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A fresh look at factorization breaking in diffractive photoproduction of dijets at HERA at next-to-leading order QCD

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Abstract We calculate the cross section of diffractive dijet photoproduction in ep scattering at next-to-leading order (NLO) of perturbative QCD (pQCD), which we supplement by a model of factorization breaking for the resolved-photon contribution. In this model, the suppression depends on the flavor and momentum fraction of the partons in the photon. We show that within experimental and theoretical uncertainties, the resulting approach provides a good description of the available HERA data in most of the bins. Hence, taken together with the observation that NLO pQCD explains well the data on diffractive photoproduction of open charm in ep scattering, our model of factorization breaking presents a viable alternative to the scheme based on the global suppression factor.

1 Introduction

One of the highlights of the physics results obtained at HERA was the measurement of inclusive diffraction $\gamma^* p \rightarrow Xp$ in lepton–proton deep-inelastic scattering (DIS). Combined analyses of the H1 and ZEUS experiments have been published in Ref. [1]. Contrary to the expectations that the probability of large rapidity gap events in DIS should be very small [2], it was found that diffractive events in DIS constitute approximately 10–15 % of the total cross section over a wide range of Q^2. Furthermore, the QCD collinear factorization theorem for hard inclusive diffraction [3] allowed one to treat diffractive DIS on the same footing as inclusive DIS. One can thus first introduce universal diffractive parton distribution functions (dPDFs) and then determine them by fitting to the measured diffractive structure function [4–6]. The universality of the resulting dPDFs is confirmed by

good agreement between the perturbative QCD (pQCD) calculations and the data on diffractive production of dijets [7] and open charm [8] in DIS.

At the same time, in diffractive photoproduction of dijets in ep scattering, based on the well-known factorization breaking in diffractive dijet production in $\bar{p}p$ collisions at the Tevatron [9–11], collinear factorization is generally not expected to hold [12–14]. However, the pattern of this factorization breaking remains an open question [15]; for recent reviews see, e.g., Refs. [16–18]. To summarize, while the recent H1 [19,20] and ZEUS [21] data on diffractive dijet photoproduction are largely consistent with each other after the renormalization of the ZEUS data, the next-to-leading order (NLO) perturbative QCD calculations overestimate the data by approximately 40–50 %. The theory and the data can be made consistent by introducing either a global suppression factor of 0.5 or a suppression factor of approximately 0.4 only for the resolved-photon contribution. The most recent H1 measurement of diffractive dijet photoproduction with a leading proton [22] is also consistent with the observation that NLO pQCD globally overestimates the data by the factor of 0.5.

Factorization breaking in diffractive dijet photoproduction is a result of soft inelastic photon interactions with the proton, which populate and thus partially destroy the final-state rapidity gap. This effect is usually described in the literature by a rapidity gap survival factor $S^2 \leq 1$. Since the magnitude of S^2 decreases with an increase of the interaction strength between the probe and the target, the pattern of the factorization breaking can be related to various components of the photon [17]. In the laboratory reference frame, the high-energy photon interacts with hadronic targets by fluctuating into various configurations (components) interacting with the target with different cross sections. These fluctuations contain both weakly interacting (the so-called point-like) components and the components interacting with large cross sections, which are of the order of

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the vector meson–proton cross sections. This general space-time picture of photon–hadron interactions at high energies is usually realized in the framework of such approaches as the vector meson dominance (VMD) model and its generalizations [23,24] or the color dipole model [25,26]. It is also used in the language of collinear factorization, where the photon structure function and parton distribution functions (PDFs) are given by a sum of the resolved-photon contribution corresponding to the VMD part of the photon wave function and the point-like (inhomogeneous) term originating from the $\gamma \rightarrow q\bar{q}$ splitting; see, e.g., Ref. [27]. Note that the direct-photon contribution to diffractive dijet photoproduction corresponds to the configurations interacting with very small cross sections of the order of $1/E_T^2$ (E_T is the transverse jet energy), which preserves factorization.

Let us recall that the hadron (VMD) part of the photon PDFs contributes only for small x_γ, whereas the point-like term gives the dominant contribution for large x_γ. Here, x_γ is the light-cone momentum fraction of a parton in the photon. Based on the arguments presented above, it is then natural to expect $S^2 = 1$ for the direct-photon contribution localized near $x_\gamma = 1$, $S^2 \approx 0.34$ for the hadron-like component of the photon at small x_γ, and $S^2 \approx 0.53$–0.75 for the gluon and quark contributions at large x_γ corresponding to small, but non-negligible factorization breaking due to the point-like component of the resolved photon [17,18]. Note that in pQCD the separation of the direct and the resolved-photon contributions is unambiguous only at leading order. At NLO, it becomes a matter of convention depending on the factorization scheme and the factorization scale [28–30].

Another important observation relevant to the possible pattern of factorization breaking is that the HERA data on diffractive photoproduction of open charm [31] agree with the NLO pQCD calculations [32,33], and hence no factorization breaking is required. This calls into question the assumption of a global suppression factor modeling factorization breaking and indicates that S^2 for the resolved-photon contribution may depend on the parton flavor. In particular, $S^2 \approx 1$ for the charm quark distribution in the photon, which agrees with the observation that in the VMD model the J/ψ-proton cross section is of the order of a few mbarn, i.e. much smaller than the ρ–nucleon cross section [23,24].

In the present work we revisit the issue of factorization breaking in diffractive dijet photoproduction in ep scattering and perform NLO pQCD calculations of the corresponding cross sections, which we combine with a new flavor-dependent and momentum fraction-dependent scheme of factorization breaking for the resolved-photon contribution. We demonstrate that the results of our calculations provide a good description of the H1 [19,20] and

ZEUS [21] data on diffractive dijet photoproduction in ep scattering at HERA, while simultaneously, by construction, not conflicting with the good pQCD description of diffractive photoproduction of open charm in ep scattering. Thus, next-to-leading order perturbative QCD coupled with the physically motivated assumption as regards the rapidity gap survival probability for the resolved-photon contribution and the effect of hadronization corrections provide a good description of all available HERA data on diffractive dijet photoproduction. This result reinforces the conclusion of Ref. [17].

2 New scenario for factorization breaking in diffractive photoproduction of dijets in ep

We performed next-to-leading order (NLO) pQCD calculations [18] of the cross sections of diffractive photoproduction of dijets in ep scattering $ep \rightarrow e + 2\text{jets} + X' + Y$ using the kinematic conditions and cuts of the H1 [19,20] and ZEUS [21] measurements of this process. This process is illustrated in Fig. 1. As is well known, jet production with real photons involves direct interactions of the photon with quarks or gluons from the proton (or in our case from the pomeron) as well as resolved-photon contributions, leading to parton–parton interactions and an additional remnant jet coming from the photon. For the direct interactions, factorization is expected to be valid as in the case of DIS, whereas we expect it to fail for the resolved process as in hadron-hadron scattering. For this part of photoproduction one would therefore naively expect a similar suppression factor due to rescattering effects of the hadronic fluctuations of the photon. The expression for the cross section reads

$$
\begin{aligned}
&\mathrm{d}\sigma(ep \rightarrow e + 2\text{jets} + X' + Y) \\
&= \sum_{i,j} \int \mathrm{d}t \int \mathrm{d}x_{I\!P} \int \mathrm{d}z_{I\!P} \int \mathrm{d}y \int \mathrm{d}x_\gamma \\
&\quad \times S_i^2(x_\gamma) f_{\gamma/e}(y) f_{i/\gamma}(x_\gamma, \mu^2) f_{j/p}^{D(4)} \\
&\quad \times (x_{I\!P}, z_{I\!P}, t, \mu^2) \mathrm{d}\hat{\sigma}_{ij \rightarrow \text{jets}}^{(n)},
\end{aligned}
\tag{1}
$$

where X' denotes the pomeron (and possibly photon) remnant jet(s); Y denotes either a proton or a low-mass proton excitation; $S_i^2(x_\gamma)$ is the factor modeling factorization breaking; $f_{\gamma/e}(y)$ is the photon flux of the electron depending on the photon light-cone momentum fraction y; $f_{i/\gamma}(x_\gamma, \mu^2)$ is the PDF of parton i in the photon depending on the momentum fraction x_γ and the factorization scale μ; $f_{j/p}^{D(4)}(x_{I\!P}, z_{I\!P}, t, \mu^2)$ is the diffractive PDF of the proton, which depends on $x_{I\!P}$ (the momentum fraction carried by the diffractive exchange or "Pomeron") and $z_{I\!P}$ (the momen-

Fig. 1 Diffractive production of dijets with invariant mass M_{12} in direct (*left*) and resolved (*right*) photon–pomeron collisions, leading to the production of one or two additional remnant jets
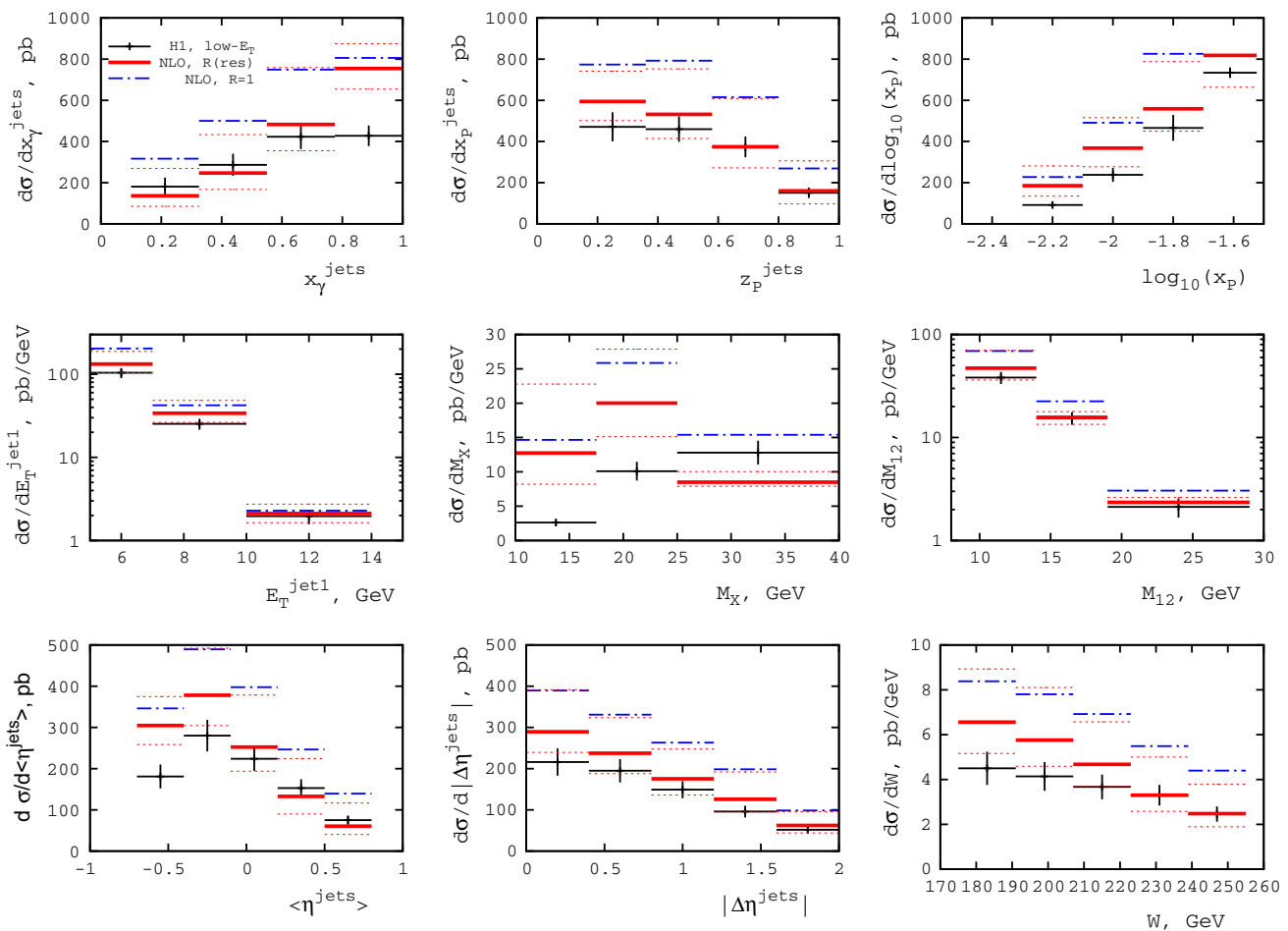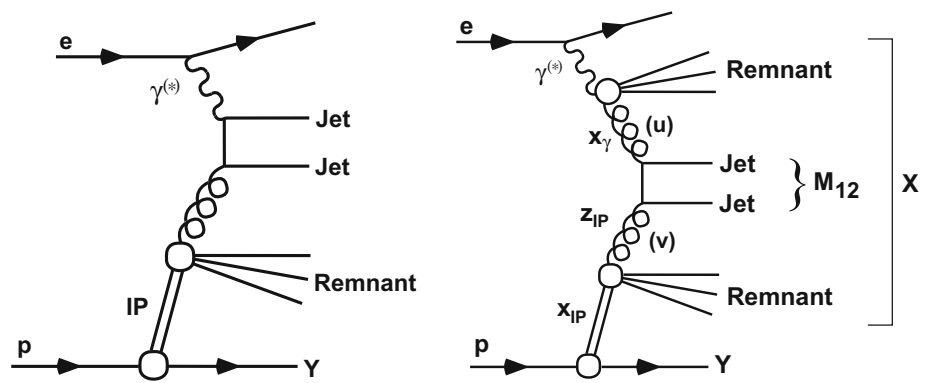

Fig. 2 Cross section of diffractive dijet photoproduction in *ep* scattering: comparison of the NLO pQCD predictions combined with the model of factorization breaking of Eq. (2) (*red solid lines*) to the H1 data with the low-E_T^{jet} cut [19]; the theoretical uncertainty due to the variation of the normalization and factorization scales is shown by the *red dotted lines*. Also, the NLO pQCD results without the effect of factorization breaking are given by the *blue dot-dashed lines* labeled "NLO, $R = 1$". Note that the pQCD predictions include the hadronization corrections

tum fraction of parton j with respect to the "Pomeron" momentum), the invariant momentum transfer squared t, and μ^2; and $d\hat{\sigma}_{ab \to jets}^{(n)}$ is the elementary pQCD cross section for the production of an n-parton final state in the inter-

action of partons i and j. The sum over i involves both quarks and gluons (resolved-photon contribution) and the photon (direct-photon contribution). For input in Eq. (1), we used the GRV photon PDFs transformed to the $\overline{\text{MS}}$
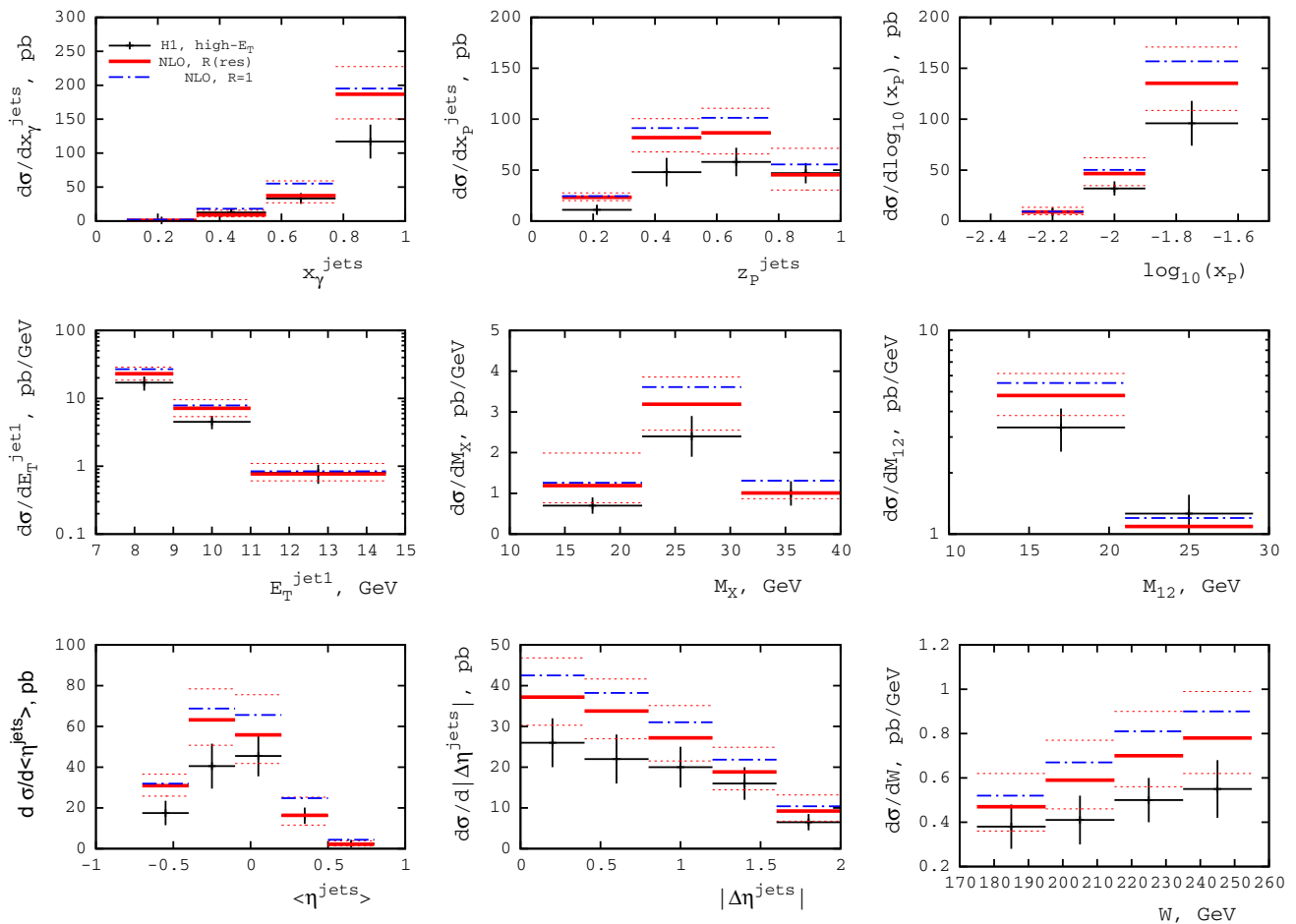
Fig. 3 The same as in Fig. 2, but for the H1 data with the high-E_T^{jet} cut [20]

scheme [27] and the 2006 H1 proton diffractive PDFs (fit B) [5].

We explained in the Introduction that the space-time picture of high-energy photon–proton interactions suggests that in diffractive dijet photoproduction on the proton, QCD diffractive factorization holds for the direct-photon contribution and is broken for the resolved-photon contribution. Moreover, for the latter contribution, the factorization breaking is strongest at small x_γ, small, but non-negligible, for large x_γ, and depends on the parton flavor. In the framework of collinear QCD factorization, the decrease of factorization-breaking effects in the resolved-photon contribution with an increase of x_γ can be explained by the observation that based on the factorization of the collinear singularity, there should be a smooth transition from the resolved-photon contribution at large x_γ to the direct-photon contribution.

Therefore, we model the effect of factorization breaking by introducing the following suppression factor of $S_i^2(x_\gamma)$ for the resolved-photon contribution (i.e. for the photon PDFs) in Eq. (1):

$$S_i^2(x_\gamma) \rightarrow \begin{cases} 1, & i = c, \\ A_q\, x_\gamma + 0.34, & i = u, d, s, \\ A_g\, x_\gamma + 0.34, & i = g, \end{cases} \quad (2)$$

where i is the parton flavor; $A_q = 0.37$–0.41 and $A_g = 0.19$–0.24. The given ranges of values take into account the possible effective dependence of $S_i^2(x_\gamma)$ on the hard resolution scale, where the first and the second values correspond to $E_T^{jet1} = 5$ and 7.5 GeV, respectively. Thus, the factor of $S_i^2(x_\gamma)$ in Eq. (2) represents a linear interpolation between the domain of small x_γ dominated by the hadronic contribution to photon PDFs, where $S_i^2(x_\gamma) = 0.34$, and the regime of large-x_γ dominated by the point-like contribution to photon PDFs, where $S_q^2(x_\gamma) = 0.71$–0.75 for quarks and $S_g^2(x_\gamma) = 0.53$–0.58 for gluons; see Ref. [17]. Note that the model of Eq. (2) assumes no factorization breaking in the charm quark channel according to the observation that NLO pQCD describes well diffractive photoproduction of open charm in ep scattering; see Sect. 1.

The comparison of the results of our calculations to the H1 and ZEUS data is shown in Figs. 2, 3 and 4. The kinematic
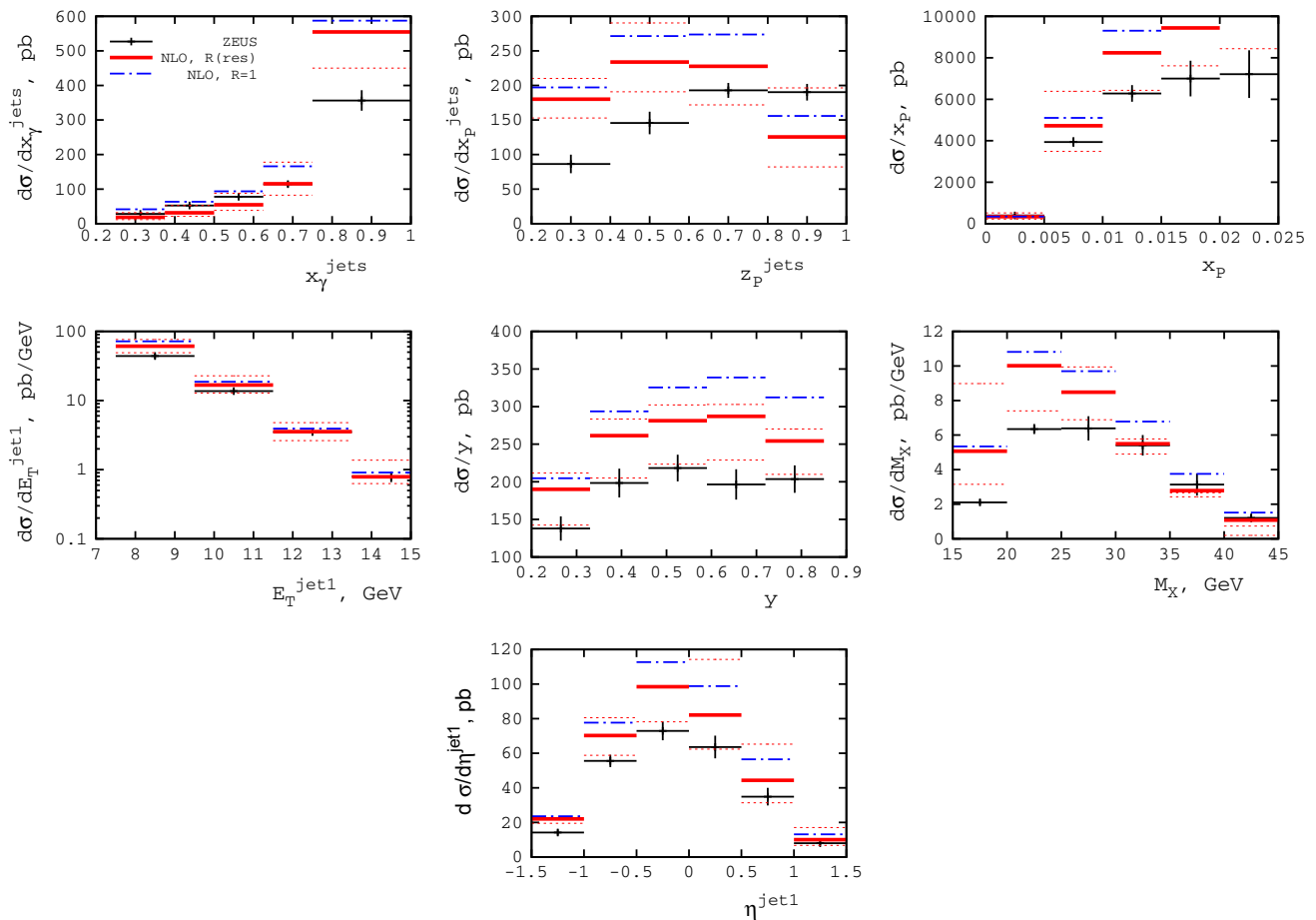
Fig. 4 The same as in Fig. 2, but for the ZEUS data [21]

cuts of the experimental analyses are summarized in Table 1, where Q^2 refers to the photon virtuality, y its momentum fraction in the electron, $E_T^{\text{jet1(2)}}$ are the leading and subleading transverse jet energies, and $\eta^{\text{jet1(2)}}$ their rapidities. In Figs. 2, 3, and 4, the thick red solid lines correspond to the calculation, when the renormalization and factorization scale μ is identified with the transverse energy of jet 1, $\mu = E_T^{\text{jet1}}$. The thin red dotted lines quantify the scale uncertainty of our NLO calculations and correspond to $\mu = 2E_T^{\text{jet1}}$ (lower) and $\mu = E_T^{\text{jet1}}/2$ (upper). For comparison, we also show the unsuppressed predictions (assuming no factorization breaking) by the blue dot-dashed lines labeled "NLO, $R = 1$". Note that our theoretical calculations have been multiplied by the hadronization corrections in each bin [19–21]. In the different panels, the values of the cross section are shown as functions of the following variables; see, e.g., Ref. [19]: $x_\gamma^{\text{jets}} = \sum_{\text{jets}} (E_i - P_{i,z})/(E_X - P_{X,z})$ is the hadron-level estimator of the parton momentum fraction in the photon, where the sum runs over the hadronic final states i included in the jets and X refers to the full diffractive final state; $z_{IP}^{\text{jets}} = \sum_{\text{jets}} (E_i + P_{i,z})/(E_X + P_{X,z})$ is

the estimator of the "Pomeron" momentum fraction carried by a parton; $x_{IP} = (E_X + P_{X,z})/(2E_p^{\text{beam}})$ is the measured "Pomeron" momentum fraction, where E_p^{beam} is the proton beam energy; E_T^{jet1} is the transverse energy of jet 1; M_X is the invariant mass of the diffractive final state (two jets plus diffractively produced remnants of the photon and the "Pomeron"); M_{12} is the invariant mass of the dijet system; $\langle \eta^{\text{jets}} \rangle = (\eta_1 + \eta_2)/2$ and $\Delta \eta^{\text{jets}}| = |\eta_1 - \eta_2|$ are the average and the relative jet rapidities, and W is the invariant photon–proton energy.

One can see from Figs. 2, 3, and 4 that within theoretical uncertainties, NLO perturbative QCD combined with the model of factorization breaking of Eq. (2) provides a good description of the data for most of the bins. The quality of the description of the data is similar to that of Ref. [18], where factorization breaking is realized either by the global or the resolved-only, flavor- and x-independent suppression factors.

An inspection of Figs. 2, 3, and 4 shows that NLO pQCD correctly reproduces the shape of almost all considered distributions and only fails to explain the normaliza-

Table 1 Kinematic cuts applied in the most recent H1 [19,20] and ZEUS [21] analyses of diffractive dijet photoproduction

H1 low-E_T^{jet} cuts	H1 high-E_T^{jet} cuts	ZEUS cuts
$Q^2 < 0.01$ GeV^2	$Q^2 < 0.01$ GeV^2	$Q^2 < 1$ GeV^2
$0.3 < y < 0.65$	$0.3 < y < 0.65$	$0.2 < y < 0.85$
$E_T^{\mathrm{jet1}} > 5$ GeV	$E_T^{\mathrm{jet1}} > 7.5$ GeV	$E_T^{\mathrm{jet1}} > 7.5$ GeV
$E_T^{\mathrm{jet2}} > 4$ GeV	$E_T^{\mathrm{jet2}} > 6.5$ GeV	$E_T^{\mathrm{jet2}} > 6.5$ GeV
$-1 < \eta^{\mathrm{jet1(2)}} < 2$	$-1.5 < \eta^{\mathrm{jet1(2)}} < 1.5$	$-1.5 < \eta^{\mathrm{jet1(2)}} < 1.5$
$z_{I\!P} < 0.8$	$z_{I\!P} < 1$	
$x_{I\!P} < 0.03$	$x_{I\!P} < 0.025$	$x_{I\!P} < 0.025$
$|t| < 1$ GeV^2	$|t| < 1$ GeV^2	$|t| < 5$ GeV^2
$M_Y < 1.6$ GeV	$M_Y < 1.6$ GeV	

Table 2 The theoretical and experimental values of the total integrated cross sections of diffractive dijet photoproduction in ep scattering at HERA, $\sigma_{\mathrm{NLO}}^{\mathrm{tot}}$ and $\sigma_{\mathrm{data}}^{\mathrm{tot}}$, and their ratios R; see Eq. (3)

H1, low-E_T cut	H1, high-E_T cut	ZEUS
$\sigma_{\mathrm{data}}^{\mathrm{tot}} = 295 \pm 6(\mathrm{stat.}) \pm 58(\mathrm{syst.})$ pb	$\sigma_{\mathrm{data}}^{\mathrm{tot}} = 37 \pm 2(\mathrm{stat.}) \pm 8(\mathrm{syst.})$ pb	$\sigma_{\mathrm{data}}^{\mathrm{tot}} = 124^{+11}_{-5}$ pb
$\sigma_{\mathrm{tot}}^{\mathrm{NLO}} = 375^{+157}_{-81}$ pb	$\sigma_{\mathrm{tot}}^{\mathrm{NLO}} = 51^{+15}_{-11}$ pb	$\sigma_{\mathrm{tot}}^{\mathrm{NLO}} = 165^{+46}_{-34}$ pb
$R = 1.27^{+0.46}_{-0.29}$	$R = 1.38^{+0.37}_{-0.31}$	$R = 1.33^{+0.29}_{-0.21}$

tion in some bins receiving a significant contribution from the direct-photon contribution, which is unsuppressed in our factorization-breaking scheme. The most notable example is the distribution at high x_γ^{jets} at low E_T^{jet}. It was hypothesized in Ref. [17] that additional sizable hadronization corrections along with bin migration effects, which are not included in our analysis, might help to improve the agreement between theory and data at large x_γ^{jets}. This hypothesis is supported by very similar observations in inclusive photoproduction at low E_T^{jet} [30].

Integrating our results for $d\sigma/dE_T^{\mathrm{jet1}}$ over E_T^{jet1}, we obtain the theoretical prediction for the integrated cross section $\sigma_{\mathrm{NLO}}^{\mathrm{tot}}$. Table 2 gives our results for $\sigma_{\mathrm{NLO}}^{\mathrm{tot}}$, the corresponding experimental cross sections $\sigma_{\mathrm{data}}^{\mathrm{tot}}$, and the ratios

$$R = \frac{\sigma_{\mathrm{NLO}}^{\mathrm{tot}}}{\sigma_{\mathrm{data}}^{\mathrm{tot}}} \tag{3}$$

of the theoretical predictions to the measured values. In the presented values for R, we have added in quadrature the experimental and theoretical uncertainties. One can see from the results shown in the table that within combined experimental and theoretical uncertainties, NLO pQCD with our factorization-breaking scheme gives a good description of the integrated cross section of diffractive dijet photoproduction in ep scattering measured at HERA.

Note that the central value of R for the high-E_T cut data is somewhat larger than that for the low-E_T cut, which can be explained by the fact that the unsuppressed charm quark con-

tribution becomes more prominent due to the QCD evolution of the photon PDFs.

3 Conclusions

We calculated the cross sections of diffractive dijet photoproduction in ep scattering in HERA kinematics using NLO perturbative QCD and a scenario of factorization breaking, which assumes that only the resolved-photon contribution is suppressed. The suppression depended on the parton flavor and the light-cone momentum fraction of partons in the photon. It was absent for charm quarks, larger for gluons than for light quarks, and decreased with an increase of the parton momentum. This model for factorization breaking in diffractive QCD is based on the space-time picture of photon–hadron interactions and complies with the good pQCD description of diffractive photoproduction of open charm in ep scattering. We compared our results with the available H1 and ZEUS data and found that various measured distributions and the integrated cross sections can be reproduced by our calculations with good accuracy. This agreement allows us to advocate our model as a viable alternative to the purely phenomenological scheme based on a global suppression factor.

References

1. F.D. Aaron et al. [H1 and ZEUS Collaborations], Eur. Phys. J. C **72**, 2175 (2012). arXiv:1207.4864 [hep-ex]
2. A. Levy, in *Lectures on QCD** ed. by F. Lenz, pp. 347–477
3. J.C. Collins, Phys. Rev. D **57**, 3051 (1998). [Erratum: Phys. Rev. D **61**, 019902 (2000)]. arXiv:hep-ph/9709499
4. S. Chekanov et al. [ZEUS Collaboration], Eur. Phys. J. C **38**, 43 (2004). arXiv:hep-ex/0408009
5. A. Aktas et al. [H1 Collaboration], Eur. Phys. J. C **48**, 715 (2006). arXiv:hep-ex/0606004
6. A. Aktas et al. [H1 Collaboration], Eur. Phys. J. C **48**, 749 (2006). arXiv:hep-ex/0606003
7. A. Aktas et al. [H1 Collaboration], Eur. Phys. J. C **51**, 549 (2007). arXiv:hep-ex/0703022
8. A. Aktas et al. [H1 Collaboration], Eur. Phys. J. C **50**, 1 (2007). arXiv:hep-ex/0610076
9. T. Affolder et al. [CDF Collaboration], Phys. Rev. Lett. **84**, 5043 (2000)
10. A.B. Kaidalov, V.A. Khoze, A.D. Martin, M.G. Ryskin, Eur. Phys. J. C **21**, 521 (2001). arXiv:hep-ph/0105145
11. M. Klasen, G. Kramer, Phys. Rev. D **80**, 074006 (2009). arXiv:0908.2531 [hep-ph]
12. A.B. Kaidalov, V.A. Khoze, A.D. Martin, M.G. Ryskin, Phys. Lett. B **567**, 61 (2003). arXiv:hep-ph/0306134
13. M. Klasen, G. Kramer, Eur. Phys. J. C **38**, 93 (2004). arXiv:hep-ph/0408203
14. M. Klasen, G. Kramer, Phys. Rev. Lett. **93**, 232002 (2004). arXiv:hep-ph/0410105
15. M. Klasen, G. Kramer, J. Phys. G **31**, 1391 (2005). arXiv:hep-ph/0506121
16. M. Klasen, G. Kramer, Mod. Phys. Lett. A **23**, 1885 (2008). arXiv:0806.2269 [hep-ph]
17. A.B. Kaidalov, V.A. Khoze, A.D. Martin, M.G. Ryskin, Eur. Phys. J. C **66**, 373 (2010). arXiv:0911.3716 [hep-ph]
18. M. Klasen, G. Kramer, Eur. Phys. J. C **70**, 91 (2010). arXiv:1006.4964 [hep-ph]
19. F.D. Aaron et al. [H1 Collaboration], Eur. Phys. J. C **70**, 15 (2010). arXiv:1006.0946 [hep-ex]
20. K. Cerny, Ph.D. thesis, Charles University, Prague, H1 thesis 493 (2008). http://www-h1.desy.de/publications/theseslist.html
21. S. Chekanov et al. [ZEUS Collaboration], Eur. Phys. J. C **55**, 177 (2008). arXiv:0710.1498 [hep-ex]
22. V. Andreev et al. [H1 Collaboration], JHEP **1505**, 056 (2015). arXiv:1502.01683 [hep-ex]
23. T.H. Bauer, R.D. Spital, D.R. Yennie, F.M. Pipkin, Rev. Mod. Phys. **50**, 261 (1978)
24. T.H. Bauer, R.D. Spital, D.R. Yennie, F.M. Pipkin, Rev. Mod. Phys. **51**, 407 (1979)
25. J. Nemchik, N.N. Nikolaev, E. Predazzi, B.G. Zakharov, Phys. Lett. B **374**, 199 (1996). arXiv:hep-ph/9604419
26. H. Kowalski, L. Motyka, G. Watt, Phys. Rev. D **74**, 074016 (2006). arXiv:hep-ph/0606272
27. M. Gluck, E. Reya, A. Vogt, Phys. Rev. D **46**, 1973 (1992)
28. M. Klasen, G. Kramer, S.G. Salesch, Z. Phys. C **68**, 113 (1995)
29. M. Klasen, G. Kramer, Z. Phys. C **72**, 107 (1996). arXiv:hep-ph/9511405
30. M. Klasen, Rev. Mod. Phys. **74**, 1221 (2002). arXiv:hep-ph/0206169
31. S. Chekanov et al. [ZEUS Collaboration], Eur. Phys. J. C **51**, 301 (2007)
32. S. Frixione, P. Nason, G. Ridolfi, Nucl. Phys. B **454**, 3 (1995). arXiv:hep-ph/9506226
33. M. Klasen, G. Kramer, Eur. Phys. J. C **71**, 1774 (2011). arXiv:1104.0095 [hep-ph]