dS/CFT at uniform energy density and a de Sitter "bluewall"

Diptarka Das,[a] **Sumit R. Das**[a] **and K. Narayan**[b]

[a]*Department of Physics and Astronomy,*
 University of Kentucky, Lexington, KY 40506, U.S.A.

[b]*Chennai Mathematical Institute,*
 SIPCOT IT Park, Siruseri 603103, India

 E-mail: diptarka.das@uky.edu, das@pa.uky.edu, narayan@cmi.ac.in

ABSTRACT: We describe a class of spacetimes that are asymptotically de Sitter in the Poincare slicing. Assuming that a dS/CFT correspondence exists, we argue that these are gravity duals to a CFT on a circle leading to uniform energy-momentum density, and are equivalent to an analytic continuation of the Euclidean AdS black brane. These are solutions with a complex parameter which then gives a real energy-momentum density. We also discuss a related solution with the parameter continued to a real number, which we refer to as a de Sitter "bluewall". This spacetime has two asymptotic de Sitter universes and Cauchy horizons cloaking timelike singularities. We argue that the Cauchy horizons give rise to a blue-shift instability.

Contents

1   Introduction and summary

One version of the dS/CFT correspondence [1–3] states that quantum gravity in de Sitter space is dual to a Euclidean CFT living on the boundary $\mathcal{I}^+$ or $\mathcal{I}^-$. More specifically, the partition function of the CFT with specified sources $\phi_{i0}(\vec{x})$ coupled to operators $\mathcal{O}_i$ is identified with the wavefunctional of the bulk theory as a functional of the boundary values of the fields dual to $\mathcal{O}_i$ given by $\phi_{i0}(\vec{x})$. In the semiclassical regime this becomes

$$\Psi[\phi_{i0}(\vec{x})] = \exp\left[iI_{cl}(\phi_{i0})\right] \tag{1.1}$$

where we need to impose regularity conditions on the cosmological horizon. This has been developed further in [4]. Unlike AdS/CFT, there are few concrete realizations of dS/CFT (for a recent proposal see [5] and e.g. [6–13] for related work). Nevertheless, it is interesting to explore the consequences of such a correspondence, assuming it exists.

In this note we address the question of what the bulk dual is of a euclidean CFT with *constant* spatially uniform energy-momentum density. One way to achieve this is to put the CFT on a circle. It is well known that the dual CFT to de Sitter space cannot be a usual unitary (more precisely reflection-positive) quantum field theory. The bulk dual of such a theory would be euclidean AdS. Such a CFT on a circle has a uniform energy-momentum density, describing the corresponding Lorentzian theory in a thermal state. The dual of this is a Euclidean AdS black brane, not a Lorentzian geometry.

In this context, consider a class of asymptotically de Sitter spacetimes

$$ds^2 = -\frac{R_{dS}^2 d\tau^2}{\tau^2(1 + \frac{C}{\tau^d})} + \frac{\tau^2}{R_{dS}^2}\left(1 + \frac{C}{\tau^d}\right)dw^2 + \frac{\tau^2}{R_{dS}^2}dx_i^2, \qquad C \propto \tau_0^d, \tag{1.2}$$

with C a general complex parameter and τ_0 is real. This metric should be regared as a (generally complex) saddle point in a functional integral. Motivated by this, we impose a requirement that the euclidean metric obtained by Wick rotation of the time coordinate τ is real and regular — this fixes the parameter $C = -i^d\tau_0^d$, and requires w to be periodic. However, as will be clear in the following, the Lorentzian metric can become singular for even d. Our solutions are similar to those in [9] who considered solutions with $S^{d-1} \times S^1$

boundaries. In fact (1.2) can be obtained as a limit of the solution [9] when the radius of the S^1 is much smaller than the radius of S^{d-1} (or equivalently, as S^{d-1} decompactifies).

The resulting spacetime can equivalently be obtained from the Euclidean AdS black brane by the analytic continuation from AdS to dS familiar in dS/CFT [1, 3]. Clearly this leads to a *complex* solution for odd d. This is in fact quite common in the dS/CFT correspondence [3, 4]. Indeed we will show that the energy-momentum tensor $T_{ij} \sim \frac{\delta \Psi}{\delta h^{ij}}$ in the CFT which follows from (1.1) is real for odd d. This is consistent with known results for correlators in pure dS.[1] For example in $d = 3$ we get $\langle T_{ij} \rangle \propto \frac{\tau_0^3}{G_4 R_{dS}^4}$, which is exactly what we need. For even d, the solution (1.2) is real and the boundary energy-momentum tensor is purely imaginary.

While real energy momentum tensors are thus obtained only for complex solutions, it is interesting to consider the geometry of the solutions with real parameters. The geometry is bounded by asymptotically de Sitter spacelike $\mathcal{I}^\pm$ and time-like singularities at the two ends of space. The null lines $\tau = \tau_0$ are Cauchy horizons. In fact the geometry bears some resemblance to the interior of the Reissner-Nordstrom black hole [14, 15]. The physics of physical observers is also quite similar. Timelike geodesics originating from $\mathcal{I}^-$ are repelled by the singularities. As an observer approaches the horizon, light coming from $\mathcal{I}^-$ is infinitely blueshifted, just as in the RN interior. It is natural to expect that this blueshift signals an instability, preserving cosmic censorship and distinguishing these from naked singularities. It is intriguing to note that from a dS/CFT perspective, the energy-momentum tensor is purely imaginary. It is tempting to think of this imaginary T_{ij} as a possible dual signature of the Cauchy horizon blue-shift instability that we have seen. It would be interesting to explore this and more generally cosmic censorship in dS/CFT. We dub these solutions "bluewalls".

2 dS/CFT at uniform energy-momentum density

The CFT correlation functions in dS/CFT correspondence follow from analytic continuation from euclidean AdS (or double analytic continuation from lorentzian AdS), with the interpretation that the wavefunctional is the generating functional of correlators. One half of dS_{d+1}, e.g. the upper patch being $\mathcal{I}^+$ at $\tau = \infty$ with a coordinate horizon at $\tau = 0$ is described in the planar coordinate foliation by the metric

$$ds^2 = -R_{dS}^2 \frac{d\tau^2}{\tau^2} + \frac{\tau^2}{R_{dS}^2} \delta_{ij} dx^i dx^j \ . \tag{2.1}$$

This may be obtained by analytic continuation of a Poincare slicing of $EAdS$,

$$r \to -i\tau \ , \qquad R_{\text{AdS}} \to -i R_{dS} \ . \tag{2.2}$$

In fact the analytic continuation of the smooth euclidean solutions lead to Bunch-Davies initial conditions on the cosmological horizon.

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[1]One may wonder if there could be an additional factor of i in this relation. However, the requirement that the n-point correlator does not have a n-dependent phase rules this out [5].

Consider the asymptotically de Sitter spacetime

$$ds^2 = -\frac{R_{dS}^2 d\tau^2}{\tau^2(1 + \frac{C}{\tau^d})} + \frac{\tau^2}{R_{dS}^2}\Big(1 + \frac{C}{\tau^d}\Big)dw^2 + \frac{\tau^2}{R_{dS}^2}dx_i^2 \,, \tag{2.3}$$

with C a general complex parameter. This is a *complex* metric which satisfies Einstein's equation with a positive cosmological constant

$$R_{MN} = \frac{d}{R_{dS}^2}g_{MN} \,, \qquad \Lambda = \frac{d(d-1)}{2R_{dS}^2} \,. \tag{2.4}$$

With a view to requiring an analog of regularity in the interior for an asymptotically AdS solution, consider a Wick rotation of the time coordinate τ above. Then (2.3) becomes

$$\tau = il \quad \Rightarrow \quad ds_E^2 = -\frac{R_{dS}^2 dl^2}{l^2(1 + \frac{C}{i^d l^d})} - \frac{l^2}{R_{dS}^2}\Big(1 + \frac{C}{i^d l^d}\Big)dw^2 - \frac{l^2}{R_{dS}^2}dx_i^2 \,. \tag{2.5}$$

With a further continuation $R_{dS} \to iR'$, this is in general a complex euclidean metric. We require that this euclidean spacetime is real and regular in the interior, by which we demand that the spacetime in the interior approaches flat Euclidean space in the (l, w)-plane with no conical singularity. This is true if

$$C = -i^d \tau_0^d \,, \qquad l \geq \tau_0 \,, \qquad w \simeq w + \frac{4\pi}{(d-1)\tau_0} \,, \tag{2.6}$$

where τ_0 is some real parameter of dimension length, and the w-coordinate is compactified with the periodicity fixed by demanding that there is no conical singularity.

This requirement of regularity is similar to the one we use in an asymptotically AdS spacetime, where e.g. Wick rotating the time coordinate renders the resulting Euclidean space regular if the time coordinate is regarded as compact with a periodicity that removes any conical singularity (thus rendering it sensible for a Euclidean path integral). A sharp difference in the asymptotically de Sitter case is that we Wick rotate the asymptotic bulk time coordinate but the absence of a conical singularity fixes the w-coordinate to be compact with appropriate periodicity. This, however, is at odds with the regularity of the real time metric with this value of C when d is even. In that case, a periodic w leads to a conical type singularity at $\tau = \tau_0$ pretty much like the Milne universe with a compact spatial direction. In this regard, it is interesting to consider the asymptotically dS_5 solution above: then the above Wick rotation procedure fixes $C = -\tau_0^4$ and the periodicity of the w-coordinate and the solution is

$$ds^2 = -\frac{R_{dS}^2 d\tau^2}{\tau^2(1 - \frac{\tau_0^4}{\tau^4})} + \frac{\tau^2}{R_{dS}^2}\Big(1 - \frac{\tau_0^4}{\tau^d}\Big)dw^2 + \frac{\tau^2}{R_{dS}^2}dx_i^2 \,. \tag{2.7}$$

The metric in the vicinity of $l = \tau_0$ is $ds^2 \sim -dT^2 + T^2 dw^2 + \tau_0^2 dx_i^2$, where $T \sim l - \tau_0$. This is Milne space in the (T, w)-plane, with w compact (and thus a resulting singularity). We note that Wick rotating the coordinate T does not give a Euclidean space and is not equivalent to the above procedure of Wick rotating the asymptotic time coordinate τ.

As expected, this entire procedure is equivalent to analytically continuing from the Euclidean AdS black brane

$$ds^2 = R_{\text{AdS}}^2 \frac{dr^2}{r^2(1 - \frac{r_0^d}{r^d})} + \frac{r^2}{R_{\text{AdS}}^2}\Big(1 - \frac{r_0^d}{r^d}\Big)d\theta^2 + + \frac{r^2}{R_{\text{AdS}}^2}\sum_{i=1}^{d-1} dx^i dx^i \,, \qquad (2.8)$$

where $\theta \sim \theta + \frac{4\pi}{(d-1)r_0}$, to the asymptotically de Sitter spacetime (2.3) using (2.2) and we identify $r_0 \equiv \tau_0$. The phase obtained by this analytic continuation is $\frac{-1}{(-i)^d}$ which can be seen as identical to $-i^d$ in (2.6). The regularity criterion (2.6) itself is then seen to simply be the analog of regularity of the $EAdS$ black brane. The condition $l \geq \tau_0$ is equivalent to the radial coordinate having the range $r \geq r_0$. In the Lorentzian signature spacetime (2.3), the time τ-coordinate extends to $\tau \to 0$. The curvature invariant $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ diverges as $\tau \to 0$.

Near $\mathcal{I}^+$, i.e. $\tau \to \infty$, the metric (2.3) approaches that of de Sitter space with a Fefferman-Graham expansion

$$ds^2 = -\frac{R_{dS}^2}{\tau^2}d\tau^2 + h_{ij}dy^i dy^j = -\frac{R_{dS}^2}{\tau^2}d\tau^2 + \frac{\tau^2}{R_{dS}^2}\Big[g_{ij}^{(0)}(y^i) + \frac{R_{dS}^2}{\tau^2}g_{ij}^{(2)}(y^i) + \dots\Big]dy^i dy^j \,. \quad (2.9)$$

It is clear from (2.9) that "normalizable" metric pieces are turned on in (2.3).[2] We then expect a nonzero expectation value for the energy-momentum tensor here, as in the AdS context [16–19]. For concreteness, let us consider the asymptotically dS_4 solution (2.3) with the regularity conditions (2.6),

$$ds^2 = -\frac{R_{dS}^2 d\tau^2}{\tau^2(1 + \frac{i\tau_0^3}{\tau^3})} + \frac{\tau^2}{R_{dS}^2}\Big(1 + \frac{i\tau_0^3}{\tau^3}\Big)dw^2 + \frac{\tau^2}{R_{dS}^2}dx_i^2 \,. \qquad (2.10)$$

The calculation of the energy momentum tensor proceeds in a way entirely analogous to that in AdS. The total action, obtained by adding suitable Gibbons-Hawking surface terms and counterterms to the bulk action is

$$I = \frac{1}{16\pi G_4}\int_{\mathcal{M}} d\tau d^3x \sqrt{-g}\,(R - 2\Lambda) + \frac{1}{8\pi G_4}\int_{\partial\mathcal{M}} d^3x \sqrt{h}\,\Big(K + \frac{2}{R_{dS}}\Big) \qquad (2.11)$$

The counterterms have been engineered to remove divergences in the bulk action coming from the boundary at $\tau \to \infty$. Here h_{ij} is the boundary metric and K is the trace of the extrinsic curvature. This renormalized action appears in (1.1). This leads to the energy momentum tensor[3]

$$T_{ij} = \lim_{\tau \to \infty} \frac{\tau}{R_{dS}}\frac{2}{\sqrt{h}}\frac{\delta\Psi}{\delta h^{ij}} \sim \lim_{\tau \to \infty} \frac{\tau}{R_{dS}}\frac{i}{G_4}\Big(K_{ij} - Kh_{ij} - \frac{2}{R_{dS}}h_{ij}\Big), \qquad (2.12)$$

We have used the standard relationship $\sqrt{h^B}h_{\mu\nu}^B T^{\nu\rho} = \sqrt{h}h_{\mu\nu}\tau^{\nu\rho}$ between the energy momentum tensor of the boundary theory and the quasi-local stress tensor $\tau^{\mu\nu}$, with $h_{\mu\nu}^B =$

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[2]Scalar modes in dS_{d+1} near the boundary are $\phi \sim \tau^\Delta$, with $\Delta(\Delta - d) = -m^2R^2$. For $m^2 = 0$, we have $\Delta = d$ as analogous to a "normalizable" mode (in AdS): this is the mode turned on in (2.10).

[3]Note that our definition is consistent with [3] (also [5]), but differs from e.g. [1, 20] which use a derivative of the action rather than the wavefunction.

$\lim_{\tau\to\infty}\frac{R_{dS}^2}{\tau^2}h_{\mu\nu}$ the boundary metric. The above energy-momentum tensor vanishes for pure dS_4 as expected. For the spacetime (2.10), we obtain

$$T_{ww} = -2T_{ii} \sim \frac{i}{G_4}\Big(\frac{i\tau_0^3}{R_{dS}^4}\Big) = -\frac{\tau_0^3}{G_4 R_{dS}^4}\,, \tag{2.13}$$

which is a real and spatially uniform energy-momentum density. Since (2.10) is a complex solution, its conjugate is also a solution (obtained by analytically continuing the opposite way), giving T_{ij} of the opposite sign as above. In the AdS case, spacetimes of this sort which are solutions in pure gravity have $I_{bulk} = \frac{1}{16\pi G_4}\int_{\mathcal{M}} dr d^3x \sqrt{-g}\,(R - 2\Lambda) = \frac{1}{16\pi G_4}\int dr d^3x \frac{R^4}{r^4}(\frac{-6}{R^2})$. Under the analytic continuation (2.2), we have

$$I_{EAdS} \;\to\; \frac{1}{16\pi G_4}\int(-id\tau)d^3x\frac{R_{dS}^4}{\tau^4}\left(\frac{-6}{-R_{dS}^2}\right) = -iI_{dS}\,, \tag{2.14}$$

where I_{dS} is the action for the asymptotically de Sitter solution.[4] Thus the energy-momentum tensor is continued as $\frac{2}{\sqrt{h}}\frac{\delta(-I_{EAdS})}{\delta h^{\mu\nu}} \to \frac{2}{\sqrt{h}}\frac{\delta(iI_{dS})}{\delta h^{ij}}$. Note that T_{ij} in (2.13) is traceless ($T_{ww} + 2T_{ii} = 0$) as expected for a CFT.

Thus this asymptotically dS_4 complex solution is dual to a euclidean CFT with spatially uniform energy-momentum density, i.e. uniform T_{ij} expectation value (2.13) in the Euclidean CFT dual to dS_4 (in Poincare slicing). Loosely speaking, this Euclidean partition function corresponds to a thermal state of the would-be corresponding Lorentzian theory (on $R \times R^2$), analogous to the AdS_4 Schwarzschild black brane dual to a thermal state in the SYM CFT with uniform $T_{\mu\nu}$.

Similar arguments apply in other dimensions but with different results. Using the Fefferman-Graham expansion (2.9) for an asymptotically dS_{d+1} spacetime, we see that "normalizable" metric modes $g_{\mu\nu}^{(d)}$ turned on give rise to a nonzero expectation value for the holographic energy-momentum tensor

$$T_{ij} = \lim_{\tau\to\infty}\frac{\tau^{d-2}}{R_{dS}^{d-2}}\frac{2}{\sqrt{h}}\frac{\delta\Psi}{\delta h^{ij}} \sim \lim_{\tau\to\infty}\frac{\tau^{d-2}}{R_{dS}^{d-2}}\frac{i}{G_{d+1}}\Big(K_{ij} - Kh_{ij} - \frac{d-1}{R_{dS}}h_{ij}\Big) \propto \frac{i}{G_{d+1}R_{dS}}g_{ij}^{(d)}\,, \tag{2.15}$$

where the form of (2.9) shows $g_{ij}^{(d)}$ to be the dimensionless coefficient of the normalizable $\frac{1}{\tau^{d-2}}$ term, and the i arises from Ψ, the wavefunction of the universe. In effect, this dS/CFT energy-momentum tensor can be thought of as the analytic continuation of the $EAdS$ one, with the i arising from $R_{AdS} \to -iR_{dS}$, and the metric modes also continuing correspondingly. The spacetime (2.3) with the parameter $C = -i^d\tau_0^d$ in (2.6) gives

$$g_{ww}^{(d)} \sim -\frac{i^d\tau_0^d}{R_{dS}^d} \qquad \Rightarrow \qquad T_{ww} \sim -\frac{i^{d+1}\tau_0^d}{G_{d+1}R_{dS}^{d+1}} = \frac{i^{d-1}\tau_0^d}{G_{d+1}R_{dS}^{d+1}}\,, \tag{2.16}$$

with $T_{ww} + (d-1)T_{ii} = 0$. The phase i^{d-1} is equivalent to that in general dS/CFT correlation functions arising from the analytic continuation (2.2) from $EAdS$ correlators [11],

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[4]As we saw, the divergent terms in I_{dS} cancel: this gives a single new term from the τ-location where h_{ij} departs from the dS_4 value. The on-shell wavefunction for these solutions becomes $\Psi \sim \Psi_{dS}\,exp[\frac{V_3\tau_0^3}{8\pi G_4 R_{dS}^4}]$.

following the arguments of [5]. For even d the energy momentum tensor is imaginary — this is also the case when the Lorentzian signature metric is singular at $\tau = \tau_0$.

We thus see that a real energy-momentum density must arise from a metric mode $g^{(d)}$ that is pure imaginary: in other words, the spacetime (2.3) with a pure imaginary parameter C is dual to a CFT with real spatially uniform energy-momentum density. For the dS_5 case, we see that the regularity criterion for the Euclidean solution (or equivalently the analytic continuation from the $EAdS$ black brane) gives the spacetime (2.7) which is real: this metric which is singular gives an imaginary T_{ij} above.

In summary, we have described asymptotically de Sitter spacetimes (2.3) which under a Wick rotation are regular in the interior for certain values of the general complex parameter (2.6). The resulting spacetime can then be equivalently obtained by analytic continuation (2.2) from the Euclidean AdS black brane (2.8). These spacetimes give rise to a spatially uniform holographic energy-momentum density (2.15), which is real if the spacetime is complex (for odd d). Conversely, given a T_{ij} expectation value in dS/CFT, we could ask what the gravity dual is. An asymptotically de Sitter spacetime with the Fefferman-Graham series expansion (2.9) and thus corresponding T_{ij} then in fact sums to the closed form expression (2.3).

3 Real parameter C: a de Sitter "bluewall"

Even though the metric needs to be complex to yield a real energy momentum tensor, it is interesting to explore the properties of metrics of the form (2.3), (2.6), but with the parameter τ_0^d also continued to be real, i.e.

$$ds^2 = -\frac{d\tau^2}{f(\tau)} + f(\tau)dw^2 + \tau^2 dx_i^2, \qquad f(\tau) = \tau^2\Big(1 - \frac{\tau_0^d}{\tau^d}\Big), \tag{3.1}$$

with a nonzero parameter τ_0, and x_i are $d-1$ of the d spatial dimensions. The w-coordinate here has the range $-\infty \le w \le \infty$. This can be recast in FRW form as an asymptotically deSitter cosmology with anisotropy in the w-direction. The metric (3.1) is simply the analytic continuation of AdS-Schwarzschild with a further continuation of the mass parameter. We do not speculate about the significance of this real solution for dS/CFT for odd d.

The lines $\tau = \tau_0$ are coordinate singularities whose nature will be explored below. For concreteness, we focus on $d = 3$. The maximally extended geometry in Kruskal type coordinates (appendix A, eq. (A.2)) is

$$ds^2 = \tau^2\left[-\frac{4}{9}\Big(1 + \frac{\tau_0}{\tau} + \frac{\tau_0^2}{\tau^2}\Big)^{3/2} e^{-\sqrt{3}\tan^{-1}(\frac{2\frac{\tau}{\tau_0}+1}{\sqrt{3}})} d\tilde{u}d\tilde{v} + dx_i^2\right]. \tag{3.2}$$
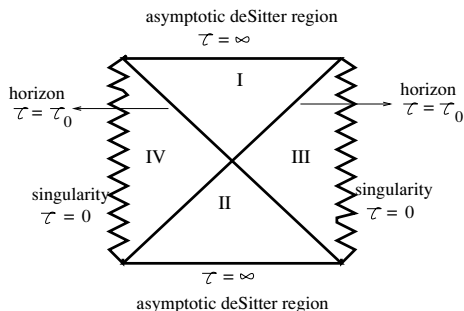
The Penrose diagram[5] figure 1 shows the following key features of the geometry.

Two asymptotic dS-regions: $v^2 - u^2 = \tilde{u}\tilde{v} > 0$ both map to $\tau \gg \tau_0$, using (A.2).

Cauchy horizons: $\tau = \tau_0 \Rightarrow \tilde{u}\tilde{v} = 0$, *i.e.* $u = \pm v$.

Using (A.2), we see that $\tanh\frac{3w\tau_0}{2} = \frac{\tilde{u}-\tilde{v}}{\tilde{u}+\tilde{v}}$ so that the two horizons are $\tilde{u} = 0 \Rightarrow \tau =$

Figure 1. de Sitter "bluewall" Penrose diagram. This resembles the Penrose diagram of the AdS Schwarzschild black brane rotated by $\frac{\pi}{2}$.

τ_0, $w = -\infty$, and $\tilde{v} = 0 \Rightarrow \tau = \tau_0$, $w = +\infty$. These are Cauchy horizons, as we discuss later. We refer to the intersection of the horizons $\tilde{u} = 0 = \tilde{v}$, i.e. $u = 0 = v$ or $\tau = \tau_0$ as the **bifurcation region**: the w-coordinate can take any value here.

Timelike singularities: $\qquad \tau = 0 \quad \Rightarrow \quad \tilde{u}\tilde{v} = -e^{\frac{\pi}{2\sqrt{3}}} \sim v^2 - u^2$

In the Kruskal diagram these are hyperbolae with $u^2 > v^2$. There are two singularity loci $\tilde{v} = -\frac{c}{\tilde{u}}$ with $\tilde{u} > 0$ and $\tilde{u} < 0$. The curvature invariants for (3.1) are

$$R = d(d+1), \quad R_{\mu\nu}R^{\mu\nu} = d^2(d+1), \quad R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = 2d\Big(d+1+\frac{(d-2)(d-1)^2}{2}\frac{\tau_0^{2d}}{\tau^{2d}}\Big).$$

The divergence in $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ implies a curvature singularity as $\tau \to 0$: this is what the Schwarzschild interior singularity becomes after analytic continuation. Interestingly, the dS_3 solution is singularity-free: the metric in this case is isomorphic to dS_3 in static coordinates.

Near $\tau \to 0$, the metric approaches $ds^2 \sim -\frac{dw^2}{\tau^{d-2}} + \tau^{d-2}d\tau^2 + \frac{dx_i^2}{\tau^2} \sim \frac{1}{\tau}d\tilde{u}d\tilde{v} + \tau^2 dx_i^2$. For $\tau < \tau_0$, the τ-coordinate is spacelike while w is timelike. Then the singularity which occurs on a constant-τ slice is timelike (metric approaching $ds^2 \sim -\frac{dw^2}{\tau^{d-2}}$).

Note that these features (Cauchy horizons, timelike singularities) resemble the interior of the Reissner-Nordstrom black hole or "wormhole" (discussed in e.g. [15]). Recall that the latter geometry is of the form

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \qquad f(r) = (r - r_+)(r - r_-). \tag{3.3}$$

Near the inner horizon r_- this can be approximated as $ds^2 \sim -\frac{dr^2}{k(r-r_-)} + k(r-r_-)dt^2 + r^2 d\Omega^2$, where $k = r_+ - r_-$. In the region $r_- < r < r_+$, the radial coordinate r is timelike. Thus we see that the geometry near the inner horizon r_- in fact resembles the geometry $ds^2 \sim -\frac{d\tau^2}{\tau - \tau_0} + (\tau - \tau_0)dw^2 + \tau^2 dx_i^2$ near the horizon τ_0 in the present dS-case. Thus it is not surprising that the Penrose diagram and associated physics are similar in both cases.

For general timelike geodesic trajectories the momenta satisfy $p_\mu p^\mu = -m^2$ and the action is $S = \int d\tau \frac{m}{2} g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu$, with λ the affine parameter. Sinc ∂_w is a Killing vector the associated momentum p_w is conserved

$$\frac{p_w}{m} = \Big(\tau^2 - \frac{\tau_0^3}{\tau}\Big)\frac{dw}{d\lambda}, \quad \frac{\dot{\tau}^2 - p_w^2/m^2}{\tau^2 - \frac{\tau_0^3}{\tau}} = 1, \quad \frac{dw}{d\tau} = \frac{\pm\frac{p_w}{m}}{\tau^2(1 - \frac{\tau_0^3}{\tau^3})\sqrt{\frac{p_w^2}{m^2} + \tau^2(1 - \frac{\tau_0^3}{\tau^3})}}. \tag{3.4}$$
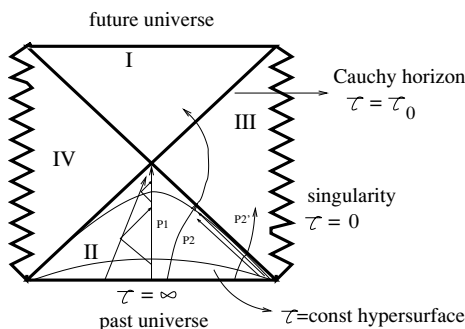
Figure 2. Trajectories in the de Sitter bluewall and the Cauchy horizon. Observers P_1 are static while P_2 has w-momentum p_w, crosses the horizon, turns around inside and appears to re-emerge in the future universe. Also shown are incoming lightrays from infinity which "crowd near" the Cauchy horizon.

In these coordinates static observers are timelike geodesics at const-w, x_i with $p_w = 0$, $g_{\tau\tau}(\frac{d\tau}{d\lambda})^2 = -1$. Using (A.2), these are $\frac{u}{v} = -\tanh\frac{3w\tau_0}{2} = const$ i.e. straight lines crossing from the past universe II to the future one I through the bifurcation region. Generic observers have $p_w \neq 0$: as $\tau \to \tau_0$, they approach the horizon with increasing coordinate speed $|\frac{dw}{d\tau}| \to \infty$ and fall through the horizon. They do not hit the singularity however: the singularity appears to be repulsive. This can be seen from (3.4) by noting that $\dot\tau^2 = \frac{p_w^2}{m^2} + \tau^2 - \frac{\tau_0^3}{\tau} > 0$ implies a turning point $\tau_{\min} = \frac{\tau_0^3}{p_w^2}$, and likewise $(\frac{dw}{d\tau})^2 > 0 \Rightarrow \tau \geq \tau_{\min}$, so that timelike geodesic trajectories never reach the singularity. We see that in this "deSitter bluewall" solution, particles can apparently pass from the past universe through the horizons avoiding the timelike singularities behind the horizons and emerge in the future universe. Whether such trajectories can actually go across is unclear due to a blue-shift instability stemming from the Cauchy horizons, as we discuss now.

Bluewalls: We now discuss the role of the Cauchy horizons and the possibility of traversing from the past universe to the future one. First recall that the horizons are $\tilde{u} = 0$ at $\tau = \tau_0, w = -\infty$ and $\tilde{v} = 0$ at $\tau = \tau_0, w = +\infty$. Thus the horizons are actually infinitely far away in the w-direction. As we have seen, trajectories from the past universe (beginning at some point on $\mathcal{I}^-$) can pass through the horizons into the interior regions: however there are timelike and null geodesics which begin in the interior regions alone and thus cannot be obtained by time development of any Cauchy data on $\mathcal{I}^-$. Thus the past horizons are future Cauchy horizons for Cauchy data on $\mathcal{I}^-$. Likewise the future horizons are causal boundaries for the future universe, so that these are past Cauchy horizons for data on $\mathcal{I}^+$.

Two static observers at different w-locations communicating by lightray signals are always in contact with each other. Consider observers P_1, P_2, with P_1 a static observer while P_2 is a geodesic infalling observer with some w-momentum. P_2 falls freely through the horizon, turns around somewhere in the interior and then appears to re-emerge in the future universe. From the point of view of P_1, the observer P_2 appears to be going to $|w| \to \infty$. Eventually P_2 sends, from $|w| \to \infty$, a "final" lightray which is the generator of the corresponding horizon. Similarly one can consider signals received by infalling observers

P_2 at late times, sent by infalling observers $P_{2'}$ at early times. Such observers $P_2, P_{2'}$ have $\dot{\tau}^2 \equiv (\frac{d\tau}{d\lambda})^2 = \frac{p_w^2}{m^2} + \tau^2 - \frac{\tau_0^3}{\tau}$, using (3.4), so that we have the proper time intervals

$$\Delta\lambda_{P_2} \sim \frac{1}{p_w}\Delta\tau \qquad (\text{near } \tau_0), \qquad \Delta\lambda_{P_{2'}} \sim \frac{\Delta\tau}{\tau} \qquad (\text{early times}). \qquad (3.5)$$

Ingoing lightray congruences of the form $U = w - \tau_* = c$ have a cross-sectional vector $v = w + \tau_*$. To analyse these transmitting-receiving events further, it is convenient to use Eddington-Finkelstein-type coordinates here: defining the ingoing coordinate $v = w + \tau_*$, the metric (3.1) becomes

$$ds^2 = f(\tau)dv^2 - 2dvd\tau + \tau^2 dx_i^2, \qquad (3.6)$$

and infalling geodesic observers at const-x_i (i.e. τ, w decreasing with proper time λ) are

$$f\dot{v}^2 - 2\dot{v}\dot{\tau} = -1, \qquad -f\dot{v} + \dot{\tau} = -f\dot{w} = p_v > 0 \qquad (\text{i.e. } \dot{w} < 0), \qquad (3.7)$$

$$\Rightarrow \quad \frac{d\tau}{d\lambda} = -\sqrt{f(\tau) + p_v^2} < 0, \quad \frac{dv}{d\lambda} = -\frac{p_v + \sqrt{f(\tau) + p_v^2}}{f(\tau)}, \quad \frac{dv}{d\tau} = \frac{p_v + \sqrt{f(\tau) + p_v^2}}{f(\tau)\sqrt{f(\tau) + p_v^2}}. \qquad (3.8)$$

figure 2 shows infalling observers P_2 approaching the horizon, receiving at late times ($\tau \sim \tau_0$) light signals that emanate from early times ($\tau \sim \infty$): the latter can be thought of as signals transmitted by infalling observers $P_{2'}$ at early times. It can be seen from figure 2 that such events (transmission-reception of such light signals) are consistent with the causal (lightcone) structure of the spacetime. The light rays in question have constant v which is very large and negative.

Let us denote the conserved momenta for the two geodesics P_2 and $P_{2'}$ by p_v and p_v' respectively. Suppose $P_{2'}$ sends out successive light signals along constant v and constant $v + dv$, at coordinate times τ' and $\tau' + d\tau'$, and these are received by P_2 at coordinate times τ and $\tau + d\tau$. The proper time between emission these signals is $d\lambda_{P_{2'}}$ while the proper time between reception of the same two signals by P_2 is $d\lambda_{P_2}$. Then equations (3.8) yield

$$\frac{d\lambda_{P_2}}{d\lambda_{P_{2'}}} = \frac{f(\tau)}{f(\tau')}\frac{p_v' + \sqrt{f(\tau') + (p_v')^2}}{p_v + \sqrt{f(\tau) + p_v^2}} \qquad (3.9)$$

When both observers are at rest, $p_v = p_v' = 0$ this leads to the standard formula for the gravitaional redshift/blueshift. In our setup $\tau \sim \tau_0$ while $\tau' \to \infty$, so that

$$f(\tau) \sim 3\tau_0(\tau - \tau_0), \qquad f(\tau') \sim (\tau')^2, \qquad (3.10)$$

which leads to

$$\frac{d\lambda_{P_2}}{d\lambda_{P_{2'}}} \sim \frac{3\tau_0(\tau - \tau_0)}{2p_v\tau'}. \qquad (3.11)$$

We now need to express the ratio in (3.11) in terms of v. It is clear from the last equation in (3.8) that at early times the geodesic $P_{2'}$ is described by

$$v(\tau') = -\frac{1}{\tau'} + c', \qquad (3.12)$$

where c' is a constant. Note that we are considering light rays which have $v \sim -\infty$, so that the integration constant c' must be large and negative (since $(v - c') = -\frac{1}{\tau'}$ is small as $\tau' \to \infty$). The trajectory P_2 is described in the vicinity of $\tau = \tau_0$ by

$$\tau - \tau_0 \sim A \exp\left[\frac{3\tau_0}{2}v\right], \tag{3.13}$$

where A is a finite constant of integration. Substituting (3.12) and (3.13) in (3.11) we get

$$\frac{d\lambda_{P_2}}{d\lambda_{P'_2}} \sim -\frac{3A\tau_0}{2p_v}(v - c') \exp\left[\frac{3\tau_0}{2}v\right]. \tag{3.14}$$

Thus for a fixed proper time between the signals during emission, the proper time interval for reception becomes *exponentially small* as $v \to -\infty$.

It is interesting to compare the situation with pure de Sitter. Here $f(\tau) = \tau^2$ for all τ and the cosmological horizon is at $\tau = 0$. In this case the ratio of the proper time interval (3.9) for the observers P_2 and $P_{2'}$ becomes, instead of (3.11),

$$\frac{d\lambda_{P_2}}{d\lambda_{P'_2}} \sim \frac{\tau^2}{2p_v\tau'}. \tag{3.15}$$

Near $\tau = 0$, the trajectory P_2 can be obtained by solving the third equation in (3.8) with $f(\tau) = \tau^2 \sim 0$. Since U is finite it is easy to see that one needs $p_v \neq 0$ and one gets

$$v(\tau) = -\frac{2}{\tau} + a, \tag{3.16}$$

where the constant of integration a is finite. The trajectory $P_{2'}$ is exactly the same as (3.12). Using this, the equation (3.15) becomes

$$\frac{d\lambda_{P_2}}{d\lambda_{P'_2}} \sim -\frac{2(v - c')}{(v - a)^2 p_v} \sim -\frac{2(v - c')}{v^2 p_v}, \tag{3.17}$$

where in the second equation above we have used finiteness of a. Once again $\frac{d\lambda_{P_2}}{d\lambda_{P'_2}} \to 0$, however in a power law fashion. This is a much milder blueshift than what is experienced for our bluewall solution.

This exponentially vanishing blueshift is a reflection of the "crowding" of lightrays near the horizon. The energy flux that the infalling observer measures is $T_{\mu\nu}v^\mu v^\nu \sim T_{vv}\dot{v}^2$. From above, we see that the infalling observer thus crosses a diverging flux of incoming lightrays in finite proper time as he approaches the horizon,[6] suggesting an instability. This is somewhat akin to the Reissner-Nordstrom black hole inner horizon (see e.g. [23]) where an infalling observer receives signals from the exterior region in vanishingly small proper time ("seeing entire histories in a flash"). However note that here, this occurs for the late time infalling observer only as he approaches the horizon and only from signals emanating at early times from "infinity" ($|w| \to \infty$). Now applying the energy-momentum calculation earlier gives an imaginary energy density $\langle T_{ij} \rangle$: it is interesting to ask if this is the dual CFT signature of the blue-shift instability. It would be interesting to explore these further, perhaps keeping in mind black holes, firewalls and entanglement [24–26].

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[6]It is a reasonable assumption that T_{vv} for the lightrays follows a power law in v, akin to [22]. From (3.8), (3.13), we have $\dot{v} \sim e^{-(3\tau_0/2)v}|_{v \to -\infty} \to \infty$. Thus $T_{vv}\dot{v}^2$ diverges as $\tau \to \tau_0$.

Acknowledgments

A de Sitter "bluewall" details

We give some details on the dS_4-"bluewall" $\frac{ds^2}{R^2} = -\frac{d\tau^2}{\tau^2(1-\tau_0^3/\tau^3)} + \tau^2(1-\tau_0^3/\tau^3)dw^2 + \tau^2 dx_i^2$ here. We can analyse the vicinity of $\tau = \tau_0$ as for the Schwarzschild black hole, defining a "tortoise" τ-coordinate: for the dS_4-solution, this is

$$\tau_* = \int \frac{d\tau}{\tau^2(1-\frac{\tau_0^3}{\tau^3})} = \frac{1}{3\tau_0}\Big(\log\frac{\tau-\tau_0}{\sqrt{\tau^2+\tau\tau_0+\tau_0^2}} \ + \ \sqrt{3}\tan^{-1}\frac{2\frac{\tau}{\tau_0}+1}{\sqrt{3}}\Big). \qquad (A.1)$$

Analogs of Kruskal-Szekeres coordinates can then be defined as

$$\tilde{u} = e^{3(\tau_*-w)\tau_0/2}, \quad \tilde{v} = e^{3(\tau_*+w)\tau_0/2}, \qquad \tilde{u}\tilde{v} = e^{3\tau_*\tau_0} = \frac{\tau-\tau_0}{\sqrt{\tau^2+\tau\tau_0+\tau_0^2}} \, e^{\sqrt{3}\tan^{-1}\frac{2\frac{\tau}{\tau_0}+1}{\sqrt{3}}},$$
$$(A.2)$$

and $u = \tilde{u} - \tilde{v} = -2e^{3\tau_*\tau_0/2}\sinh\frac{3w\tau_0}{2}$, $v = \tilde{u} + \tilde{v} = 2e^{3\tau_*\tau_0/2}\cosh\frac{3w\tau_0}{2}$, giving (3.2) and the Penrose diagram figure 1. With $T = \int d\tau/(\tau\sqrt{1-\tau_0^3/\tau^3})$, this is recast in FRW-form as an accelerating cosmology $ds^2 = -dT^2 + e^{-2T}(e^{3T}+\tau_0^3)^{4/3}\frac{(e^{3T}-\tau_0^3)^2}{(e^{3T}+\tau_0^3)^2}dw^2 + e^{-2T}(e^{3T}+\tau_0^3)^{4/3}dx_i^2$ with w-anisotropy. Further redefining $T = \log\eta$, we can obtain a Fefferman-Graham expansion for this asymptotically-dS_4 spacetime near the boundary $\tau \to \infty$. Following Einstein-Rosen's description [27] of the "bridge" in the Schwarzschild black hole (using $\rho^2 = r - 2m$), define $t^2 = \tau - \tau_0$. This coordinate has the range $t : -\infty \to \infty$ as $\tau : \infty \to \tau_0$ and then $\tau : \tau_0 \to \infty$, giving two t-sheets of the asymptotic deSitter region,

$$ds^2 = \frac{-4(t^2+\tau_0)dt^2}{(t^2+\tau_0)^2+\tau_0(t^2+\tau_0)+\tau_0^2} + \Big(\frac{(t^2+\tau_0)^2+\tau_0(t^2+\tau_0)+\tau_0^2}{t^2+\tau_0}\Big)t^2 dw^2 + (t^2+\tau_0)^2 dx_i^2.$$

Thus the two asymptotic universes are connected by a timelike Einstein-Rosen bridge. At $\tau = \tau_0$, we have $g_{ww} = 0$ so the w-direction shrinks to vanishing size. Near $\tau = \tau_0$, the metric is approximated as $ds^2 \sim -\frac{d\tau^2}{k(\tau-\tau_0)} + (\tau-\tau_0)\tau_0^2 dw^2 + \tau_0^2 dx_i^2 \sim -dt^2 + t^2 d\tilde{w}^2 + d\tilde{x}_i^2$, which is flat space with the (t,w)-plane in Milne coordinates.

Null geodesics $ds^2 = 0$, in the (τ, w)-plane defining lightcones and causal structure, are $dw = \pm d\tau_*$, $\frac{dw}{d\tau} = \pm\frac{1}{\tau^2(1-\tau_0^3/\tau^3)}$, with τ_* given in (A.1). Near the horizon, the trajectories approach $w = \pm\tau_* + const \sim \pm\frac{1}{3\tau_0}\log\frac{|\tau-\tau_0|}{3\tau_0}$, i.e. $w \to \pm\infty$. These null rays intersect the horizon and hit the singularity in the interior at $w_0 \pm \frac{1}{3\tau_0}\log 3$.

Note that $\tau = const$ surfaces are spacelike hyperbolic hypersurfaces with $v^2 - u^2 = const$ in the region outside the horizons, using (A.2). In these exterior regions,

$$w = const \ \text{path} \quad \Rightarrow \quad \frac{u}{v} = -\tanh\frac{3w\tau_0}{2} = const \,, \quad i.e. \quad \frac{\tilde{u}}{\tilde{v}} = \frac{1 - \tanh\frac{3w\tau_0}{2}}{1 + \tanh\frac{3w\tau_0}{2}} \equiv k \,, \ \text{(A.3)}$$

i.e. straight lines passing through the bifurcation region, crossing over from the past asymptotic region II to the future one I. The induced worldline metric on such a $w, x_i = const$ trajectory and associated proper time are $dl^2 = \frac{d\tau^2}{\tau^2 - \tau_0^3/\tau} \equiv dT^2$, $T = \frac{2}{3}\log(\tau^{3/2} + \sqrt{\tau^3 - \tau_0^3})$. The spatial metric on a $\tau = const$ hypersurface orthogonal to these constant-w, x_i trajectories is $\frac{d\sigma^2}{R^2} = \tau^2\left(1 - \frac{\tau_0^3}{\tau^3}\right)dw^2 + \tau^2 dx_i^2$. We see that at the bridge $\tau \to \tau_0$, the spatial metric degenerates and the cross-sectional 3-area $V_{w,x_1,x_2} = \Delta w \Delta^2 x_i \tau^3 \sqrt{1 - \tau_0^3/\tau^3}$ vanishes. The proper time T is consistent with the equations for timelike geodesics at constant x_i in the (τ, w)-plane, and is finite along such geodesic paths between the horizon $\tau = \tau_0$ and any point $\tau < \infty$. It can be seen by studying geodesic deviation for a congruence of such timelike geodesic static observers with const-w, x_i that there are no diverging tidal forces as one crosses the bifurcation region from the past universe to the future one.

Consider now the spacetime in Kruskal form (3.2) written as $ds^2 = -2f(\tilde{u}, \tilde{v})d\tilde{u}d\tilde{v} + g(\tilde{u}, \tilde{v})dx_i^2$. A family of generic timelike paths in the Penrose diagram is $\tilde{u} = k\tilde{v} + c$, which are obtained by translating sideways the $w = const$ paths (A.3). For $c = 0$, these are geodesics passing through the bifurcation region (without intersecting the horizon). Parametrizing these timelike paths as $x^\alpha(\lambda)$ in the $\tilde{u}, \tilde{v}$-plane ($x_i = const$), it can be shown that the acceleration components $a^{\tilde{u}} = \ddot{\tilde{u}} + \Gamma^{\tilde{u}}_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta$ and similarly $a^{\tilde{v}}$ are finite as $\tau \to \tau_0$, as is the covariant acceleration norm $g_{\mu\nu}a^\mu a^\nu$. Any arbitrary smooth timelike trajectory can be approximated as a straight line in the neighbourhood of any point, in particular near the horizon. Thus the acceleration vanishes for any timelike path crossing the horizons. This is perhaps not surprising since the near horizon geometry is essentially Milne.

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