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Letter

Quark-gluon plasma connected to finite heat bath

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Abstract. We derive entropy formulas for finite reservoir systems, S_q, from universal thermostat independence and obtain the functional form of the corresponding generalized entropy-probability relation. Our result interprets thermodynamically the subsystem temperature, T_1, and the index q in terms of the temperature, T, entropy, S, and heat capacity, C of the reservoir as $T_1 = T\exp(-S/C)$ and $q = 1-1/C$. In the infinite C limit, irrespective of the value of S, the Boltzmann-Gibbs approach is fully recovered. We apply this framework for the experimental determination of the original temperature of a finite thermostat, T, from the analysis of hadron spectra produced in high-energy collisions, by analyzing frequently considered simple models of the quark-gluon plasma.

1 Motivation

A nonlinear entropy formula has been suggested by Rényi long ago and been applied to several areas in physics [1–7]. Another formula, the Tsallis entropy, has more recently been promoted as the keystone for a generalized thermodynamics, treating correlated physical systems [8–11] with intrinsic, either statistical or dynamical fluctuations [12, 13]. A respectable amount of papers applying this idea to one or the other area in physics appeared [14–19]. Since from this entropy the canonical energy distribution is power law tailed in place of the Boltzmann-Gibbs exponential, numerous high-energy distributions have been fitted using the Tsallis formula [12,20–28]. Its independence from the thermostat and the thermodynamical foundation behind the use of such a formula are interesting questions. This is even true for such a simple system like the ideal gas in a generalized form [29], which can also provide power law distribution [30–33]. It has been recently proven that —considering in the traditional way— the thermodynamical and conditional probability of an ideal gas and its part, respectively, provide Rényi and Tsallis q-entropy formulas [34].

In our earlier works we investigated some general mathematical properties of alternative entropy formulas via their pairwise composition rules, and established that a scaled repetition of an arbitrary composition rule leads to an associative asymptotic composition rule of large subsystems [35]. All such rules are uniquely defined by a strict monotonic function, their formal logarithm denoted by L (see eq. (3)). Recently we have also observed that —in connection to the zero-th law of thermodynamics— the factorizability condition on the common entropy maximum [36] allows only for such rules [37]. We seek in this paper for the thermodynamical meaning of the q parameter generalizing the classical entropy formula, valid for $q = 1$. Some $q \neq 1$ parameter were calculated theoretically [38–40].

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2 Classical thermodynamical and statistical fundaments

The total entropy is expressed by the Planck formula,

$$S\left[P_i\right] := \lim_{N\to\infty} \frac{\Omega}{N} = -\sum_i P_i \ln P_i, \qquad (1)$$

with $\Omega = \ln N!/\prod_{i=1}^{r} N_i!$, considering altogether N states in r classes and in each class N_i indistinguishable states. The Boltzmann constant is set to one ($k_B = 1$). The probability to find our system in a given class (macrostate) is approaching the ratio $P_i = N_i/N$ in the large N limit. This entropy is Boltzmann's permutation measure per state in the large number of states limit. Conventional atomic statistical physics is based on the enormous largeness of N. Modern nanotechnology and attoscale high-energy research, on the other hand, have reached a point where N can be rather small in experiments. Furthermore the conjecture that the largeness of N would suffice to derive the classical formula holds only if the assumptions, most prominently the statistical independence assumption, hold [11].

In this paper we found the thermodynamical interpretation of the entropy formula and its parameters on the analysis of the two-body thermodynamics of a single observed subsystem and a reservoir. The binary system will be regarded in its maximal probability state, with the individual contributions $S_i = -\ln P_i$. The knowledge gained from this analysis will be generalized by a Gibbs ensemble, treated as the extension from a sum of two to a weighted sum of many. In this way a result of the two-body analysis in the form $L(S_1) = L(-\ln P_1)$ will generalize the classical formula $S = -\sum_i P_i \ln P_i$ to

$$L(S) = \sum_i P_i L(-\ln P_i). \tag{2}$$

This *L-additive* version of the additive entropy is our starting point.

3 Derivation

We found the thermodynamical interpretation of the entropy formula and its parameters on the analysis of the two-body thermodynamics of a single observed subsystem and a reservoir. For finite systems the microcanonical approach is the key to the physical interpretation. In the classical treatment subleading terms in a finite-energy expansion of the microcanonical entropy maximum are often ignored, the reservoir is treated as constant in the canonical limit. A notable exception is the analysis of statistical fluctuations and their scaling in the thermodynamical limit [41–47]. We aim to compensate the correlation between subsystem and reservoir induced by the conservation of total energy by maximizing a monotonic function of the Boltzmann-Gibbs entropy, $L(S)$. We seek for that very function, L, which *counteracts* finite-size effects beyond the usual linear term, $-\beta E_i$, in the Taylor expansion of the $L(S) = \max$ principle.

We discuss now the thermal equilibrium of two systems, one with energy E_1 (subsystem) and the other with energy $E - E_1$ (reservoir), while their respective entropy contributions are combined by the general rule satisfying

$$L(S_{12}) = L(S_1) + L(S_2). \tag{3}$$

Here we do not assume that the deviation from the simple additive rule be small. For the sake of simplicity we consider here homogeneous rules, relevant for the cases when subsystem and reservoir are composed from the same matter (for details see ref. [37]). The microcanonical condition for a maximal entropy state then defines the thermodynamical inverse temperature, requiring

$$L\left(S(E_1)\right) + L\left(S(E - E_1)\right) = \max. \tag{4}$$

Varying the subsystem energy, E_1, while keeping the total energy E fixed, we describe the thermal contact between subsystem and reservoir. This means that the derivative with respect to E_1 of the above expression (4) vanishes. Owing to the two E_1-dependent contributions, it is equivalent to the statement that

$$\begin{aligned}\beta_1 &= L'\left(S(E_1)\right) \cdot S'(E_1) \\ &= L'\left(S(E - E_1)\right) \cdot S'(E - E_1).\end{aligned} \tag{5}$$

This equality, when taken in the $E \gg E_1$ limit, usually defines the canonical approach. Now we would like to take into account effects to higher order in E_1/E, and require that their leading term vanishes on the r.h.s. The reservoir's entropy on the r.h.s. is Taylor-expanded: $S(E - E_1) = S(E) - S'(E) \cdot E_1 + \dots$ Collecting the coefficients of E_1 we arrive at

$$\begin{aligned}\beta_1 = {} & L'\left(S(E)\right) \cdot S'(E) \\ & - \left[S'(E)^2 L''(S(E)) + S''(E) L'(S(E))\right] E_1 + \dots\end{aligned} \tag{6}$$

Here the first term on the r.h.s. is the familiar canonical (E_1-independent) Lagrange multiplier,

$$\beta = L'\left(S(E)\right) \cdot S'(E) = L'(S) \cdot \frac{1}{T}, \tag{7}$$

constituting the $\beta_1 = \beta$ relation. Our key addition to the usual treatment is to require that the coefficient of the linear term in eq. (6) vanishes: This is a constraint for the $L(S)$ function in general. Obviously, without considering $L(S)$, the whole coefficient consisted only of $S''(E)$ as in the traditional approach, and nothing further could be done. We obtain the following condition:

$$\frac{L''(S)}{L'(S)} = -\frac{S''(E)}{S'(E)^2}. \tag{8}$$

Since the l.h.s. of eq. (8) is a function of S, while the r.h.s. is a function of E, the l.h.s. must be treated as an S-independent constant by solving eq. (8) for $L(S)$, if we want it to hold for arbitrary $S(E)$. This *Universal Thermostat Independence* (UTI) reads

$$\frac{L''(S)}{L'(S)} = a. \tag{9}$$

The natural requirement that $L(S) \approx S$ for small S, a specific aspect of the third law of thermodynamics leads to the particular conditions with $L'(0) = 1$ and $L(0) = 0$. The solution of eq. (9) becomes

$$L(S) = \frac{\mathrm{e}^{aS} - 1}{a}. \tag{10}$$

The derivatives of the $S(E)$ equation of state do have physical meaning: $S'(E) = 1/T$ and $S''(E) = -1/CT^2$ are related to the traditional temperature and heat capacity of the reservoir. By using this we obtain

$$a = 1/C. \tag{11}$$

The non-additivity parameter of the entropy composition rule, a, defined by eq. (3) *is simply the inverse heat* *capacity* of the reservoir[1]. For $C \to \infty$ one has $a \to 0$ and $L(S) \to S$, so the Boltzmann-Gibbs formula is included by this limit. A connection between Tsallis entropy and constant heat capacity of the reservoir has been observed years ago [30,48]. Our result shows the background for this observation. The philosophy behind our approach is first to decide on the entropy composition formula by choosing $L(S)$ generally and then to solve the maximization problem in terms of subsystem energies and corresponding probabilities.

The knowledge gained from this analysis now will be generalized. Similarly to a Gibbs ensemble, we extend a sum of two to a weighted sum of many. Based on the two-body analysis in the form $L(S_1)$, the result generalizes the classical entropy formula, $S_{BG} = -\sum_i P_i \ln P_i$, to eq. (2). This *L-additive* form of the generally non-additive entropy leads to the Tsallis entropy formula when applying eq. (10). In this way one obtains

$$L\left(S(E_1)\right) - \beta E_1 = \frac{1}{a}\left(\mathrm{e}^{aS(E_1)} - 1\right) - \beta E_1 = \max. \quad (12)$$

In this equation the coefficient of the second-order correction, $\mathcal{O}\left(E_1^2/E^2\right)$ vanishes for $a = 1/C(E)$. By this we are led to the following entropy expression:

$$L\left(S(E_1)\right) = L\left(-\ln P_1\right) = \frac{1}{a}\left(P_1^{-a} - 1\right). \quad (13)$$

Our result applied to a Gibbs ensemble with the relative occurrence frequency P_i of states with energy E_i, hence, reads

$$\sum_i P_i L(-\ln P_i) - \beta \sum_i P_i E_i - \alpha \sum_i P_i = \max. \quad (14)$$

Substituting eq. (13) we finally arrive at

$$\frac{1}{a}\sum_i \left(P_i^{1-a} - P_i\right) - \beta \sum_i P_i E_i - \alpha \sum_i P_i = \max. \quad (15)$$

With the widespread notation $q = 1 - a$ one obtains the Tsallis entropy formula,

$$S_{\text{Tsallis}} := L(S) = \frac{1}{q-1}\sum_i \left(P_i - P_i^q\right). \quad (16)$$

It is suggestive to consider its inverse function according to eq. (2). This delivers the Rényi entropy,

$$S_{\text{Rényi}} := S = \frac{1}{1-q}\ln \sum_i P_i^q. \quad (17)$$

Now the parameters β and a, defined in eqs. (7) and (11), are set by the physics of the finite-energy reservoir. The sign of the heat capacity, C, determines whether q is smaller or larger than one. It may possibly carry an interesting message for the description of gravitating systems, with $C < 0$.

[1] The meaning of non-additivity parameter, a, is given in ref. [35], however this can be converted to the so-called Tsallis q parameter by $a = 1 - q$.

We proceed by noting that, maximizing S_{Tsallis} with respect to the P_i weights of system instances with energy E_i, one obtains the canonical cut power law distribution of energies

$$P_i = \left(Z^{1-q} + (1-q)\frac{\beta}{q}E_i\right)^{\frac{1}{q-1}}. \quad (18)$$

Using eqs. (7), (10), and (11) we rewrite this in the equivalent form,

$$P_i = \frac{1}{Z}\left(1 + \frac{Z^{-1/C}\,\mathrm{e}^{S/C}}{C-1}\frac{E_i}{T}\right)^{-C}, \quad (19)$$

expressing the energy distribution in terms of the temperature, T, entropy, S and heat capacity, C of the ideal reservoir. The partition sum Z, obtained from normalization, is related to the Tsallis entropy, $L(S_1)$, and energy, E_1, of the subsystem via its deformed logarithm,

$$\ln_q Z := C\left(Z^{1/C} - 1\right) = L\left(S_1\right) - \frac{1}{1-1/C}\beta E_1. \quad (20)$$

In the infinite heat capacity limit, irrespective of the value of S, formula (19) recovers the exponential distribution. The inverse logarithmic slope of the energy distribution, derived from it, is linear,

$$T_{\text{slope}}(E_i) = \left(-\frac{\mathrm{d}}{\mathrm{d}E_i}\ln P_i\right)^{-1} = T_0 + E_i/C, \quad (21)$$

with $T_0 = T\mathrm{e}^{-S/C}Z^{1/C}(1-1/C)$. One concludes that the generalized entropy formula leads to a cut power law energy distribution, based on a finite heat capacity reservoir. For an ideal gas, with equation of state

$$S(E) = C_{0,sys}\ln\left(1 + \frac{E}{C_{0,sys}T_0}\right), \quad (22)$$

where the heat capacity of the isolated system is $C_{0,sys}$. The microcanonical energy distribution of the subsystem,

$$w(E_1) \propto \frac{\mathrm{e}^{S(E-E_1)}}{\mathrm{e}^{S(E)}} = \left(1 - \frac{E_1}{C_{0,res}T_0 + E}\right)^{C_{0,res}}, \quad (23)$$

approaches the canonical exponential with the reservoir's heat capacity, $C_{0,res}$, approaching infinity,

$$P(E_1) \propto \frac{\mathrm{e}^{L(S(E-E_1))}}{\mathrm{e}^{L(S(E))}} = \mathrm{e}^{-E_1/T}. \quad (24)$$

Thus the assumption on L-additivity of the entropy, cf. eq. (3), factorizes the microcanonical probability of an ideal gas. Microcanonical distribution of non-quantum ideal gases provide an example when UTI principle is not an approximation, the correction is exact. Several further aspects of this important example are developed in [34].

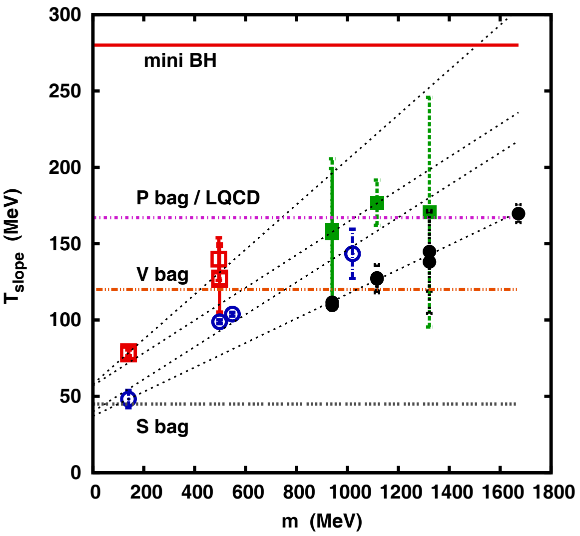

Fig. 1. Extrapolated inverse slopes from RHIC AuAu data at 200 GeV (circles), LHC *pp* data at 900 GeV (boxes) and for different theoretical models of a QGP thermostat with temperature $T = 167\,\mathrm{MeV}$ (horizontal lines). Open symbols correspond to mesons, filled symbols to baryons. The skew lines indicate the respective valence quark assumptions: $T^{\mathrm{meson}}_{\mathrm{slope}}(p_T = 0) = T_0 + m_i/2C$ and $T^{\mathrm{baryon}}_{\mathrm{slope}}(p_T = 0) = T_0 + m_i/3C$, for the LHC (upper two lines) and for RHIC (lower two lines).

4 Application

We demonstrate the usefulness of the above general results on a particular thermal model to heavy-ion collisions. Experimental data from RHIC AuAu collision at 200 GeV deliver different T_{slope}'s extrapolated to $p_T = 0$ for different hadrons [20,49]. Considering that the energy at zero momentum is the rest mass in $c = 1$ units, a linear trend shows in the $T_{\mathrm{slope}}(m_i)$ values, as can be seen in fig. 1. The open circles correspond to mesons, the filled ones to baryons in this figure. The steepness for mesons and baryons seem to be in the proportion 2 : 3, suggesting a quark coalescence hadronization picture, compatible with the factorization assumption $P_{\mathrm{hadron}}(E) = P_i^K(E/K)$ with $K = 2$ and $K = 3$ for mesons and baryons, respectively. This scaling is acceptable and leads to

$$T^{\mathrm{hadron}}_{\mathrm{slope}}(E) = T^{\mathrm{quark}}_{\mathrm{slope}}(E/K), \tag{25}$$

with the common $T_0 \approx 48\,\mathrm{MeV}$ intersect in the formula (21). The valence quark matter heat capacity at RHIC AuAu collision tends to be $C \approx 4.5$. Similar trends can be extracted from the analysis of fits to the ALICE data in 900 GeV *pp* collisions done in ref. [50]: the values for the Tsallis slope parameters, T_0, are much lower than the canonical QCD phase transition temperature. Here we replotted the tabulated values given in ref. [50], using the coalescence quark assumption, denoting mesons by open square boxes while baryons by filled boxes. We note, however, that these fits were performed in the very low p_T range ($p_T < 2.5\,\mathrm{GeV}/c$) only, therefore the uncertainty of the fitted parameters is large.

In order to interpret this surprisingly low value for T_0, we have to consider physical models of a finite thermostat, and calculate

$$T_1 = 1/\beta_1 = T\mathrm{e}^{-S/C} = \frac{\mathrm{e}^{S(E_1)/C}}{1 + 0\frac{E_1}{CT} + \alpha\frac{E_1^2}{C^2T^2} + \dots}, \tag{26}$$

based on eq. (6). The coefficient "zero" is due to the UTI principle eq. (8). The next order, $\alpha\, E_1^2/C^2T^2$, is just indicated. Here, $\lim_{C\to\infty} T_0 = T_1$, *i.e.* for small subsystems in large reservoirs $E_1 \ll CT$. The usual treatment of finite-size effects, on the other hand, does not annul the coefficient of the term E_1/CT. In this way with the UTI principle we improve the approximation for finite C.

In order to model the reservoir physically, we study the Stefan – Boltzmann formula supplemented with a bag constant, $E/V = \sigma T^4 + B$ in a volume V. Since the pressure is given by $p = \frac{1}{3}\sigma T^4 - B$, the entropy is $S = \frac{4}{3}\sigma V T^3$. The heat capacity is the derivative of the energy with respect to the temperature,

$$C = \frac{\mathrm{d}E}{\mathrm{d}T} = 4\sigma V T^3 + \left(\sigma T^4 + B\right)\frac{\mathrm{d}V}{\mathrm{d}T}. \tag{27}$$

At constant volume, V, this gives $C_V = 4\sigma V T^3 = 3S$ and $T_{1V} = T\mathrm{e}^{-1/3}$. At constant pressure the temperature cannot change in this model, so $C_p = \infty$ and $T_{1P} = T$. Furthermore, considering an adiabatically expanding reservoir, a more realistic scenario in high-energy experiments, one deals with the heat capacity at constant entropy, $C_S = 3S(1 - T_*^4/T^4)/4$, with T_* being the temperature where the pressure vanishes. In this case $C_S \le 3S/4$ and $T_{1S} \le T\mathrm{e}^{-4/3}$ is the theoretical prediction.

Figure 1 presents the inverse logarithmic slope, T_{slope}, as a function of hadron masses and T_1 lines for different physical models of the thermostat. Besides the three above-described bag model approaches we also indicate the classical Schwarzschild black hole, having $C = -2S$ and $T_1 = T\mathrm{e}^{1/2}$, marked as "mini BH". One inspects that this possibility is far from all experimental observations.

We note that theoretically a really constant heat capacity, C_0, stems from the equation of state eq. (22). The latter is a good ansatz for an effective equation of state of classical non-Abelian gauge field systems on the lattice [51] and represents the high-E limit of Planck's $S''(E)$ formula for thermal radiation.

Considering the heat capacity in the above scenarios and a standard numerical value of $T \approx 167$ MeV for the reservoir temperature, conjectured for the QGP at hadronization phenomenology and determined by lattice QCD calculations, one obtains $T_{1P} = T = 167\,\mathrm{MeV}$, $T_{1V} = T\mathrm{e}^{-1/3} \approx 120\,\mathrm{MeV}$ and $T_{1S} \le T\mathrm{e}^{-4/3} \approx 45\,\mathrm{MeV}$ characterizing the Tsallis distribution of valence quarks.

The conjecture that in heavy-ion collisions a statistical power law energy distribution due to finite phase space availability corrections to the traditional canonical distribution may appear is further supported by the observation that the measure of non-additivity, $a = 1/C$, expressed by the inverse power in the fitted power law tail, is reduced for increasing participant number [52].

The fitted power C is also tendentiously smaller in e^+e^- or pp than in heavy-ion collisions [22]. Finally, we realize that only the adiabatic scenario for the quark matter thermostat leads to T_0 values near to the ones extracted from experimental analysis by the coalescence assumption, $T_{1S} \approx T_0 \approx 45$–$55\,\text{MeV}$.

5 Conclusion

In conclusion, the Tsallis entropy formula is derived as the consequence of the following requirement: we seek for that non-additive entropy composition rule, which cancels linearly energy-dependent corrections due to the finite $E - E_1$ energy in the reservoir to a subsystem's thermodynamical inverse temperature. This determines the composition rule and the entropy formula uniquely turns out to be the Tsallis entropy. This derivation explains the particular functional form of the Tsallis and Rényi formulas as generalized entropy expressions satisfying the UTI principle. With regard to the physical interpretation we have obtained $q = 1 - 1/C$, with C being the heat capacity of the total system with the conserved energy E. The canonical temperature of the subsystem becomes $T_1 = \mathrm{e}^{-S/C}\,T$ with $T(E)$ and $S(E)$ being the traditional temperature and entropy of the finite reservoir, respectively. A preliminary analysis of experimental data on particle production seems to be sensitive to different physical assumptions about a QGP thermostat. Here the isentropic scenario performs best.

At last we note that in the case of non-constant heat capacity, $a = 1/C(S)$, the UTI principle (9) can be integrated to a more general $L(S)$ function [34]. In this case the Tsallis and Rényi formulas do not apply, a more general entropy-probability relation emerges from eq. (2).

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