Critical points of maximal $D = 8$ gauged supergravities

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ABSTRACT: We study the general deformations of maximal eight-dimensional supergravity by using the embedding tensor approach. The scalar potential induced by these gaugings is determined. Subsequently, by combining duality covariance arguments and algebraic geometry techniques, we find the complete set of critical points of the scalar potential. Remarkably, up to $\mathrm{SO}(2) \times \mathrm{SO}(3)$ rotations there turns out to be a unique theory admitting extrema. The gauge group of the theory is $\mathrm{CSO}(2, 0, 1)$.

Contents

1 Introduction

In the last decade a new formalism has been constructed in extended supergravity theories which is able to comprise all the consistent gaugings of a theory in a single universal formulation [1–5]. This goes under the name of embedding tensor formalism and it describes deformations of extended supergravities in a duality covariant way. Indeed the duality group $\mathrm{E}_{d(d)}$ of the ungauged theory in $D = 11 - d$ dimensions obtained from the compactification of eleven-dimensional supergravity on a $d-$torus turns out to determine all the possible deformations (gaugings) thereof.

The theories in our interest are maximal gauged supergravities in $D = 8$. These theories present, in analogy with half-maximal supergravity in $D = 4$, an SL(2) factor in the global symmetry group which allows for gaugings at angles, i.e., gaugings in which the gauge generators point in different SL(2) directions. This feature seems to play the role of the so-called duality angles [6] in half-maximal supergravity in $D = 4$, even though the interpretation of this SL(2) symmetry as electromagnetic duality is different in $D = 8$, since now the 3-forms rather than the vectors build SL(2) doublets with their Hodge duals.

Some interesting gaugings in $D = 8$ were already studied in the literature, e.g., the SO(3) gauging found in ref. [7] resulting from compactifying eleven-dimensional supergravity on S^3; furthermore, all the gaugings in $D = 8$ without non-trivial SL(2) phases have been classified in terms of their eleven-dimensional origin [8–10] by means of a compactification on a group manifold of dimension 3. They are divided into two categories: in the

first one we find all the gaugings of the type $\mathrm{CSO}(p,q,r)$ [11] with $p+q+r=3$, which arise from a compactification on class A group manifolds according to the Bianchi classification; in the second one, we find a set of gaugings which are peculiar because of their lack of an action principle formulation (class B group manifold reductions). These theories might stem from the procedure of gauging the so-called trombone symmetry (see e.g., ref. [12] where this has been investigated in the maximal $D=4$ case).

In the context of the embedding tensor, one gives a complete duality covariant classification of all the gaugings; a natural question to address is then which of those gaugings actually have a well-understood eleven-dimensional origin. In contrast with the case of maximal gauged supergravity theories in $D=9$, where all the consistent deformations turn out to come from higher dimensions [13], in $D=8$ there are gaugings for which no higher-dimensional origin is known yet, e.g., gaugings at angles.

The main goal of the paper is to derive the scalar potential for the most general gauging in $D=8$ compatible with maximal supersymmetry and to study the set of its critical points. The paper is organised as follows. In section 2 we briefly present the embedding tensor formalism in $D=8$, we describe the general deformation by means of group theory and we give the quadratic constraints. In section 3 we observe that any consistent gauging in $D=8$ is mapped into a consistent gauging in $D=7$ upon reduction over an S^1 and subsequently we derive the scalar potential of the eight-dimensional theory by making use of the scalar potential of the seven-dimensional theory studied in ref. [3]. Finally, in section 4, we make use of some algebraic geometry techniques in order to study the complete landscape of vacua that these theories have. The main result of this paper is that there is a unique $\mathrm{SO}(2)\times\mathrm{SO}(3)$ orbit of gaugings of maximal $D=8$ supergravity allowing for critical points of the scalar potential. Each of these corresponds to a $\mathrm{CSO}(2,0,1)$ gauging admitting a non-supersymmetric Minkowski extremum.

2 Overview on maximal $D=8$ supergravities

2.1 The ungauged theory

The maximal (ungauged) supergravity in $D=8$ can be obtained by reducing eleven-dimensional supergravity on a T^3. The global symmetry group of this theory is $G_0 = \mathrm{SL}(2)\times\mathrm{SL}(3)$. The full field content consists of the following objects (which arrange themselves into irrep's of G_0):

$$8\mathrm{D}: \qquad \underbrace{e_\mu{}^a,\, A_\mu{}^{Im},\, B_{\mu\nu m},\, C_{\mu\nu\rho},\, L_m{}^i,\, \phi,\, \chi}_{\text{bosonic dof's}}\; ;\; \underbrace{\psi_\mu,\, \chi_i}_{\text{fermionic dof's}}, \qquad (2.1)$$

where $\mu,\nu,\cdots$ denote eight-dimensional curved spacetime , $a,b,\cdots$ eight-dimensional flat spacetime, $m,n,\cdots$ fundamental SL(3), $i,j,\cdots$ fundamental SO(3) and $I,J,\cdots$ fundamental SL(2) indices respectively.

The bosonic sector consists of the eight-dimensional vielbein $e_\mu{}^a$, a set of vector fields $A_\mu{}^{Im}$, an SL(3) triplet of two-forms $B_{\mu\nu m}$, a three-form $C_{\mu\nu\rho}$ and the scalars $L_m{}^i$ and (ϕ,χ) spanning the cosets SL(3)/SO(3) and SL(2)/SO(2) respectively. The fermionic

sector, instead, is made out of a doublet of symplectic-Majorana (SM) gravitini ψ_μ and a set of dilatini χ_i.

Let us introduce the following parametrisations in the scalar sector

$$W_{IJ} = \begin{pmatrix} e^{-\phi} + \chi^2 e^\phi & \chi e^\phi \\ \chi e^\phi & e^\phi \end{pmatrix} , \qquad M_{mn} = L_m{}^i L_n{}^j \delta_{ij} . \qquad (2.2)$$

The gravity/scalar part of the action reads [8]

$$S = \frac{1}{16\pi G_8} \int d^8x \; e \left(R + \frac{1}{4}\mathrm{Tr}(\partial M \partial M^{-1}) + \frac{1}{4}\mathrm{Tr}(\partial W \partial W^{-1}) \right) , \qquad (2.3)$$

where e is the determinant of the vielbein. The full bosonic action, in addition to the terms in (2.3), contains kinetic terms for the vector fields, the two- and three-forms and finally Chern-Simons terms.

2.2 Embedding tensor deformations

When gauging a subgroup of the global symmetry group, the embedding tensor is turned on, via which in the gauge-covariant derivative the vectors become coupled to group generators. The embedding tensor parameterizes the most general deformations consistent with the global symmetries and supersymmetry. It is an object of the form $\Theta_v{}^\alpha$, where the indices v and α live in the dual of the representation of the vectors and in the adjoint representation of the global symmetry group, respectively.

In the maximal $D = 8$ case, there are six vector fields $A_\mu{}^{Im}$ transforming in $V' = (\mathbf{2}, \mathbf{3'})$, the dual of the fundamental representation of G_0. And there are eleven group generators, which can be expressed in the adjoint representation $\mathfrak{g}_0$:[1]

$$\left[t_I{}^J \right]_K{}^L = \delta_I{}^L \delta_K{}^J - \frac{1}{2}\delta_I{}^J \delta_K{}^L , \qquad (2.4a)$$

$$\left[t_m{}^n \right]_p{}^q = \delta_m{}^q \delta_p{}^n - \frac{1}{3}\delta_m{}^n \delta_p{}^q. \qquad (2.4b)$$

The embedding tensor Θ then lives in the representation $\mathfrak{g}_0 \otimes V$, which can be decomposed into irreducible representations as

$$\mathfrak{g}_0 \otimes V = 2 \cdot (\mathbf{2}, \mathbf{3}) \oplus (\mathbf{2}, \mathbf{6'}) \oplus (\mathbf{2}, \mathbf{15}) \oplus (\mathbf{4}, \mathbf{3}) . \qquad (2.5)$$

Consistency and supersymmetry restrict the embedding tensor to the $(\mathbf{2}, \mathbf{3}) \oplus (\mathbf{2}, \mathbf{6'})$ [5]. This restriction goes under the name of linear constraint. It is worth noticing that there are two copies of the $(\mathbf{2}, \mathbf{3})$ irrep in the above composition; the linear constraint imposes a relation between them [14]. This shows that, for consistency, gauging some SL(2) generators implies the necessity of gauging some SL(3) generators as well. Let us denote the allowed embedding tensor irrep's by ξ_{Im} and $f_I{}^{(mn)}$ respectively. Then the following

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[1]A traceless pair of SL(2) indices $_I{}^J$ lives in its adjoint representation. So does a traceless pair $_m{}^n$ for SL(3).

parametrisation holds

$$\Theta_{Im,J}{}^K = \delta_I{}^K \xi_{Jm} - \frac{1}{2} \delta_J{}^K \xi_{Im} \ , \tag{2.6a}$$

$$\Theta_{Im,n}{}^p = \epsilon_{mnq} f_I{}^{qp} - \frac{3}{4} \left(\delta_m{}^p \xi_{In} - \frac{1}{3} \delta_n{}^p \xi_{Im} \right) \ . \tag{2.6b}$$

Furthermore, one can prove that the generators of the gauge group can be expressed in the same way:

$$(X_{Im})_J{}^K = \Theta_{Im,J'}{}^{K'} \left[t_{K'}{}^{J'} \right]_J{}^K = \delta_I{}^K \xi_{Jm} - \frac{1}{2} \delta_J{}^K \xi_{Im} \ , \tag{2.7a}$$

$$(X_{Im})_n{}^p = \Theta_{Im,n'}{}^{p'} \left[t_{p'}{}^{n'} \right]_n{}^p = \epsilon_{mnq} f_I{}^{qp} - \frac{3}{4} \left(\delta_m{}^p \xi_{In} - \frac{1}{3} \delta_n{}^p \xi_{Im} \right) \ . \tag{2.7b}$$

For closure of the algebra, the following quadratic constraints [15] should be imposed on the embedding tensor:

$$\epsilon^{IJ} \, \xi_{Ip} \xi_{Jq} = 0 \ , \qquad \qquad (\mathbf{1}, \mathbf{3'}) \tag{2.8a}$$

$$f_{(I}{}^{np} \xi_{J)p} = 0 \ , \qquad \qquad (\mathbf{3}, \mathbf{3'}) \tag{2.8b}$$

$$\epsilon^{IJ} \left(\epsilon_{mqr} f_I{}^{qn} f_J{}^{rp} + f_I{}^{np} \xi_{Jm} \right) = 0 \ . \qquad (\mathbf{1}, \mathbf{3'}) \oplus (\mathbf{1}, \mathbf{15}) \tag{2.8c}$$

In this paper, we are mostly interested in the scalar potential in the Lagrangian, which is quadratic in the embedding tensor. One can write down an Ansatz for such a potential:[2]

$$V = W^{IJ} \left[f_I{}^{mn} f_J{}^{pq} \left(a \, M_{mp} M_{nq} + b \, M_{mn} M_{pq} \right) + c \, \xi_{Im} \xi_{Jn} M^{mn} \right] \ , \tag{2.9}$$

where W_{IJ} and M_{mn} are elements of the scalar cosets introduced in (2.2), whereas W^{IJ} and M^{mn} denote their inverse matrices and a, b and c are coefficients that are going to be determined. The most convenient way of fixing these coefficients is to use the scalar potential in maximal $D = 7$ supergravity, which was already well studied in [3].

3 Gaugings of $D = 8$ supergravity as truncations of gaugings in $D = 7$

3.1 Review of maximal $D = 7$ supergravity

The general deformations of seven-dimensional maximal supergravity are constructed and presented in ref. [3]. For the sake of clarity, we briefly summarise the results obtained there. The global symmetry group is SL(5), which has an adjoint representation $\mathbf{24}$. The vectors[3] $A_\mu{}^{MN} = A_\mu{}^{[MN]}$ of the theory transform in the $\mathbf{10'}$ of SL(5). Then the embedding tensor Θ will take values in the following irrep's of SL(5)

$$\mathbf{10} \otimes \mathbf{24} = \mathbf{10} \oplus \mathbf{15} \oplus \mathbf{40'} \oplus \mathbf{175} . \tag{3.1}$$

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[2]The corresponding term to be added to the Lagrangian (2.3) should be $\mathcal{L}_V = -e g^2 V$, where g is the coupling strength.

[3]Here we denote by M a fundamental SL(5) index.

After imposing the linear constraint, the parametrization of the embedding tensor is restricted to only two irreducible components $\mathbf{15} \oplus \mathbf{40'}$:

$$Y_{MN} = Y_{(MN)} \qquad\qquad \mathbf{15}\;: \qquad \Box\Box \;, \qquad\qquad (3.2a)$$

$$Z^{MN,P} = Z^{[MN],P} \text{ with } Z^{[MN,P]} = 0 \qquad \mathbf{40'}\;: \; \begin{matrix}\Box\\\Box\end{matrix} \otimes \Box = \bcancel{\begin{matrix}\Box\\\Box\\\Box\end{matrix}} \oplus \begin{matrix}\Box\Box\\\Box\end{matrix} \;, \quad (3.2b)$$

where M, N and P represent fundamental SL(5) indices. Furthermore, supersymmetry and the consistency of the gauging require the following quadratic constraints to hold

$$Y_{MQ} Z^{QN,P} + 2\,\epsilon_{MRSTU}\, Z^{RS,N}\, Z^{TU,P} = 0 \;. \qquad (3.3)$$

Any embedding tensor configuration satisfying (3.3) identifies a gauging of a certain (at most) ten-dimensional group suitably embedded in SL(5). The expression of the gauge generators is given by [3]

$$(X_{MN})_P{}^Q = \delta_{[M}{}^Q Y_{N]P} - 2\,\epsilon_{MNPRS} Z^{RS,Q} \;, \qquad (3.4)$$

where the pair of indices $_P{}^Q$ is in the adjoint representation of SL(5) once the linear constraint is satisfied.

The scalar sector is described by the SL(5)/SO(5) coset geometry parametrised by the symmetric matrix $\mathcal{M}_{MN}$ with inverse $\mathcal{M}^{MN}$. This divides the isometry group of the scalar manifold SL(5) into unphysical scalar degrees of freedom (generating the adjoint representation of SO(5)) and physical scalar fields completing them to the $\mathbf{24}$, i.e., the adjoint representation of SL(5). Maximal supersymmetry completely and uniquely determines the scalar potential to be of the form

$$V = \frac{1}{64}\left(2\mathcal{M}^{MN} Y_{NP} \mathcal{M}^{PQ} Y_{QM} - (\mathcal{M}^{MN} Y_{MN})^2\right) +$$
$$+ Z^{MN,P} Z^{QR,S}\left(\mathcal{M}_{MQ}\mathcal{M}_{NR}\mathcal{M}_{PS} - \mathcal{M}_{MQ}\mathcal{M}_{NP}\mathcal{M}_{RS}\right). \qquad (3.5)$$

3.2 From $D = 7$ to $D = 8$

Every gauging in $D = 8$ must be an at most six-dimensional subgroup of the global symmetry group $SL(2) \times SL(3)$. After dimensional reduction to $D = 7$, the global symmetry group gets enhanced with respect to what one would naively expect;[4] for this reason, one would certainly expect any consistent gauging of the eight-dimensional theory to be reduced to a consistent gauging of the seven-dimensional theory where the gauge group, though, undergoes an enlargement just in the same way as for the global symmetry group. This statement implies that the irreducible components of the embedding tensor in eight dimensions must be obtained as a truncation of the embedding tensor in $D = 7$. This implies the possibility of deriving the scalar potential of maximal $D = 8$ gauged supergravity from the expression of the seven-dimensional scalar potential given in (3.5), after understanding

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[4]One would expect $\mathbb{R}^+ \times SL(2) \times SL(3)$, whereas it turns out to be enlarged to an SL(5).

how the eight-dimensional degrees of freedom associated with internal symmetries sit inside
SL(5) irrep's. To this end, we need the branching of some relevant irrep's of SL(5) with
respect to irrep's of SL(2) × SL(3), which is a maximal subgroup thereof. The embedding
turns out to be unique and it gives rise to the following decompositions

$$\mathbf{5} \quad \longrightarrow \quad (\mathbf{2}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}), \tag{3.6a}$$

$$\mathbf{15} \quad \longrightarrow \quad (\mathbf{1}, \mathbf{6}) \oplus (\mathbf{2}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{1}), \tag{3.6b}$$

$$\mathbf{24} \quad \longrightarrow \quad (\mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8}) \oplus (\mathbf{2}, \mathbf{3}) \oplus (\mathbf{2}, \mathbf{3'}) \oplus (\mathbf{3}, \mathbf{1}), \tag{3.6c}$$

$$\mathbf{40'} \quad \longrightarrow \quad (\mathbf{1}, \mathbf{3'}) \oplus (\mathbf{1}, \mathbf{8}) \oplus (\mathbf{2}, \mathbf{1}) \oplus (\mathbf{2}, \mathbf{6'}) \oplus (\mathbf{2}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{3'}). \tag{3.6d}$$

The decomposition (3.6a) essentially tells that the fundamental SL(5) index $M = 1, 2, 3, 4, 5$
goes into $(I \, ; m)$, where $I = +, -$ and $m = 1, 2, 3$ represent fundamental SL(2) and SL(3)
indices respectively. The decomposition (3.6c) tells us how the SL(2) × SL(3) scalar degrees
of freedom (living in the $(\mathbf{1}, \mathbf{8}) \oplus (\mathbf{3}, \mathbf{1})$) are embedded in the adjoint of SL(5). It is worth
mentioning at this point that we are losing a Cartan generator in the branching procedure;
such an abelian generator is realised as an extra $\mathbb{R}^+$ factor corresponding to a dilaton in
the seven-dimensional theory, with respect to which any eight-dimensional object should
have a scaling weight which we are omitting. This extra scalar exactly accounts for the
$(\mathbf{1}, \mathbf{1})$ irrep appearing in (3.6c). The truncation that we need consists then in switching
off all the off-diagonal axionic excitations (spanning the $(\mathbf{2}, \mathbf{3})$ and $(\mathbf{2}, \mathbf{3'})$ terms in (3.6c)),
thus resulting in the following parametrisation

$$\mathcal{M}_{MN} = \left(\begin{array}{c|c} e^{3\sigma} \, W_{IJ} & 0 \\ \hline 0 & e^{-2\sigma} \, M_{mn} \end{array} \right), \tag{3.7}$$

where σ is the extra dilaton corresponding to $\mathbb{R}^+$, whereas W_{IJ} and M_{mn} parametrise the
SL(2)/SO(2) and SL(3)/SO(3) cosets respectively. It has been checked explicitly that the
scaling weights of all the terms in the $D = 8$ scalar potential with respect to the extra $\mathbb{R}^+$
are all equal such that it is perfectly consistent to set $\sigma = 0$ in the rest of our derivation,
since any other constant value can be seen as a change of normalisation of the potential
energy in the lagrangian.

As has been mentioned, the embedding tensor in maximal $D = 8$ supergravity lives in
the [5] $(\mathbf{2}, \mathbf{3}) \oplus (\mathbf{2}, \mathbf{6'})$, which are parametrised by ξ_{Im} and $f_I{}^{mn} = f_I{}^{(mn)}$, respectively.
After taking a look at the decompositions given in (3.6b) and in (3.6d), one can infer that
ξ will in general source non-vanishing components of both Y and Z, whereas f will only
turn on components of[5] Z. This results in the following general Ansatz

$$Z^{Im,n} = -Z^{mI,n} = \lambda_1 \, \epsilon^{IJ} f_J{}^{mn} + \lambda_2 \, \epsilon^{mnp} \epsilon^{IJ} \xi_{Jp}, \tag{3.8a}$$

$$Z^{mn,I} = \lambda_3 \, \epsilon^{mnp} \epsilon^{IJ} \xi_{Jp}, \tag{3.8b}$$

$$Y_{Im} = Y_{mI} = \lambda_4 \, \xi_{Im}, \tag{3.8c}$$

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[5]This is due to the fact that a $(\mathbf{2}, \mathbf{3})$ irrep appears in the branching of both the **15** and the **40'**, whereas
a $(\mathbf{2}, \mathbf{6'})$ is only present in the decomposition of the **40'**.

where all the other components of Y and Z vanish and the parameters $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ will be fixed by some consistency requirements. First of all, the linear constraint implies in particular that Z lives in the **40'**, which means, as explained in (3.2b), that the three-form must vanish

$$Z^{[MN,P]} = 0 \ , \tag{3.9}$$

which yields the condition

$$\lambda_3 = -2\,\lambda_2 \ . \tag{3.10}$$

Secondly, we will substitute the Ansatz (3.8) into (3.4), which translates into the following expression for the gauge generators[6]

$$(X_{Im})_n{}^p = 4\lambda_1 \epsilon_{mnq} f_I{}^{qp} + \left(4\lambda_2 - \frac{1}{2}\lambda_4 \right) \delta_m{}^p \xi_{In} - 4\lambda_2 \delta_n{}^p \xi_{Im} \ , \tag{3.11a}$$

$$(X_{Im})_J{}^K = \left(4\lambda_3 + \frac{1}{2}\lambda_4 \right) \delta_I{}^K \xi_{Jm} - 4\lambda_3 \delta_J{}^K \xi_{Im} \ , \tag{3.11b}$$

$$(X_{IJ})_m{}^K = (-8\lambda_3 + \lambda_4) \, \delta_{[I}{}^K \xi_{J]m} \ , \tag{3.11c}$$

$$(X_{mn})_I{}^p = 4\lambda_1 \epsilon_{mnq} f_I{}^{qp} + (8\lambda_2 + \lambda_4) \, \delta_{[m}{}^p \xi_{|I|n]} \ , \tag{3.11d}$$

the remaining components being all zero. Now one has to make sure that the expression of the eight-dimensional gauge generators given in (2.7) is correctly obtained.

Therefore, by comparing (2.7) with (3.11a) and (3.11b),[7] while also taking (3.10) into account, one can consistently fix all λ's as:

$$\lambda_1 = \frac{1}{4}, \ \lambda_2 = -\frac{1}{16}, \ \lambda_3 = \frac{1}{8}, \ \lambda_4 = 1 \ . \tag{3.12}$$

By substituting these values into (3.8), the decomposition rules on the embedding tensor are obtained:

$$Z^{Im,n} = -Z^{mI,n} = \frac{1}{4}\,\epsilon^{IJ} f_J{}^{mn} - \frac{1}{16}\,\epsilon^{mnp} \epsilon^{IJ} \xi_{Jp} \ , \tag{3.13a}$$

$$Z^{mn,I} = \frac{1}{8}\,\epsilon^{mnp} \epsilon^{IJ} \xi_{Jp} \ , \tag{3.13b}$$

$$Y_{Im} = Y_{mI} = \xi_{Im} \ , \tag{3.13c}$$

$$\text{other components} = 0 \ . \tag{3.13d}$$

Furthermore, one can check that substituting (3.13) into the $D = 7$ quadratic constraints (3.1) exactly leads to the ones in $D = 8$ as shown in (2.8).

Finally in this section, let's come back to the scalar potential. One can apply the decomposition rules (3.7) and (3.13) on the $D = 7$ scalar potential (3.5), so that the relative coefficients in (2.9) can be determined, and by taking the normalisation of the action (2.3) into account one can further fix the overall factor of (2.9). Then the $D = 8$ scalar potential is fully derived:

$$V = \frac{1}{2}\,W^{IJ}\,\left[f_I{}^{mn} f_J{}^{pq} \left(2M_{mp}M_{nq} - M_{mn}M_{pq} \right) + \xi_{Im}\xi_{Jn}M^{mn} \right] \ . \tag{3.14}$$

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[6]We use the convention that $\epsilon_{IJmnp} = \epsilon_{IJ}\epsilon_{mnp}$.

[7](3.11c) and (3.11d) are some extra non-vanishing gauge generators due to the enlargement of the gauge group we already mentioned when compactifying from $D = 8$ to $D = 7$.

4 Investigating the vacua

4.1 Extrema of the potential

In the previous sections we have presented the quadratic constraints (2.8) and the scalar potential (3.14). With these formulae in hand we can now investigate the vacua of the maximal $D = 8$ supergravity.

In total there are 7 scalars for the coset $\frac{\text{SL}(2)}{\text{SO}(2)} \times \frac{\text{SL}(3)}{\text{SO}(3)}$. In (2.2) we already gave a parametrisation for the SL(2) scalars; now we also specify a parametrisation of the vielbein L appearing in (2.2) containing the information about the SL(3) scalars, which is given by

$$L_m{}^i = \begin{pmatrix} e^{-\phi_1} & \chi_1 e^{\frac{\phi_1 - \phi_2}{2}} & \chi_2 e^{\frac{\phi_1 + \phi_2}{2}} \\ 0 & e^{\frac{\phi_1 - \phi_2}{2}} & \chi_3 e^{\frac{\phi_1 + \phi_2}{2}} \\ 0 & 0 & e^{\frac{\phi_1 + \phi_2}{2}} \end{pmatrix} . \tag{4.1}$$

Subsequently, by substituting such a parametrisation into the scalar potential (3.14) and requiring that

$$\frac{\delta V}{\delta \,(\text{scalars})} = 0 , \tag{4.2}$$

one obtains 7 equations which represent the extremality condition for the scalar potential. Since the full theory enjoys a global $\text{SL}(2) \times \text{SL}(3)$ duality symmetry, one can choose to solve these equations in the origin of moduli space (setting all 7 scalars to zero.[8]). This can always be done without loss of generality by performing a non-compact duality transformation. This will translate the 7 equations of motion for the scalars into a set of 7 quadratic conditions in the embedding tensor components. Furthermore the quadratic constraints (2.8) give another 30 equations in the embedding tensor components which need to be satisfied for the solution to be consistent. This set of 37 equations appears in the form an ideal consisting of homogeneous polynomial equations which can be solved for the components ξ_{Im} and $f_I{}^{(mn)}$.

As explained in the footnote 8, we still have compact duality transformations that we can use in order to simplify the general form of ξ and f without spoiling the choice of solving the equations of motion in the origin. For instance, we can make use of an SO(3) transformation in order to diagonalise $f_-{}^{mn}$, whereas for the moment we don't need to exploit SO(2) transformations.

In the next step, we will exploit an algebraic geometry tool called the Gianni-Trager-Zacharias (GTZ) algorithm [16]. This algorithm has been computationally implemented by the SINGULAR project [17] and such an implementation has been used recently in ref. [18] for a purpose similar to the one discussed here.

We find in the end only one $\text{SO}(2) \times \text{SO}(3)$ orbit of solutions,[9] in which the simplest

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[8]This translates into $W = \mathbb{1}_2$ and $M = \mathbb{1}_3$, from which it becomes manifest that the origin still presents a residual $\text{SO}(2) \times \text{SO}(3)$ invariance.

[9]It is worth mentioning that an $\text{SO}(2) \times \text{SO}(3)$ rotation has been used in order to reduce some apparently inequivalent solutions to the form (4.3).

representative is given by

$$f_+{}^{mn} = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & 0 \end{pmatrix} \, , \quad f_-{}^{mn} \; = \; \xi_{+m} \; = \; \xi_{-m} \; = \; 0 \, , \tag{4.3}$$

where λ represents an arbitrary real parameter. This orbit of solutions represents a $CSO(2,0,1)$ gauging, which was obtained in ref. [10] as eleven-dimensional supergravity compactified on an $ISO(2)$ manifold, with structure constants given by $f_{mn}{}^p = \epsilon_{mnq} f_+{}^{qp}$.

4.2 Supersymmetry breaking analysis

Let's now see whether we can say something about the fraction of supersymmetry preserved by this class of solutions. Using the expression in ref. [8] for the variation of the gravitino[10]

$$\delta \psi_\mu = -\frac{g}{48} e^{\phi/2} f_{mn}{}^p \Gamma^{mn}{}_p \Gamma_\mu \varepsilon \, , \tag{4.4}$$

and choosing the following parametrisation for eleven-dimensional Dirac matrices [7]

$$\Gamma^\mu = \gamma^\mu \otimes \mathbb{1}_2 \qquad \text{and} \qquad \Gamma^m = \gamma^9 \otimes \sigma^m \, , \tag{4.5}$$

one finds

$$\delta \psi_\mu = -\frac{g}{24} e^{\phi/2} f_+{}^{mn} M_{mn} \gamma^9 \gamma_\mu \varepsilon \; \propto \; 2\lambda \gamma^9 \gamma_\mu \varepsilon \neq 0 \, , \tag{4.6}$$

which implies that these solutions are always non-supersymmetric, whereas for $\lambda = 0$ the standard supersymmetric Minkowski vacuum of the ungauged theory is recovered.

4.3 Stability analysis

To this end, we need to compute the seven eigenvalues of the mass matrix at the solution. We would like to point out that the scalars parametrised in (2.2) and (4.1) are not canonically normalised, i.e., the kinetic terms read

$$\mathcal{L}_{\text{kin}} = \frac{1}{2} K_{ij} \left(\partial \Phi^i \right) \left(\partial \Phi^j \right) \, , \tag{4.7}$$

$$K_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{2\phi} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{3\phi_1 - \phi_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{3\phi_1 + \phi_2} & -e^{3\phi_1 + \phi_2} \chi_1 \\ 0 & 0 & 0 & 0 & 0 & -e^{3\phi_1 + \phi_2} \chi_1 & e^{3\phi_1 + \phi_2} \chi_1^2 + e^{2\phi_2} \end{pmatrix} \, . \tag{4.8}$$

This means that the physical mass matrix is given by

$$\left(m^2 \right)^i{}_j = K^{ik} \, \partial_k \, \partial_j V \, , \tag{4.9}$$

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[10]This expression is valid for a maximally symmetric solution of a theory obtained from the reduction of eleven-dimensional supergravity on a three-dimensional group manifold with structure constants $f_{mn}{}^p$.

where K^{ij} denotes the inverse of the matrix K in (4.8).

Computing the mass matrix defined in (4.9) for the solutions (4.3), one finds the following eigenvalues

$$0 \ (\times 5), \quad 8\lambda^2 \ (\times 2). \tag{4.10}$$

These solutions present five flat directions. In fact, the underlying CSO gauging doesn't have any non-trivial SL(2) phases ($f_+ \neq 0$, and $f_- = 0$) and hence the potential given in (3.14) has an overall e^ϕ and no dependence at all on χ. Only the vanishing of V itself at the solution saves it from the run-away. This explains why the SL(2) scalars are massless.

Furthermore, in any theory in which a bosonic symmetry is gauged, one expects a number of Goldstone bosons corresponding to unbroken generators of the gauge group. This explains the presence of some extra flat directions. Discussion of the stability of all flat directions would require an analysis of higher-order derivatives.

5   Conclusions

In the present paper we considered the general deformations of maximal $D = 8$ supergravity and we have derived the scalar potential for the general case. Subsequently, by combining duality covariance arguments with algebraic geometry techniques, we were able to study the set of extremality conditions for the general gauging. The remarkable outcome is that there is only a unique $SO(2) \times SO(3)$ orbit of Minkowski solutions corresponding with a $CSO(2,0,1)$ gauging. As discussed above, they are all non-supersymmetric with no possibility of an intermediate case of partial supersymmetry breaking from $\mathcal{N} = 2$ to $\mathcal{N} = 1$. Moreover, these solutions have the good feature of being free of tachyons at a quadratic level. There are, though, a number of flat directions which might require a further analysis at higher perturbative orders.

Acknowledgments

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