{1.22}$$

where $\lambda \in (0, 2\alpha)$ and $\{\alpha_n\}$ is a sequence in $[0, 1]$ satisfying the following conditions:

(1) $\lim_{n\to\infty}\alpha_n = 0$, $\sum_{n=1}^{\infty} \alpha_n = \infty$;

(2) $\sum_{n=0}^{\infty} |\alpha_{n+1} - \alpha_n| < \infty$.

Then $\{x_n\}$ converges strongly to $P_{F(S)\cap \mathrm{VI}(H,A,M)}x_0$.

In this paper, motivated by Ceng et al. [12], Cho and Qin [13], Cho et al. [8], Hao [9], Iiduka and Takahashi [14], Noor [15], Qin et al. [16], Takahashi and Toyoda [17], Y. Yao and J. C. Yao [18], Zhao et al. [19], and Zhang et al. [11], we consider a relaxed extragradient-type method for finding common elements of the set of solutions to a general system of variational inclusions with inverse-strongly accretive mappings and common set of fixed points for a λ-strict pseudocontraction. Note that no Banach space is q-uniformly smooth for $q > 2$ (see [20] for more details). Strong convergence theorems are established in uniformly convex and 2-uniformly smooth Banach spaces by some authors. The results presented in this paper improve and extend the corresponding results announced by many others.

In order to prove our main results, we need the following lemmas.

Lemmas 1.5 and 1.6 can be obtained from Aoyama et al. [21]; see also Zhang et al. [11].

Lemma 1.5. *The resolvent operator $J_{(M,\rho)}$ associated with M is single valued and nonexpansive for all $\rho > 0$.*

Lemma 1.6. *$u \in E$ is a solution of variational inclusion* (1.12) *if and only if $u = J_{(M,\rho)}(u - \rho Au)$, for all $\rho > 0$, that is,*

$$\mathrm{VI}(E, A, M) = F\big(J_{(M,\rho)}(I - \rho A)\big), \quad \forall \rho > 0, \tag{1.23}$$

where $\mathrm{VI}(E, A, M)$ *denotes the set of solutions to problem* (1.12).

Lemma 1.7. *For any $(x^*, y^*) \in E \times E$, where $y^* = J_{(M_2,\rho_2)}(x^* - \rho_2 A_2 x^*)$, (x^*, y^*) is a solution of problem* (1.10) *if and only if x^* is a fixed point of the mapping Q defined by*

$$Q(x) = J_{(M_1,\rho_1)}\big[J_{(M_2,\rho_2)}(x - \rho_2 A_2 x) - \rho_1 A_1 J_{(M_2,\rho_2)}(x - \rho_2 A_2 x)\big]. \tag{1.24}$$

Proof. Note that

$$\begin{gathered} 0 \in x^* - y^* + \rho_1\big(A_1 y^* + M_1 x^*\big), \\ 0 \in y^* - x^* + \rho_2\big(A_2 x^* + M_2 y^*\big) \\ \Updownarrow \\ x^* = J_{(M_1,\rho_1)}\big(y^* - \rho_1 A_1 y^*\big), \\ y^* = J_{(M_2,\rho_2)}\big(x^* - \rho_2 A_2 x^*\big) \\ \Updownarrow \\ Q(x^*) = J_{(M_1,\rho_1)}\big[J_{(M_2,\rho_2)}(x^* - \rho_2 A_2 x^*) - \rho_1 A_1 J_{(M_2,\rho_2)}(x^* - \rho_2 A_2 x^*)\big] = x^*. \end{gathered} \tag{1.25}$$

This completes the proof. □

The following lemma is a corollary of Bruck's result in [22].

Lemma 1.8. *Let E be a strictly convex Banach space. Let T_1 and T_2 be two nonexpansive mappings from E into itself with a common fixed point. Define a mapping $S : E \to E$ by*

$$Sx = \lambda T_1 x + (1 - \lambda) T_2 x, \quad \forall x \in E, \tag{1.26}$$

where λ is a constant in $(0, 1)$. Then S is nonexpansive and $F(S) = F(T_1) \cap F(T_2)$.

Proof. It is obvious that $F(T_1) \cap F(T_2) \subset F(S)$. Fixing $x^* \in F(S)$ and $y \in F(T_1) \cap F(T_2)$, we see that

$$\begin{aligned}
\|x^* - y\| &= \|\lambda T_1 x^* + (1-\lambda) T_2 x^* - y\| \\
&\le \lambda \|T_1 x^* - y\| + (1-\lambda)\|T_2 x^* - y\| \\
&\le \lambda \|x^* - y\| + (1-\lambda)\|x^* - y\| \\
&= \|x^* - y\|.
\end{aligned} \tag{1.27}$$

Since E is strictly convex, it follows that

$$x^* = \lambda T_1 x^* + (1-\lambda) T_2 x^* = T_1 x^* = T_2 x^*, \tag{1.28}$$

that is, $x^* \in F(T_1) \cap F(T_2)$. This implies that $F(S) = F(T_1) \cap F(T_2)$. On the other hand, it is easy to see that S is also nonexpansive. This completes the proof. □

Lemma 1.9 (see [23]). *Let E be a uniformly convex Banach space and S a nonexpansive mapping on E. Then $I - S$ is demiclosed at zero.*

Lemma 1.10 (see [24]). *Assume that $\{\alpha_n\}$ is a sequence of nonnegative real numbers such that*

$$\alpha_{n+1} \le (1-\gamma_n)\alpha_n + \delta_n, \tag{1.29}$$

where $\{\gamma_n\}$ is a sequence in $(0,1)$ and $\{\delta_n\}$ is a sequence such that

(a) $\sum_{n=1}^{\infty} \gamma_n = \infty$;

(b) $\limsup_{n\to\infty} \delta_n/\gamma_n \le 0$ *or* $\sum_{n=1}^{\infty} |\delta_n| < \infty$,

then $\lim_{n\to\infty} \alpha_n = 0$.

Lemma 1.11 (see [25]). *Let $\{x_n\}$ and $\{y_n\}$ be bounded sequences in a Banach space E and $\{\beta_n\}$ a sequence in $[0,1]$ with $0 < \liminf_{n\to\infty}\beta_n \le \limsup_{n\to\infty}\beta_n < 1$. Suppose that $x_{n+1} = (1-\beta_n)y_n + \beta_n x_n$ for all $n \ge 0$ and*

$$\limsup_{n\to\infty} \left(\|y_{n+1} - y_n\| - \|x_{n+1} - x_n\|\right) \le 0, \tag{1.30}$$

then $\lim_{n\to\infty} \|y_n - x_n\| = 0$.

Lemma 1.12 (see [26]). *Let E be a real 2-uniformly smooth Banach space and $T : E \to E$ a λ-strict pseudocontraction. Then $S := (1 - \lambda/K^2)I + \lambda/K^2 T$ is nonexpansive and $F(T) = F(S)$.*

Lemma 1.13 (see [20]). *Let E be a real 2-uniformly smooth Banach space with the best smooth constant K. Then the following inequality holds:*

$$\|x + y\|^2 \le \|x\|^2 + 2\langle y, Jx\rangle + 2\|Ky\|^2, \quad \forall x, y \in E. \tag{1.31}$$

2. Main Results

Now, we are ready to give our main results in this paper.

Theorem 2.1. *Let E be a uniformly convex and 2-uniformly smooth Banach space with the smooth constant K. Let $M_i : E \to 2^E$ be a maximal monotone mapping and $A_i : E \to E$ a γ_i-inverse-strongly accretive mapping, respectively, for each $i = 1, 2$. Let $T : E \to E$ be a λ-strict pseudocontraction with a fixed point. Define a mapping S by $Sx = (1 - (\lambda/K^2))x + (\lambda/K^2)Tx$, for all $x \in E$. Assume that $\Omega = F(T) \cap F(Q) \neq \emptyset$, where Q is defined as Lemma 1.7. Let $x_1 = u \in E$ and $\{x_n\}$ be a sequence generated by*

$$\begin{aligned} z_n &= J_{(M_2,\rho_2)}(x_n - \rho_2 A_2 x_n), \\ y_n &= J_{(M_1,\rho_1)}(z_n - \rho_1 A_1 z_n), \\ x_{n+1} &= \alpha_n u + \beta_n x_n + (1 - \beta_n - \alpha_n)[\mu S x_n + (1-\mu) y_n], \quad \forall n \geq 1, \end{aligned} \tag{γ}$$

where $\mu \in (0,1)$, $\rho_1 \in (0, \gamma_1/K^2]$, $\rho_2 \in (0, \gamma_2/K^2]$, and $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in (0,1). If the control consequences $\{\alpha_n\}$ and $\{\beta_n\}$ satisfy the following restrictions:

(C1) $0 < \liminf_{n\to\infty}\beta_n \leq \limsup_{n\to\infty}\beta_n < 1$

(C2) $\lim_{n\to\infty}\alpha_n = 0$ *and* $\sum_{n=1}^{\infty}\alpha_n = \infty$,

then $\{x_n\}$ converges strongly to $x^* = P_\Omega u$, where P_Ω is the sunny nonexpansive retraction from E onto Ω and (x^*, y^*), where $y^* = J_{(M_2,\rho_2)}(x^* - \rho_2 A_2 x^*)$, is a solution to problem (1.10).

Proof. First, we show that the mappings $I - \rho_1 A_1$ and $I - \rho_2 A_2$ are nonexpansive. Indeed, for all $x, y \in E$, from the condition $\rho_1 \in (0, \gamma_1/K^2)$ and Lemma 1.13, one has

$$\begin{aligned} &\|(I-\rho_1 A_1)x - (I-\rho_1 A_1)y\|^2 \\ &\quad = \|(x-y) - \rho_1(A_1x - A_1y)\|^2 \\ &\quad \leq \|x-y\|^2 - 2\rho_1\langle A_1x - A_1y, J(x-y)\rangle + 2K^2\rho_1^2\|A_1x - A_1y\|^2 \\ &\quad \leq \|x-y\|^2 - 2\rho_1\gamma_1\|A_1x - A_1y\|^2 + 2K^2\rho_1^2\|A_1x - A_1y\|^2 \\ &\quad = \|x-y\|^2 - 2\rho_1\left(\gamma_1 - K^2\rho_1\right)\|A_1x - A_1y\|^2 \\ &\quad \leq \|x-y\|^2, \end{aligned} \tag{2.1}$$

which implies the mapping $I - \rho_1 A_1$ is nonexpansive and so is $I - \rho_2 A_2$. Taking $\overline{x} \in \Omega$, one has

$$\overline{x} = J_{(M_1,\rho_1)}\left[J_{(M_2,\rho_2)}(\overline{x} - \rho_2 A_2 \overline{x}) - \rho_1 A_1 J_{(M_2,\rho_2)}(\overline{x} - \rho_2 A_2 \overline{x})\right]. \tag{2.2}$$

Putting $\overline{y} = J_{(M_2,\rho_2)}(\overline{x} - \rho_2 A_2 \overline{x})$, one sees that

$$\overline{x} = J_{(M_1,\rho_1)}(\overline{y} - \rho_1 A_1 \overline{y}). \tag{2.3}$$

It follows from Lemma 1.5 that

$$\begin{aligned}\|z_n - \overline{y}\| &= \left\|J_{(M_2,\rho_2)}(x_n - \rho_2 A_2 x_n) - J_{(M_2,\rho_2)}(\overline{x} - \rho_2 A_2 \overline{x})\right\| \\ &\leq \left\|(x_n - \rho_2 A_2 x_n) - (\overline{x} - \rho_2 A_2 \overline{x})\right\| \\ &\leq \|x_n - \overline{x}\|.\end{aligned} \tag{2.4}$$

This implies that

$$\begin{aligned}\|y_n - \overline{x}\| &= \left\|J_{(M_1,\rho_1)}(z_n - \rho_1 A_1 z_n) - J_{(M_1,\rho_1)}(\overline{y} - \rho_1 A_1 \overline{y})\right\| \\ &\leq \left\|(z_n - \rho_1 A_1 z_n) - (\overline{y} - \rho_1 A_1 \overline{y})\right\| \\ &\leq \|z_n - \overline{y}\| \\ &\leq \|x_n - \overline{x}\|.\end{aligned} \tag{2.5}$$

Set $t_n = \mu S x_n + (1-\mu) y_n$. It follows from Lemma 1.12 that S is nonexpansive. This implies that

$$\begin{aligned}\|t_n - \overline{x}\| &= \left\|\mu S x_n + (1-\mu) y_n - \overline{x}\right\| \\ &\leq \mu\|S x_n - \overline{x}\| + (1-\mu)\left\|y_n - \overline{x}\right\| \\ &\leq \|x_n - \overline{x}\|,\end{aligned} \tag{2.6}$$

from which it follows that

$$\begin{aligned}\|x_{n+1} - \overline{x}\| &= \left\|\alpha_n u + \beta_n x_n + (1-\beta_n-\alpha_n) t_n - \overline{x}\right\| \\ &\leq \alpha_n\|u - \overline{x}\| + \beta_n\|x_n - \overline{x}\| + (1-\beta_n-\alpha_n)\|t_n - \overline{x}\| \\ &\leq \alpha_n\|u - \overline{x}\| + (1-\alpha_n)\|x_n - \overline{x}\| \\ &\leq \max\{\|u - \overline{x}\|, \|x_1 - \overline{x}\|\}.\end{aligned} \tag{2.7}$$

This shows that the sequence $\{x_n\}$ is bounded, so are $\{y_n\}$, $\{z_n\}$, and $\{t_n\}$.

On the other hand, from the nonexpansivity of the mappings $J_{(M_2,\rho_2)}$, one sees that

$$\begin{aligned}\|y_{n+1} - y_n\| &= \left\|J_{(M_1,\rho_1)}(z_{n+1} - \rho_1 A_1 z_{n+1}) - J_{(M_1,\rho_1)}(z_n - \rho_1 A_1 z_n)\right\| \\ &\leq \left\|(z_{n+1} - \rho_1 A_1 z_{n+1}) - (z_n - \rho_1 A_1 z_n)\right\| \\ &\leq \|z_{n+1} - z_n\|.\end{aligned} \tag{2.8}$$

In a similar way, one can obtain that

$$\|z_{n+1} - z_n\| \leq \|x_{n+1} - x_n\|. \tag{2.9}$$

It follows that

$$\|y_{n+1} - y_n\| \leq \|x_{n+1} - x_n\|. \tag{2.10}$$

This implies that

$$\begin{aligned}\|t_{n+1} - t_n\| &= \|\mu S x_{n+1} + (1-\mu) y_{n+1} - [\mu S x_n + (1-\mu) y_n]\| \\ &\leq \mu \|S x_{n+1} - S x_n\| + (1-\mu) \|y_{n+1} - y_n\| \\ &\leq \mu \|x_{n+1} - x_n\| + (1-\mu) \|x_{n+1} - x_n\| \\ &= \|x_{n+1} - x_n\|.\end{aligned} \tag{2.11}$$

Setting

$$x_{n+1} = (1-\beta_n) e_n + \beta_n x_n, \quad \forall n \geq 1, \tag{2.12}$$

one sees that

$$\begin{aligned}e_{n+1} - e_n &= \frac{\alpha_{n+1} u + (1 - \beta_{n+1} - \alpha_{n+1}) t_{n+1}}{1 - \beta_{n+1}} - \frac{\alpha_n u + (1 - \beta_n - \alpha_n) t_n}{1 - \beta_n} \\ &= \frac{\alpha_{n+1}}{1 - \beta_{n+1}} (u - t_{n+1}) + t_{n+1} - \frac{\alpha_n}{1 - \beta_n} (u - t_n) - t_n,\end{aligned} \tag{2.13}$$

and so it follows that

$$\|e_{n+1} - e_n\| \leq \frac{\alpha_{n+1}}{1 - \beta_{n+1}} \|u - t_{n+1}\| + \frac{\alpha_n}{1 - \beta_n} \|u - t_n\| + \|t_{n+1} - t_n\|, \tag{2.14}$$

which combined with (2.11) yields that

$$\|e_{n+1} - e_n\| - \|x_{n+1} - x_n\| \leq \frac{\alpha_{n+1}}{1 - \beta_{n+1}} \|u - t_{n+1}\| + \frac{\alpha_n}{1 - \beta_n} \|u - t_n\|. \tag{2.15}$$

It follows from the conditions (C1) and (C2) that

$$\limsup_{n \to \infty} (\|e_{n+1} - e_n\| - \|x_{n+1} - x_n\|) \leq 0. \tag{2.16}$$

Hence, from Lemma 1.11, it follows that

$$\lim_{n \to \infty} \|e_n - x_n\| = 0. \tag{2.17}$$

From (2.12), it follows that

$$\|x_{n+1} - x_n\| = (1 - \beta_n) \|e_n - x_n\|. \tag{2.18}$$

Using condition (C1), one sees that

$$\lim_{n\to\infty} \|x_{n+1} - x_n\| = 0. \tag{2.19}$$

On the other hand, one has

$$x_{n+1} - x_n = \alpha_n(u - t_n) + (1 - \beta_n)(t_n - x_n). \tag{2.20}$$

It follows that

$$(1 - \beta_n)\|t_n - x_n\| \le \|x_{n+1} - x_n\| + \alpha_n\|u - t_n\|. \tag{2.21}$$

From the conditions (C1), (C2), and (2.19), one sees that

$$\lim_{n\to\infty} \|t_n - x_n\| = 0. \tag{2.22}$$

Next, we prove that

$$\limsup_{n\to\infty} \langle u - x^*, J(x_n - x^*)\rangle \le 0, \tag{2.23}$$

where $x^* = P_\Omega u$, and P_Ω is the sunny nonexpansive retraction from E onto Ω. Define a mapping W by

$$Wy = \mu Sy + (1 - \mu)J_{(M_1,\rho_1)}(I - \rho_1 A_1)J_{(M_2,\rho_2)}(I - \rho_2 A_2)y, \quad \forall y \in E. \tag{2.24}$$

In view of Lemmas 1.7 and 1.8, we see that W is nonexpansive such that

$$F(W) = F(S) \cap F\left(J_{(M_1,\rho_1)}(I - \rho_1 A_1)J_{(M_2,\rho_2)}(I - \rho_2 A_2)\right) = F(T) \cap F(Q). \tag{2.25}$$

From (2.22), it follows that

$$\lim_{n\to\infty} \|Wx_n - x_n\| = 0. \tag{2.26}$$

Let z_t be the fixed point of the contraction $z \mapsto tu + (1 - t)Wz$, where $t \in (0, 1)$. That is,

$$z_t = tu + (1 - t)Wz_t. \tag{2.27}$$

It follows that

$$\|z_t - x_n\| = \|(1 - t)(Wz_t - x_n) + t(u - x_n)\|. \tag{2.28}$$

On the other hand, we have

$$\begin{aligned}\|z_t - x_n\|^2 &\le (1-t)^2\|Wz_t - x_n\|^2 + 2t\langle u - x_n, J(z_t - x_n)\rangle \\ &\le \left(1 - 2t + t^2\right)\|z_t - x_n\|^2 + f_n(t) \\ &\quad + 2t\langle u - z_t, J(z_t - x_n)\rangle + 2t\|z_t - x_n\|^2,\end{aligned} \tag{2.29}$$

where

$$f_n(t) = (2\|z_t - x_n\| + \|x_n - Wx_n\|)\|x_n - Wx_n\| \longrightarrow 0 \quad \text{as } n \to 0. \tag{2.30}$$

It follows that

$$\langle z_t - u, J(z_t - x_n)\rangle \le \frac{t}{2}\|z_t - x_n\|^2 + \frac{1}{2t}f_n(t). \tag{2.31}$$

In view of (2.30), we arrive at

$$\limsup_{n\to\infty}\langle z_t - u, J(z_t - x_n)\rangle \le \frac{t}{2}M, \tag{2.32}$$

where $M > 0$ is an appropriate constant such that $M \ge \|z_t - x_n\|^2$ for all $t \in (0,1)$ and $n \ge 1$. Letting $t \to 0$ in (2.32), we have

$$\limsup_{t\to 0}\limsup_{n\to\infty}\langle z_t - u, J(z_t - x_n)\rangle \le 0. \tag{2.33}$$

So, for any $\epsilon > 0$, there exists a positive number δ_1 with $t \in (0, \delta_1)$ such that

$$\limsup_{n\to\infty}\langle z_t - u, J(z_t - x_n)\rangle \le \frac{\epsilon}{2}. \tag{2.34}$$

On the other hand, we see that $P_{F(W)}u = \lim_{t\to 0} z_t$ and $F(W) = \Omega$. It follows that $z_t \to x^* = P_\Omega u$ as $t \to 0$. There exists $\delta_2 > 0$, for $t \in (0, \delta_2)$, such that

$$\begin{aligned}&|\langle u - x^*, J(x_n - x^*)\rangle - \langle z_t - u, J(z_t - x_n)\rangle| \\ &\quad\le |\langle u - x^*, J(x_n - x^*)\rangle - \langle u - x^*, J(x_n - z_t)\rangle| \\ &\qquad + |\langle u - x^*, J(x_n - z_t)\rangle - \langle z_t - u, J(z_t - x_n)\rangle| \\ &\quad\le |\langle u - x^*, J(x_n - x^*) - J(x_n - z_t)\rangle| + |\langle z_t - x^*, J(x_n - z_t)\rangle| \\ &\quad\le \|u - x^*\|\|J(x_n - x^*) - J(x_n - z_t)\| + \|z_t - x^*\|\|x_n - z_t\| \\ &\quad< \frac{\epsilon}{2}.\end{aligned} \tag{2.35}$$

Choosing $\delta = \min\{\delta_1, \delta_2\}$, it follows that, for each $t \in (0, \delta)$,

$$\langle u - x^*, J(x_n - x^*) \rangle \le \langle z_t - u, J(z_t - x_n) \rangle + \frac{\epsilon}{2}, \tag{2.36}$$

which implies that

$$\limsup_{n\to\infty} \langle u - x^*, J(x_n - x^*) \rangle \le \limsup_{n\to\infty} \langle z_t - u, J(z_t - x_n) \rangle + \frac{\epsilon}{2}. \tag{2.37}$$

It follows from (2.34) that

$$\limsup_{n\to\infty} \langle u - x^*, J(x_n - x^*) \rangle \le \epsilon. \tag{2.38}$$

Since ϵ is chosen arbitrarily, we have

$$\limsup_{n\to\infty} \langle u - x^*, J(x_n - x^*) \rangle \le 0. \tag{2.39}$$

Finally, we show that $x_n \to x^*$ as $n \to \infty$. Indeed,

$$\begin{aligned}
&\|x_{n+1} - x^*\|^2 \\
&\quad = \left\langle \alpha_n u + \beta_n x_n + (1 - \beta_n - \alpha_n) t_n - x^*, J(x_{n+1} - x^*) \right\rangle \\
&\quad = \alpha_n \langle u - x^*, J(x_{n+1} - x^*) \rangle + \beta_n \langle x_n - x^*, J(x_{n+1} - x^*) \rangle \\
&\qquad + \langle (1 - \beta_n - \alpha_n)(t_n - x^*), J(x_{n+1} - x^*) \rangle \\
&\quad \le \alpha_n \langle u - x^*, J(x_{n+1} - x^*) \rangle \\
&\qquad + \beta_n \|x_n - x^*\| \|x_{n+1} - x^*\| + (1 - \beta_n - \alpha_n) \|x_n - x^*\| \|x_{n+1} - x^*\| \\
&\quad \le \alpha_n \langle u - x^*, J(x_{n+1} - x^*) \rangle + \frac{(1 - \alpha_n)}{2} \left(\|x_n - x^*\|^2 + \|x_{n+1} - x^*\|^2 \right),
\end{aligned} \tag{2.40}$$

which implies that

$$\|x_{n+1} - x^*\|^2 \le (1 - \alpha_n) \|x_n - x^*\|^2 + 2\alpha_n \langle u - x^*, J(x_{n+1} - x^*) \rangle. \tag{2.41}$$

Therefore, from condition (C2), (2.39), and Lemma 1.10, we see that

$$\lim_{n\to\infty} \|x_n - x^*\| = 0. \tag{2.42}$$

This completes the proof. □

Remark 2.2. Theorem 2.1 which includes Ceng et al. [12], Y. Yao and J.C Yao [18] as special cases mainly improves Theorem 2.1 of Zhang et al. [11] in the following respects:

(a) from Hilbert spaces to Banach spaces;

(b) from a single variational inclusion to a system of variational inclusions;

(c) from nonexpansive mappings to strict pseudocontractions.

As some applications of Theorem 2.1, we have the following results.

Lemma 2.3. *For given $(x^*, y^*) \in E$, where $y^* = J_{(M,\rho_2)}(x^* - \rho_2 Ax^*)$, (x^*, y^*) is a solution of problem* (1.11) *if and only if x^* is a fixed point of the mapping Q' defined by*

$$Q'(x) = J_{(M,\rho_1)}\left[J_{(M,\rho_2)}(x - \rho_2 Ax) - \rho_1 A J_{(M,\rho_2)}(x - \rho_2 Ax)\right]. \tag{2.43}$$

Corollary 2.4. *Let E be a uniformly convex and 2-uniformly smooth Banach space with the smooth constant K. Let $M : E \to 2^E$ be a maximal monotone mapping and $A : E \to E$ a γ-inverse-strongly accretive mapping. Let $T : E \to E$ be a λ-strict pseudocontraction with a fixed point. Define a mapping S by $Sx = (1 - (\lambda/K^2))x + (\lambda/K^2)Tx$ for all $x \in E$. Assume that $\Omega = F(T) \cap F(Q') \neq \emptyset$, where Q' is defined as Lemma 2.3. Let $x_1 = u \in E$ and $\{x_n\}$ be a sequence generated by*

$$\begin{aligned} z_n &= J_{(M,\rho_2)}(x_n - \rho_2 Ax_n), \\ y_n &= J_{(M,\rho_1)}(z_n - \rho_1 Az_n), \\ x_{n+1} &= \alpha_n u + \beta_n x_n + (1 - \beta_n - \alpha_n)\left[\mu Sx_n + (1-\mu)y_n\right], \quad \forall n \geq 1, \end{aligned} \tag{2.44}$$

where $\mu \in (0,1)$, ρ_1, $\rho_2 \in (0, \gamma/K^2]$, $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in $(0,1)$. If the control consequences $\{\alpha_n\}$ and $\{\beta_n\}$ satisfy the following restrictions:

(C1) $0 < \liminf_{n\to\infty}\beta_n \leq \limsup_{n\to\infty}\beta_n < 1$,

(C2) $\lim_{n\to\infty}\alpha_n = 0$ *and* $\sum_{n=1}^{\infty}\alpha_n = \infty$,

then $\{x_n\}$ converges strongly to $x^* = P_\Omega u$, where P_Ω is the sunny nonexpansive retraction from E onto Ω and (x^*, y^*), where $y^* = J_{(M_2,\rho_2)}(x^* - \rho_2 Ax^*)$, is a solution to problem (1.11).

If E is a Hilbert space, then Corollary 2.4 is reduced to the following.

Corollary 2.5. *Let E be a real Hilbert space. Let $M : E \to 2^E$ be a maximal monotone mapping and $A : E \to E$ a γ-inverse-strongly monotone mapping. Let $T : E \to E$ be a λ-strict pseudocontraction with a fixed point. Define a mapping S by $Sx = (1 - 2\lambda)x + 2\lambda Tx$ for all $x \in E$. Assume that $\Omega = F(T) \cap F(Q') \neq \emptyset$, where Q' is defined as Lemma 2.3. Let $x_1 = u \in E$ and $\{x_n\}$ be a sequence generated by*

$$\begin{aligned} z_n &= J_{(M,\rho_2)}(x_n - \rho_2 Ax_n), \\ y_n &= J_{(M,\rho_1)}(z_n - \rho_1 Az_n), \\ x_{n+1} &= \alpha_n u + \beta_n x_n + (1 - \beta_n - \alpha_n)\left[\mu Sx_n + (1-\mu)y_n\right], \quad \forall n \geq 1, \end{aligned} \tag{2.45}$$

where $\mu \in (0,1)$*,* $\rho_1, \rho_2 \in (0, 2\gamma]$*,* $\{\alpha_n\}$ *and* $\{\beta_n\}$ *are sequences in* $(0,1)$*. If the control consequences* $\{\alpha_n\}$ *and* $\{\beta_n\}$ *satisfy the following restrictions:*

(C1) $0 < \liminf_{n\to\infty}\beta_n \le \limsup_{n\to\infty}\beta_n < 1$,

(C2) $\lim_{n\to\infty}\alpha_n = 0$ *and* $\sum_{n=1}^{\infty}\alpha_n = \infty$,

then $\{x_n\}$ *converges strongly to* $x^* = P_\Omega u$*, where* P_Ω *is the metric projection from* E *onto* Ω *and* (x^*, y^*)*, where* $y^* = J_{(M_2,\rho_2)}(x^* - \rho_2 A x^*)$*, is a solution to problem* (1.11)*.*

Further, if T is a nonexpansive mapping, then Corollary 2.5 is reduced to the following result.

Corollary 2.6. *Let* E *be a real Hilbert space. Let* $M : E \to 2^E$ *be a maximal monotone mapping and* $A : E \to E$ *a* γ*-inverse-strongly monotone mapping. Let* $T : E \to E$ *be a nonexpansive mapping with a fixed point. Assume that* $\Omega = F(T) \cap F(Q') \ne \emptyset$*, where* Q' *is defined as Lemma 2.3. Let* $x_1 = u \in E$ *and* $\{x_n\}$ *be a sequence generated by*

$$\begin{aligned} z_n &= J_{(M,\rho_2)}(x_n - \rho_2 A x_n), \\ y_n &= J_{(M,\rho_1)}(z_n - \rho_1 A z_n), \\ x_{n+1} &= \alpha_n u + \beta_n x_n + (1-\beta_n-\alpha_n)\left[\mu T x_n + (1-\mu) y_n\right], \quad \forall n \ge 1, \end{aligned} \tag{2.46}$$

where $\mu \in (0,1)$*,* $\rho_1, \rho_2 \in (0, 2\gamma]$*,* $\{\alpha_n\}$ *and* $\{\beta_n\}$ *are sequences in* $(0,1)$*. If the control consequences* $\{\alpha_n\}$ *and* $\{\beta_n\}$ *satisfy the following restrictions:*

(C1) $0 < \liminf_{n\to\infty}\beta_n \le \limsup_{n\to\infty}\beta_n < 1$,

(C2) $\lim_{n\to\infty}\alpha_n = 0$ *and* $\sum_{n=1}^{\infty}\alpha_n = \infty$,

then $\{x_n\}$ *converges strongly to* $x^* = P_\Omega u$*, where* P_Ω *is the metric projection from* E *onto* Ω *and* (x^*, y^*)*, where* $y^* = J_{(M_2,\rho_2)}(x^* - \rho_2 A x^*)$*, is a solution to problem* (1.11)*.*

Acknowledgment

The authors are grateful to two anonymous referees for useful suggestions that improved the contents of the paper. This work was supported by the Korea Research Foundation Grant funded by the Korean Government (KRF-2008-313-C00050).

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