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Intersecting D6-branes on the $\mathbb{Z}_{12}$-II orientifold

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Abstract: Much work has been done by a number of authors with the aim of constructing the supersymmetric Standard Model in type IIA intersecting-brane theories compactified on an orientifold with various $\mathbb{Z}_N$ or $\mathbb{Z}_M \times \mathbb{Z}_N$ point groups. Here we consider the $\mathbb{Z}_{12}$ point group which has previously received comparatively little attention. We consider intersecting D6-branes that wrap 3-cycles consisting of a 2-cycle of the 4-dimensional lattice upon which the $\mathbb{Z}_{12}$ is realised times a 1-cycle of the remaining 2-torus. Our discussion is restricted to the case when these 2-cycles are "factorisable" in the sense discussed in section 3. Although it is possible to find models with the correct supersymmetric Standard Model quark-doublet content, we have not found it possible to obtain the correct quark-singlet content.

Keywords: Intersecting branes models, D-branes

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Contents

1 Introduction

The use of intersecting D6-branes in Type IIA string theory offers an attractive route to constructing the Standard Model in string theory [1, 2], and indeed an attractive model having just the spectrum of the (non-supersymmetric) Standard Model has been obtained by Ibañéz et al. [3]. In this approach one starts with two stacks a, with $N_a = 3$ D6-branes, and b with $N_b = 2$ D6-branes, each wrapping the three large spatial dimensions plus 3-cycles of the six-demensional compactified space Y. Open strings beginning and ending on the stack a generate the gauge group $\mathrm{U}(3) = \mathrm{SU}(3)_{\mathrm{colour}} \times \mathrm{U}(1)_a$, while those that begin and end on the stack b generate the gauge group $\mathrm{U}(2) = \mathrm{SU}(2)_L \times \mathrm{U}(1)_b$. Thus the non-Abelian component of the Standard Model gauge group is immediately assured. Further, (four-dimensional) chiral fermions in the bi-fundamental $(\mathbf{N}_a, \bar{\mathbf{N}}_b) = (\mathbf{3}, \bar{\mathbf{2}})$ representation of $\mathrm{U}(3) \times \mathrm{U}(2)$ appear at the multiple intersections of the two stacks. (Here the $\mathbf{3}$ representation of $\mathrm{U}(3)$ has charge $Q_a = +1$ with respect to $\mathrm{U}(1)_a$, and the $\bar{\mathbf{2}}$ representation of $\mathrm{U}(2)$ has charge $Q_b = -1$ with respect to $\mathrm{U}(1)_b$.) This is just the representation needed for the Standard Model quark doublet Q_L. However, non-supersymmetric intersecting-brane models lead to flavour-changing neutral-current (FCNC) processes that can only be suppressed to levels consistent with the current bounds by making the string scale rather high, of order 10^4 TeV, which in turn leads to fine-tuning problems [4]. Further, in non-supersymmetric theories, the complex structure moduli are generally unstable [5]. Both of these problems are avoided if instead we seek intersecting-brane models that yield the *supersymmetric* Standard Model. This is the strategy that we shall pursue in this paper.

To ensure that we obtain $\mathcal{N}=1$ supersymmetry in the four space-time dimensions, it is necessary that the compactified space Y should be a Calabi-Yau 3-fold or a toroidal orbifold $\Omega = T^6/P$, where the (discrete) point group P must be a subgroup of SU(3) [6]. (We shall only consider the latter possibility.) The requirement that the point-group generator θ acts crystallographically on the lattice Γ that defines the torus T^6 then restricts P to be either $\mathbb{Z}_N$, with $N = 3,4,6,7,8,12$, or $\mathbb{Z}_M \times \mathbb{Z}_N$, with N a multiple of M and $N = 2,3,4,6$ [7, 8]. The first question is whether one can find stacks a and b, as above, whose intersections yield just the three Standard Model quark doublets. However, before proceeding further it should be noted that both of these stacks are positively charged with respect to the Ramond-Ramond (RR) 7-form gauge field to which they are "electrically" coupled. Since Y is a compact space, the electrical flux lines associated with the RR charges must close, which can only happen if the RR charges sum to zero. This in turn requires the introduction of negative RR charge. Anti D-branes, $\bar{\text{D}}$6-branes, annihilate D6-branes, and the only feasible alternative is to use the O6-planes. These are topological defects that arise when Y is an orientifold, i.e. $Y = \Omega/\mathcal{R}$, where $\mathcal{R}$ is the embedding of the world-sheet parity operator in the compactified space. This means that every stack $\kappa = a, b, \ldots$ has an orientifold image $\kappa' = \mathcal{R}\kappa$, and that the stack a will in general intersect with both b and its orientifold image b'. As with the intersections of a with b, the intersections of a with b' also yield chiral fermions but they are now in the representation $(\mathbf{N}_a, \mathbf{N}_b) = (\mathbf{3}, \mathbf{2})$ representation of U(3) $\times$ U(2), where the $\mathbf{2}$ of U(2) has charge $Q_b = +1$ with respect to U$(1)_b$. Then in order to get just the $3Q_L$ quark doublets, we require that the numbers of intersections, $a \circ b$ of a with b, and $a \circ b'$ of a with b', satisfy

$$a \circ b + a \circ b' = 3 \tag{1.1}$$

Of course, we must also ensure that these states have weak hypercharge $Y(Q_L) = 1/6$. In general, Y is a linear combination

$$Y = \sum_{\kappa} y_\kappa Q_\kappa \tag{1.2}$$

of all of the U$(1)_\kappa$ charges Q_κ. A quark doublet arising as a $(\mathbf{3}, \bar{\mathbf{2}})$ representation of U(3) $\times$ U(2) has $Y(\mathbf{3}, \bar{\mathbf{2}}) = y_a - y_b$, whereas the alternative has $Y(\mathbf{3}, \mathbf{2}) = y_a + y_b$. If quark doublets of *both* types occur, then $y_a = 1/6$ and $y_b = 0$. However, if there is only one type then, depending upon which, all we know is that $y_a \mp y_b = 1/6$.

There have been many attempts to construct the supersymmetric Standard Model, or something like it, using a variety of orientifolds [9]–[23]. None has been completely successful, but the closest approach has probably come using the $\mathbb{Z}_6'$ orientifold. The question then arises as to whether one can do better with a different orientifold. In this paper, we address that question using the $\mathbb{Z}_{12}$-II orientifold. This orbifold (and the $\mathbb{Z}_{12}$-I orbifold) is not completely factorisable; that is, it cannot be realised on $T^2 \times T^2 \times T^2$. Some of the technical problems associated with such orbifolds have been discussed in [24]. In that paper the authors determine the non-chiral solutions of the RR tadpole cancellation conditions when the D6-branes lie on top of the orientifold O6-planes, the whole system satisfying (twisted) sector-by-sector RR tadpole cancellation; this is more stringent than necessary, as the vanishing of RR flux just requires overall tadpole cancellation. In what follows we consider more general configurations of intersecting (fractional) D6-branes, and attempt to construct the chiral quark, lepton and Higgs spectrum of the supersymmetric Standard Model, with the strategy of imposing overall tadpole cancellation at the end to constrain any such configurations that generate the required spectrum.

2 The $\mathbb{Z}_{12}$ orbifolds

The generator θ of any abelian point group P may be diagonalised using three complex coordinates z_k $(k = 1, 2, 3)$ for T^6 such that

$$\theta z_k = e^{2\pi i v_k} z_k \tag{2.1}$$

with $0 \leq v_k < 1$ and $v_1 \pm v_2 \pm v_3 = 0$ so that $P \subset \mathrm{SU}(3)$. For the $\mathbb{Z}_{12}$ point group, there are two essentially different ways to ensure the SU(3) holonomy:

$$\mathbb{Z}_{12}\text{–I}: \quad (v_1, v_2, v_3) = \frac{1}{12}(1, -5, 4) \tag{2.2}$$

$$\mathbb{Z}_{12}\text{–II}: \quad (v_1, v_2, v_3) = \frac{1}{12}(1, 5, -6) \tag{2.3}$$

Both of these may be realised as Coxeter orbifolds. That is to say, θ acts on the (six-dimensional) lattice of simple roots of a Lie algebra as a (possibly generalised) Coxeter element. For the $\mathbb{Z}_{12}$-I case we may use the lattice $\mathrm{SO}(8) \times \mathrm{SU}(3)$, and for $\mathbb{Z}_{12}$-II case $\mathrm{SO}(8) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$. The SO(8) lattice is generated by the four simple roots α_a $(a = 1, 2, \ldots, 4)$ of the SO(8) Lie algebra, which satisfy $\alpha_a^2 = 2$ and $\alpha_1.\alpha_2 = -1 = \alpha_2.\alpha_3 = \alpha_2.\alpha_4$; the other scalar products $\alpha_1.\alpha_3 = 0 = \alpha_3.\alpha_4 = \alpha_4.\alpha_1$ are all zero. The order 12 generalised Coxeter element is given by

$$C_{\mathrm{SO}(8)^{[3]}} := s_1 s_2 s_{134} \tag{2.4}$$

where the Weyl reflection s_a in α_a acts on a general vector x as

$$s_a(x) := x - (x.\alpha_a)\alpha_a \tag{2.5}$$

and s_{134} is the automorphism of the SO(8) Dynkin diagram that cyclically permutes the outer roots $\alpha_1 \to \alpha_3 \to \alpha_4 \to \alpha_1$. ($\alpha_2$ is the central root.) Then

$$s_{134}(x) := x - \frac{1}{2}[(x.\alpha_1)(\alpha_1 - \alpha_3) + (x.\alpha_3)(\alpha_3 - \alpha_4) + (x.\alpha_4)(\alpha_4 - \alpha_1)] \tag{2.6}$$

$C_{\mathrm{SO}(8)^{[3]}}$ determines the action of θ on the four basis 1-cycles π_a $(a = 1, 2, \ldots 4)$ of the SO(8) lattice:

$$\theta\pi_1 = \pi_1 + \pi_2 + \pi_3 \tag{2.7}$$

$$\theta\pi_2 = -\pi_1 - \pi_2 \tag{2.8}$$

$$\theta\pi_3 = \pi_1 + \pi_2 + \pi_4 \tag{2.9}$$

$$\theta\pi_4 = \pi_2 \tag{2.10}$$

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The F_4 lattice is generated by the simple roots β_a $(a = 1, 2, \ldots 4)$ of the F_4 Lie algebra. They satisfy $\beta_1^2 = 2 = \beta_2^2$, $\beta_3^2 = 4 = \beta_4^2$ and $\beta_1.\beta_2 = -1$, $\beta_2.\beta_3 = -2 = \beta_3.\beta_4$; the other scalar products $\beta_1.\beta_3 = 0 = \beta_2.\beta_4 = \beta_1.\beta_4$ are all zero. . The (ordinary) Coxeter element is

$$C_{F_4} := s_1 s_2 s_3 s_4 \tag{2.11}$$

where the Weyl reflection is now given by

$$s_a(x) := x - 2\frac{(x.\beta_a)}{(\beta_a.\beta_a)}\beta_a \tag{2.12}$$

C_{F_4} also acts as the generator of $\mathbb{Z}_{12}$. However, it is easy to verify that the SO(8) and F_4 lattices are identical. It follows that the orbifolds $F_4 \times \mathrm{SU}(3)$ for $\mathbb{Z}_{12}$-I and $F_4 \times \mathrm{SU}(2) \times \mathrm{SU}(2)$ for $\mathbb{Z}_{12}$-II respectively are identical to the corrsponding SO(8) orbifolds, so we shall not pursue them further. The action of θ on the remaining two basis 1-cycles, π_5 and π_6, is different for the two $\mathbb{Z}_{12}$ orbifolds.

$$\mathbb{Z}_{12}\text{–I}: \quad \theta\pi_5 = \pi_6 - \pi_5 \quad \text{and} \quad \theta\pi_6 = -\pi_5 \tag{2.13}$$

$$\mathbb{Z}_{12}\text{–II}: \quad \theta\pi_5 = -\pi_5 \qquad \text{and} \quad \theta\pi_6 = -\pi_6 \tag{2.14}$$

There are six independent 2-cycles $\pi_{a,b}$ on the SO(8) lattice. They are defined as $\pi_{a,b} := \pi_a \otimes \pi_b$ with $a, b = 1, 2, 3, 4$ and $a < b$. So for both orbifolds there are twelve independent 3-cycles $\pi_{a,b,k} := \pi_{a,b} \otimes \pi_k$ with $k = 5, 6$.

Invariant 3-cycles are constructed by evaluating the independent combinations of the form $(1+\theta+\theta^2+\ldots+\theta^{11})\pi_{a,b,k}$. In the $\mathbb{Z}_{12}$-I case there are only *two* independent invariant 3-cycles

$$\rho_1 := (1+\theta+\theta^2+\ldots+\theta^{11})\pi_{2,4,6} = 4(\pi_{1,2,5} - \pi_{2,4,5} - \pi_{3,4,5} + \pi_{1,3,6} + \pi_{2,3,6} + \pi_{2,4,6}) \tag{2.15}$$

$$\rho_2 := (1+\theta+\theta^2+\ldots+\theta^{11})\pi_{3,4,6} = 4(\pi_{1,3,5} + \pi_{2,3,5} + \pi_{2,4,5} - \pi_{1,2,6} - \pi_{1,3,6} - \pi_{2,3,6} + \pi_{3,4,6}) \tag{2.16}$$

However, for the $\mathbb{Z}_{12}$-II case there are *four*:

$$\rho_1 := (1 + \theta + \theta^2 + \ldots + \theta^{11})\pi_{2,3,5} = 6(\pi_{1,4,5} + \pi_{2,3,5} + \pi_{2,4,5}) \tag{2.17}$$

$$\rho_2 := (1 + \theta + \theta^2 + \ldots + \theta^{11})\pi_{2,4,5} = 6(-\pi_{1,3,5} - \pi_{2,3,5} + \pi_{2,4,5} + \pi_{3,4,5}) \tag{2.18}$$

$$\rho_3 := (1 + \theta + \theta^2 + \ldots + \theta^{11})\pi_{2,3,6} = 6(\pi_{1,4,6} + \pi_{2,3,6} + \pi_{2,4,6}) \tag{2.19}$$

$$\rho_4 := (1 + \theta + \theta^2 + \ldots + \theta^{11})\pi_{2,4,6} = 6(-\pi_{1,3,6} - \pi_{2,3,6} + \pi_{2,4,6} + \pi_{3,4,6}) \tag{2.20}$$

Both of these are consistent with the cohomology of these orbifolds in the untwisted sector. Because of the smaller number of independent invariant 3-cycles, the former case has the property, also posessed by the $\mathbb{Z}_6$ orbifold, that any supersymmetric bulk 3-cycle is automatically invariant under the orientifold action $\mathcal{R}$. The action of $\mathcal{R}$ is derived for the $\mathbb{Z}_{12}$-II case in section 5. (The corresponding results for the $\mathbb{Z}_{12}$-I orientifold are given in the appendix.) Then, up to an overall multiplicative factor, all supersymmetric 3-cycles have a common bulk part, and the differing intersection numbers needed to construct the Standard Model must derive solely from their differing exceptional parts. Previous experience

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with the the $\mathbb{Z}_6$ orbifold [12], as opposed to the $\mathbb{Z}_6'$ case [15], suggests that such a structure is not rich enough to permit construction of the Standard Model. In any case, as also shown in the appendix, the $\mathbb{Z}_{12}$-I orbifold only has six exceptional 3-cycles, whereas there are ten in the $\mathbb{Z}_6$ case. Accordingly we have not studied the $\mathbb{Z}_{12}$-I case further. Henceforth we consider only the $\mathbb{Z}_{12}$-II case. A general 3-cycle π_κ is specified by the eight integer wrapping numbers $n^\kappa_{a,b}, n^\kappa_3, m^\kappa_3$

$$\pi_\kappa := \sum_{a,b} (n^\kappa_{a,b}\pi_{a,b}) \otimes (n^\kappa_3 \pi_5 + m^\kappa_3 \pi_6) \tag{2.21}$$

Then the invariant bulk 3-cycle constructed from this is

$$\Pi^{\rm bulk}_\kappa := 2(1+\theta+\theta^2+\ldots+\theta^5)\pi_\kappa \tag{2.22}$$

$$= \sum_{p=1}^{4} A^\kappa_p \rho_p \tag{2.23}$$

where

$$A^\kappa_1 = n^\kappa_3 a^\kappa_1 \tag{2.24}$$

$$A^\kappa_2 = n^\kappa_3 a^\kappa_2 \tag{2.25}$$

$$A^\kappa_3 = m^\kappa_3 a^\kappa_1 \tag{2.26}$$

$$A^\kappa_4 = m^\kappa_3 a^\kappa_2 \tag{2.27}$$

with

$$a^\kappa_1 := -n^\kappa_{1,3} + n^\kappa_{1,4} + n^\kappa_{2,3} \tag{2.28}$$

$$a^\kappa_2 := n^\kappa_{1,2} - n^\kappa_{1,3} - n^\kappa_{1,4} + n^\kappa_{2,4} \tag{2.29}$$

The intersection number $\Pi^{\rm bulk}_\kappa \circ \Pi^{\rm bulk}_\lambda$ of two bulk 3-cycles is defined as

$$\Pi^{\rm bulk}_\kappa \circ \Pi^{\rm bulk}_\lambda := \frac{1}{12}\left(\sum_{k=0}^{11}\theta^k \pi_\kappa\right) \circ \left(\sum_{\ell=0}^{11}\theta^\ell \pi_\lambda\right) \tag{2.30}$$

with π_κ and π_λ one of the basis 3-cycles $\pi_{a,b,k}$. Then

$$\rho_1 \circ \rho_2 = 0 = \rho_3 \circ \rho_4 \tag{2.31}$$

$$\rho_1 \circ \rho_3 = 6 = \rho_2 \circ \rho_4 \tag{2.32}$$

$$\rho_1 \circ \rho_4 = 0 = \rho_2 \circ \rho_3 \tag{2.33}$$

and for two general bulk 3-cycles of the form (2.21) we get

$$\Pi^{\rm bulk}_\kappa \circ \Pi^{\rm bulk}_\lambda = 6(A^\kappa_1 A^\lambda_3 - A^\kappa_3 A^\lambda_1 + A^\kappa_2 A^\lambda_4 - A^\kappa_4 A^\lambda_2) \tag{2.34}$$

$$= 6(a^\kappa_1 a^\lambda_1 + a^\kappa_2 a^\lambda_2)(n^\kappa_3 m^\lambda_3 - m^\kappa_3 n^\lambda_3) \tag{2.35}$$

As with other orbifolds, it is evident that in order to get *odd* intersection numbers, as required by eq. (1.1), we shall need to make use of exceptional 3-cycles, constructed using the collapsed 2-cycles that arise in the θ^6-twisted sector.

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In the θ^6-twisted sector there are 16 fixed tori T_3^2 at the $\mathbb{Z}_2$ fixed points $f_{\sigma_1,\sigma_2,\sigma_3,\sigma_4}$ on the SO(8) lattice, where

$$f_{\sigma_1,\sigma_2,\sigma_3,\sigma_4} := \frac{1}{2}\sum_{a=1}^{4}\sigma_a\alpha_a \tag{2.36}$$

with $\sigma_a = 0,1$. For ease of reference, we use the same notation as in the $\mathbb{Z}_6'$ case [15], denoting the fixed points by $f_{i,j}$ with the pairs (σ_1,σ_2) and (σ_3,σ_4) given the labels $i,j = 1,4,5,6$ respectively for the values $(0,0),(1,0),(0,1),(1,1)$. Under the action of the point-group the 16 fixed points split into four sets, each set transforming into itself as follows:

$$f_{1,1} \ \text{invariant} \tag{2.37}$$
$$f_{4,4} \to f_{1,6} \to f_{4,5} \to f_{4,4} \tag{2.38}$$
$$f_{4,1} \to f_{6,4} \to f_{6,6} \to f_{4,6} \to f_{5,6} \to f_{5,5} \to f_{4,1} \tag{2.39}$$
$$f_{5,1} \to f_{6,1} \to f_{1,4} \to f_{6,5} \to f_{5,4} \to f_{1,5} \to f_{5,1} \tag{2.40}$$

There are then four non-zero invariant exceptional 3-cycles:

$$\epsilon_1 := (1+\theta+\theta^2+\ldots+\theta^5)f_{4,1}\otimes\pi_5 = (f_{4,1}-f_{6,4}+f_{6,6}-f_{4,6}+f_{5,6}-f_{5,5})\otimes\pi_5 \tag{2.41}$$
$$\tilde{\epsilon}_1 := (1+\theta+\theta^2+\ldots+\theta^5)f_{4,1}\otimes\pi_6 = (f_{4,1}-f_{6,4}+f_{6,6}-f_{4,6}+f_{5,6}-f_{5,5})\otimes\pi_6 \tag{2.42}$$
$$\epsilon_2 := (1+\theta+\theta^2+\ldots+\theta^5)f_{5,1}\otimes\pi_5 = (f_{5,1}-f_{6,1}+f_{1,4}-f_{6,5}+f_{5,4}-f_{1,5})\otimes\pi_5 \tag{2.43}$$
$$\tilde{\epsilon}_2 := (1+\theta+\theta^2+\ldots+\theta^5)f_{5,1}\otimes\pi_6 = (f_{5,1}-f_{6,1}+f_{1,4}-f_{6,5}+f_{5,4}-f_{1,5})\otimes\pi_6 \tag{2.44}$$

which is consistent with the cohomology of the θ^6-twisted sector. The self-intersection number of a ($\mathbb{Z}_2$) collapsed 2-cycle is, as before, given by

$$f_{i,j}\circ f_{k,\ell} = -2\delta_{i,k}\delta_{j,\ell} \tag{2.45}$$

Then,

$$\epsilon_i\circ\tilde{\epsilon}_j = 2\delta_{ij} = -\tilde{\epsilon}_i\circ\epsilon_j \quad i,j=1,2 \tag{2.46}$$

(The corresponding results for the $\mathbb{Z}_{12}$-I case are given in the appendix.) The general exceptional brane Π_κ^{ex} is then given by

$$\Pi_\kappa^{\mathrm{ex}} = \sum_{i=1}^{2} e_i^\kappa(n_3^\kappa\epsilon_i + m_3^\kappa\tilde{\epsilon}_i) \tag{2.47}$$

where the coefficients e_i^κ are determined by the fixed points wrapped by the 2-cycle used to construct $\Pi_\kappa^{\mathrm{bulk}}$, as we shall see in the following section. For two general exceptional branes of this form

$$\Pi_\kappa^{\mathrm{ex}}\circ\Pi_\lambda^{\mathrm{ex}} = 2(e_1^\kappa e_1^\lambda + e_2^\kappa e_2^\lambda)(n_3^\kappa m_3^\lambda - m_3^\kappa n_3^\lambda) \tag{2.48}$$

Exceptional cycles also arise in other twisted sectors. For example, in the θ^4-sector there are 9 fixed tori at the $\mathbb{Z}_3$ fixed points

$$g_{m,p} := \frac{1}{3}[m(\alpha_4-\alpha_1-\alpha_3)+p(\alpha_2-\alpha_3)] \tag{2.49}$$

with $m, p = 0, 1, 2$, and, as above, collapsed 2-cycles at these fixed points may be combined with 1-cycles in T_3^2 to construct further twisted 3-cycles. However, only bulk cycles and exceptional cycles at $\mathbb{Z}_2$ fixed points have a known interpretation in terms of partition functions [25] . In what follows we have therefore only considered the exceptional 3-cycles defined in eqs. (2.41)–(2.44).

3 Factorisable 2-cycles

The general 2-cycle on the SO(8) lattice that appears in eq. (2.21) has the form

$$\Pi_2 = \sum_{a<b} n_{a,b}\pi_{a,b} \tag{3.1}$$

with $a, b = 1, 2, \ldots, 4$ and $n_{a,b}$ six arbitrary integers. Now suppose that Π_2 is the product of two 1-cycles $\sum_a n_a\pi_a$ and $\sum_b m_b\pi_b$, where n_a and m_b are integers. In this case the six integers $n_{a,b}$ are expressible in terms of the eight integers n_a and m_b as

$$n_{a,b} = n_a m_b - m_a n_b \tag{3.2}$$

They then satisfy the constraint

$$n_{1,2}n_{3,4} + n_{1,4}n_{2,3} = n_{1,3}n_{2,4} \tag{3.3}$$

A general set of six wrapping numbers $n_{a,b}$ will generally *not* satisfy this constraint, and even if they do it is not sufficient to ensure that Π_2 is "factorisable" in this way. If it is, it is straightforward to identify the four fixed points $f_{i,j}$ that are wrapped by Π_2. For example, if such a factorisable 2-cycle has $(n_{1,2}, n_{1,3}, n_{1,4}, n_{2,3}, n_{2,4}, n_{3,4}) = (1, 0, 0, 0, 0, 0)$ mod 2, then $(n_3, n_4) = (0, 0)$ mod $2 = (m_3, m_4)$ and either $(n_1, n_2) = (1, 0)$ mod 2 and $(m_1, m_2) = (0, 1)$ or $(1, 1)$ mod 2, or vice versa. Evidently Π_2, like $\pi_{1,2}$, wraps the four fixed points $f_{1,j}, f_{4,j}, f_{5,j}, f_{6,j}$ with $j = 1, 4, 5, 6$ arbitrary. Henceforth we shall only consider such factorisable 2-cycles.

A priori, there are 2^6 cases to consider for the set $(n_{1,2}, n_{1,3}, n_{1,4}, n_{2,3}, n_{2,4}, n_{3,4})$ mod 2. However, the case in which all $n_{i,j}$ are even is of no physical interest, since we require the wrapping numbers to have no common factor. The action of θ splits the remaining 63 cases into sets as follows:

$$63 = 3(1) + 6(2) + 4(3) + 6(6) \tag{3.4}$$

and we only need to keep one representative of each of the 19 sets. In fact, only 9 of these can satisfy the factorisation constraint given in eq. (3.3). They are listed in table 1 together with the associated values of $a_{1,2}$ mod 2; these are defined in eqs. (2.28) and (2.29).

Each of these classes is associated with four sets of four fixed points, as illustrated above. The bulk part Π_κ^{bulk} of a fractional brane κ, where

$$\kappa = \frac{1}{2}\Pi_\kappa^{\text{bulk}} + \frac{1}{2}\Pi_\kappa^{\text{ex}}, \tag{3.5}$$

is determined by the 3-cycle given in eq. (2.21). Supersymmetry requires that it wraps the four fixed points that determimine the exceptional part Π_κ^{ex} as follows. The four fixed points

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$(n_{1,2}, n_{1,3}, n_{1,4}, n_{2,3}, n_{2,4}, n_{3,4})$ mod 2	(a_1, a_2) mod 2
$(0, 1, 1, 0, 0, 1)$	$(0, 0)$
$(0, 0, 0, 1, 0, 0)$	$(1, 0)$
$(1, 1, 1, 0, 0, 0)$	$(0, 1)$
$(0, 0, 0, 0, 0, 1)$	$(0, 0)$
$(0, 1, 1, 0, 0, 0)$	$(0, 0)$
$(1, 0, 0, 0, 0, 0)$	$(0, 1)$
$(0, 1, 0, 0, 0, 0)$	$(1, 1)$
$(0, 0, 1, 0, 0, 0)$	$(1, 1)$
$(1, 1, 0, 0, 0, 0)$	$(1, 0)$

Table 1. Representatives of the 9 potentially factorisable classes of 2-cycles.

contribute with a sign determined by the Wilson lines $t_0^\kappa, t_1^\kappa, t_2^\kappa = \pm 1$. In the example given above, the four fixed points $f_{1,1}, f_{4,1}, f_{5,1}, f_{6,1}$ are associated with the invariant exceptional 3-cycle generated by $t_0^\kappa(f_{1,1} + t_2^\kappa f_{4,1} + t_1^\kappa f_{5,1} + t_1^\kappa t_2^\kappa f_{6,1}) \otimes (n_3^\kappa \pi_5 + m_3^\kappa \pi_6)$, which gives

$$\Pi_\kappa^{\text{ex}} = \sum_{i=1}^{2} (\alpha_i^\kappa \epsilon_i + \tilde{\alpha}_i^\kappa \tilde{\epsilon}_i) \tag{3.6}$$

where

$$\alpha_i^\kappa = n_3^\kappa e_i^\kappa \tag{3.7}$$

$$\tilde{\alpha}_i^\kappa = m_3^\kappa e_i^\kappa \tag{3.8}$$

and in this example

$$e_1^\kappa = t_0^\kappa t_2^\kappa \tag{3.9}$$

$$e_2^\kappa = t_0^\kappa t_1^\kappa (1 - t_2^\kappa) \tag{3.10}$$

The fixed points for all 9 classes, together with the corresponding values for e_1^κ and e_2^κ, are listed in table 2.

4 Supersymmetric bulk 3-cycles

The action of the point group generator given in eq. (2.3) ensures that the closed-string sector is supersymmetric, but to avoid supersymmetry breaking in the open-string sector the D6-branes must wrap special Lagrange cycles. That is to say, we require that

$$X^\kappa := \text{Re}\ \Omega|_{\Pi^\kappa} > 0 \tag{4.1}$$

$$Y^\kappa := \text{Im}\ \Omega|_{\Pi^\kappa} = 0 \tag{4.2}$$

where

$$\Omega := dz_1 \wedge dz_2 \wedge dz_3 \tag{4.3}$$

$n^{\kappa}_{a,b}$ mod 2	$f_{i,j}$	a^{κ}_1 mod 2	a^{κ}_2 mod 2	e^{κ}_1	e^{κ}_2
$(1,0,0,0,0,0)$	$f_{1,1}, f_{4,1}, f_{5,1}, f_{6,1}$	0	1	t_2	$t_1(1-t_2)$
	$f_{1,4}, f_{4,4}, f_{5,4}, f_{6,4}$			$-t_1 t_2$	$1+t_1$
I	$f_{1,5}, f_{4,5}, f_{5,5}, f_{6,5}$			$-t_1$	$-(1+t_1 t_2)$
	$f_{1,6}, f_{4,6}, f_{5,6}, f_{6,6}$			$t_1 t_2 + t_1 - t_2$	0
$(0,1,0,0,0,0)$	$f_{1,1}, f_{4,1}, f_{1,4}, f_{4,4}$	1	1	t_2	t_1
	$f_{5,1}, f_{6,1}, f_{5,4}, f_{6,4}$			$-t_1 t_2$	$1+t_1-t_2$
II	$f_{1,5}, f_{4,5}, f_{1,6}, f_{4,6}$			$-t_1 t_2$	-1
	$f_{5,5}, f_{6,5}, f_{5,6}, f_{6,6}$			$t_1 t_2 + t_1 - 1$	$-t_2$
$(0,0,1,0,0,0)$	$f_{1,1}, f_{4,1}, f_{1,5}, f_{4,5}$	1	1	t_2	$-t_1$
	$f_{5,1}, f_{6,1}, f_{5,5}, f_{6,5}$			$-t_1$	$1-t_2-t_1 t_2$
III	$f_{1,4}, f_{4,4}, f_{1,6}, f_{4,6}$			$-t_1 t_2$	1
	$f_{5,4}, f_{6,4}, f_{5,6}, f_{6,6}$			$t_1 t_2 + t_1 - t_2$	1
$(0,0,0,1,0,0)$	$f_{1,1}, f_{5,1}, f_{1,4}, f_{5,4}$	1	0	0	$t_1+t_2+t_1 t_2$
	$f_{4,1}, f_{6,1}, f_{4,4}, f_{6,4}$			$1-t_1 t_2$	$-t_2$
IV	$f_{1,5}, f_{5,5}, f_{1,6}, f_{5,6}$			$t_2(t_1-1)$	-1
	$f_{4,5}, f_{6,5}, f_{4,6}, f_{6,6}$			$t_1(t_2-1)$	$-t_2$
$(0,0,0,0,0,1)$	$f_{1,1}, f_{1,4}, f_{1,5}, f_{1,6}$	0	0	0	t_2-t_1
	$f_{4,1}, f_{4,4}, f_{4,5}, f_{4,6}$			$1-t_1 t_2$	0
V	$f_{5,1}, f_{5,4}, f_{5,5}, f_{5,6}$			$t_1(t_2-1)$	$1+t_2$
	$f_{6,1}, f_{6,4}, f_{6,5}, f_{6,6}$			$t_2(t_1-1)$	$-(1+t_1)$
$(1,1,0,0,0,0)$	$f_{1,1}, f_{4,1}, f_{5,4}, f_{6,4}$	1	0	$t_2(1-t_1)$	t_1
	$f_{1,5}, f_{4,5}, f_{5,6}, f_{6,6}$			$t_1(1+t_2)$	-1
VI	$f_{5,1}, f_{6,1}, f_{1,4}, f_{4,4}$			0	$1+t_1-t_2$
	$f_{5,5}, f_{6,5}, f_{1,6}, f_{4,6}$			$-(1+t_1 t_2)$	$-t_2$
$(0,1,1,0,0,0)$	$f_{1,1}, f_{4,1}, f_{4,6}, f_{5,4}$	0	0	$t_2(1-t_1)$	0
	$f_{5,1}, f_{6,1}, f_{5,6}, f_{6,6}$			$t_1(1+t_2)$	$1-t_2$
VII	$f_{1,4}, f_{4,4}, f_{1,5}, f_{4,5}$			0	$1-t_1$
	$f_{5,4}, f_{6,4}, f_{5,5}, f_{6,5}$			$-(t_1+t_2)$	$1-t_1 t_2$
$(1,1,1,0,0,0)$	$f_{1,1}, f_{4,1}, f_{5,6}, f_{6,6}$	0	1	$t_1+t_2+t_1 t_2$	0
	$f_{5,1}, f_{6,1}, f_{1,6}, f_{4,6}$			$-t_1 t_2$	$1-t_2$
VIII	$f_{1,4}, f_{4,4}, f_{5,5}, f_{6,5}$			$-t_1$	$1-t_1 t_2$
	$f_{5,4}, f_{6,4}, f_{1,5}, f_{4,5}$			$-t_1 t_2$	t_1-1
$(0,1,1,0,0,1)$	$f_{1,1}, f_{1,6}, f_{4,5}, f_{4,4}$	0	0	0	0
	$f_{5,1}, f_{5,6}, f_{6,5}, f_{6,4}$			$t_2(1-t_1)$	$1-t_1$
IX	$f_{4,1}, f_{4,6}, f_{1,5}, f_{1,4}$			$1-t_2$	$t_1(1-t_2)$
	$f_{6,1}, f_{6,6}, f_{5,5}, f_{5,4}$			t_2-t_1	$t_1 t_2 - 1$

Table 2. The fixed points and coefficients e^{κ}_i of the exceptional cycles associated with the 9 classes of factorisable 2-cycles; an overall factor of t_0 is omitted.

is the holomorphic 3-form. The complex coordinates z_1 and z_2 are those which diagonalise the action of θ as in eq. (2.1) with v_1 and v_2 as given in eq. (2.3). The 2-cycle $\pi_{a,b}$ may be

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parametrised as

$$\pi_{a,b} = \lambda\pi_a + \mu\pi_b \quad \text{with} \quad 0 \leq \lambda, \mu < 1 \tag{4.4}$$

so to evaluate $dz_1 \wedge dz_2$ on $\pi_{a,b}$ we need a representation of the four simple roots α_a in this complex basis:

$$\alpha_a = (w_1^{(a)}, w_2^{(a)}) \tag{4.5}$$

Defining the central root by the general form

$$\alpha_2 = \sqrt{2}(e^{i\phi_1}\cos\theta, e^{i\phi_2}\sin\theta) \quad \text{with} \quad 0 \leq \theta \leq \pi/2 \quad \text{and} \quad 0 \leq \phi_{1,2} < 2\pi \tag{4.6}$$

so that $\alpha_2.\alpha_2 = 2$, it is easy to verify that the remaining roots are given by

$$\alpha_1 = -\sqrt{2}(e^{i\phi_1}\cos\theta(1+\beta), e^{i\phi_2}\sin\theta(1-\beta^{-1})) \tag{4.7}$$

$$\alpha_3 = \sqrt{2}(-e^{i\phi_1}\cos\theta\ \beta^2, e^{i\phi_2}\sin\theta\ \beta^4) \tag{4.8}$$

$$\alpha_4 = \sqrt{2}(e^{i\phi_1}\cos\theta\ \beta^{-1}, -e^{i\phi_2}\sin\theta\ \beta) \tag{4.9}$$

where $\beta := e^{i\pi/6}$ and $\cos 2\theta = -1/\sqrt{3}$. We parametrise the 1-cycle in T_3^2 by

$$z_3 = \nu(n_3^\kappa e_5 + m_3^\kappa e_6) \quad \text{with} \quad 0 \leq \nu < 1 \tag{4.10}$$

where e_5 and e_6 define the SU(2) $\times$ SU(2) lattice. Then, with π_κ as defined in eq. (2.21), we find

$$\Omega|_{\pi_\kappa} = \sum_{a,b} n_{a,b}^\kappa (w_1^{(a)} w_2^{(b)} - w_1^{(b)} w_2^{(a)})(n_3^\kappa + m_3^\kappa \tau_3) e_5\ d\lambda \wedge d\mu \wedge d\nu \tag{4.11}$$

$$= \sqrt{2} e^{i(\phi_1+\phi_2)}\ e_5 [iA_1^\kappa - A_2^\kappa + \tau_3(iA_3^\kappa - A_4^\kappa)]\ d\lambda \wedge d\mu \wedge d\nu \tag{4.12}$$

where $\tau_3 := e_6/e_5$ is the complex structure of T_3^2. The phases of e_5 and e_6 as well as ϕ_1 and ϕ_2 are constrained by the requirement that the orientifold embedding of the world-sheet parity operator also acts as an automorphism of the lattice.

5 The $\mathbb{Z}_{12}$-II orientifold

The embedding $\mathcal{R}$ of the world-sheet parity operator acts on the three complex coordinates z_k as complex conjugation

$$\mathcal{R} z_k = \bar{z}_k \quad (k = 1, 2, 3) \tag{5.1}$$

In particular, since we require that $\mathcal{R}$ acts crystallographically on the root lattice, this requires that

$$\mathcal{R}\alpha_a = \bar{\alpha}_a = \sum_b N_{ab}\alpha_b \tag{5.2}$$

where $N_{ab} \in \mathbb{Z}$. This leads to six independent solutions which are displayed in table 3. For the bulk 3-cycles ρ_p $(p = 1, 2, \ldots, 4)$ defined in eqs. (2.17)–(2.20), only two combinations $\sigma_{1,2}$ of 2-cycles enter the invariant bulk 3-cycles:

$$\sigma_1 := \pi_{1,4} + \pi_{2,3} + \pi_{2,4} \tag{5.3}$$

$$\sigma_2 := -\pi_{1,3} - \pi_{2,3} + \pi_{2,4} + \pi_{3,4} \tag{5.4}$$

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Lattice	$\mathcal{R}\alpha_1$	$\mathcal{R}\alpha_2$	$\mathcal{R}\alpha_3$	$\mathcal{R}\alpha_4$	$e^{-2i\phi_1}$	$e^{-2i\phi_2}$
a	$-(\alpha_2+\alpha_4)$	α_2	$-(\alpha_2+\alpha_3)$	$-(\alpha_1+\alpha_2)$	1	1
b	$-(\alpha_1+\alpha_2+\alpha_3+\alpha_4)$	$\alpha_1+\alpha_2+\alpha_4$	$-(\alpha_1+\alpha_2)$	$\alpha_2+\alpha_3$	$-\beta^3$	$-\beta^3$
c	$-\alpha_1$	$\alpha_1+\alpha_2$	α_4	α_3	$-\beta$	β^{-1}
d	$-(\alpha_2+\alpha_3+\alpha_4)$	α_4	$-(\alpha_1+\alpha_2+\alpha_4)$	α_2	β^{-1}	$-\beta$
e	$-(\alpha_1+\alpha_2+\alpha_3)$	α_3	α_2	$\alpha_1+\alpha_2+\alpha_4$	$-\beta^2$	$-\beta^{-2}$
f	$-(\alpha_1+2\alpha_2+\alpha_3+\alpha_4)$	$\alpha_2+\alpha_3$	$-\alpha_3$	α_4	β^{-2}	β^2

Table 3. The action of $\mathcal{R}$ and the phases ϕ_1 and ϕ_2 for crystallographic action of $\mathcal{R}$ on α_a ($a = 1,2,3,4$); an overall sign of $\epsilon = \pm 1$ is undisplayed.

Lattice	$\mathcal{R}\rho_1$	$\mathcal{R}\rho_2$	$\mathcal{R}\rho_3$	$\mathcal{R}\rho_4$
(a,e,f)A	$-\rho_1$	ρ_2	ρ_3	$-\rho_4$
(a,e,f)B	$-\rho_1$	ρ_2	$-\rho_1+\rho_3$	$\rho_2-\rho_4$
(b,c,d)A	ρ_1	$-\rho_2$	$-\rho_3$	ρ_4
(b,c,d)B	ρ_1	$-\rho_2$	$\rho_1-\rho_3$	$-\rho_2+\rho_4$

Table 4. The action of $\mathcal{R}$ on the invariant 3-cycles.

It is easy to verify that the six different lattices reduce to just two classes when acting on these combinations:

$$(\mathbf{a},\mathbf{e},\mathbf{f}): \qquad \mathcal{R}\sigma_1 = -\sigma_1, \qquad \mathcal{R}\sigma_2 = \sigma_2 \tag{5.5}$$

$$(\mathbf{b},\mathbf{c},\mathbf{d}): \qquad \mathcal{R}\sigma_1 = \sigma_1, \qquad \mathcal{R}\sigma_2 = -\sigma_2 \tag{5.6}$$

Note too that, independently of the overall sign ϵ, the product of the phases given in table 3 restricts the hitherto unknown phase in eq. (4.12)

$$(\mathbf{a},\mathbf{e},\mathbf{f}): \qquad e^{i(\phi_1+\phi_2)} = \pm 1 \tag{5.7}$$

$$(\mathbf{b},\mathbf{c},\mathbf{d}): \qquad e^{i(\phi_1+\phi_2)} = \pm i \tag{5.8}$$

As in the $\mathbb{Z}_6'$ case, the action of $\mathcal{R}$ on the basis 1-cycles $\pi_{5,6}$ in T_3^2 is given by

$$\mathbf{A}: \qquad \mathcal{R}\pi_5 = \pi_5, \qquad \mathcal{R}\pi_6 = -\pi_6 \tag{5.9}$$

$$\mathbf{B}: \qquad \mathcal{R}\pi_5 = \pi_5, \qquad \mathcal{R}\pi_6 = \pi_5 - \pi_6 \tag{5.10}$$

Thus, in both cases e_5 is real and chosen to be positive, and the complex structure of T_3^2 is given by

$$\tau_3 = b + i\mathrm{Im}\ \tau_3 \tag{5.11}$$

with $b = 0$ or $b = 1/2$ respectively for the **A** and **B** lattices. Hence there are just four different classes of behaviour of the bulk 3-cycles under the action of $\mathcal{R}$. The results are displayed in table 4. Choosing the lower signs in eqs. (5.7) and (5.8), the functions X^κ and Y^κ defined in eqs. (4.1) and (4.2) are then given in table 5.

Lattice	X^κ	Y^κ
(a,e,f) A	$A_2^\kappa + \text{Im}\ \tau_3 A_3^\kappa$	$-A_1^\kappa + \text{Im}\ \tau_3 A_4^\kappa$
(a,e,f) B	$A_2^\kappa + \frac{1}{2}A_4^\kappa + \text{Im}\ \tau_3 A_3^\kappa$	$-A_1^\kappa - \frac{1}{2}A_3^\kappa + \text{Im}\ \tau_3 A_4^\kappa$
(b,c,d) A	$A_1^\kappa - \text{Im}\ \tau_3 A_4^\kappa$	$A_2^\kappa + \text{Im}\ \tau_3 A_3^\kappa$
(b,c,d) B	$A_1^\kappa + \frac{1}{2}A_3^\kappa - \text{Im}\ \tau_3 A_4^\kappa$	$A_2^\kappa + \frac{1}{2}A_4^\kappa + \text{Im}\ \tau_3 A_3^\kappa$

Table 5. The functions X^κ and Y^κ. (A global positive factor of $\sqrt{2}e_5$ for each entry is omitted).

Lattice	Invariant	1-cycle(s)
SO(8)**a**	$\mathcal{R}$	$\pi_2,\ \pi_1 - \pi_4$
	$\theta\mathcal{R}$	$\pi_1,\ \pi_3 - \pi_4$
SO(8)**b**	$\mathcal{R}$	$\pi_1 + \pi_2 - \pi_3,\ \pi_2 + \pi_3 + \pi_4$
	$\theta\mathcal{R}$	$\pi_4,\ 2\pi_2 + \pi_3$
SO(8)**c**	$\mathcal{R}$	$\pi_1 + 2\pi_2,\ \pi_3 + \pi_4$
	$\theta\mathcal{R}$	$\pi_1 - \pi_3 + 2\pi_4,\ \pi_2 + \pi_3$
SO(8)**d**	$\mathcal{R}$	$\pi_1 - \pi_3,\ \pi_2 + \pi_4$
	$\theta\mathcal{R}$	$\pi_2,\ \pi_1 - \pi_4$
SO(8)**e**	$\mathcal{R}$	$\pi_1 - \pi_3 + 2\pi_4,\ \pi_2 + \pi_3$
	$\theta\mathcal{R}$	$\pi_1 + \pi_2 - \pi_3,\ \pi_2 + \pi_3 + \pi_4$
SO(8)**f**	$\mathcal{R}$	$\pi_4,\ 2\pi_2 + \pi_3$
	$\theta\mathcal{R}$	$\pi_1 - \pi_3,\ \pi_2 + \pi_4$
T_3^2**A**	$\mathcal{R}$	π_5
	$\theta\mathcal{R}$	π_6
T_3^2**B**	$\mathcal{R}$	π_5
	$\theta\mathcal{R}$	$\pi_5 - \pi_6$

Table 6. $\mathcal{R}$- and $\theta\mathcal{R}$-invariant 1-cycles.

As already noted, the orientifold action leads to the formation of O6-planes. To determine these we must first identify the two $\mathcal{R}$- and two $\theta\mathcal{R}$-invariant 1-cycles on each configuration of the SO(8) lattice. These are displayed in table 6, as is the single $\mathcal{R}$- and single $\theta\mathcal{R}$-invariant 1-cycle on T_3^2. The corresponding $\mathcal{R}$- and $\theta\mathcal{R}$-invariant 3-cycles then generate the bulk 3-cycles displayed in table 7; the overall sign is fixed by the supersymmetry requirement that X^κ is positive. The O6-plane is then the sum of the two orbits, which gives:

$$(\mathbf{a},\mathbf{e},\mathbf{f})\mathbf{A}: \quad \pi_{\text{O6}} = 2(\rho_2 + s\rho_3) \tag{5.12}$$

$$(\mathbf{a},\mathbf{e},\mathbf{f})\mathbf{B}: \quad \pi_{\text{O6}} = 2[\rho_2 + s(-\rho_1 + 2\rho_3)] \tag{5.13}$$

Lattice	Invariant	$(n_{1,2}, n_{1,3}, n_{1,4}, n_{2,3}, n_{2,4}, n_{3,4})(n_3, m_3)$	3-cycle
aA	$\mathcal{R}$	$(1,0,0,0,1,0)(1,0)$	$2\rho_2$
	$\theta\mathcal{R}$	$(0,1,-1,0,0,0)(0,1)$	$2s\rho_3$
aB	$\mathcal{R}$	$(1,0,0,0,1,0)(1,0)$	$2\rho_2$
	$\theta\mathcal{R}$	$(0,1,-1,0,0,0)(1,-1)$	$2s(-\rho_1+2\rho_3)$
bA	$\mathcal{R}$	$(1,1,1,1,1,-1)(1,0)$	$2\rho_1$
	$\theta\mathcal{R}$	$(0,0,0,0,2,1)(0,1)$	$-2s\rho_4$
bB	$\mathcal{R}$	$(1,1,1,1,1,-1)(1,0)$	$2\rho_1$
	$\theta\mathcal{R}$	$(0,0,0,0,2,1)(1,-1)$	$2s(\rho_2-2\rho_4)$
cA	$\mathcal{R}$	$(0,1,1,2,2,0)(1,0)$	$2\rho_1$
	$\theta\mathcal{R}$	$(1,1,0,1,-2,-2)(0,1)$	$-2s\rho_4$
cB	$\mathcal{R}$	$(0,1,1,2,2,0)(1,0)$	$2\rho_1$
	$\theta\mathcal{R}$	$(1,1,0,1,-2,-2)(1,-1)$	$2s(\rho_2-2\rho_4)$
dA	$\mathcal{R}$	$(1,0,1,1,0,-1)(1,0)$	$2\rho_1$
	$\theta\mathcal{R}$	$(1,0,0,0,1,0)(0,1)$	$-2s\rho_4$
dB	$\mathcal{R}$	$(1,0,1,1,0,-1)(1,0)$	$2\rho_1$
	$\theta\mathcal{R}$	$(1,0,0,0,1,0)(1,-1)$	$2s(\rho_2-2\rho_4)$
eA	$\mathcal{R}$	$(1,1,0,1,-2,-2)(1,0)$	$2\rho_2$
	$\theta\mathcal{R}$	$(1,1,1,2,1,-1)(0,1)$	$2s\rho_3$
eB	$\mathcal{R}$	$(1,1,0,1,-2,-2)(1,0)$	$2\rho_2$
	$\theta\mathcal{R}$	$(1,1,1,2,1,-1)(1,-1)$	$2s(-\rho_1+2\rho_3)$
fA	$\mathcal{R}$	$(0,0,0,0,2,1)(1,0)$	$2\rho_2$
	$\theta\mathcal{R}$	$(1,0,1,1,0,-1)(0,1)$	$2s\rho_3$
fB	$\mathcal{R}$	$(0,0,0,0,2,1)(1,0)$	$2\rho_2$
	$\theta\mathcal{R}$	$(1,0,1,1,0,-1)(1,-1)$	$2s(-\rho_1+2\rho_3)$

Table 7. Supersymmetric $\mathcal{R}$- and $\theta\mathcal{R}$-invariant bulk 3-cycles of the $\mathbb{Z}_{12}$-II orientifold; $s = \pm 1$ is the sign of Im τ_3.

$$(\mathbf{b}, \mathbf{c}, \mathbf{d})\mathbf{A}: \quad \pi_{\mathrm{O6}} = 2(\rho_1 - s\rho_4) \tag{5.14}$$

$$(\mathbf{b}, \mathbf{c}, \mathbf{d})\mathbf{B}: \quad \pi_{\mathrm{O6}} = 2[\rho_1 + s(\rho_2 - 2\rho_4)] \tag{5.15}$$

where s is the sign of Im τ_3.

We also need the action of $\mathcal{R}$ on the exceptional cycles ϵ_j and $\tilde{\epsilon}_j$, which in turn depends upon the action of $\mathcal{R}$ on the sixteen $\mathbb{Z}_2$ fixed points $f_{i,j}$ $(i,j = 1,4,5,6)$ in the θ^6-twisted sector. This may be determined using the action of $\mathcal{R}$ on the simple roots α_a of the SO(8) lattice, which is displayed in table 3. On all six lattices there are 4 invariant fixed points and 6 pairs that transform into each other under the action of $\mathcal{R}$. These are displayed in table 8.

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Lattice	Invariants	Pairs
a	$f_{1,1}, f_{5,1}, f_{4,5}, f_{6,5}$	$(f_{4,1}, f_{5,5}), (f_{6,1}, f_{1,5}), (f_{1,4}, f_{5,4}), (f_{1,6}, f_{4,4}), (f_{6,4}, f_{5,6}), (f_{6,6}, f_{4,6})$
b	$f_{1,1}, f_{5,6}, f_{4,5}, f_{6,4}$	$(f_{4,1}, f_{6,6}), (f_{5,1}, f_{6,5}), (f_{6,1}, f_{1,4}), (f_{1,6}, f_{4,4}), (f_{4,6}, f_{5,5}), (f_{1,5}, f_{5,4})$
c	$f_{1,1}, f_{4,1}, f_{1,6}, f_{4,6}$	$(f_{1,4}, f_{1,5}), (f_{4,4}, f_{4,5}), (f_{5,4}, f_{6,5}), (f_{5,5}, f_{6,4}), (f_{5,6}, f_{6,6}), (f_{5,1}, f_{6,1})$
d	$f_{1,1}, f_{4,4}, f_{5,5}, f_{6,6}$	$(f_{1,4}, f_{6,5}), (f_{1,5}, f_{5,1}), (f_{1,6}, f_{4,5}), (f_{4,1}, f_{5,6}), (f_{6,1}, f_{5,4}), (f_{4,6}, f_{6,4})$
e	$f_{1,1}, f_{4,4}, f_{5,4}, f_{6,1}$	$(f_{1,4}, f_{5,1}), (f_{1,5}, f_{6,5}), (f_{1,6}, f_{4,5}), (f_{4,1}, f_{6,4}), (f_{5,6}, f_{4,6}), (f_{5,5}, f_{6,6})$
f	$f_{1,1}, f_{1,4}, f_{1,5}, f_{1,6}$	$(f_{4,1}, f_{4,6}), (f_{5,1}, f_{5,4}), (f_{6,1}, f_{6,5}), (f_{4,5}, f_{4,4}), (f_{5,5}, f_{5,6}), (f_{6,6}, f_{6,4})$

Table 8. Action of $\mathcal{R}$ on the θ^6-sector fixed points $f_{i,j}$ $(i, j = 1, 4, 5, 6)$.

Lattice	$\mathcal{R}\epsilon_1$	$\mathcal{R}\epsilon_2$	$\mathcal{R}\tilde{\epsilon}_1$	$\mathcal{R}\tilde{\epsilon}_2$
(a,e,f)A	ϵ_1	$-\epsilon_2$	$-\tilde{\epsilon}_1$	$\tilde{\epsilon}_2$
(a,e,f)B	ϵ_1	$-\epsilon_2$	$\epsilon_1 - \tilde{\epsilon}_1$	$-\epsilon_2 + \tilde{\epsilon}_2$
(b,c,d)A	$-\epsilon_1$	ϵ_2	$\tilde{\epsilon}_1$	$-\tilde{\epsilon}_2$
(b,c,d)B	$-\epsilon_1$	ϵ_2	$-\epsilon_1 + \tilde{\epsilon}_1$	$\epsilon_2 - \tilde{\epsilon}_2$

Table 9. Action of $\mathcal{R}$ on the invariant exceptional 3-cycles ϵ_j and $\tilde{\epsilon}_j$.

The action of $\mathcal{R}$ on the exceptional cycles then follows from their definition in eqs. (2.41)–(2.44) using eqs. (5.9) and (5.10). It is important to include also the further minus sign as detailed in eqn (4.3) of Blumenhagen et al. [25]; this is most easily seen by considering the action of $\mathcal{R}$ on the Kähler form $J := idz_k \wedge d\bar{z}_k$. The results are displayed in table 9.

6 Fractional branes

As noted earlier, in order to obtain stacks which intersect at an *odd* number of points it is necessary to use fractional branes of the form given in eq. (3.5), where the bulk part $\Pi^{\rm bulk}_\kappa$ is of the form given in eq. (2.23), and determined by the 2-cycle wrapping numbers $n^\kappa_{a,b}$ and the 1-cycle wrapping numbers (n^κ_3, n^κ_3) on T^2_3. The exceptional part $\Pi^{\rm ex}_\kappa$ is of the form given in eq. (2.47), in which, to ensure supersymmetry, the coefficients e^κ_i are determined in the manner described in section 3 by the fixed points $f^\kappa_{i,j}$ on the SO(8) lattice that are wrapped by the bulk 2-cycle. It follows from eqs. (2.35) and (2.48) that

$$a \circ b = \left[\frac{3}{2}(a^a_1 a^b_1 + a^a_2 a^b_2) + \frac{1}{2}(e^a_1 e^b_1 + e^a_2 e^b_2)\right](n^a_3 m^b_3 - m^a_3 n^b_3) \tag{6.1}$$

Similarly, using the results given in tables 4 and 9, on the **(a,e,f)A** lattice we find that

$$a \circ b' = \left[\frac{3}{2}(a^a_1 a^b_1 - a^a_2 a^b_2) + \frac{1}{2}(-e^a_1 e^b_1 + e^a_2 e^b_2)\right](n^a_3 m^b_3 + m^a_3 n^b_3) \tag{6.2}$$

Hence

$$a \circ b - a \circ b' = n^a_3 m^b_3 (3 a^a_2 a^b_2 + e^a_1 e^b_1) - m^a_3 n^b_3 (3 a^a_1 a^b_1 + e^a_2 e^b_2) \tag{6.3}$$

Now, by inspection of table 2 we see that in all cases

$$e^\kappa_1 = a^\kappa_2 \bmod 2 \quad \text{and} \quad e^\kappa_2 = a^\kappa_1 \bmod 2 \tag{6.4}$$

Thus, on the **(a,e,f)A** lattice

$$a \circ b - a \circ b' = 0 \bmod 2 \tag{6.5}$$

Since $a \circ b + a \circ b' = (a \circ b - a \circ b') \bmod 2$, we can*not* satisfy eq. (1.1). It is apparent from tables 4 and 9 that on the **(b,c,d)A** lattice the orientifold image b' differs only by an overall sign from that on the **(a,e,f)A** lattice. Thus the expression on the right-hand side of eq. (6.3) applies to $a \circ b + a \circ b'$ on the **(b,c,d)A** lattice. Hence we can*not* satisfy eq. (1.1) on this lattice either.

Proceeding similarly, on the **(a,e,f)B** lattice we find instead that

$$a \circ b - a \circ b' = -\frac{1}{2} m_3^a m_3^b (a_1^a a_1^b - a_2^a a_2^b + e_1^a e_1^b - e_2^a e_2^b) \bmod 2 \tag{6.6}$$

It follows from eq. (6.4) that

$$X_{a,b} := a_1^a a_1^b - a_2^a a_2^b + e_1^a e_1^b - e_2^a e_2^b = 0 \bmod 2 \tag{6.7}$$

so to ensure that $a \circ b - a \circ b' = 1 \bmod 2$, we require that

$$m_3^a = 1 \bmod 2 = m_3^b \tag{6.8}$$

$$X_{a,b} = 2 \bmod 4 \tag{6.9}$$

For the reasons given above, the same conclusions apply in the case of the **(b,c,d)B** lattice. The general solution of eq. (6.9) is given by

$$(a_1^a a_1^b, a_2^a a_2^b, e_1^a e_1^b, e_2^a e_2^b) = (x, y, y, x+2) \quad \text{or} \quad (x, y, y+2, x) \bmod 4 \tag{6.10}$$

with $x, y = 0, 1, 2, 3 \bmod 4$.

Besides the requirements of supersymmetry and factorisability discussed earlier, there are two further constraints that must be imposed upon the non-abelian stacks a and b. The first derives from the fact that on an orientifold chiral matter in the symmetric $\mathbf{S}_\kappa$ and antisymmetric $\mathbf{A}_\kappa$ representations of the gauge group may arise at the interesections of any stack κ with its orientifold image κ'. The dimensionality of these is given by

$$[\mathbf{S}_\kappa] := (\mathbf{N}_\kappa \times \mathbf{N}_\kappa)_{\text{symm}} = \frac{1}{2} N_\kappa (N_\kappa + 1) \tag{6.11}$$

$$[\mathbf{A}_\kappa] := (\mathbf{N}_\kappa \times \mathbf{N}_\kappa)_{\text{antisymm}} = \frac{1}{2} N_\kappa (N_\kappa - 1) \tag{6.12}$$

Thus, on the U(3) stack a, this gives unobserved symmetric 6-dimensional representations. Likewise, on the U(2) stack b unobserved 3-dimensional chiral representations may arise. Clearly, we must demand the absence of such symmetric representations on both of these stacks. The antisymmetric representation on the a stack is the $\bar{\mathbf{3}}$ representation. In principle such states are acceptable as quark singlets q_L^c states, provided that the hypercharge $Y(q_L^c) = 2y_a$ is right. Evidently, this require that $y_a = 1/6$ or $-1/3$, corresponding respectively to d_L^c and u_L^c states. On the b stack the antisymmetric representation is the singlet representation. Again, such states are acceptable as charged lepton singlets ℓ_L^c, provided that $y_b = 1/2$, or as neutrino singlets ν_L^c, if $y_b = 0$. It follows from the considerations at the end of section 1 that only $(y_a, y_b) = (1/6, 0)$ or $(-1/3, 1/2)$ are consistent with getting the correct weak hypercharge for the quark doublets. The numbers of such chiral representations are given by

$$\#(\mathbf{S}_\kappa) = \frac{1}{2}(\kappa \circ \kappa' - \kappa \circ \pi_{O6}) \tag{6.13}$$

$$\#(\mathbf{A}_\kappa) = \frac{1}{2}(\kappa \circ \kappa' + \kappa \circ \pi_{O6}) \tag{6.14}$$

Since we must demand the absence of the symmetric $\mathbf{S}_a$ and $\mathbf{S}_b$ representations, the numbers of surviving anti-symmetric representations are

$$\#(\mathbf{A}_\kappa) = \kappa \circ \pi_{O6} \quad \kappa = a, b \tag{6.15}$$

So the first additional constraint is that

$$|\#(\mathbf{A}_\kappa)| \leq 3 \quad \kappa = a, b \tag{6.16}$$

since there are only 3 quark singlets and 3 lepton singlets of each flavour in the Standard Model. It follows from eqs. (5.13) and (5.15), using the supersymmetry constraint $Y^\kappa = 0$, with the forms of Y^κ as displayed in table 5, that

$$(\mathbf{a}, \mathbf{e}, \mathbf{f})\mathbf{B} \qquad \#(\mathbf{A}_\kappa) = 6[s(A_3^\kappa + 2A_1^\kappa) - A_4^\kappa] = 6(2|\mathrm{Im}\ \tau_3| - 1)A_4^\kappa \tag{6.17}$$

$$(\mathbf{b}, \mathbf{c}, \mathbf{d})\mathbf{B} \qquad = -6[s(A_4^\kappa + 2A_2^\kappa) + A_3^\kappa] = 6(2|\mathrm{Im}\ \tau_3| - 1)A_3^\kappa \tag{6.18}$$

Since the bulk wrapping numbers A_p^κ are all integers, it is evident from the middle equations that $\#(\mathbf{A}_\kappa) = 0 \bmod 6$. Thus, we can*not* satisfy eq. (6.16) unless $\#(\mathbf{A}_a) = 0 = \#(\mathbf{A}_b)$. On both lattices and both stacks this requires that $A_3^\kappa = A_4^\kappa \bmod 2$. It follows from eq. (6.8) that this in turn requires that

$$a_1^\kappa = a_2^\kappa \bmod 2 \quad \kappa = a, b \tag{6.19}$$

on both lattices. If $|\mathrm{Im}\ \tau_3| \neq 1/2$, then on both stacks and on both lattices $(a_1^\kappa, a_2^\kappa) = (0, 0) \bmod 2$, and all terms on the left-hand side of eq. (6.10) are 0 mod 4 so cannot satisfy eq. (6.9). The alternative is to require that

$$|\mathrm{Im}\ \tau_3| = \frac{1}{2} \tag{6.20}$$

The solutions given in eq. (6.10) are now restricted to the form

$$(a_1^a a_1^b, a_2^a a_2^b, e_1^a e_1^b, e_2^a e_2^b) = (\underline{x, x, x, x+2}) \bmod 4 \tag{6.21}$$

with $x = 0, 1, 2, 3 \bmod 4$; the underlining signifies any permutation of the underlined entries. This can only be satisfied if at most one of $\kappa = a$ or b has $(a_1^\kappa, a_2^\kappa) = (0, 0) \bmod 2$. Furthermore, if, say, $(a_1^a, a_2^a) = (0, 0) \bmod 2$, and $(a_1^b, a_2^b) = (1, 1) \bmod 2$, then

$$(a_1^a a_1^b, a_2^a a_2^b, e_1^a e_1^b, e_2^a e_2^b) = (a_1^a, a_2^a, e_1^a, e_2^a) \bmod 4 \tag{6.22}$$

and eq. (6.21) requires that only an *odd* number of $a^a_1, a^a_2, e^a_1, e^a_2$ can be 2 mod 4. However, in this case it is easy to verify that $a \circ a' \neq 0$, and hence $\#(\mathbf{S}_a) \neq 0$. The conclusion is that only if $(a^\kappa_1, a^\kappa_2) = (1, 1)$ mod 2 for both stacks $\kappa = a$ and b can this constraint be satisfied if we allow only the Standard Model spectrum.

Should we succeed in finding supersymmetric (factorisable) stacks a and b satisfying the constraints detailed above, it is desirable that the the (four-dimensional) SU(3) and SU(2) gauge couplings strengths unify, i.e.

$$\alpha_a = \alpha_b \tag{6.23}$$

although we do not impose this as a constraint. For the gauge group $\mathrm{U}(N_\kappa)$, the four-dimensional fine structure constant α_κ of a stack κ of N_κ D6-branes wrapping a 3-cycle π_κ is given by [26, 27]

$$\frac{1}{\alpha_\kappa} = \frac{m_{\mathbb{P}}}{2\sqrt{2}m_{\text{string}}} \frac{\mathrm{Vol}(\pi_\kappa)}{\sqrt{\mathrm{Vol}(Y)}} \tag{6.24}$$

where $m_{\mathbb{P}}$ is the Planck mass, and $Y = T^6/\mathcal{R} \times \mathbb{Z}_{12}$-II is the compactified space in this case. For fractional branes κ as defined in eq. (3.5)

$$\mathrm{Vol}(\kappa) = \frac{1}{2}\mathrm{Vol}(\Pi^{\text{bulk}}_\kappa) + \frac{1}{2}\mathrm{Vol}(\Pi^{\text{ex}}_\kappa) \simeq \frac{1}{2}\mathrm{Vol}(\Pi^{\text{bulk}}_\kappa) \tag{6.25}$$

since the consistency of the supergravity approximation requires that the contribution of the bulk part is large compared to the contribution from the exceptional part. Then, as shown in [21], for supersymmetric stacks

$$\frac{\alpha_a}{\alpha_b} = \frac{\mathrm{Vol}(\Pi^{\text{bulk}}_b)}{\mathrm{Vol}(\Pi^{\text{bulk}}_a)} \tag{6.26}$$

$$= \frac{X^b}{X^a} \tag{6.27}$$

where X^κ is defined in eq. (4.1) and for the various lattices takes the values displayed in table 5.

7 Computations

We have shown in section 6 that the only way that we might satisfy all of the constraints is if a^κ_1 and a^κ_2 are both odd for both stacks, i.e. if they are of type II or III in table 2; then x in eq. (6.21) is odd. The numerical search produced no solutions satisfying the constraints in which $(a \circ b, a \circ b') = (1, 2)$ or $(2, 1)$. The only solutions that satisfy eq. (1.1) (with $(a \circ b, a \circ b') = (0, 3)$ or $(3, 0)$) and the constraints have the wrapping numbers (n^κ_3, m^κ_3) of T^2_3 equal to $(0, \pm 3)$ for one of the stacks, i.e. the wrapping numbers are not coprime; such solutions are unacceptable. The conclusion is that the $\mathbb{Z}_{12}$-II orientifold cannot yield *just* the spectrum of the supersymmetric Standard Model.

Since there are no solutions with just the supersymmetric Standard Model spectrum, it is of interest to study models that approximate to it. Instead of demanding that $\#(\mathbf{A}_\kappa) = 0$

for both stacks, suppose that we allow just one, a say, to have $|\#(\mathbf{A}_a)| = |a \circ \pi_{O6}| = 6$, the minimal non-zero number. On the **(a,e,f)B** lattice, it then follows from eq. (6.17) that

$$|\text{Im } \tau_3| = \frac{A_4^a + \epsilon}{2A_4^a} \tag{7.1}$$

where $\epsilon = \pm 1$. Further, since $A_3^a - A_4^a = 1 \bmod 2$, it follows that $(a_1, a_2) = (1, 0)$ or $(0, 1) \bmod 2$. Thus a is of type I/VIII or of type IV/VI in table 2. For the other stack, it follows that

$$\#(\mathbf{A}_b) = b \circ \pi_{O6} = \frac{A_4^b}{A_4^a}\epsilon \tag{7.2}$$

So if there are no antisymmetric representations on this stack, we require that

$$A_4^b = 0 = a_2^b \tag{7.3}$$

Hence $A_2^b = 0$ too. Also, since $2A_1^b + A_3^b = 0 = (2n_3^b + m_3^b)a_1^b$, it follows that $A_1^b = 0 = A_3^b$. This means that $X^b = 0$, which gives an infinite value for the gauge coupling strength α_b. We are therefore compelled to have antisymmetric matter on *both* stacks. If we also require the minimal amount on b too, then the stack b must be of the same type as a with

$$|A_4^b| = |A_4^a| \tag{7.4}$$

Similarly, on the **(b,c,d)B** lattice, if $\#(\mathbf{A}_a) = 6\epsilon$, then

$$s(2A_4^a + A_2^a) + A_3^a = (1 - 2|\text{Im } \tau_3|)A_3^a = -\epsilon \tag{7.5}$$

Hence

$$|\text{Im } \tau_3| = \frac{A_3^a + \epsilon}{2A_3^a} \tag{7.6}$$

Again, if we demand that $\#(\mathbf{A}_b) = 0$, then $A_p^b = 0$ $(p = 1, 2, 3, 4)$, and α_b is infinite. Likewise, if instead we require the minimal amount on b too, then it must be of the same type as a with

$$|A_3^b| = |A_3^a| \tag{7.7}$$

Solutions for a and b satisfying even these weaker constraints are fairly limited. For example, on the **(a,e,f)B** lattice, when both a and b are of type I, we find solutions of the required type with

$$(a_1^a, a_2^a) = (2x^a, y^a), \qquad (n_3^a, m_3^a) = (0, y^a), \qquad (e_1^a, e_2^a) = (z^a, 2t^a) \tag{7.8}$$

$$(a_1^b, a_2^b) = (2x^b, y^b), \qquad (n_3^b, m_3^b) = (y^b, -y^b), \qquad (e_1^a, e_2^a) = (z^b, 2t^b) \tag{7.9}$$

where $x^\kappa, y^\kappa, z^\kappa, t^\kappa = \pm 1$. Then

$$A_p^a = (0, 0, 2x^a y^a, 1), \qquad (\alpha_i^a, \tilde{\alpha}_i^a) = (0, 0, y^a z^a, 2y^a t^a) \tag{7.10}$$

$$A_p^b = (2x^b y^b, 1, -2x^b y^b, -1), \qquad (\alpha_i^b, \tilde{\alpha}_i^b) = (y^b z^b, 2y^b t^b, -y^b z^b, -2y^b t^b) \tag{7.11}$$

and

$$x^a y^a = \text{Im } \tau_3 = -x^b y^b \tag{7.12}$$

$$X^a = \frac{5}{2} = X^b \tag{7.13}$$

Then from eq. (6.17), it follows that

$$\#(\mathbf{A}_a) = 6 = -\#(\mathbf{A}_b) \tag{7.14}$$

and the required intersection numbers $(a \circ b, a \circ b') = (3, 0)$ arise provided that

$$x^a x^b = -y^a y^b = z^a z^b = -t^a t^b \tag{7.15}$$

Similarly, on the **(a,e,f)B** lattice, when both a and b are of type IV, there are solutions of the form

$$A_p^a = (x^a y^a, 2, -x^a y^a, -2), \qquad (\alpha_i^a, \tilde{\alpha}_i^a) = (2y^a z^a, y^a t^a, -2y^a z^a, -y^a t^a) \tag{7.16}$$
$$A_p^b = (0, 0, -x^b y^b, 2), \qquad (\alpha_i^b, \tilde{\alpha}_i^b) = (0, 0, -2y^b z^b, -y^b t^b) \tag{7.17}$$

when

$$\frac{x^a y^a}{4} = -\mathrm{Im}\tau_3 = \frac{x^b y^b}{4} \tag{7.18}$$
$$X^a = \frac{5}{4} = X^b \tag{7.19}$$

These too satisfy eqs. (7.14) and have the required intersection numbers when

$$x^a x^b = y^a y^b = z^a z^b = -t^a t^b \tag{7.20}$$

Without loss of generality, we identify a as the SU(3) stack, and b as the SU(2) stack. To avoid further non-abelian gauge symmetries, all remaining stacks λ must consist of a single $D6$-brane with $N_\lambda = 1$. Given the fairly limited number of solutions for a and b, the intersection numbers $(a \circ \lambda, a \circ \lambda')$ and $(b \circ \lambda, b \circ \lambda')$ with an arbitrary (supersymmetric) stack λ are also limited in number and highly correlated. As already noted, unavoidably we have $6q_L^c$ states arising in the antisymmetric $\bar{\mathbf{3}}$ representation of SU(3) on the stack a; if $y_a = 1/6$ these are $6d_L^c$, whereas if $y_a = -1/3$ they are $6u_L^c$. Thus in these models the minimal quark-singlet spectrum arising from the intersections of a with other stacks λ, and their orientifold images λ', is $3\bar{d}_L^c + 3u_L^c$ when $y_a = 1/6$, and $3\bar{u}_L^c + 3d_L^c$ when $y_a = -1/3$. In both cases we must therefore impose the constraint $|a \circ \lambda| + |a \circ \lambda'| \leq 6$ on any one of the other stacks. The intersections of the b with other stacks λ yield doublets that must be identified either as lepton L and Higgs H_d doublets, if $Y = -1/2$, or H_u doublets if $Y = 1/2$. The supersymmetric Standard Model has $3L + H_u + H_d$, so we should also impose the constraint $|b \circ \lambda| + |b \circ \lambda'| \leq 5$ on any single stack. With a and b both of the same type, I or IV, and on both the **(a,e,f)B** and **(b,c,d)B** lattices, the allowed intersection numbers, subject to the constraints described above, are displayed in table 10.

In both cases, since the only negative intersection numbers for $a \circ \lambda$ are invariably accompanied by negative intersection numbers $a \circ \lambda'$, and vice versa, it is clear that we can never get *just* the required $3(\bar{\mathbf{3}}) + 3(\mathbf{3})$ quark-singlet states. When a and b are both of type IV, this conclusion is true even if we do not impose the latter constraint

$(a \circ \lambda, a \circ \lambda')$	$(b \circ \lambda, b \circ \lambda')$
$(-1,-1)$	$(2,2)$
$(-2,-2)$	$(1,1)$
$(0,6)$	$(-3,0)$
$(6,0)$	$(0,-3)$

Table 10. Correlations between intersection numbers of the SU(3) stack a and those of the SU(2) stack b when $(a \circ b, a \circ b') = (3,0)$.

$|b \circ \lambda| + |b \circ \lambda'| \leq 5$. However, if they are both of type I, then it can be satisfied, but only at the expense of having at least 12 doublets at the intersections of b with λ and λ'. The conclusion is that, at least within the range of parameters searched, we can*not* get the quark-singlet spectrum even of this Standard-like model.

8 Discussion

We have investigated whether there is scope to construct supersymmetric Standard Models in type IIA intersecting-brane theories compactified on an orientifold with a $\mathbb{Z}_{12}$ point group. We focussed on the $\mathbb{Z}_{12}$-II case because, as discussed in section 2, the $\mathbb{Z}_{12}$-I case does not have enough independent 3-cycles to make a viable model likely. The SO(8) $\times$ SU(2) $\times$ SU(2) lattice has been used; the $F_4 \times$ SU(2) $\times$ SU(2) case is equivalent. A bulk 3-cycle then consists of a 2-cycle on the SO(8) lattice times a 1-cycle on the SU(2) $\times$ SU(2) torus T^2_3, and we have restricted attention to the case when the 2-cycle is factorisable in the sense discussed in section 3. It is possible to find models with the correct supersymmetric Standard Model quark-doublet content. All examples have $(a \circ b, a \circ b') = (3,0)$ or $(0,3)$ and possess 6 copies of either d^c_L or u^c_L quark singlets, depending on the values of y_a. Thus, some vector-like matter is inevitable. All examples have non-abelian gauge coupling constant unification in the sense that $\alpha_a = \alpha_b$ at the string scale, but we have not found it possible to obtain the minimal quark-singlet structure described in the previous section..

A The $\mathbb{Z}_{12}$-I orientifold

The six independent invariant exceptional 3-cycles on the $\mathbb{Z}_{12}$-I orbifold may be chosen as follows:

$$\epsilon_1 := (1+\theta+\theta^2+\ldots+\theta^5)f_{4,4} \otimes \pi_5 = 2[(f_{4,4}-f_{1,6}) \otimes \pi_5 + (f_{1,6}-f_{4,5}) \otimes \pi_6] \quad \text{(A.1)}$$

$$\tilde{\epsilon}_1 := (1+\theta+\theta^2+\ldots+\theta^5)f_{4,4} \otimes \pi_6 = 2[(f_{4,5}-f_{1,6}) \otimes \pi_5 + (f_{4,4}-f_{4,5}) \otimes \pi_6] \quad \text{(A.2)}$$

$$\begin{aligned}\epsilon_2 := (1+\theta+\theta^2+\ldots+\theta^5)f_{4,1} \otimes \pi_5 &= (f_{4,1}-f_{6,4}+f_{4,6}-f_{5,6}) \otimes \pi_5 + \\ &\quad +(f_{6,4}-f_{6,6}+f_{5,6}-f_{5,5}) \otimes \pi_6 \qquad \text{(A.3)}\\ \tilde{\epsilon}_2 := (1+\theta+\theta^2+\ldots+\theta^5)f_{4,1} \otimes \pi_6 &= (-f_{6,4}+f_{6,6}-f_{5,6}+f_{5,5}) \otimes \pi_5 + \\ &\quad +(f_{4,1}-f_{6,6}+f_{4,6}-f_{5,5}) \otimes \pi_6 \qquad \text{(A.4)}\end{aligned}$$

Lattice	$\mathcal{R}\alpha_1$	$\mathcal{R}\alpha_2$	$\mathcal{R}\alpha_3$	$\mathcal{R}\alpha_4$	$e^{-2i\phi_1}$	$e^{-2i\phi_2}$
a	$-(\alpha_2+\alpha_4)$	α_2	$-(\alpha_2+\alpha_3)$	$-(\alpha_1+\alpha_2)$	1	1
b	$\alpha_1+\alpha_2+\alpha_3$	$-\alpha_3$	$-\alpha_2$	$-(\alpha_1+\alpha_2+\alpha_4)$	β^2	β^2
c	$-(\alpha_1+2\alpha_2+\alpha_3+\alpha_4)$	$\alpha_2+\alpha_3$	$-\alpha_3$	α_4	β^{-2}	β^{-2}
d	α_1	$-(\alpha_1+\alpha_2)$	$-\alpha_4$	$-\alpha_3$	β	$-\beta$
e	$-(\alpha_2+\alpha_3+\alpha_4)$	α_4	$-(\alpha_1+\alpha_2+\alpha_4)$	α_2	β^{-1}	$-\beta^{-1}$
f	$\alpha_1+\alpha_2+\alpha_3+\alpha_4$	$-(\alpha_1+\alpha_2+\alpha_4)$	$\alpha_1+\alpha_2$	$-(\alpha_2+\alpha_3)$	i	$-i$

Table 11. The phases ϕ_1 and ϕ_2 for crystallographic action of $\mathcal{R}$ on α_i $(i=1,2,3,4)$; an overall sign of $\epsilon=\pm1$ is undisplayed.

$$\begin{aligned}\epsilon_3 := (1+\theta+\theta^2+\ldots+\theta^5)f_{5,1}\otimes\pi_5 &= (f_{5,1}-f_{6,1}+f_{6,5}-f_{5,4})\otimes\pi_5+\\ &\quad+(f_{6,1}-f_{1,4}+f_{5,4}-f_{1,5})\otimes\pi_6 \end{aligned}\tag{A.5}$$

$$\begin{aligned}\tilde{\epsilon}_3 := (1+\theta+\theta^2+\ldots+\theta^5)f_{5,1}\otimes\pi_6 &= (-f_{6,1}+f_{1,4}-f_{5,4}+f_{1,5})\otimes\pi_5+\\ &\quad+(f_{5,1}-f_{1,4}+f_{6,5}-f_{1,5})\otimes\pi_6 \end{aligned}\tag{A.6}$$

Then

$$\epsilon_j\circ\epsilon_k = 0 = \tilde{\epsilon}_j\circ\tilde{\epsilon}_k \qquad j,k=1,2,3 \tag{A.7}$$

$$\epsilon_j\circ\tilde{\epsilon}_k = -12E_j\delta_{j,k} \qquad \text{(no summation)} \tag{A.8}$$

where

$$E_1=2, \quad E_2=1=E_3 \tag{A.9}$$

assuming, as in eq. (2.45), that the self-intersection of a fixed point $f_{i,j}$ is -2.

In this case the action of the point group generator θ is given in eq. (2.2). Then, with the central root α_2 of the SO(8) lattice parametrised as in eq. (4.6), the remaining roots are given by

$$\alpha_1 = -\sqrt{2}(e^{i\phi_1}\cos\theta(1+\beta), e^{i\phi_2}\sin\theta(1-\beta^{-1})) \tag{A.10}$$

$$\alpha_3 = -\sqrt{2}\beta^2(e^{i\phi_1}\cos\theta, e^{i\phi_2}\sin\theta) \tag{A.11}$$

$$\alpha_4 = \sqrt{2}\beta^{-1}(e^{i\phi_1}\cos\theta, -e^{i\phi_2}\sin\theta) \tag{A.12}$$

With $\mathcal{R}$ acting as complex conjugation, as in eq. (5.1), it acts crystallographically on this lattice in the 6 orientations displayed in table 11. $\mathcal{R}$ acts crystallographically on the basis 1-cycles $\pi_{5,6}$ of the SU(3) lattice in T^2_3 in 2 orientations:

$$\mathbf{A}: \qquad \mathcal{R}\pi_5=\pi_5, \qquad \mathcal{R}\pi_6=\pi_5-\pi_6 \tag{A.13}$$

$$\mathbf{B}: \qquad \mathcal{R}\pi_5=\pi_6, \qquad \mathcal{R}\pi_6=\pi_5 \tag{A.14}$$

Then the action of $\mathcal{R}$ on the invariant bulk 3-cycles defined in eqs. (2.15) and (2.16) is given in table 12. In this case, instead of eq. (4.10), we parametrise the 1-cycle on T^2_3 by

$$dz_3 = e_5(n_3^\kappa+m_3^\kappa\beta^2)d\nu \tag{A.15}$$

Lattice	$\mathcal{R}\rho_1$	$\mathcal{R}\rho_2$
(a,f)A	$\rho_1+\rho_2$	$-\rho_2$
(a,f)B	ρ_1	$-(\rho_1+\rho_2)$
(b,e)A	$-\rho_2$	$-\rho_1$
(b,e)B	$-(\rho_1+\rho_2)$	ρ_2
(c,d)A	$-\rho_1$	$\rho_1+\rho_2$
(c,d)B	ρ_2	ρ_1

Table 12. The action of $\mathcal{R}$ on the invariant 3-cycles.

Lattice	X^κ	Y^κ
(a,f)A	$\sqrt{3}A_1^\kappa$	$A_1^\kappa - 2A_2^\kappa$
(a,f)B	$2A_1^\kappa - A_2^\kappa$	$-\sqrt{3}A_2^\kappa$
(b,e)A	$\sqrt{3}(A_1^\kappa - A_2^\kappa)$	$-(A_1^\kappa + A_2^\kappa)$
(b,e)B	$A_1^\kappa - 2A_2^\kappa$	$-\sqrt{3}A_1^\kappa$
(c,d)A	$\sqrt{3}A_2^\kappa$	$2A_1^\kappa - A_2^\kappa$
(c,d)B	$A_1^\kappa + A_2^\kappa$	$\sqrt{3}(A_1^\kappa - A_2^\kappa)$

Table 13. The functions X^κ and Y^κ. (A global positive factor of $R_5 \sin 2\theta_2$ for each entry is omitted).

which gives

$$\begin{aligned} \Omega|_{\Pi^\kappa} &= -2\sin 2\theta_2 e_5 e^{i(\phi_1+\phi_2)}[(A_1^\kappa - A_2^\kappa)\beta + A_2^\kappa\beta^{-1}]d\lambda\wedge d\mu\wedge d\nu && \text{(A.16)} \\ &:= (X^\kappa + iY^\kappa)d\lambda\wedge d\mu\wedge d\nu && \text{(A.17)} \end{aligned}$$

where now the bulk wrapping numbers are given by

$$\begin{aligned} A_1^\kappa &:= a_1^\kappa n_3^\kappa + a_2^\kappa(n_3^\kappa + m_3^\kappa) && \text{(A.18)} \\ A_2^\kappa &:= -a_1^\kappa m_3^\kappa + a_2^\kappa n_3^\kappa && \text{(A.19)} \end{aligned}$$

with

$$\begin{aligned} a_1^\kappa &:= n_{1,2}^\kappa - n_{1,3}^\kappa - n_{3,4}^\kappa && \text{(A.20)} \\ a_2^\kappa &:= n_{1,3}^\kappa - n_{1,4}^\kappa + n_{2,4}^\kappa && \text{(A.21)} \end{aligned}$$

The bulk brane is now given by

$$\Pi^\kappa = A_1^\kappa\rho_1 + A_2^\kappa\rho_2 \qquad \text{(A.22)}$$

The functions X^κ and Y^κ are as displayed in table 13. Evidently, as claimed in section 2, up to an overall scale, all supersymmetric stacks have the same ($\mathcal{R}$-invariant) bulk part.

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