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More on non-supersymmetric asymmetric orbifolds with vanishing cosmological constant

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Abstract: We explore various non-supersymmetric type II string vacua constructed based on asymmetric orbifolds of tori with vanishing cosmological constant at the one loop. The string vacua we present are modifications of the models studied in [14], of which orbifold group is just generated by a single element. We especially focus on two types of modifications: **(i)** the orbifold twists include different types of chiral reflections not necessarily removing massless Rarita-Schwinger fields in the 4-dimensional space-time, **(ii)** the orbifold twists do not include the shift operator. We further discuss the unitarity and stability of constructed non-supersymmetric string vacua, with emphasizing the common features of them.

Keywords: Conformal Field Models in String Theory, Superstring Vacua

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Contents

1 Introduction and summary

Much attention has been currently focused on the string theories on non-geometric backgrounds. A simple and interesting class of such backgrounds are constructed due to the asymmetric orbifolds, in which the orbifold twists act asymmetrically on the left and right movers [1]. Although they look beyond our intuitive picture of space-time, they are well-described as done for geometrical ones by the approach of world-sheet conformal field theory (CFT) in the α'-exact fashion.

Above all, one of the natural purposes to study the type II string on asymmetric orbifolds would be the construction of non-supersymmetric (SUSY) string vacua *with vanishing cosmological constant* motivated by phenomenological or theoretical interests. It seems evident that the SUSY-breaking realized in any geometric or symmetric orbifolds inevitably gives rise to a non-vanishing cosmological constant already at the one-loop. In this sense, the bose-fermi cancellation without SUSY would only be possible in the suitable non-geometric compactification in superstring theory. The attempts of construction of non-SUSY vacua have been initiated by the works [2–4] based on some non-abelian orbifolds, followed by closely related studies e.g. in [5–8]. Moreover, sharing similar motivations, non-SUSY vacua in heterotic string theory have been investigated e.g. in [9–13].

Recently, in our previous paper [14], we have presented a simple new realization of non-SUSY string vacua with the bose-fermi cancellation based on a *cyclic orbifold*, that is, the relevant orbifold group is generated by a single element. Hence, this construction looks rather simpler than the previous ones given in the papers quoted above. The crucial point in this construction is the fact that 'chiral reflection' (or the T-duality twist) along the T^4-directions;[1]

$$\mathcal{R} \equiv (-\mathbf{1}_R)^{\otimes 4} \ : \ (X^\mu_L, X^\mu_R) \longmapsto (X^\mu_L, -X^\mu_R), \quad (\psi^\mu_L, \psi^\mu_R) \longmapsto (\psi^\mu_L, -\psi^\mu_R), \qquad (\mu = 6,7,8,9), \tag{1.1}$$

is *not* necessarily involutive when acting on the world-sheet fermions, even in the untwisted sector.[2] Indeed, as illustrated in [14], while it is always involutive on the (right-moving) NS-fermions in the untwisted sector, we still have two possibilities **(i)** $\mathcal{R}^2 = \mathbf{1}$, **(ii)** $\mathcal{R}^2 = -\mathbf{1}$ for the R-sector. In other words, even though $\mathcal{R}^2$ obviously commutes with all the world-sheet coordinates;

$$\mathcal{R}^2 X^\mu_R \mathcal{R}^{-2} = X^\mu_R, \qquad \mathcal{R}^2 \psi^\mu_R \mathcal{R}^{-2} = \psi^\mu_R,$$

it may still act on the Ramond vacua (or spin fields) as a sign flip. The case **(ii)** means that $\mathcal{R}^2 = (-1)^{F_R}$, where F_R (F_L) denotes the 'space-time fermion number' from the right(left)-mover. If taking the second one, which we often call the '$\mathbb{Z}_4$-chiral reflection', one finds that the type II string vacuum constructed as the $\mathbb{Z}_4$-orbifold by $\sigma \equiv (-1)^{F_L} \otimes (-\mathbf{1}_R)^{\otimes 4}$ possesses the next properties;

- All the space-time supercharges arising from the untwisted sector are eliminated by the $\mathbb{Z}_4$-projection $\frac{1}{4}\sum_{r\in\mathbb{Z}_4} \sigma^r$, since any supercharges in the unorbifolded theory do not commute with both of $(-1)^{F_L}$ and $(-1)^{F_R}$.

- All the partition sums in the untwisted sector vanish under the insertion of σ^r for ${}^\forall r \in \mathbb{Z}_4$. Namely, we find ($q \equiv e^{2\pi i \tau}$);

  $$\mathrm{Tr}_{\text{untwisted}}\left[\sigma q^{L_0 - \frac{c}{24}} \bar{q}^{\tilde{L}_0 - \frac{c}{24}}\right] = \mathrm{Tr}_{\text{untwisted}}\left[\sigma^3 q^{L_0 - \frac{c}{24}} \bar{q}^{\tilde{L}_0 - \frac{c}{24}}\right] = 0,$$

  due to the cancellation in the right moving fermions caused by $(-\mathbf{1}_R)^{\otimes 4}$, while

  $$\mathrm{Tr}_{\text{untwisted}}\left[q^{L_0 - \frac{c}{24}} \bar{q}^{\tilde{L}_0 - \frac{c}{24}}\right] = \mathrm{Tr}_{\text{untwisted}}\left[\sigma^2 q^{L_0 - \frac{c}{24}} \bar{q}^{\tilde{L}_0 - \frac{c}{24}}\right] = 0,$$

  holds because σ^2 trivially acts on the left-mover, yielding the familiar vanishing factor $\theta_3^4 - \theta_4^4 - \theta_2^4$.

They are surely nice features for the purpose to realize the non-SUSY string vacua with the bose-fermi cancellation. However, as addressed in [14] and will be demonstrated in section 2 for a detail, it turns out that 8 supercharges eventually emerge *in the twisted sector*. We thus adopted in [14] the (infinite order) orbifold group generated by the operator

$$g = \mathcal{T}_{2\pi R} \otimes \sigma \equiv \mathcal{T}_{2\pi R} \otimes (-1)^{F_L} \otimes (-\mathbf{1}_R)^{\otimes 4}, \tag{1.2}$$

[1]Through this paper, $X^\mu \equiv (X^\mu_L, X^\mu_R)$ ($\psi^\mu \equiv (\psi^\mu_L, \psi^\mu_R)$) denotes the world-sheet bosonic (fermionic) fields in the RNS formalism of type II string theory. The directions $\mu = 0, \ldots, 3$ are always identified as the 4-dim. Minkowski space-time M^4, and we mainly focus on the transverse part $\mu = 2, \ldots, 9$. In addition, we will often use the notations $\lambda^i \equiv (\lambda^i_L, \lambda^i_R)$ ($i = 1, \ldots, 2N$) to express the free fermions describing the N-dim. torus with the SO($2N$)-symmetry enhancement, which will be denoted as $T^N[\mathrm{SO}(2N)]$ in the text.

[2]It is well-known that the chiral reflections often define order $N \geq 4$ orbifolds rather than order 2 due to the non-trivial phase factors appearing in the *twisted sectors*, even though they act as an involution on the untwisted sector. See e.g. [15].

in place of σ, following the spirit of Scherk-Schwarz type compactification [16, 17]. Here, $\mathcal{T}_{2\pi R}$ denotes the shift by $2\pi R$ along the 'base' direction, originally identified as a real line $\mathbb{R}_{\text{base}}$. The inclusion of shift into (1.2) enables us to naturally identify the twisted sectors with the winding sectors of the 'Scherk-Schwarz circle'. More significantly, it plays the role of removing potential supercharges which might arise from the twisted sectors.[3] We also note that this model would be interpreted as a modification of the simple realizations of the 'T-folds' [18–24], that is, the orbifolds by the chiral reflection (or the T-duality twist) combined with the shift in the base space. These types of non-geometric backgrounds have been studied by the approach of world-sheet CFT e.g. in [25–32].

Now, in this paper, we would like to explore a variety of non-SUSY string vacua of this type. We shall especially focus on the next two modifications of (1.2):

(i) We replace $(-1)^{F_L}$ with $(-\mathbf{1}_L)^{\otimes 2}$, which acts along the various directions of back-gorund tori, and plays the role of breaking the left-moving SUSY.

(ii) We do not include the shift operator $\mathcal{T}_{2\pi R}$. Instead, we assume that $\mathcal{R} \equiv (-\mathbf{1}_R)^{\otimes 4}$ acts as the $\mathbb{Z}_4$-chiral reflection also for the world-sheet *bosons*. This is achieved by utilizing the fermionization of bosonic coordinates X^μ, and plays the role of preventing the twisted sectors from providing additional supercharges.

Stated more concretely, the models that we shall study in this paper are displayed in tables 1 and 2. In section 2, we briefly review on the 'previous' one studied in [14], which would be helpful to readers. We then investigate the new six models (the 'models I to VI') in section 3. We exhibit the relevant orbifold actions in table 1, while the original backgrounds that we orbifold are summarized in table 2. In all the models the orbifold groups are generated by a single element denoted as g in table 1. In table 2, M^4 expresses the four-dimensional Minkowski space-time. The orbifold twists do not act on $\left[M^4 \times \cdots\right]$ in each row. The shift $\mathcal{T}_{2\pi R}$ always acts along $\mathbb{R}_{\text{base}}$. Throughout this paper, we use the notation '$T^N[\mathrm{SO}(2N)]$' to express the N-dimensional torus at the symmetry enhancement point of $\mathrm{SO}(2N)$. In other words, they can be described in terms of $2N$ Majorana fermions (denoted as '$\lambda^i \equiv (\lambda^i_L, \lambda^i_R)$').

Let us summarize the aspects of models I to VI on which we will elaborate in section 3. The models I and II are defined by including $(-\mathbf{1}_L)^{\otimes 2}$ instead of $(-1)^{F_L}|_\psi$. Combining it with $(-\mathbf{1}_R)^{\otimes 4}$, some directions of tori are eventually orbifolded by the non-chiral reflection: $(X^\mu_L, X^\mu_R) \rightarrow (-X^\mu_L, -X^\mu_R)$, and we simply denote '$T^2$' and '$S^1$' for the corresponding directions. It will be shown that these models are indeed the non-SUSY string vacua with the bose-fermi cancellation as expected. We do not have any tachyonic instability in all the untwisted and twisted sectors, while some winding massless modes emerge at particular values of the Scherk-Schwarz radius R. These features are quite similar to the previous one. However, the physical spectra significantly differ from it. Some Rarita-Schwinger fields

[3] At first glance, this fact would look obvious, since the inclusion of shift $\mathcal{T}_{2\pi R}$ generically makes all the Ramond states lying in the twisted sectors massive. However, we often find that additional Ramond massless states appear when choosing the Scherk-Schwarz radius R suitably. Nevertheless, one can show that the space-time SUSY is completely broken for an arbitrary value of R. See [14] for the detail.

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model	g	g^2
previous	$\mathcal{T}_{2\pi R} \otimes (-1)^{F_L}\vert_\psi \otimes (-\mathbf{1}_R)^{\otimes 4}$	$\mathcal{T}_{4\pi R} \otimes (-1)^{F_R}\vert_\psi$
I, II	$\mathcal{T}_{2\pi R} \otimes (-\mathbf{1}_L)^{\otimes 2} \otimes (-\mathbf{1}_R)^{\otimes 4}$	$\mathcal{T}_{4\pi R} \otimes (-1)^{F_R}\vert_\psi$
III	$(-1)^{F_L}\vert_\psi \otimes (-\mathbf{1}_R)^{\otimes 4}$	$(-1)^{F_R}\vert_\lambda \otimes (-1)^{F_R}\vert_\psi$
IV, V, VI	$(-\mathbf{1}_L)^{\otimes 2} \otimes (-\mathbf{1}_R)^{\otimes 4}$	$(-1)^{F_R}\vert_\lambda \otimes (-1)^{F_R}\vert_\psi$

Table 1. The orbifold actions.

models	original backgrounds
previous	$\left[M^4 \times S^1\right] \times \mathbb{R}_{\text{base}} \times T^4[\mathrm{SO}(8)]$
I	$\left[M^4 \times S^1\right] \times \mathbb{R}_{\text{base}} \times T^2 \times T^2[\mathrm{SO}(4)]$
II	$\left[M^4\right] \times \mathbb{R}_{\text{base}} \times S^1 \times T^4[\mathrm{SO}(8)]$
III	$\left[M^4 \times T^2\right] \times T^4[\mathrm{SO}(8)]$
IV	$\left[M^4 \times T^2\right] \times T^2 \times T^2[\mathrm{SO}(4)]$
V	$\left[M^4 \times S^1\right] \times S^1 \times T^4[\mathrm{SO}(8)]$
VI	$\left[M^4\right] \times T^6[\mathrm{SO}(12)]$

Table 2. The original backgrounds.

survive in the 4-dim. massless spectrum in the models I and II, although not interpreted as the gravitini due to the absence of space-time SUSY. We recall that, in the previous model, the twist by $(-1)^{F_L}$ eliminates all the massless spin 3/2 states in the untwisted sector.

The models III–VI are those not including the shift operator. Instead, we shall modify the right-moving chiral reflections so that their squares yield $(-1)^{F_R}|_\lambda$, that is, the sign flip on the Ramond sector of fermions λ_R^i that describe $T^N[\mathrm{SO}(2N)]$. The left-moving space-time SUSY is broken by $(-1)^{F_L}|_\psi$ in the model III as in the previous one, while $(-\mathbf{1}_L)^{\otimes 2}$ acts on the various directions of tori in the cases of models IV–VI. By the effect of $(-1)^{F_R}|_\lambda$, the twisted sectors gain extra zero point energies despite the absence of shift operator, thereby preventing additional right-moving supercharges from arising. It then turns out that we achieve the desired non-SUSY vacua. They are simpler than the models I and II for the computations of the torus partition functions. Once again, we do not face any tachyonic instabilities, and massless states appear in the twisted sectors as well as the untwisted sector. Note that these models do not include the modulus R as opposed to the cases of models I and II.

The partition functions for all the models in this paper are found manifestly modular invariant and q-expanded in the way compatible with unitarity. Moreover, they are always free from tachyonic instabilities. These would be common features of the toroidal asymmetric orbifolds of these types, as we will discuss in section 4.

2 Notes on the non-SUSY asymmetric orbifold of [14]

In this section, we make a brief sketch of the non-SUSY model constructed in [14] to clarify several points that we will discuss for the new models.

Let us introduce the type II string vacuum in the ten-dimensional flat background;

$$\left[M^4 \times S^1\right] \times \mathbb{R}_{\text{base}} \times T^4[\text{SO}(8)], \tag{2.1}$$

where M^4 ($X^{0,1,2,3}$ -directions) denotes the 4-dimensional Minkowski space-time, and S^1 (X^4 -direction) is a circle that plays no role in this model. $\mathbb{R}_{\text{base}}$ (X^5 -direction) is just a real line, identified as the 'base space' of the twisted compactification like Scherk-Schwarz [16, 17], and, as already mentioned, $T^4[\text{SO}(8)]$ ($X^{6,7,8,9}$ -directions) is the 4-dimensional torus with the SO(8)-symmetry enhancement.

Then, as was introduced in section 1, we define the asymmetric orbifold generated by the operator

$$g = \mathcal{T}_{2\pi R} \otimes \sigma \equiv \mathcal{T}_{2\pi R} \otimes (-1)^{F_L}|_{\psi} \otimes (-\mathbf{1}_R)^{\otimes 4}|_{T^4}, \tag{2.2}$$

acting on the background (2.1). Recall that $\mathcal{T}_{2\pi R}$ denotes the shift operator along $\mathbb{R}_{\text{base}}$; $X^5 \rightarrow X^5 + 2\pi R$, and the operator $(-1)^{F_L}|_{\psi}$ $((-1)^{F_R}|_{\psi})$ acts as the sign flip of the left (right) moving Ramond sector. $(-\mathbf{1}_R)^{\otimes 4}|_{T^4}$ denotes the chiral reflection along T^4 given in (1.1). To complete the definition of the operator σ (or $(-\mathbf{1}_R)^{\otimes 4}|_{T^4}$), we still need to specify the construction of Ramond vacua (or spin fields) of right-moving world-sheet fermions ψ_R^{μ} and how σ should act on them. Here, we define the Ramond vacua as $|s_1, \ldots, s_4\rangle_R \equiv e^{i\sum_{a=1}^4 s_a H_R^a}\,|0\rangle_R$, $(s_a \equiv \pm\frac{1}{2})$, where $e^{i\sum_{a=1}^4 s_a H_R^a}$ denotes the SO(8)-spin fields associated to the transverse fermions $\psi_R^2, \ldots, \psi_R^9$ by the bosonization

$$\begin{aligned} \psi_R^2 \pm i\psi_R^3 &= \sqrt{2}e^{\pm iH_R^1}, \quad & \psi_R^4 \pm i\psi_R^6 &= \sqrt{2}e^{\pm iH_R^2}, \\ \psi_R^5 \pm i\psi_R^7 &= \sqrt{2}e^{\pm iH_R^3}, \quad & \psi_R^8 \pm i\psi_R^9 &= \sqrt{2}e^{\pm iH_R^4}. \end{aligned} \tag{2.3}$$

We then obtain

$$\sigma\,|s_1, s_2, s_3, s_4\rangle_R = e^{i\pi s_4}\,|s_1, -s_2, -s_3, s_4\rangle_R\,, \tag{2.4}$$

since σ acts as the sign flip of $\psi_R^6, \ldots \psi_R^9$. Thus we readily find

$$\sigma^2 = (-1)^{F_R}|_{\psi}, \tag{2.5}$$

which plays a crucial role in the following discussions. See [14] for more detail.

Let us focus on the partition function on the world-sheet torus to investigate the one-loop cosmological constant and the space-time supersymmetry. The relevant partition function is schematically written in the form as

$$Z(\tau,\bar{\tau}) = \sum_{w,m\in\mathbb{Z}} Z_{(w,m)}(\tau,\bar{\tau}) \equiv \sum_{w,m\in\mathbb{Z}} Z^X_{(w,m)}(\tau,\bar{\tau}) Z^{\psi_L}_{(w,m)}(\tau)\overline{Z^{\psi_R}_{(w,m)}(\tau)}, \tag{2.6}$$

where the integer w labels the twisted sectors, while m indicates the g^m-insertions into the trace. As already suggested in section 1, they are identified as the spatial and temporal winding numbers on the base space (or the Scherk-Schwarz circle) because of the inclusion of shift $\mathcal{T}_{2\pi R}$ into (2.2). $Z^{X}_{(w,m)}(\tau,\bar{\tau})$ denotes the partition functions of the bosonic sectors, while $Z^{\psi_L}_{(w,m)}(\tau)$, $Z^{\psi_R}_{(w,m)}(\tau)$ are the partition functions of the left- and right-moving fermionic sectors.

Each partition sum $Z_{(w,m)}(\tau,\bar{\tau})$ is evaluated in the easiest way as follows. We first calculate the trace over the untwisted sector ($w=0$),[4]

$$\begin{aligned} Z_{(0,m)}(\tau,\bar{\tau}) &= \mathrm{Tr}_{w=0}\left[g^m q^{L_0-\frac{c}{24}}\bar{q}^{\tilde{L}_0-\frac{c}{24}}\right] \\ &= Z_{R,(0,m)}(\tau,\bar{\tau})\,\mathrm{Tr}_{w=0}\left[\sigma^m q^{L_0-\frac{c}{24}}\bar{q}^{\tilde{L}_0-\frac{c}{24}}\right], \end{aligned} \tag{2.7}$$

$$Z_{R,(w,m)}(\tau,\bar{\tau}) \equiv \frac{R}{\sqrt{\tau_2}|\eta(\tau)|^2}e^{-\frac{\pi R^2}{\tau_2}|w\tau+m|^2}, \quad (w,m\in\mathbb{Z}), \tag{2.8}$$

and those for the general winding sectors (w,m) are uniquely determined by requiring the modular covariance

$$Z_{(w,m)}(\tau,\bar{\tau})|_S = Z_{(m,-w)}(\tau,\bar{\tau}), \tag{2.9}$$

$$Z_{(w,m)}(\tau,\bar{\tau})|_T = Z_{(w,w+m)}(\tau,\bar{\tau}), \tag{2.10}$$

where $S:\tau\to-1/\tau$, $T:\tau\to\tau+1$ are the modular transformations. We then achieve the partition function (2.6) that is manifestly modular invariant.

Note that the left and right partition sums of fermionic sectors $Z^{\psi_L}_{(w,m)}(\tau)$, $\overline{Z^{\psi_R}_{(w,m)}(\tau)}$ are generically asymmetric. The twist operator σ includes $(-1)^{F_L}|_\psi$, and we thus find $Z^{\psi_L}_{(w,m)}(\tau)\neq 0$ for ${}^\forall w\in 2\mathbb{Z}+1$ or ${}^\forall m\in 2\mathbb{Z}+1$. Similarly, by $\sigma^2=(-1)^{F_R}|_\psi$, we obtain $\overline{Z^{\psi_R}_{(w,m)}(\tau)}\neq 0$ for ${}^\forall w\in 4\mathbb{Z}+2$ or ${}^\forall m\in 4\mathbb{Z}+2$. However, one easily finds

$$\begin{aligned} &\overline{Z^{\psi_R}_{(w,m)}(\tau)} = 0,\ (w \text{ or } m\in 2\mathbb{Z}+1), \\ &Z^{\psi_L}_{(w,m)}(\tau) = 0,\ (w,m\in 2\mathbb{Z}). \end{aligned} \tag{2.11}$$

Thus the total partition function vanishes.

Let us turn our attention to the spectrum in the untwisted sector ($w=0$). As already mentioned in section 1, all the space-time supercharges are eliminated by the orbifold projection $\frac{1}{4}\sum_{n\in\mathbb{Z}_4}\sigma^n$ due to the inclusions $(-1)^{F_L}|_\psi$ and $(-1)^{F_R}|_\psi$. For all that, one can observe that the same number of bosonic and fermionic states exist at each mass level of the untwisted sector. Especially, the massless spectrum is summarized in table 3, which includes 32 bosonic and fermionic states. Note that no gravitino appears in the 4-dim. spectrum, which suggests the absence of space-time SUSY.

[4] Here we shall adopt the conventional normalization of the trace for the CFT describing $\mathbb{R}_{\text{base}}$;

$$\mathrm{Tr}\left[q^{L_0-\frac{1}{24}}\overline{q^{\tilde{L}_0-\frac{1}{24}}}\right] = \frac{R}{\sqrt{\tau_2}\,|\eta|^2},$$

so that we simply obtain

$$\mathrm{Tr}\left[(\mathcal{T}_{2\pi R})^m\ q^{L_0-\frac{1}{24}}\overline{q^{\tilde{L}_0-\frac{1}{24}}}\right] = \frac{R}{\sqrt{\tau_2}\,|\eta|^2}e^{-\frac{\pi}{\tau_2}R^2m^2}.$$

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spin structure	4D fields
(NS, NS)	graviton, 8 vectors, 14 (pseudo) scalars
(R , NS)	16 Weyl fermions

Table 3. Massless spectrum in the untwisted sector for the orbifold model defined by g.

However, this is not the whole story. It might be possible that new supercharges arise from the twisted sectors. We also note that tachyonic states would potentially emerge in the twisted sectors, as in many examples of the SUSY-breaking models of Scherk-Schwarz type. Furthermore, the unitarity of string spectrum is not necessarily self-evident because of the non-trivial phase factors appearing in the twisted sectors necessary for the modular invariance. It is surely significant to examine these issues for our purpose. A direct way to do so is to decompose the partition functions with respect to the spatial winding w and the spin structures as

$$\begin{aligned} Z(\tau,\bar{\tau}) = &\frac{1}{4}\mathcal{Z}_{M^4\times S^1}(\tau,\bar{\tau}) \\ &\times \sum_{w\in\mathbb{Z}}\left\{Z_w^{(\mathrm{NS,NS})}(\tau,\bar{\tau}) + Z_w^{(\mathrm{NS,R})}(\tau,\bar{\tau}) + Z_w^{(\mathrm{R,NS})}(\tau,\bar{\tau}) + Z_w^{(\mathrm{R,R})}(\tau,\bar{\tau})\right\}, \end{aligned} \tag{2.12}$$

where $\mathcal{Z}_{M^4\times S^1}$ denotes the bosonic transverse contribution for the $M^4 \times S^1$-sector that has nothing to do with the orbifolding. The string spectrum in each Hilbert space with winding w can be examined by making the Poisson resummation with respect to the temporal winding m. In this way the following results have been shown in [14];

- The partition function for each winding w and each spin structure is compatible with unitarity.
- The bose-fermi cancellation is observed at each mass level of the string spectrum.
- The space-time SUSY is completely broken.
- No tachyonic states appear in all the sectors.
- Massless states arise in some twisted sectors at the specific radius R (the modulus related to the shift $\mathcal{T}_{2\pi R}$).

Especially, let us focus on how one can conclude that the space-time SUSY is truly broken. It has been explicitly shown in [14] that the partition functions for the winding sectors have the relations summarized in table 4. For the odd winding sectors, we have the bose-fermi cancellation compatible only with right-moving SUSY, while the even sectors behave as if we only had left-moving supercharges. It is obvious that any supercharges can never be consistent with both of them at the same time.

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$w \in 2\mathbb{Z}+1$ $Z_w^{(\mathrm{NS,NS})} \neq -Z_{w'}^{(\mathrm{R,NS})}$ $Z_w^{(\mathrm{NS,R})} \neq -Z_{w'}^{(\mathrm{R,R})}$	$Z_w^{(\mathrm{NS,NS})} = -Z_w^{(\mathrm{NS,R})}$ $Z_w^{(\mathrm{R,NS})} = -Z_w^{(\mathrm{R,R})}$
$w \in 2\mathbb{Z}$ $Z_w^{(\mathrm{NS,NS})} = -Z_w^{(\mathrm{R,NS})}$ $Z_w^{(\mathrm{NS,R})} = -Z_w^{(\mathrm{R,R})}$	$Z_w^{(\mathrm{NS,NS})} \neq -Z_{w'}^{(\mathrm{NS,R})}$ $Z_w^{(\mathrm{R,NS})} \neq -Z_{w'}^{(\mathrm{R,R})}$

Table 4. Relations among the winding sectors in the orbifold defined by (2.2). ($^\forall w' \in \mathbb{Z}$).

Remarks on the supersymmetric cases. It would be worthwhile to figure out what happens in the closely related model with the SUSY unbroken, that is, the asymmetric orbifold defined by $\sigma \equiv (-1)^{F_L}|_\psi \otimes (-\mathbf{1}_R)^{\otimes 4}$ without including the shift. We also adopt (2.4) for the action of σ on Ramond vacua, and thus the orbifold twist is still a $\mathbb{Z}_4$-action. The partition function is then written in the form as

$$Z(\tau,\bar{\tau}) = \frac{1}{4}\sum_{a,b\in\mathbb{Z}_4} Z_{(a,b)}(\tau,\bar{\tau}) \equiv \frac{1}{4}\sum_{a,b\in\mathbb{Z}_4} Z^X_{(a,b)}(\tau,\bar{\tau}) Z^{\psi_L}_{(a,b)}(\tau)\overline{Z^{\psi_R}_{(a,b)}(\tau)}. \tag{2.13}$$

In this case, the orbifold projection still removes all the supercharges in the untwisted sector, but the right-moving supercharges revive from the $a=2$ twisted sector.

To show this fact explicitly, let us again decompose the partition functions as

$$\begin{aligned} Z(\tau,\bar{\tau}) = & \frac{1}{16}\mathcal{Z}_{M^4\times T^2}(\tau,\bar{\tau}) \\ & \times \sum_{a\in\mathbb{Z}_4}\left\{Z_a^{(\mathrm{NS,NS})}(\tau,\bar{\tau}) + Z_a^{(\mathrm{NS,R})}(\tau,\bar{\tau}) + Z_a^{(\mathrm{R,NS})}(\tau,\bar{\tau}) + Z_a^{(\mathrm{R,R})}(\tau,\bar{\tau})\right\}, \end{aligned} \tag{2.14}$$

where the overall factor $\frac{1}{16} \equiv \frac{1}{4}\times\frac{1}{4}$ is due to the $\mathbb{Z}_4$-orbifolding as well as the chiral GSO projection. Then we obtain

$$\begin{aligned} Z_0^{(\mathrm{NS,NS})}(\tau,\bar{\tau}) = & -Z_0^{(\mathrm{R,NS})}(\tau,\bar{\tau}) = Z_2^{(\mathrm{R,R})}(\tau,\bar{\tau}) = -Z_2^{(\mathrm{NS,R})}(\tau,\bar{\tau}) \\ & = Z_{1,3}^{(\mathrm{R,R})}(\tau,\bar{\tau}) = -Z_{1,3}^{(\mathrm{R,NS})}(\tau,\bar{\tau}) = \left\{\left|\frac{\theta_3}{\eta}\right|^8 + \left|\frac{\theta_4}{\eta}\right|^8 + \left|\frac{\theta_2}{\eta}\right|^8\right\}\left|\frac{\theta_2}{\eta}\right|^8, \end{aligned} \tag{2.15}$$

$$\begin{aligned} Z_{1,3}^{(\mathrm{NS,NS})}(\tau,\bar{\tau}) = & -Z_{1,3}^{(\mathrm{NS,R})}(\tau,\bar{\tau}) \\ = & \overline{\left(\frac{\theta_2}{\eta}\right)^4}\left\{\left|\frac{\theta_3}{\eta}\right|^8 - \left|\frac{\theta_4}{\eta}\right|^8\right\}\left\{\left(\frac{\theta_3}{\eta}\right)^4 + \left(\frac{\theta_4}{\eta}\right)^4\right\} \\ & + \left|\frac{\theta_2}{\eta}\right|^8\left\{\overline{\left(\frac{\theta_3}{\eta}\right)^4} + \overline{\left(\frac{\theta_4}{\eta}\right)^4}\right\}\left\{\left(\frac{\theta_3}{\eta}\right)^4 + \left(\frac{\theta_4}{\eta}\right)^4\right\}, \end{aligned} \tag{2.16}$$

Obviously, we cannot construct any left-moving supercharges since we find

$$Z_{1,3}^{(\mathrm{NS,NS})}(\tau,\bar{\tau}) \neq -Z_a^{(\mathrm{R,NS})}(\tau,\bar{\tau}), \qquad (^\forall a \in \mathbb{Z}_4).$$

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spin structure	4D fields
(NS, R)	2 gravitini, 14 Weyl fermions
(R , R)	8 vectors, 16 (pseudo)scalars

Table 5. Massless spectrum in the $a = 2$ sector for the orbifold model defined by σ.

On the other hand, there would exist some right supercharges in the $a = 2$ sector which realizes the equalities

$$Z_a^{(*,\mathrm{NS})}(\tau,\bar{\tau}) = -Z_{a+2 \bmod 4}^{(*,\mathrm{R})}(\tau,\bar{\tau}), \tag{2.17}$$

as found in (2.15), (2.16). In fact, one can explicitly confirm that the $a = 2$ sector includes the right-moving massless Ramond states, even though all of them are projected out by $(-1)^{F_R}|_\psi$ in the untwisted sector. To be more precise, if starting with the type IIA (IIB) string theory, one can construct 8 supercharges that possess the *opposite chirality as those in the type IIB (IIA) theory* from the $a = 2$ sector, as discussed e.g. in [33, 34].

The massless spectrum in the untwisted sector is the same as that displayed in table 3, while that lying in the $a = 2$ sector is summarized in table 5. These states are combined into the super-multiplets in an $N = 2$ supersymmetric theory in 4-dimension.

3 Variety of non-supersymmetric asymmetric orbifolds

In this section, we present the main analyses in this paper. As already mentioned in section 1, we especially focus on the modifications of the previous model introduced in section 2 by **(i)** replacing $(-1)^{F_L}|_\psi$ with $(-\mathbf{1}_L)^{\otimes 2}$ in (1.2), or/and **(ii)** requiring that $\mathcal{R} \equiv (-\mathbf{1}_R)^{\otimes 4}$ acts as the $\mathbb{Z}_4$-chiral reflection *also on the bosonic sector* instead of including the shift $\mathcal{T}_{2\pi R}$.

We shall start our analyses with constructing the relevant building blocks in subsection 3.1, with emphasizing the modular covariance of them. After that, we present the new six vacua composed of asymmetric orbifolds, and concretely discuss their physical aspects in subsection 3.2. The readers not interested in the technical part of this work may skip many parts of subsection 3.1, and can refer only to the definitions of building blocks.

3.1 Building blocks

3.1.1 Bosonic $T^N[\mathrm{SO}(2N)]$ sector

Firstly, we discuss the simple example $T^2[\mathrm{SO}(4)]$, identified as the X^6, X^7-directions. The torus partition function of this system is

$$Z^{T^2[\mathrm{SO}(4)]}(\tau,\bar{\tau}) = \frac{1}{2}\left\{\left|\frac{\theta_3}{\eta}\right|^4 + \left|\frac{\theta_4}{\eta}\right|^4 + \left|\frac{\theta_2}{\eta}\right|^4\right\}. \tag{3.1}$$

A convenient description is given by introducing the Majorana-Weyl fermions λ_L^i, λ_R^i ($i = 1,2,3,4$).

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In the previous section, for simplicity, it has been assumed that the twist operator σ including chiral reflection acts as an involution on the untwisted sector of the bosonic part. However, once adopting the fermionic description of $T^2[\mathrm{SO}(4)]$, we are aware of another possibility in the manner similar to the world-sheet fermions ψ_L^μ, ψ_R^μ. Namely, considering the left-mover for instance, the chiral reflection $(-\mathbf{1}_L)^{\otimes 2}|_{T^2} : (X_L^6, X_L^7) \to (-X_L^6, -X_L^7)$ is just interpretable as the sign flip of two of $\lambda_L^1, \ldots, \lambda_L^4$, say,

$$(-\mathbf{1}_L)^{\otimes 2} : (\lambda_L^1, \lambda_L^2, \lambda_L^3, \lambda_L^4) \to (\lambda_L^1, \lambda_L^2, -\lambda_L^3, -\lambda_L^4). \tag{3.2}$$

As illustrated in [14] and already mentioned in section 2 for the world-sheet fermions ψ^μ, we still need to define the Ramond vacua of this free fermion system to specify completely the action of $(-\mathbf{1}_L)^{\otimes 2}$. Here, there are essentially two different cases;

(a) $\{(-\mathbf{1}_L)^{\otimes 2}\}^2 = (-1)^{F_L}|_\lambda$:

One can introduce the spin fields as

$$\tilde{S}_{\epsilon_1, \epsilon_2, L} \equiv e^{i \sum_{i=1}^2 \epsilon_i \tilde{H}_L^i}, \quad \left(\epsilon_i = \pm \frac{1}{2}\right), \tag{3.3}$$

with the bosonization;

$$\lambda_L^1 \pm \lambda_L^2 \equiv \sqrt{2} e^{\pm i \tilde{H}_L^1}, \quad \lambda_L^3 \pm \lambda_L^4 \equiv \sqrt{2} e^{\pm i \tilde{H}_L^2}. \tag{3.4}$$

Then, (3.2) yields

$$(-\mathbf{1}_L)^{\otimes 2} : (\tilde{H}_L^1, \tilde{H}_L^2) \to (\tilde{H}_L^1, \tilde{H}_L^2 + \pi), \tag{3.5}$$

and the Ramond vacua $|\epsilon_1, \epsilon_2\rangle_L \equiv \tilde{S}_{\epsilon_1, \epsilon_2, L}(0)\, |0\rangle_L$ are transformed as

$$(-\mathbf{1}_L)^{\otimes 2}\, |\epsilon_1, \epsilon_2\rangle_L = e^{i\pi \epsilon_2}\, |\epsilon_1, \epsilon_2\rangle_L \,. \tag{3.6}$$

Thus, we find that $\{(-\mathbf{1}_L)^{\otimes 2}\}^2 = -\mathbf{1}$ holds for the R sector, while $\{(-\mathbf{1}_L)^{\otimes 2}\}^2 = \mathbf{1}$ for the NS sector of λ_L^i. Namely, we obtain $\{(-\mathbf{1}_L)^{\otimes 2}\}^2 = (-1)^{F_L}|_\lambda$.

(b) $\{(-\mathbf{1}_L)^{\otimes 2}\}^2 = \mathbf{1}$:

One may also bosonize $\lambda_L^1, \ldots, \lambda_R^4$ in a different way;

$$\lambda_L^1 \pm \lambda_L^3 \equiv \sqrt{2} e^{\pm i \tilde{H}_L'^1}, \quad \lambda_L^2 \pm \lambda_L^4 \equiv \sqrt{2} e^{\pm i \tilde{H}_L'^2}, \tag{3.7}$$

and define the spin fields as follows;

$$\tilde{S}'_{\epsilon_1, \epsilon_2, L} \equiv e^{i \sum_{i=1}^2 \epsilon_i \tilde{H}_L'^i}, \quad \left(\epsilon_i = \pm \frac{1}{2}\right). \tag{3.8}$$

This time, (3.2) yields

$$(-\mathbf{1}_L)^{\otimes 2} : (\tilde{H}_L'^1, \tilde{H}_L'^2) \to (-\tilde{H}_L'^1, -\tilde{H}_L'^2), \tag{3.9}$$

and the Ramond vacua $|\epsilon_1, \epsilon_2\rangle'_L \equiv S'_{\epsilon_1, \epsilon_2}(0)\, |0\rangle$ are transformed as

$$(-\mathbf{1}_L)^{\otimes 2}\, |\epsilon_1, \epsilon_2\rangle'_L = |-\epsilon_1, -\epsilon_2\rangle'_L \,. \tag{3.10}$$

We thus simply obtain $\{(-\mathbf{1}_L)^{\otimes 2}\}^2 = \mathbf{1}$.

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The above arguments are straightforwardly generalized to the cases of $T^N[\mathrm{SO}(2N)]$ ($N \in 2\mathbb{Z}_{>0}$) described by $2N$ Majorana-Weyl fermions λ^i_L, λ^i_R ($i = 1, \ldots, 2N$), and we always have two possibilities; **(i)** $\{(-\mathbf{1}_L)^{\otimes N}\}^2 = \mathbf{1}$, or **(ii)** $\{(-\mathbf{1}_L)^{\otimes N}\}^2 = (-1)^{F_L}|_\lambda$.

Let us describe the relevant blocks which we will utilize later. In the following, the twist parameters $a, b \in \mathbb{Z}$ in the subscript always labels the spatial and temporal boundary conditions.[5] In other words, the parameter a labels the twisted sectors, while the parameter b corresponds to the insertion of σ^b into the trace.

(i) $(-1)^{F_L}|_\lambda$-twisting in the $T^N[\mathrm{SO}(2N)]$-sector: first we consider the building blocks $\mathbf{Z}^{T^N[\mathrm{SO}(2N)]}_{(a,b)}(\tau, \bar{\tau})$, $(a, b \in \mathbb{Z})$, defined by the twisting $(-1)^{F_L}|_\lambda$ acting on $T^N[\mathrm{SO}(2N)]$. The $(0,b)$-sector is just the insertion of $\left\{(-1)^{F_L}|_\lambda\right\}^b$ into the trace, and easily evaluated as

$$\mathbf{Z}^{T^N[\mathrm{SO}(2N)]}_{(0,b)}(\tau, \bar{\tau}) = \begin{cases} Z^{T^N[\mathrm{SO}(2N)]}(\tau, \bar{\tau}), & (b \in 2\mathbb{Z}), \\ \dfrac{1}{2}\left\{\left|\dfrac{\theta_3}{\eta}\right|^{2N} + \left|\dfrac{\theta_4}{\eta}\right|^{2N} - \left|\dfrac{\theta_2}{\eta}\right|^{2N}\right\}, & (b \in 2\mathbb{Z}+1). \end{cases} \tag{3.11}$$

Then, requiring the modular covariance

$$\begin{aligned} \mathbf{Z}^{T^N[\mathrm{SO}(2N)]}_{(a,b)}(\tau, \bar{\tau})|_S &= \mathbf{Z}^{T^N[\mathrm{SO}(2N)]}_{(b,-a)}(\tau, \bar{\tau}), \\ \mathbf{Z}^{T^N[\mathrm{SO}(2N)]}_{(a,b)}(\tau, \bar{\tau})|_T &= \mathbf{Z}^{T^N[\mathrm{SO}(2N)]}_{(a,a+b)}(\tau, \bar{\tau}), \end{aligned} \tag{3.12}$$

we obtain

$$\mathbf{Z}^{T^N[\mathrm{SO}(2N)]}_{(a,b)}(\tau, \bar{\tau}) \equiv \begin{cases} \dfrac{1}{2}\left\{\left|\dfrac{\theta_3}{\eta}\right|^{2N} + \left|\dfrac{\theta_4}{\eta}\right|^{2N} + \left|\dfrac{\theta_2}{\eta}\right|^{2N}\right\}, & (a \in 2\mathbb{Z},\ b \in 2\mathbb{Z}) \\ \dfrac{1}{2}\left\{\left|\dfrac{\theta_3}{\eta}\right|^{2N} + \left|\dfrac{\theta_4}{\eta}\right|^{2N} - \left|\dfrac{\theta_2}{\eta}\right|^{2N}\right\}, & (a \in 2\mathbb{Z},\ b \in 2\mathbb{Z}+1) \\ \dfrac{1}{2}\left\{\left|\dfrac{\theta_3}{\eta}\right|^{2N} - \left|\dfrac{\theta_4}{\eta}\right|^{2N} + \left|\dfrac{\theta_2}{\eta}\right|^{2N}\right\}, & (a \in 2\mathbb{Z}+1,\ b \in 2\mathbb{Z}) \\ \dfrac{1}{2}\left\{-\left|\dfrac{\theta_3}{\eta}\right|^{2N} + \left|\dfrac{\theta_4}{\eta}\right|^{2N} + \left|\dfrac{\theta_2}{\eta}\right|^{2N}\right\}, & (a \in 2\mathbb{Z}+1,\ b \in 2\mathbb{Z}+1). \end{cases} \tag{3.13}$$

(ii) $(-\mathbf{1}_R)^{\otimes N}$-twisting in the $T^N[\mathrm{SO}(2N)]$-sector: next, we consider the building blocks corresponding to the twist operator $(-\mathbf{1}_R)^{\otimes 4}$ which acts on $T^4[\mathrm{SO}(8)]$. Yet, the twist operator is not specified. As noticed at the beginning of this section, we have two possibilities $\{(-\mathbf{1}_R)^{\otimes 4}\}^2 = \mathbf{1}$, or $\{(-\mathbf{1}_R)^{\otimes 4}\}^2 = (-1)^{F_R}|_\lambda$.

[5] Here, we shall allow the parameters of twisting a, b to be arbitrary integers just for convenience, although it is enough to restrict their range at most as $a, b \in \mathbb{Z}_4$.

The building blocks for the first case are given as follows;

$$F^{T^4[\mathrm{SO}(8)]}_{(a,b)}(\tau,\bar\tau)$$
$$\equiv \begin{cases} \frac{1}{2}\left\{\left|\frac{\theta_3}{\eta}\right|^8+\left|\frac{\theta_4}{\eta}\right|^8+\left|\frac{\theta_2}{\eta}\right|^8\right\}, & (a\in 2\mathbb{Z}, b\in 2\mathbb{Z}) \\ (-1)^{\frac{a}{2}}\overline{\left(\frac{\theta_3\theta_4}{\eta^2}\right)^2}\frac{1}{2}\left\{\left(\frac{\theta_3}{\eta}\right)^4+\left(\frac{\theta_4}{\eta}\right)^4\right\}, & (a\in 2\mathbb{Z}, b\in 2\mathbb{Z}+1) \\ (-1)^{\frac{b}{2}}\overline{\left(\frac{\theta_2\theta_3}{\eta^2}\right)^2}\frac{1}{2}\left\{\left(\frac{\theta_3}{\eta}\right)^4+\left(\frac{\theta_2}{\eta}\right)^4\right\}, & (a\in 2\mathbb{Z}+1, b\in 2\mathbb{Z}) \\ e^{-\frac{i\pi}{2}ab}\overline{\left(\frac{\theta_2\theta_4}{\eta^2}\right)^2}\frac{1}{2}\left\{\left(\frac{\theta_4}{\eta}\right)^4-\left(\frac{\theta_2}{\eta}\right)^4\right\}, & (a\in 2\mathbb{Z}+1, b\in 2\mathbb{Z}+1). \end{cases} \tag{3.14}$$

On the other hand, The building blocks corresponding to $\{(-\mathbf{1}_R)^{\otimes 4}\}^2=(-1)^{F_R}|_\lambda$ are obtained by combining (3.14) with (3.13);

$$\mathbf{F}^{T^4[\mathrm{SO}(8)]}_{(a,b)}(\tau,\bar\tau)\equiv\begin{cases}\mathbf{Z}^{T^4[\mathrm{SO}(8)]}_{(\frac{a}{2},\frac{b}{2})}(\tau,\bar\tau), & (a\in 2\mathbb{Z}, b\in 2\mathbb{Z}), \\ F^{T^4[\mathrm{SO}(8)]}_{(a,b)}(\tau,\bar\tau), & (a\in 2\mathbb{Z}+1 \text{ or } b\in 2\mathbb{Z}+1).\end{cases} \tag{3.15}$$

Similarly, the building blocks for the $(-\mathbf{1}_R)^{\otimes 2}$ -twisting on $T^2[\mathrm{SO}(4)]$ are written as

$$F^{T^2[\mathrm{SO}(4)]}_{(a,b)}(\tau,\bar\tau)$$
$$\equiv \begin{cases} \frac{1}{2}\left\{\left|\frac{\theta_3}{\eta}\right|^4+\left|\frac{\theta_4}{\eta}\right|^4+\left|\frac{\theta_2}{\eta}\right|^4\right\}, & (a\in 2\mathbb{Z}, b\in 2\mathbb{Z}) \\ e^{\frac{i\pi}{4}ab}\overline{\left(\frac{\theta_3\theta_4}{\eta^2}\right)}\frac{1}{2}\left\{\left(\frac{\theta_3}{\eta}\right)^2+(-1)^{\frac{a}{2}}\left(\frac{\theta_4}{\eta}\right)^2\right\}, & (a\in 2\mathbb{Z}, b\in 2\mathbb{Z}+1) \\ e^{-\frac{i\pi}{4}ab}\overline{\left(\frac{\theta_2\theta_3}{\eta^2}\right)}\frac{1}{2}\left\{\left(\frac{\theta_3}{\eta}\right)^2+(-1)^{\frac{b}{2}}\left(\frac{\theta_2}{\eta}\right)^2\right\}, & (a\in 2\mathbb{Z}+1, b\in 2\mathbb{Z}) \\ e^{-\frac{i\pi}{4}ab}\overline{\left(\frac{\theta_2\theta_4}{\eta^2}\right)}\frac{1}{2}\left\{\left(\frac{\theta_4}{\eta}\right)^2-i(-1)^{\frac{a+b}{2}}\left(\frac{\theta_2}{\eta}\right)^2\right\}, & (a\in 2\mathbb{Z}+1, b\in 2\mathbb{Z}+1), \end{cases} \tag{3.16}$$

and, for the case of $\{(-\mathbf{1}_R)^{\otimes 2}\}^2=(-1)^{F_R}|_\lambda$,

$$\mathbf{F}^{T^2[\mathrm{SO}(4)]}_{(a,b)}(\tau,\bar\tau)\equiv\begin{cases}\mathbf{Z}^{T^2[\mathrm{SO}(4)]}_{(\frac{a}{2},\frac{b}{2})}(\tau,\bar\tau), & (a\in 2\mathbb{Z}, b\in 2\mathbb{Z}), \\ F^{T^2[\mathrm{SO}(4)]}_{(a,b)}(\tau,\bar\tau), & (a\in 2\mathbb{Z}+1 \text{ or } b\in 2\mathbb{Z}+1).\end{cases} \tag{3.17}$$

(iii) twisting by $(-\mathbf{1}_L) \otimes (-\mathbf{1}_R)^{\otimes 3}$: for the later convenience, we also consider the building blocks corresponding to twisting

$$\sigma \equiv (-\mathbf{1}_L)|_{X^6} \otimes (-\mathbf{1}_R)^{\otimes 3}|_{X^{7,8,9}}, \tag{3.18}$$

acting on $T^4[\mathrm{SO}(8)]$. They are obtained in the same way as above. Namely, we first evaluate the trace with the twist operator inserted, and then all the building blocks are uniquely determined by requiring the modular covariance such as (3.12). The explicit computation is straightforward, but a little more cumbersome about the phase factors than those for the blocks $F_{(a,b)}^{T^N[\mathrm{SO}(2N)]}$ given above. They are summarized as

$$G_{(a,b)}^{T^4[\mathrm{SO}(8)]}(\tau,\bar{\tau})$$
$$\equiv \begin{cases} \frac{1}{2}\left\{\left|\frac{\theta_3}{\eta}\right|^8 + \left|\frac{\theta_4}{\eta}\right|^8 + \left|\frac{\theta_2}{\eta}\right|^8\right\}, & (a \in 2\mathbb{Z}, b \in 2\mathbb{Z}), \\ e^{\frac{i\pi}{4}ab}\overline{\left(\frac{\theta_3\theta_4}{\eta^2}\right)}\left|\frac{\theta_3\theta_4}{\eta^2}\right|\frac{1}{2}\left\{\left(\frac{\theta_3}{\eta}\right)^2\left|\frac{\theta_3}{\eta}\right|^2 + (-1)^{\frac{a}{2}}\left(\frac{\theta_4}{\eta}\right)^2\left|\frac{\theta_4}{\eta}\right|^2\right\}, \\ \qquad (a \in 2\mathbb{Z}, b \in 2\mathbb{Z}+1), \\ e^{-\frac{i\pi}{4}ab}\overline{\left(\frac{\theta_2\theta_3}{\eta^2}\right)}\left|\frac{\theta_2\theta_3}{\eta^2}\right|\frac{1}{2}\left\{\left(\frac{\theta_3}{\eta}\right)^2\left|\frac{\theta_3}{\eta}\right|^2 + (-1)^{\frac{b}{2}}\left(\frac{\theta_2}{\eta}\right)^2\left|\frac{\theta_2}{\eta}\right|^2\right\}, \\ \qquad (a \in 2\mathbb{Z}+1, b \in 2\mathbb{Z}), \\ e^{-\frac{i\pi}{4}ab}\overline{\left(\frac{\theta_2\theta_4}{\eta^2}\right)}\left|\frac{\theta_2\theta_4}{\eta^2}\right|\frac{1}{2}\left\{\left(\frac{\theta_4}{\eta}\right)^2\left|\frac{\theta_4}{\eta}\right|^2 - i(-1)^{\frac{a+b}{2}}\left(\frac{\theta_2}{\eta}\right)^2\left|\frac{\theta_2}{\eta}\right|^2\right\}, \\ \qquad (a \in 2\mathbb{Z}+1, b \in 2\mathbb{Z}+1), \end{cases} \tag{3.19}$$

or

$$\mathbf{G}_{(a,b)}^{T^4[\mathrm{SO}(8)]}(\tau,\bar{\tau}) \equiv \begin{cases} \mathbf{Z}_{(\frac{a}{2},\frac{b}{2})}^{T^4[\mathrm{SO}(8)]}(\tau,\bar{\tau}), & (a \in 2\mathbb{Z}, b \in 2\mathbb{Z}), \\ \mathbf{G}_{(a,b)}^{T^4[\mathrm{SO}(8)]}(\tau,\bar{\tau}), & (a \in 2\mathbb{Z}+1 \text{ or } b \in 2\mathbb{Z}+1), \end{cases} \tag{3.20}$$

in the case that the twist operator is not involutive in the untwisted sector.[6]

3.1.2 Fermionic sector

We next consider the fermionic sector. We first recall that the fermionic part of the partition function of the type II string on 10-dim. flat background is just written as

$$Z_{\text{typeII}}^{\psi,\tilde{\psi}}(\tau,\bar{\tau}) = \frac{1}{4}|\mathcal{J}(\tau)|^2, \tag{3.21}$$

[6]Stated more precisely, we have the four possibilities; (i) $\sigma^2 = \mathbf{1}$, (ii) $\sigma^2 = (-1)^{F_L}|_\lambda$, (iii) $\sigma^2 = (-1)^{F_R}|_\lambda$, (iv) $\sigma^2 = (-1)^{F_L+F_R}|_\lambda$. However, since the spin structure of λ^i is diagonal, the cases (ii) and (iii) lead us to the same building blocks (3.20), while the case (iv) yields (3.19) as well as the case (i).

where

$$\mathcal{J}(\tau) \equiv \left(\frac{\theta_3}{\eta}\right)^4 - \left(\frac{\theta_4}{\eta}\right)^4 - \left(\frac{\theta_2}{\eta}\right)^4 (\equiv 0)\,. \tag{3.22}$$

Its modular property is easily seen as[7]

$$\mathcal{J}(\tau)|_T = -e^{\frac{\pi i}{3}}\mathcal{J}(\tau), \tag{3.23}$$
$$\mathcal{J}(\tau)|_S = \mathcal{J}(\tau), \tag{3.24}$$

and thus, (3.21) is modular invariant. The desired free fermion chiral blocks are given by making the suitable modifications of $\mathcal{J}(\tau)$ caused by the orbifold twists so as to be compatible with the modular invariance.

We present the relevant chiral blocks from now on. We only focus on the left-mover, and the right-mover is completely parallel. Although the cases (i) and (ii) are already given e.g. in [14], we dare to present them for the convenience to readers.

(i) twisting by $(-1)^{F_L}|_\psi$: we first describe the twisting by $(-1)^{F_L}|_\psi$ and denote the corresponding chiral blocks as $h_{(a,b)}(\tau)$. Again it is easiest to first compute $h_{(0,b)}(\tau)$, which just means the insertion of $\left\{(-1)^{F_L}|_\psi\right\}^b$ into the trace;

$$h_{(0,b)}(\tau) = \begin{cases} \mathcal{J}(\tau), & (b \in 2\mathbb{Z}), \\ \left(\frac{\theta_3}{\eta}\right)^4 - \left(\frac{\theta_4}{\eta}\right)^4 + \left(\frac{\theta_2}{\eta}\right)^4 & (b \in 2\mathbb{Z}+1). \end{cases} \tag{3.25}$$

Requiring the modular covariance

$$\left[h_{(a,b)}(\tau)\overline{\mathcal{J}(\tau)}\right]|_S = \left[h_{(b,-a)}(\tau)\overline{\mathcal{J}(\tau)}\right],$$
$$\left[h_{(a,b)}(\tau)\overline{\mathcal{J}(\tau)}\right]|_T = \left[h_{(a,a+b)}(\tau)\overline{\mathcal{J}(\tau)}\right], \tag{3.26}$$

we obtain

$$h_{(a,b)}(\tau) = \begin{cases} \mathcal{J}(\tau), & (a \in 2\mathbb{Z},\ b \in 2\mathbb{Z}), \\ \left(\frac{\theta_3}{\eta}\right)^4 - \left(\frac{\theta_4}{\eta}\right)^4 + \left(\frac{\theta_2}{\eta}\right)^4, & (a \in 2\mathbb{Z},\ b \in 2\mathbb{Z}+1), \\ \left(\frac{\theta_3}{\eta}\right)^4 + \left(\frac{\theta_4}{\eta}\right)^4 - \left(\frac{\theta_2}{\eta}\right)^4, & (a \in 2\mathbb{Z}+1,\ b \in 2\mathbb{Z}), \\ -\left\{\left(\frac{\theta_3}{\eta}\right)^4 + \left(\frac{\theta_4}{\eta}\right)^4 + \left(\frac{\theta_2}{\eta}\right)^4\right\}, & (a \in 2\mathbb{Z}+1,\ b \in 2\mathbb{Z}+1). \end{cases} \tag{3.27}$$

We note that the left-chiral blocks have to give rise to the phase $-e^{-\frac{\pi i}{3}}$ under the T-transformation to satisfy the modular covariance relation (3.26). $h_{(a,b)}$ ($a \in 2\mathbb{Z}+1$, or $b \in 2\mathbb{Z}+1$) are non-vanishing, which implies the SUSY breaking in the left-moving sector.

[7]The equations (3.23), (3.24) or the modular covariance relations (3.26) would look slightly subtle since we know $\mathcal{J}(\tau) \equiv 0$. See e.g. [14] for more rigid statements.

(ii) twisting by $(-\mathbf{1}_L)^{\otimes 4}$: next, we look at the chiral blocks defined by the chiral reflection

$$(-\mathbf{1}_L)^{\otimes 4}: \ (\psi_L^6, \psi_L^7, \psi_L^8, \psi_L^9) \to (-\psi_L^6, -\psi_L^7, -\psi_L^8, -\psi_L^9). \tag{3.28}$$

As illustrated in [14], we have again two possibilities; $\{(-\mathbf{1}_L)^{\otimes 4}\}^2 = \mathbf{1}$, or $\{(-\mathbf{1}_L)^{\otimes 4}\}^2 = (-1)^{F_L}|_\psi$. We denote the chiral blocks for the first case as $f_{(a,b)}(\tau)$. One can similarly determine them by computing $f_{(0,b)}(\tau)$ first, and requiring the modular covariance such as (3.26). They are summarized as

$$f_{(a,b)}(\tau) = \begin{cases} \mathcal{J}(\tau), & (a \in 2\mathbb{Z}, \ b \in 2\mathbb{Z}), \\ e^{\frac{i\pi}{2}ab}\left\{\left(\frac{\theta_3}{\eta}\right)^2\left(\frac{\theta_4}{\eta}\right)^2 - \left(\frac{\theta_4}{\eta}\right)^2\left(\frac{\theta_3}{\eta}\right)^2 + 0\right\}, \\ & (a \in 2\mathbb{Z}, \ b \in 2\mathbb{Z}+1), \\ e^{\frac{i\pi}{2}ab}\left\{\left(\frac{\theta_3}{\eta}\right)^2\left(\frac{\theta_2}{\eta}\right)^2 + 0 - \left(\frac{\theta_2}{\eta}\right)^2\left(\frac{\theta_3}{\eta}\right)^2\right\}, \\ & (a \in 2\mathbb{Z}+1, \ b \in 2\mathbb{Z}), \\ -e^{\frac{i\pi}{2}ab}\left\{0 + \left(\frac{\theta_2}{\eta}\right)^2\left(\frac{\theta_4}{\eta}\right)^2 - \left(\frac{\theta_4}{\eta}\right)^2\left(\frac{\theta_2}{\eta}\right)^2\right\}, \\ & (a \in 2\mathbb{Z}+1, \ b \in 2\mathbb{Z}+1). \end{cases} \tag{3.29}$$

Note that all of them trivially vanish as is consistent with the preservation of half space-time SUSY in the left-mover. Each term from the left to the right corresponds to the spin structures; NS, $\widetilde{\text{NS}}$, and R sector, respectively, where the '$\widetilde{\text{NS}}$' denotes the NS-sector with $(-1)^f$ inserted (f is the world-sheet fermion number).

On the other hand, in the second case $\{(-\mathbf{1}_L)^{\otimes 4}\}^2 = (-1)^{F_L}|_\psi$, the relevant chiral blocks are just modified as follows;

$$\mathbf{f}_{(a,b)}(\tau) \equiv \begin{cases} h_{(\frac{a}{2},\frac{b}{2})}(\tau), \ (a \in 2\mathbb{Z}, \ b \in 2\mathbb{Z}), \\ f_{(a,b)}(\tau), \ \ (a \in 2\mathbb{Z}+1 \text{ or } b \in 2\mathbb{Z}+1). \end{cases} \tag{3.30}$$

Recall that $h_{(*,*)}(\tau)$ is given in (3.27), corresponding to $(-1)^{F_L}|_\psi$-twisting.

(iii) twisting by $(-\mathbf{1}_L)^{\otimes 2}$: we also need the chiral blocks defined by $(-\mathbf{1}_L)^{\otimes 2}$-twisting. They are determined in the parallel way as above, although the different phase factors have to be included to ensure the modular covariance.

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For the case $\{(-\mathbf{1}_L)^{\otimes 2}\}^2 = \mathbf{1}$, we obtain

$$g_{(a,b)}(\tau) \equiv \begin{cases} \mathcal{J}(\tau), & (a \in 2\mathbb{Z}, b \in 2\mathbb{Z}), \\ e^{-\frac{i\pi}{4}ab} \left\{ \left(\frac{\theta_3}{\eta}\right)^3 \left(\frac{\theta_4}{\eta}\right) - (-1)^{\frac{a}{2}} \left(\frac{\theta_4}{\eta}\right)^3 \left(\frac{\theta_3}{\eta}\right) + 0 \right\}, & \\ & (a \in 2\mathbb{Z}, b \in 2\mathbb{Z}+1), \\ e^{\frac{i\pi}{4}ab} \left\{ \left(\frac{\theta_3}{\eta}\right)^3 \left(\frac{\theta_2}{\eta}\right) + 0 - (-1)^{\frac{b}{2}} \left(\frac{\theta_2}{\eta}\right)^3 \left(\frac{\theta_3}{\eta}\right) \right\}, & \\ & (a \in 2\mathbb{Z}+1, b \in 2\mathbb{Z}), \\ -e^{\frac{i\pi}{4}ab} \left\{ 0 + \left(\frac{\theta_4}{\eta}\right)^3 \left(\frac{\theta_2}{\eta}\right) + i(-1)^{\frac{a+b}{2}} \left(\frac{\theta_2}{\eta}\right)^3 \left(\frac{\theta_4}{\eta}\right) \right\}, & \\ & (a \in 2\mathbb{Z}+1, b \in 2\mathbb{Z}+1), \end{cases} \tag{3.31}$$

and

$$\mathbf{g}_{(a,b)}(\tau) \equiv \begin{cases} h_{(\frac{a}{2},\frac{b}{2})}(\tau), & (a \in 2\mathbb{Z}, b \in 2\mathbb{Z}), \\ g_{(a,b)}(\tau), & (a \in 2\mathbb{Z}+1 \text{ or } b \in 2\mathbb{Z}+1), \end{cases} \tag{3.32}$$

for $\{(-\mathbf{1}_L)^{\otimes 2}\}^2 = (-1)^{F_L}|_{\psi}$.

3.2 Non-supersymmetric asymmetric orbifolds

We are now ready to study the six new models of non-SUSY vacua exhibited in tables 1 and 2, including the modifications introduced at the beginning of this section.

Model I: firstly, we consider the asymmetric orbifold defined by the orbifold twist

$$\begin{aligned} g &= \mathcal{T}_{2\pi R}|_{\text{base}} \otimes \sigma_{\text{I}} \equiv \mathcal{T}_{2\pi R}|_{\text{base}} \otimes (-\mathbf{1})^{\otimes 2}|_{X^{6,7}} \otimes (-\mathbf{1}_R)^{\otimes 2}|_{X^{8,9}} \\ &\equiv \mathcal{T}_{2\pi R}|_{\text{base}} \otimes (-\mathbf{1}_L)^{\otimes 2}|_{X^{6,7}} \otimes (-\mathbf{1}_R)^{\otimes 4}|_{X^{6,7,8,9}}, \end{aligned} \tag{3.33}$$

acting on

$$\left[M^4 \times S^1\right] \times \mathbb{R}_{\text{base}} \times T^2_{6,7} \times T^2_{8,9}[\text{SO}(4)]. \tag{3.34}$$

In the above expressions we explicitly indicated the directions along which the orbifold twist (3.33) acts in terms of the subscripts. Namely, the $X^{8,9}$-directions are compactified on $T^2[\text{SO}(4)]$, while the $X^{6,7}$-directions correspond to a 2-dim. torus with unspecified moduli. Note that the non-chiral reflection $(-\mathbf{1})^{\otimes 2}$: $(X^6, X^7) \longmapsto (-X^6, -X^7)$ is well-defined for any point of moduli space of T^2. $\mathcal{T}_{2\pi R}$ denotes the shift by $2\pi R$ along the $\mathbb{R}_{\text{base}}$.

As addressed in section 2, we further need to specify the Ramond vacua of world-sheet fermions and the action of σ_{I} on them. Adopting the Ramond vacua defined by the bosonization given in (2.3) both for the right and left movers, we can naturally define

$$\begin{aligned} \sigma_{\text{I}} \left| s_1, s_2, s_3, s_4 \right\rangle_R &= e^{i\pi s_4} \left| s_1, -s_2, -s_3, s_4 \right\rangle_R, \\ \sigma_{\text{I}} \left| s_1, s_2, s_3, s_4 \right\rangle_L &= \left| s_1, -s_2, -s_3, s_4 \right\rangle_L. \end{aligned} \tag{3.35}$$

spin structure	left		right	4D fields (d.o.f)
(NS, NS)	$\psi^{\mu}_{L,-1/2}\ket{0}$ $(\mu=2,\ldots,7)$	$\otimes$	$\psi^{\mu}_{R,-1/2}\ket{0}$ $(\mu=2,\ldots,5)$	graviton (2), 6 vectors (12), 10 (pseudo) scalars (10)
	$\psi^{\mu}_{L,-1/2}\ket{0}$ $(\mu=8,9)$	$\otimes$	$\psi^{\mu}_{R,-1/2}\ket{0}$ $(\mu=6,\ldots,9)$	8 scalars (8)
(R , NS)	$\left[1+(-\mathbf{1}_L)^{\otimes 2}\right]\ket{\mathbf{s}}_L$	$\otimes$	$\psi^{\mu}_{R,-1/2}\ket{0}$ $(\mu=2,\ldots,5)$	2 Rarita-Schwinger (4), 6 Weyl fermions (12)
	$\left[1-(-\mathbf{1}_L)^{\otimes 2}\right]\ket{\mathbf{s}}_L$	$\otimes$	$\psi^{\mu}_{R,-1/2}\ket{0}$ $(\mu=6,\ldots,9)$	8 Weyl fermions (16)

Table 6. Massless spectrum in untwisted sector for asymmetric orbifold I.

which implies

$$\sigma_{\mathrm{I}}^2=(-1)^{F_R}|_{\psi}. \tag{3.36}$$

We can write down the torus partition function in terms of the building blocks introduced in subsection 3.1 as

$$\begin{aligned}Z(\tau,\bar{\tau})=&\frac{1}{4}\mathcal{Z}_{M^4\times S^1}(\tau,\bar{\tau})\\&\times\sum_{w,m\in\mathbb{Z}}Z_{R,(w,m)}(\tau,\bar{\tau})Z^{T^2/\mathbb{Z}_2}_{(w,m)}(\tau,\bar{\tau})F^{T^2[\mathrm{SO}(4)]}_{(w,m)}(\tau,\bar{\tau})g_{(w,m)}(\tau)\overline{\mathbf{f}_{(w,m)}(\tau)}.\end{aligned} \tag{3.37}$$

As in section 2, we simply denote the contributions with no relations to the orbifolding as '$\mathcal{Z}_*$'. In the current case, $\mathcal{Z}_{M^4\times S^1}$ is identified as that for the bosonic transverse part of $M^4\times S^1$-sector ($X^{0,\ldots,4}$-directions). $Z_{R,(w,m)}(\tau,\bar{\tau})$ is given in (2.8), while $Z^{T^2/\mathbb{Z}_2}_{(w,m)}(\tau,\bar{\tau})$ expresses the building blocks of the symmetric $\mathbb{Z}_2$-orbifold along the $X^{6,7}$-directions. (We have an obvious $\mathbb{Z}_2$-periodicity with respect to the winding w, m.) The bosonic building blocks $F^{T^2[\mathrm{SO}(4)]}_{(w,m)}(\tau,\bar{\tau})$ are given in (3.16), while the chiral blocks for world-sheet fermions, denoted as $g_{(w,m)}(\tau)$, $\mathbf{f}_{(w,m)}(\tau)$, are presented in (3.31), (3.30), respectively. Looking at their expressions, it is easy to confirm that the partition function (3.37) indeed vanishes in the manner similar to the arguments in section 2.

As noticed in section 1, the non-SUSY chiral reflection $(-\mathbf{1}_L)^{\otimes 2}$ plays the similar role of $(-1)^{F_L}|_{\psi}$ in the 'previous model' introduced in section 2, and thus we anticipate to achieve a non-SUSY vacuum with the bose-fermi cancellation. We will later show that this is indeed the case.

Before doing so, let us study the massless spectrum lying in the untwisted sector, which we summarize in table 6. We express the left-moving Ramond vacua in terms of the spin fields for SO(8); $\ket{\mathbf{s}}_L\equiv e^{i\sum_{a=1}^{4}s_aH_L^a}\ket{0}_L$, $\left(s_a\equiv\pm\frac{1}{2}\right)$.

What is a remarkable difference from the previous model is the existence of massless Rarita-Schwinger fields. They of course originate from the gravitini in the original background (3.34), which are not removed by the relevant orbifold projection. In the same sense, some supercharges in the original background remain preserved under the orbifold group.

Nonetheless, the space-time SUSY is completely broken within the untwisted sector, at least. It is obvious not to have the right-moving supercharges due to the absence of right-moving Ramond vacua. Furthermore, even though having the left-moving Ramond vacua, we cannot still compose any left-moving supercharges acting as isomorphisms on the orbifolded Hilbert space. In fact, the presence of left supercharges should imply the existence of one to one correspondence between the (NS,NS) and (R,NS) massless states, *while fixing the right-movers.* It is, however, impossible as shown from table 6. For instance, pick up the states $\psi^{\mu}_{R,-1/2}\left|0\right\rangle$, $(\mu = 6, \dots, 9)$ from the right-mover. Then, one finds that the degrees of freedom of massless bosons amount to 8, whereas the fermionic one is 16.

One can examine the more detailed spectrum of physical states by making the Poisson resummation of the partition function (3.37). To this aim it is convenient to decompose it with respect to the spatial winding w and the spin structures as in (2.12);

$$\begin{aligned} Z(\tau,\bar{\tau}) = &\frac{1}{4}\mathcal{Z}_{M^4\times S^1}(\tau,\bar{\tau}) \\ &\times \sum_{w\in\mathbb{Z}}\left\{Z_w^{(\mathrm{NS,NS})}(\tau,\bar{\tau}) + Z_w^{(\mathrm{NS,R})}(\tau,\bar{\tau}) + Z_w^{(\mathrm{R,NS})}(\tau,\bar{\tau}) + Z_w^{(\mathrm{R,R})}(\tau,\bar{\tau})\right\}. \end{aligned} \tag{3.38}$$

After dualizing the temporal winding m into the KK momentum n, we obtain the following results;

- For $w \in 4\mathbb{Z}$:

$$\begin{aligned} Z_w^{(\mathrm{NS,NS})}(\tau,\bar{\tau}) =& -Z_w^{(\mathrm{R,NS})}(\tau,\bar{\tau}) \\ =&\frac{1}{2}\sum_{n\in\mathbb{Z}}\frac{1}{|\eta|^2}q^{\frac{1}{4}\left(Rw+\frac{n}{2R}\right)^2}\bar{q}^{\frac{1}{4}\left(Rw-\frac{n}{2R}\right)^2}Z^{T^2}Z^{T^2[\mathrm{SO}(4)]}\left|\left(\frac{\theta_3}{\eta}\right)^4-\left(\frac{\theta_4}{\eta}\right)^4\right|^2, \end{aligned} \tag{3.39}$$

$$\begin{aligned} Z_w^{(\mathrm{R,R})}(\tau,\bar{\tau}) =& -Z_w^{(\mathrm{NS,R})}(\tau,\bar{\tau}) \\ =&\frac{1}{2}\sum_{n\in\mathbb{Z}}\frac{1}{|\eta|^2}q^{\frac{1}{4}\left(Rw+\frac{n+\frac{1}{2}}{2R}\right)^2}\bar{q}^{\frac{1}{4}\left(Rw-\frac{n+\frac{1}{2}}{2R}\right)^2}Z^{T^2}Z^{T^2[\mathrm{SO}(4)]}\left|\frac{\theta_2}{\eta}\right|^8. \end{aligned} \tag{3.40}$$

- $w \in 4\mathbb{Z}+2$:

$$\begin{aligned} Z_w^{(\mathrm{NS,NS})}(\tau,\bar{\tau}) =& -Z_w^{(\mathrm{R,NS})}(\tau,\bar{\tau}) = \frac{1}{2}\sum_{n\in\mathbb{Z}}\frac{1}{|\eta|^2}q^{\frac{1}{4}\left(Rw+\frac{n+\frac{1}{2}}{2R}\right)^2}\bar{q}^{\frac{1}{4}\left(Rw-\frac{n+\frac{1}{2}}{2R}\right)^2} \\ &\times Z^{T^2}Z^{T^2[\mathrm{SO}(4)]}\left\{\left(\frac{\theta_3}{\eta}\right)^4-\left(\frac{\theta_4}{\eta}\right)^4\right\}\left\{\overline{\left(\frac{\theta_3}{\eta}\right)}^4+\overline{\left(\frac{\theta_4}{\eta}\right)}^4\right\}, \end{aligned} \tag{3.41}$$

$$\begin{aligned} Z_w^{(\mathrm{R,R})}(\tau,\bar{\tau}) =& -Z_w^{(\mathrm{NS,R})}(\tau,\bar{\tau}) \\ =&\frac{1}{2}\sum_{n\in\mathbb{Z}}\frac{1}{|\eta|^2}q^{\frac{1}{4}\left(Rw+\frac{n}{2R}\right)^2}\bar{q}^{\frac{1}{4}\left(Rw-\frac{n}{2R}\right)^2}Z^{T^2}Z^{T^2[\mathrm{SO}(4)]}\left|\frac{\theta_2}{\eta}\right|^8. \end{aligned} \tag{3.42}$$

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spin structure	massless point	sector	relevant equation
(NS, NS) / (NS, R)	$R=\frac{1}{2}$	$w=\pm1$	(3.43)
(NS, NS) / (R, NS)	$R=\frac{1}{2\sqrt{2}}$	$w=\pm2$	(3.41)

Table 7. The massless points for asymmetric orbifold I.

- For $w\in 4\mathbb{Z}+1$:

$$\begin{aligned} Z_w^{(\mathrm{NS,NS})}(\tau,\bar\tau) =& -Z_w^{(\mathrm{NS,R})}(\tau,\bar\tau) \\ =& \frac{1}{4}\sum_{a\in\mathbb{Z}_2}\sum_{n\in\mathbb{Z}}\Bigg[\frac{1}{|\eta|^2}q^{\frac{1}{4}\left(Rw+\frac{n+\frac{1}{2}}{2R}\right)^2}\bar q^{\frac{1}{4}\left(Rw-\frac{n+\frac{1}{2}}{2R}\right)^2} \\ &\times(-1)^{an}\left|\frac{\theta_2\theta_3(\frac{a}{2})}{\eta^2}\right|^4\left(\frac{\theta_3(\frac{a}{2})}{\eta}\right)^4\overline{\left(\frac{\theta_3(\frac{a}{2})\theta_2}{\eta^2}\right)}^2 \\ &+\frac{1}{|\eta|^2}q^{\frac{1}{4}\left(Rw+\frac{n}{2R}\right)^2}\bar q^{\frac{1}{4}\left(Rw-\frac{n}{2R}\right)^2}(-1)^{an}\left|\frac{\theta_2\theta_3(\frac{a}{2})}{\eta^2}\right|^8\Bigg], \end{aligned} \tag{3.43}$$

$$\begin{aligned} Z_w^{(\mathrm{R,R})}(\tau,\bar\tau) =& -Z_w^{(\mathrm{R,NS})}(\tau,\bar\tau) \\ =& \frac{1}{4}\sum_{a\in\mathbb{Z}_2}\sum_{n\in\mathbb{Z}}\Bigg[\frac{1}{|\eta|^2}q^{\frac{1}{4}\left(Rw+\frac{n+\frac{1}{2}}{2R}\right)^2}\bar q^{\frac{1}{4}\left(Rw-\frac{n+\frac{1}{2}}{2R}\right)^2} \\ &\times(-1)^{a(n+1)}\left|\frac{\theta_2\theta_3(\frac{a}{2})}{\eta^2}\right|^4\left(\frac{\theta_2}{\eta}\right)^4\overline{\left(\frac{\theta_2\theta_3(\frac{a}{2})}{\eta^2}\right)}^2 \\ &+\frac{1}{|\eta|^2}q^{\frac{1}{4}\left(Rw+\frac{n}{2R}\right)^2}\bar q^{\frac{1}{4}\left(Rw-\frac{n}{2R}\right)^2}(-1)^{an}\left|\frac{\theta_2\theta_3(\frac{a}{2})}{\eta^2}\right|^8\Bigg]. \end{aligned} \tag{3.44}$$

- For $w\in 4\mathbb{Z}+3$:

  in this case, the result is obtained by replacing $(-1)^{an}$ in the first term of (3.43) with $(-1)^{a(n+1)}$, and by replacing $(-1)^{a(n+1)}$ in the first term of (3.44) with $(-1)^{an}$.

All of these partition functions are q-expanded so as to be compatible with unitarity, and we have no tachyonic states as confirmed by looking at the conformal weights read from them. Extra massless excitations appear when the X^5-direction has some specific radii, as summarized in table 7. Moreover, it is easy to confirm that the above partition functions satisfy the same relation as given in table 4 with respect to the winding number w. This fact makes it clear that the model I is indeed a non-SUSY vacuum with the bose-fermi cancellation at each mass level.

In table 7 the 'relevant equation' indicates which partition function includes the terms corresponding to the massless states in question.

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Model II: the model II is defined by the orbifold twist

$$g = \mathcal{T}_{2\pi R}|_{\text{base}} \otimes \sigma_{\text{II}} \equiv \mathcal{T}_{2\pi R}|_{\text{base}} \otimes (-\mathbf{1})|_{X^5} \otimes (-\mathbf{1}_L)|_{X^6} \otimes (-\mathbf{1}_R)^{\otimes 3}|_{X^{7,8,9}}, \\ \equiv \mathcal{T}_{2\pi R}|_{\text{base}} \otimes (-\mathbf{1}_L)^{\otimes 2}|_{X^{5,6}} \otimes (-\mathbf{1}_R)^{\otimes 4}|_{X^{5,7,8,9}}, \tag{3.45}$$

acting on the background

$$[M^4] \times \mathbb{R}_{\text{base}} \times S^1_5 \times T^4_{6,7,8,9}[\text{SO}(8)]. \tag{3.46}$$

For the Ramond vacua, we set

$$\sigma_{\text{II}}\, |s_1, s_2, s_3, s_4\rangle_R = e^{i\pi s_4}\, |s_1, -s_2, -s_3, s_4\rangle_R\,, \\ \sigma_{\text{II}}\, |s_1, s_2, s_3, s_4\rangle_L = e^{i\pi s_3}\, |s_1, -s_2, -s_3, s_4\rangle_L\,. \tag{3.47}$$

which again implies

$$\sigma_{\text{II}}^2 = (-1)^{F_R}|_\psi. \tag{3.48}$$

The corresponding partition function is given as

$$Z(\tau,\bar\tau) = \frac{1}{4}\mathcal{Z}_{M^4}(\tau,\bar\tau) \sum_{w,m\in\mathbb{Z}} Z_{R(w,m)}(\tau,\bar\tau) Z^{S^1/\mathbb{Z}_2}_{(w,m)}(\tau,\bar\tau) G^{T^4[\text{SO}(8)]}_{(w,m)}(\tau,\bar\tau) g_{(w,m)}(\tau)\overline{\mathbf{f}_{(w,m)}(\tau)}, \tag{3.49}$$

where $Z^{S^1/\mathbb{Z}_2}_{(w,m)}(\tau,\bar\tau)$ denotes the building blocks corresponding to the ordinary reflection $-\mathbf{1}\ :\ (X^5_L, X^5_R) \to (-X^5_L, -X^5_R)$ acting on S^1_5 with an arbitrary radius. The bosonic blocks $G^{T^4[\text{SO}(8)]}_{(w,m)}(\tau,\bar\tau)$ are defined in (3.19).

This model is quite similar to the model I, although the partition function is slightly different. The massless spectrum and the massless points for the winding states are the same as that of the model I. This result is independent of the radius of the S^1_5.

Model III: from now on, we shall discuss the constructions of non-SUSY vacua without the shift operator $\mathcal{T}_{2\pi R}$ included. The simplest case, which we call model III, is defined on the background

$$[M^4 \times T^2] \times T^4_{6,7,8,9}[\text{SO}(8)], \tag{3.50}$$

and the orbifold twist is obtained simply as

$$g = \sigma' \equiv (-1)^{F_L}|_\psi \otimes (-\mathbf{1}_R)^{\otimes 4}|_{X^6,\dots,X^9}. \tag{3.51}$$

Although it looks almost the same as the supersymmetric vacua illustrated in section 2, we shall here adopt the $\mathbb{Z}_4$-action as the definition of $(-\mathbf{1}_R)^{\otimes 4}|_{X^6,\dots,X^9}$ *also for the bosonic sector* by utilizing the fermionization as given in subsection 3.1.1. Namely, introducing the free fermions $\lambda^i_{L\,(R)}$, $(i = 1,\dots,8)$ describing $T^4[\text{SO}(8)]$, we identify $(-\mathbf{1}_R)^{\otimes 4}|_{X^6,\dots,X^9}$ with the sign flip of $\lambda^5_R,\dots,\lambda^8_R$. We then determine its action on the Ramond vacua of λ^i_R as

$$(-\mathbf{1}_R)^{\otimes 4}|_{X^6,\dots,X^9}\ :\ |\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4\rangle_{\lambda,R} \longmapsto e^{i\pi\epsilon_4}\,|\epsilon_1,-\epsilon_2,-\epsilon_3,\epsilon_4\rangle_{\lambda,R}\,, \tag{3.52}$$

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with the definitions

$$|\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4\rangle_{\lambda,L\,(R)} \equiv e^{i\sum_{a=1}^{4}\epsilon_a \tilde{H}^a_{L\,(R)}}\,|0\rangle_{\lambda,L\,(R)}\,, \qquad \left(\epsilon_a = \pm\frac{1}{2}\right),$$
$$\lambda^1_{L\,(R)} \pm i\lambda^2_{L\,(R)} = \sqrt{2}e^{\pm i\tilde{H}^1_{L\,(R)}}, \qquad \lambda^3_{L\,(R)} \pm i\lambda^5_{L\,(R)} = \sqrt{2}e^{\pm i\tilde{H}^2_{L\,(R)}},$$
$$\lambda^4_{L\,(R)} \pm i\lambda^6_{L\,(R)} = \sqrt{2}e^{\pm i\tilde{H}^3_{L\,(R)}}, \qquad \lambda^7_{L\,(R)} \pm i\lambda^8_{L\,(R)} = \sqrt{2}e^{\pm i\tilde{H}^4_{L\,(R)}}. \tag{3.53}$$

We also assume that $(-\mathbf{1}_R)^{\otimes 4}|_{X^6,\ldots,X^9}$ acts on the Ramond vacua of the world-sheet fermions ψ^μ_R in the same way as (2.4). In total, we obtain

$$(\sigma')^2 = (-1)^{F_R}|_\lambda \otimes (-1)^{F_R}|_\psi, \tag{3.54}$$

rather than $\sigma^2 = (-1)^{F_R}|_\psi$.

As we emphasized before, the shift operator $\mathcal{T}_{2\pi R}$ plays an important role of SUSY breaking, that is, it prevents the twisted sectors from providing new supercharges. However, we here show that other types of non-SUSY vacua are realized as long as (3.54) is satisfied.

The partition function is just written as

$$Z(\tau,\bar\tau) = \frac{1}{16}\mathcal{Z}_{M^4\times T^2}(\tau,\bar\tau)\sum_{a,b\in\mathbb{Z}_4}\mathbf{F}^{T^4[\mathrm{SO}(8)]}_{(a,b)}(\tau,\bar\tau)h_{(a,b)}(\tau)\overline{\mathbf{f}_{(a,b)}(\tau)}, \tag{3.55}$$

where $\mathbf{F}^{T^4[\mathrm{SO}(8)]}_{(a,b)}(\tau,\bar\tau)$, $h_{(a,b)}(\tau)$ and $\mathbf{f}_{(a,b)}(\tau)$ are presented respectively in (3.17), (3.27), and (3.30). This partition function (3.55) also vanishes as is readily checked.

Let us decompose (3.55) with respect to the twisted sectors as well as the spin structures as

$$\begin{aligned}Z(\tau,\bar\tau) = &\frac{1}{16}\mathcal{Z}_{M^4\times T^2}(\tau,\bar\tau)\\ &\times\sum_{a\in\mathbb{Z}_4}\left\{Z^{(\mathrm{NS,NS})}_a(\tau,\bar\tau)+Z^{(\mathrm{NS,R})}_a(\tau,\bar\tau)+Z^{(\mathrm{R,NS})}_a(\tau,\bar\tau)+Z^{(\mathrm{R,R})}_a(\tau,\bar\tau)\right\}.\end{aligned} \tag{3.56}$$

Then, we obtain

$$\begin{aligned}Z^{(\mathrm{NS,NS})}_0(\tau,\bar\tau) &= -Z^{(\mathrm{R,NS})}_0(\tau,\bar\tau) = \left\{\left|\frac{\theta_3}{\eta}\right|^8+\left|\frac{\theta_4}{\eta}\right|^8\right\}\left|\frac{\theta_2}{\eta}\right|^8,\\ Z^{(\mathrm{NS,NS})}_2(\tau,\bar\tau) &= -Z^{(\mathrm{R,NS})}_2(\tau,\bar\tau)\\ &= \left\{\left|\frac{\theta_3}{\eta}\right|^8-\left|\frac{\theta_4}{\eta}\right|^8\right\}\left(\frac{\theta_2}{\eta}\right)^4\left\{\overline{\left(\frac{\theta_3}{\eta}\right)^4}+\overline{\left(\frac{\theta_4}{\eta}\right)^4}\right\},\\ Z^{(\mathrm{R,R})}_{0,2}(\tau,\bar\tau) &= -Z^{(\mathrm{NS,R})}_{0,2}(\tau,\bar\tau) = \left|\frac{\theta_2}{\eta}\right|^{16},\end{aligned} \tag{3.57}$$

for the even sectors, and

$$
\begin{aligned}
Z_{1,3}^{(\mathrm{NS,NS})}(\tau,\bar{\tau}) = -Z_{1,3}^{(\mathrm{NS,R})}(\tau,\bar{\tau}) &= \overline{\left(\frac{\theta_2}{\eta}\right)^4} \left\{\left|\frac{\theta_3}{\eta}\right|^8 - \left|\frac{\theta_4}{\eta}\right|^8\right\} \left\{\left(\frac{\theta_3}{\eta}\right)^4 + \left(\frac{\theta_4}{\eta}\right)^4\right\}, \\
&\quad + \left|\frac{\theta_2}{\eta}\right|^8 \left\{\overline{\left(\frac{\theta_3}{\eta}\right)^4} + \overline{\left(\frac{\theta_4}{\eta}\right)^4}\right\} \left\{\left(\frac{\theta_3}{\eta}\right)^4 + \left(\frac{\theta_4}{\eta}\right)^4\right\}, \\
Z_{1,3}^{(\mathrm{R,R})}(\tau,\bar{\tau}) = -Z_{1,3}^{(\mathrm{R,NS})}(\tau,\bar{\tau}) &= \left\{\left|\frac{\theta_3}{\eta}\right|^8 + \left|\frac{\theta_4}{\eta}\right|^8 + \left|\frac{\theta_2}{\eta}\right|^8\right\} \left|\frac{\theta_2}{\eta}\right|^8,
\end{aligned} \tag{3.58}
$$

for the odd sectors.

These relations should be compared with those for the supersymmetric case (2.15) and (2.16). Here we never obtain the equalities such as (2.17), rather find the cancellations as depicted in table 4. Namely, we see that the left-moving NS-R cancellations for the even sectors, whereas the right-moving ones for the odd sectors. This fact clearly shows that the space-time SUSY is completely broken. Recall that, in the supersymmetric case with $g = \sigma$, the right-moving SUSY is unbroken, and the supercharges arise from the $a = 2$ twisted sector. In the current case, however, the same does not happen because the partition functions $Z_2^{(*,\mathrm{R})}(\tau,\bar{\tau})$ do not contain any massless states. This is the crucial difference caused by the relation (3.54). In this way, we have successfully achieved a desired non-SUSY vacuum without the shift.

The massless spectrum in the untwisted sector is the same as the model introduced in the previous section. In the twisted sectors, on the other hand, there are additional massless states, while no tachyonic states appear.

Model IV: we next consider the background

$$
\left[M^4 \times T^2\right] \times T^2_{6,7} \times T^2_{8,9}[\mathrm{SO}(4)], \tag{3.59}
$$

and adopt the modification of (3.33);

$$
g = \sigma'_{\mathrm{I}} \equiv (-\mathbf{1})^{\otimes 2}|_{X^{6,7}} \otimes (-\mathbf{1}_R)^{\otimes 2}|_{X^{8,9}} \equiv (-\mathbf{1}_L)^{\otimes 2}|_{X^{6,7}} \otimes (-\mathbf{1}_R)^{\otimes 2}|_{X^{6,7,8,9}}, \tag{3.60}
$$

as the relevant orbifold twisting. σ'_{I} again acts by (3.35) for the Ramond vacua of world-sheet fermions ψ^μ_R, ψ^μ_L. On the other hand, introducing the free fermions $\lambda^i_{L\,(R)}$, $(i = 1,\ldots,4)$ describing $T^2_{8,9}[\mathrm{SO}(4)]$, we define its action on the Ramond vacua of λ^i_R as that given in **(a)** of subsection 3.1.1, that is,

$$
\sigma'_I \left|\epsilon_1,\epsilon_2\right\rangle_{\lambda,R} = e^{i\pi\epsilon_2} \left|\epsilon_1,\epsilon_2\right\rangle_{\lambda,R}, \tag{3.61}
$$

with

$$
\begin{aligned}
&\left|\epsilon_1,\epsilon_2\right\rangle_{\lambda,R} \equiv e^{i\sum_{a=1}^2 \epsilon_a \tilde{H}^a_R} \left|0\right\rangle_{\lambda,R}, \qquad \left(\epsilon_a = \pm\frac{1}{2}\right), \\
&\lambda^1_{L\,(R)} \pm i\lambda^2_{L\,(R)} = \sqrt{2}e^{\pm i\tilde{H}^1_{L\,(R)}}, \quad \lambda^3_{L\,(R)} \pm i\lambda^4_{L\,(R)} = \sqrt{2}e^{\pm i\tilde{H}^2_{L\,(R)}}.
\end{aligned} \tag{3.62}
$$

We thus obtain the crucial relation $(\sigma'_{\mathrm{I}})^2 = (-1)^{F_R}|_\lambda \otimes (-1)^{F_R}|_\psi$.

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The partition function is then written as

$$Z(\tau,\bar{\tau}) = \frac{1}{16}\mathcal{Z}_{M^4\times T^2}(\tau,\bar{\tau}) \sum_{a,b\in\mathbb{Z}_4} Z^{T^2/\mathbb{Z}_2}_{(a,b)}(\tau,\bar{\tau})\mathbf{F}^{T^2[\mathrm{SO}(4)]}_{(a,b)}(\tau,\bar{\tau})g_{(a,b)}(\tau)\overline{\mathbf{f}_{(a,b)}(\tau)}, \tag{3.63}$$

where $\mathbf{F}^{T^2[\mathrm{SO}(4)]}_{(a,b)}(\tau,\bar{\tau})$, $g_{(a,b)}(\tau)$, and $\mathbf{f}_{(a,b)}(\tau)$ are given respectively by (3.17), (3.31) and (3.30). $Z^{T^2/\mathbb{Z}_2}_{(a,b)}(\tau,\bar{\tau})$ denotes the building blocks corresponding to an ordinary $\mathbb{Z}_2$-orbifold for the reflection acting $T^2_{6,7}$ in (3.59). This partition function also vanishes, and the supersymmetry is completely broken at least in the untwisted sector, as confirmed in the same way as the case of model I.

The spectrum of physical states in each twisted sector is read off from the decomposition of partition function in the manner similar to (3.56). After a short computation, one finds the relations of the partition functions between all of the sectors as follows;

$$\begin{aligned}
Z^{(\mathrm{NS,NS})}_0(\tau,\bar{\tau}) &= -\,Z^{(\mathrm{R,NS})}_0(\tau,\bar{\tau}) = Z^{T^2}\left\{\left|\frac{\theta_3}{\eta}\right|^4+\left|\frac{\theta_4}{\eta}\right|^4\right\}\left|\frac{\theta_2}{\eta}\right|^8,\\
Z^{(\mathrm{R,R})}_{0,2}(\tau,\bar{\tau}) &= -\,Z^{(\mathrm{NS,R})}_{0,2}(\tau,\bar{\tau}) = Z^{T^2}\left|\frac{\theta_2}{\eta}\right|^{12},\\
Z^{(\mathrm{NS,NS})}_2(\tau,\bar{\tau}) &= -\,Z^{(\mathrm{R,NS})}_2(\tau,\bar{\tau}) = Z^{T^2}\left\{\left|\frac{\theta_3}{\eta}\right|^4-\left|\frac{\theta_4}{\eta}\right|^4\right\}\left(\frac{\theta_2}{\eta}\right)^4\left\{\overline{\left(\frac{\theta_3}{\eta}\right)^4}+\overline{\left(\frac{\theta_4}{\eta}\right)^4}\right\},
\end{aligned} \tag{3.64}$$

$$\begin{aligned}
Z^{(\mathrm{NS,NS})}_{1,3}(\tau,\bar{\tau}) &= -\,Z^{(\mathrm{NS,R})}_{1,3}(\tau,\bar{\tau}) = -Z^{(\mathrm{R,NS})}_{1,3}(\tau,\bar{\tau}) = Z^{(\mathrm{R,R})}_{1,3}(\tau,\bar{\tau})\\
&= \left|\frac{\theta_2\theta_3}{\eta^2}\right|^6 Z^{T^2/\mathbb{Z}_2}_{(1,0)}+\left|\frac{\theta_2\theta_4}{\eta^2}\right|^6 Z^{T^2/\mathbb{Z}_2}_{(1,1)} \equiv \left|\frac{\theta_2}{\eta}\right|^8\left\{\left|\frac{\theta_3}{\eta}\right|^8+\left|\frac{\theta_4}{\eta}\right|^8\right\}.
\end{aligned} \tag{3.65}$$

These relations manifestly show that we do not have any right-moving supercharges. It is however curious that we have the 'accidental' equalities

$$Z^{(\mathrm{NS,NS})}_a = -Z^{(\mathrm{R,NS})}_a,\quad Z^{(\mathrm{R,R})}_a = -Z^{(\mathrm{NS,R})}_a,\quad (\text{for } {}^\forall a\in\mathbb{Z}_4)$$

in spite of the absence of left-moving supercharges that originate from the unorbifolded theory. It would be actually possible to make up some operators that realize these equalities *by combining the spin fields of* ψ^μ *and* λ^i. However, it turns out that such 'fake' supercharges do not respect the super-Poincare symmetry in M^4 of the original background (3.59). Furthermore, any left-moving supercharges cannot be constructed also from the twisted sectors, because we have

$$Z^{(\mathrm{NS,NS})}_a \neq -Z^{(\mathrm{R,NS})}_{a'},\quad Z^{(\mathrm{R,R})}_a \neq -Z^{(\mathrm{NS,R})}_{a'},\quad (\text{for } {}^\forall a'\in\mathbb{Z}_4 \text{ s.t. } a'\neq a).$$

In this way, we conclude that the model IV is still a non-SUSY vacuum with the bose-fermi cancellation. Again we find additional massless states in the twisted sectors, while no tachyons appear.

Model V: the model V is defined similarly to the model IV on the background;

$$\left[M^4 \times S^1\right] \times S^1_5 \times T^4_{6,7,8,9}[\mathrm{SO}(8)], \tag{3.66}$$

with the twist operator $g = \sigma'_{\mathrm{II}}$ which is the modification of σ_{II} given in (3.45) as $(\sigma'_{\mathrm{II}})^2 = (-1)^{F_R}|_\lambda \otimes (-1)^{F_R}|_\psi$. Namely, σ'_{II} acts on the world-sheet fermions ψ^μ_R, ψ^μ_L in the same way as (3.45), (3.47), and acts as the sign flip of λ^5_L, λ^i_R $(i = 6,7,8)$, where $\lambda^i_{L\,(R)}$ $(i = 1,\dots,8)$ are the free fermions describing $T^4_{6,7,8,9}[\mathrm{SO}(8)]$. Moreover, its action on the Ramond vacua of λ^i_R, λ^i_L is given as

$$\begin{aligned}
&\sigma'_{\mathrm{II}}\,|\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4\rangle_{\lambda,R} = e^{i\pi\epsilon_4}\,|\epsilon_1,\epsilon_2,-\epsilon_3,\epsilon_4\rangle_{\lambda,R}\,,\\
&\sigma'_{\mathrm{II}}\,|\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4\rangle_{\lambda,L} = |\epsilon_1,-\epsilon_2,\epsilon_3,\epsilon_4\rangle_{\lambda,L}\,,
\end{aligned} \tag{3.67}$$

under the definitions (3.53).

The partition function is just given as

$$Z(\tau,\bar\tau) = \frac{1}{16}\mathcal{Z}_{M^4\times S^1}(\tau,\bar\tau)\sum_{a,b\in\mathbb{Z}_4} Z^{S^1/\mathbb{Z}_2}_{(a,b)}(\tau,\bar\tau)\mathbf{G}^{T^4[\mathrm{SO}(8)]}_{(a,b)}(\tau,\bar\tau)g_{(a,b)}(\tau)\overline{\mathbf{f}_{(a,b)}(\tau)}, \tag{3.68}$$

where $g_{(a,b)}(\tau)$, $\mathbf{f}_{(a,b)}(\tau)$ are as above, while $\mathbf{G}^{T^4[\mathrm{SO}(8)]}_{(a,b)}(\tau,\bar\tau)$ are presented in (3.20).

Since the fermionic building blocks are common with the model IV, we can likewise make the decomposition such as (3.56);

$$\begin{aligned}
Z^{(\mathrm{NS,NS})}_0(\tau,\bar\tau) &= -\,Z^{(\mathrm{R,NS})}_0(\tau,\bar\tau) = Z^{S^1}\left\{\left|\frac{\theta_3}{\eta}\right|^8 + \left|\frac{\theta_4}{\eta}\right|^8\right\}\left|\frac{\theta_2}{\eta}\right|^8,\\
Z^{(\mathrm{R,R})}_{0,2}(\tau,\bar\tau) &= -\,Z^{(\mathrm{NS,R})}_{0,2}(\tau,\bar\tau) = Z^{S^1}\left|\frac{\theta_2}{\eta}\right|^{16},\\
Z^{(\mathrm{NS,NS})}_2(\tau,\bar\tau) &= -\,Z^{(\mathrm{R,NS})}_2(\tau,\bar\tau),\\
&= Z^{S^1}\left\{\left|\frac{\theta_3}{\eta}\right|^8 - \left|\frac{\theta_4}{\eta}\right|^8\right\}\left(\frac{\theta_2}{\eta}\right)^4\left\{\overline{\left(\frac{\theta_3}{\eta}\right)^4} + \overline{\left(\frac{\theta_4}{\eta}\right)^4}\right\},
\end{aligned} \tag{3.69}$$

for the even sectors, and

$$\begin{aligned}
Z^{(\mathrm{NS,NS})}_{1,3}(\tau,\bar\tau) &= -\,Z^{(\mathrm{NS,R})}_{1,3}(\tau,\bar\tau) = \left|\frac{\theta_2}{\eta}\right|^{10}\left\{\left|\frac{\theta_3}{\eta}\right|^8 + \left|\frac{\theta_4}{\eta}\right|^8\right\},\\
Z^{(\mathrm{R,R})}_{1,3}(\tau,\bar\tau) &= -\,Z^{(\mathrm{R,NS})}_{1,3}(\tau,\bar\tau) = \left|\frac{\theta_2}{\eta}\right|^{8}\left\{\left|\frac{\theta_3}{\eta}\right|^{10} + \left|\frac{\theta_4}{\eta}\right|^{10}\right\},
\end{aligned} \tag{3.70}$$

for the odd sectors.

The partition functions for even sectors (3.69) coincide with those for the model III (3.57) up to the common factor Z^{S^1}, implying that the right SUSY is completely broken. They are also quite similar to the model IV (3.64), as anticipated.

Even though the odd sectors are also similar to (3.65), we here have a crucial difference. Namely, we find $Z^{(\mathrm{NS,NS})}_{1,3} \neq -Z^{(\mathrm{R,NS})}_{1,3}$, just leading to the fact that the left SUSY is obviously broken.

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The massless spectra in the twisted sectors are different from the previous two models. For example, massless (NS,NS) and (NS, R) states do not appear in the odd sectors as opposed to the case of model IV. No tachyons appear as in the models so far.

Model VI: finally, we briefly mention on the model defined by the orbifold twist $(-\mathbf{1}_L)^{\otimes 2}|_{X^{4,5}} \otimes (-\mathbf{1}_R)^{\otimes 4}|_{X^{6,7,8,9}}$, which is again organized to be a $\mathbb{Z}_4$-action both on the fermionic and bosonic sectors, acting on the background $\left[M^4\right] \times T^6_{4,5,\ldots,9}[\mathrm{SO}(12)]$. This is similar to the model V. The partition functions are almost the same as the previous two models;

$$\begin{aligned}
Z_0^{(\mathrm{NS,NS})}(\tau,\bar{\tau}) &= -Z_0^{(\mathrm{R,NS})}(\tau,\bar{\tau}) = \left\{\left|\frac{\theta_3}{\eta}\right|^{12} + \left|\frac{\theta_4}{\eta}\right|^{12}\right\}\left|\frac{\theta_2}{\eta}\right|^{8}, \\
Z_0^{(\mathrm{R,R})}(\tau,\bar{\tau}) &= -Z_0^{(\mathrm{NS,R})}(\tau,\bar{\tau}) = \left|\frac{\theta_2}{\eta}\right|^{20}, \\
Z_2^{(\mathrm{NS,NS})}(\tau,\bar{\tau}) &= -Z_2^{(\mathrm{R,NS})}(\tau,\bar{\tau}) = \left\{\left|\frac{\theta_3}{\eta}\right|^{12} - \left|\frac{\theta_4}{\eta}\right|^{12}\right\}\left(\frac{\theta_2}{\eta}\right)^{4}\left\{\overline{\left(\frac{\theta_3}{\eta}\right)^{4}} + \overline{\left(\frac{\theta_4}{\eta}\right)^{4}}\right\}, \\
Z_2^{(\mathrm{R,R})}(\tau,\bar{\tau}) &= -Z_2^{(\mathrm{NS,R})}(\tau,\bar{\tau}) = \left|\frac{\theta_2}{\eta}\right|^{20}, \\
Z_{1,3}^{(\mathrm{NS,NS})}(\tau,\bar{\tau}) &= -Z_{1,3}^{(\mathrm{NS,R})}(\tau,\bar{\tau}) = \left|\frac{\theta_2}{\eta}\right|^{12}\left\{\left|\frac{\theta_3}{\eta}\right|^{8} + \left|\frac{\theta_4}{\eta}\right|^{8}\right\}, \\
Z_{1,3}^{(\mathrm{R,R})}(\tau,\bar{\tau}) &= -Z_{1,3}^{(\mathrm{R,NS})}(\tau,\bar{\tau}) = \left|\frac{\theta_2}{\eta}\right|^{8}\left\{\left|\frac{\theta_3}{\eta}\right|^{12} + \left|\frac{\theta_4}{\eta}\right|^{12}\right\}.
\end{aligned} \tag{3.71}$$

This model shares basic features with the model V as expected, although the mass spectrum in each sector is slightly different. Once again, we find $Z_{1,3}^{(\mathrm{NS,NS})} \neq -Z_{1,3}^{(\mathrm{R,NS})}$, and no massless (NS,NS) and (NS, R) states appear in the odd sectors as opposed to the model IV.

4 Discussions about the unitarity and stability

We have studied various non-SUSY string vacua realized as asymmetric orbifolds in section 3. The right-moving part of the twist operators always include the reflection $(-\mathbf{1}_R)^{\otimes 4}$, with the non-trivial property $\left\{(-\mathbf{1}_R)^{\otimes 4}\right\}^2 = (-1)^{F_R}|_\psi$, or $\left\{(-\mathbf{1}_R)^{\otimes 4}\right\}^2 = (-1)^{F_R}|_\psi \otimes (-1)^{F_R}|_\lambda$. The torus partition functions for all of these vacua have been computed in a way showing manifestly the modular invariance, and they are properly q-expanded as to be consistent with unitarity. Moreover, by examining the spectra of physical states read off from the partition functions, we have found all of them to be stable, namely, any tachyonic states do not appear in all the untwisted and twisted sectors. These are likely to be common nice features of the non-SUSY string vacua of these types. In this section, we shall try to clarify why this is the case. There are still various extensions or modifications of the non-SUSY vacua studied in this paper, and the arguments given here would be applicable to them rather generally.

We first recall some non-trivial points that are specific in our models of asymmetric orbifolds. First of all, as we emphasized several times, the building blocks given in subsection 3.1 includes various phase factors. They are necessary to assure the modular covariance, and make the orbifold projections in the twisted sectors to differ non-trivially from that for the untwisted sector. As we already mentioned in section 2, this is a main reason why it would not be self-evident whether our models are unitary.

Secondly, needless to say, the absence of tachyonic instability is not obvious for generic non-SUSY vacua. It is a common feature that non-SUSY orbifolds involving our models would include the 'wrong GSO' NS states in the twisted sectors, which are expressed typically as $\sim \left(\frac{\theta_3}{\eta}\right)^4 + \left(\frac{\theta_4}{\eta}\right)^4$ and would be potentially tachyonic.

Now, let us start our discussions. For the time being, we focus on the models without the shift operator $\mathcal{T}_{2\pi R}|_{\text{base}}$, which are $\mathbb{Z}_4$-asymmetric orbifolds. The partition functions are decomposed with respect to the twisted sectors labeled by $a \in \mathbb{Z}_4$ in the form as, say, (3.56). Let us schematically denote the relevant partition functions as

$$Z_a^{(s_L,s_R)}(\tau,\bar{\tau}) = \frac{1}{4}\sum_{b\in\mathbb{Z}_4} Z_{(a,b)}^{(s_L,s_R)}(\tau,\bar{\tau}), \tag{4.1}$$

where s_L, (s_R) expresses the left-moving (right-moving) spin structure. We are only interested in the twisted sectors $a \neq 0$, since the unitarity and stability for the untwisted sector are obvious by construction.

As addressed above, the building blocks we utilized involve various phase factors. Consequently, it would be useful to reinterpret the b-summation in (4.1) as that for the modular T-transformation $\tau \mapsto \tau + 1$;

$$Z_a^{(s_L,s_R)}(\tau,\bar{\tau}) = \begin{cases} \dfrac{1}{4}\left[Z_{(a,0)}^{(s_L,s_R)}(\tau,\bar{\tau}) + Z_{(a,0)}^{(s_L,s_R)}(\tau+1,\bar{\tau}+1)\right], & (a=2), \\ \dfrac{1}{4}\displaystyle\sum_{\ell\in\mathbb{Z}_4} Z_{(a,0)}^{(s_L,s_R)}(\tau+\ell,\bar{\tau}+\ell), & (a=1,3). \end{cases} \tag{4.2}$$

Here, we made use of the modular covariance of the building blocks and the fact that the fermion chiral blocks $\overline{\mathbf{f}_{(2,b)}(\tau)}$ given in (3.30) vanishes for $b = 1,3$ *for each spin structure*. In the end, one finds that the existence of non-trivial phase factors mentioned above eliminates the terms including the fractional level mismatch $h_L - h_R \notin \mathbb{Z}$. This observation makes it simpler to check the unitarity of the q-expansions of partition functions. All we have to do is just to examine whether the level matching terms $h_L - h_R \in \mathbb{Z}$ in the function $\frac{1}{4}Z_{(a,0)}^{(s_L,s_R)}(\tau,\bar{\tau})$ possess suitable q-expansions with positive integer coefficients.[8] This is indeed the case for all the models given in subsection 3.2, as can be readily confirmed from the explicit forms of the building blocks. We note that, actually, *all* the terms appearing in $Z_{(a,0)}^{(s_L,s_R)}(\tau,\bar{\tau})$ are q-expanded in this way.

How about the stability of the vacua? Namely, we would like to understand why no tachyon appears in all the twisted sectors in spite of the complete SUSY violations. We note

[8]The factor $\frac{1}{4}$ is necessary due to the chiral GSO projection.

- The leading term of each $Z^{(s_L,s_R)}_{(a,0)}(\tau,\bar{\tau})$ always has a non-negative conformal weight, as is obvious from the building blocks presented in subsection 3.1.

- Each $Z^{(s_L,s_R)}_{(a,0)}(\tau,\bar{\tau})$ includes the terms that originate from the 'supersymmetric' chiral blocks $\mathcal{J}(\tau)$ or $\overline{f_{(a,0)}(\tau)}$ $\left(\equiv \overline{\mathbf{f}_{(a,0)}(\tau)}\right)$ with $a=1,3$, and the leading term of $\mathcal{J}(\tau)$ possesses the conformal weight $\frac{1}{2}$. On the other hand, $\overline{f_{(a,0)}(\tau)}$ itself has the weight $\frac{1}{4}$, while the bosonic part of $(-\mathbf{1}_R)^{\otimes 4}$ always adds the zero-point energy $\frac{1}{4}$.

Therefore, the minimum conformal weight of the *T-invariant terms* appearing in $Z^{(s_L,s_R)}_{(a,0)}(\tau,\bar{\tau})$ has to be equal $h=\frac{1}{2}+n$, $(n\in\mathbb{Z}_{\geq 0})$. This fact is sufficient to conclude that no tachyonic states emerge due to the observation given above.

We next consider the models including the shift operator $\mathcal{T}_{2\pi R}|_{\text{base}}$. For our purpose it would be useful to *partially* make the Poisson resummation of $Z_{R,(w,m)}(\tau,\bar{\tau})$ (2.8) with respect to the temporal winding $m\in 4\mathbb{Z}$ and to sum up over ${}^{\forall}w\in a+4\mathbb{Z}$. Then, we can obtain a schematic decomposition

$$Z^{(s_L,s_R)}_{a}(\tau,\bar{\tau})=\frac{1}{4}\sum_{b\in\mathbb{Z}_4}\widetilde{Z}^{(s_L,s_R)}_{(a,b)}(\tau,\bar{\tau}), \tag{4.3}$$

in place of (4.1). Here, $\widetilde{Z}^{(s_L,s_R)}_{(a,b)}(\tau,\bar{\tau})$ includes the contributions with the zero-mode part as

$$\sim q^{\frac{1}{4}\left(\frac{n}{4R}+Rw\right)^2}\bar{q}^{\frac{1}{4}\left(\frac{n}{4R}-Rw\right)^2}, \qquad (n\in\mathbb{Z},\ w\in a+4\mathbb{Z}), \tag{4.4}$$

which give rise to the phase $e^{2\pi i\frac{nw}{4}}\equiv e^{\frac{i\pi}{2}na}$ under the T-transformation. It is now very easy to repeat the above considerations about the unitarity and stability by just replacing $Z^{(s_L,s_R)}_{(a,0)}(\tau,\bar{\tau})$ with $\widetilde{Z}^{(s_L,s_R)}_{(a,0)}(\tau,\bar{\tau})$, leading to the same conclusion.

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A Theta functions

In this appendix we summarize the conventions of theta functions we use in this paper $(q\equiv e^{2\pi i\tau},\ y\equiv e^{2\pi iz}\quad {}^{\forall}\tau\in\mathbb{H}^+, {}^{\forall}z\in\mathbb{C})$;

$$\begin{aligned}
\theta_1(\tau,z)&\equiv i\sum_{n=-\infty}^{\infty}(-1)^n q^{\frac{1}{2}(n-\frac{1}{2})^2}y^{n-\frac{1}{2}}\equiv 2\sin(\pi z)q^{\frac{1}{8}}\prod_{m=1}^{\infty}(1-q^m)(1-yq^m)(1-y^{-1}q^m),\\
\theta_2(\tau,z)&\equiv \sum_{n=-\infty}^{\infty}q^{\frac{1}{2}(n-\frac{1}{2})^2}y^{n-\frac{1}{2}}\equiv 2\cos(\pi z)q^{\frac{1}{8}}\prod_{m=1}^{\infty}(1-q^m)(1+yq^m)(1+y^{-1}q^m),\\
\theta_3(\tau,z)&\equiv \sum_{n=-\infty}^{\infty}q^{\frac{1}{2}n^2}y^n\equiv\prod_{m=1}^{\infty}(1-q^m)(1+yq^{m-\frac{1}{2}})(1+y^{-1}q^{m-\frac{1}{2}}),
\end{aligned}$$

$$\theta_4(\tau,z) \equiv \sum_{n=-\infty}^{\infty} (-1)^n q^{\frac{1}{2}n^2} y^n \equiv \prod_{m=1}^{\infty} (1-q^m)(1-yq^{m-\frac{1}{2}})(1-y^{-1}q^{m-\frac{1}{2}}), \tag{A.1}$$

$$\eta(\tau) \equiv q^{\frac{1}{24}} \prod_{m=1}^{\infty} (1-q^m). \tag{A.2}$$

We often use the abbreviations; $\theta_i \equiv \theta_i(\tau,0)$, $\theta_i(z) \equiv \theta_i(\tau,z)$, and $\eta \equiv \eta(\tau)$.

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